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The case for the cosmological constant

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Abstract. I present a short overview of current observational results and theoretical models for a cosmological constant. The main motivation for invoking a small cosmological constant (or Λ-term) at the present epoch has to do with observations of high redshift Type Ia supernovae which suggest an accelerating universe. A flat accelerating universe is strongly favoured by combining supernovae observations with observations of CMB anisotropies on degree scales which give the 'best-fit' values $\Omega_\Lambda \simeq 0.7$ and $\Omega_m \simeq 0.3$. A time dependent cosmological Λ-term can be generated by scalar field models with exponential and power law potentials. Some of these models can alleviate the 'fine tuning' problem which faces the cosmological *constant.*

Keywords. Cosmology; supernovae; cosmic microwave background; cosmological constant; vacuum fluctuations.

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1. Introduction

Recent observations demonstrating that high redshift supernovae are fainter than expected in standard FRW cosmology has presented dramatic evidence in favour of an accelerating universe. As the case for an accelerating universe continues to build, attempts are on to find a logically compelling theoretical framework within which the acceleration of the universe can be understood. Currently the most popular scenario invokes a cosmological Λ-term whose energy density exceeds that of any other form of matter, luminous or dark. Variants of this scenario in which the Λ-term decays with time can also explain most current observations. In this talk I shall review attempts to understand the accelerating universe at both observational and theoretical levels.

2. The observational case for an accelerating universe

2.1 *Type Ia supernovae and the acceleration of the universe*

Type Ia supernovae are widely regarded as explosions arising when a white dwarf star has accreted enough matter from its binary companion to cross the Chandrasekhar limit. Three crucial properties of Type Ia supernovae establish them as useful standard candles with which to probe the curvature and expansion rate of the universe:

1. Their high absolute luminosity ($M_B \simeq -19.5$ mag) ensures that they can be seen out to very large cosmological distances.
2. The dispersion in their luminosity at maximum light is very small (≤ 0.3 mag).
3. The decline in the luminosity of a Type Ia supernova is strongly correlated with its intrinsic luminosity: more luminous supernovae fade more slowly than their less luminous counterparts.

Both (2) and (3) ensure that the scatter in the absolute luminosity of Type Ia supernovae can be reduced to less than 10%, making them excellent standard candles.

In an expanding homogeneous and isotropic FRW universe the flux of light received from a source with absolute luminosity L is given by the relation

$$F = \frac{L}{4\pi d_L^2}, \tag{1}$$

where d_L is the *luminosity distance* to the object. In a multicomponent universe consisting of matter and a cosmological term

$$d_L(z) = \frac{(1+z)cH_0^{-1}}{|\Omega_{\rm total} - 1|^{\frac{1}{2}}} S(\eta_0 - \eta), \tag{2}$$

where

$$\eta_0 - \eta = |\Omega_{\rm total} - 1|^{\frac{1}{2}} \int_0^z \frac{{\rm d}z'}{h(z')}. \tag{3}$$

The dimensionless Hubble parameter $h(z)$ at a cosmological redshift z is

$$h(z) = \frac{H(z)}{H_0} = \left[(1-\Omega_{\rm total})(1+z)^2 + \Omega_m(1+z)^3 + \Omega_\Lambda\right]^{\frac{1}{2}} \tag{4}$$

and $S(x)$ is defined as follows: $S(x) = \sin(x)$ if $\kappa = 1$ ($\Omega_{\rm total} > 1$), $S(x) = \sinh(x)$ if $\kappa = -1$ ($\Omega_{\rm total} < 1$), $S(x) = x$ if $\kappa = 0$ ($\Omega_{\rm total} = 1$). Ω_m is the dimensionless energy density of matter $\Omega_m = 8\pi G\rho_m/3H_0^2$, and Ω_Λ is the dimensionless energy density of a cosmological constant $\Omega_\Lambda = \Lambda/3H_0^2$. $\Omega_{\rm total} = \Omega_m + \Omega_\Lambda$. (A cosmological constant has an equation of state $P = -\rho = -\Lambda/8\pi G$ which distinguishes it from normal matter with non-negative pressure.) It is easy to see that for a given value of the matter density a positive Λ-term leads to an *increase* in the luminosity distance and hence to a decline in the observed luminosity of high redshift supernovae. Exactly such an effect has been observed for several dozen Type Ia high z supernovae ($z_{\rm max} \leq 0.83$) by two teams: the supernova cosmology project [25] and the high-z supernova search team [28]. The observations of Perlmutter *et al* (1999) indicate that the joint probability distribution of $\{\Omega_m, \Omega_\Lambda\}$ is well fitted by

$$0.8\Omega_m - 0.6\Omega_\Lambda \simeq -0.2 \pm 0.1.$$

The best-fit confidence region shown in figure 1 strongly favours a positive energy density for the cosmological constant $\Omega_\Lambda > 0$.

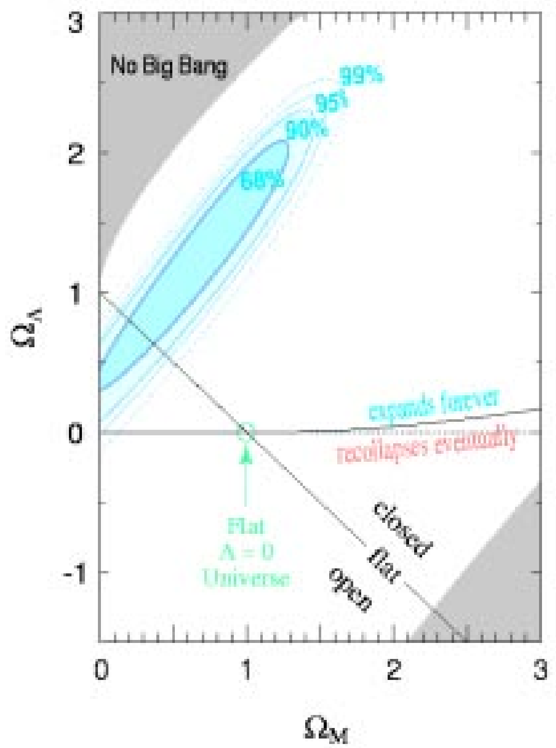

Figure 1. Most likely values of $\{\Omega_m, \Omega_\Lambda\}$ from an analysis of Type 1a high redshift supernovae by Perlmutter *et al* (1999).

2.2 *CMB anisotropies and the value of* Λ

Much tighter constraints on the parameter pair $\{\Omega_m, \Omega_\Lambda\}$ can be obtained by combining supernovae observations with those of the cosmic microwave background (CMB) on intermediate angular scales.

In a FRW universe the horizon at last scattering ($z \simeq 1100$) subtends an angle $\theta \simeq 2^\circ$ at our present location. As a result fluctuations in the CMB on large angular scales $\theta \gg 2^\circ$ probe the *primordial* spectrum of density perturbations and gravity waves. Observations made by the COBE satellite have shown that the primordial spectrum of fluctuations has an approximately scale invariant form $|\delta_k|^2 \propto k^n$, $n \simeq 1$ which is in good agreement with predictions made by the inflationary scenario. In addition to fluctuations of primordial origin, coherent acoustic oscillations existing in the photon-baryon plasma are imprinted in the CMB at the time of decouping between matter and radiation. (This takes place roughly at the time of the cosmological recombination of hydrogen.) These oscillations give rise to 'Doppler peaks' in the angular power spectrum C_l on *intermediate* angular scales $\theta \sim 1^\circ$ where C_l is defined as follows, the CMB temperature distribution on the celestial sphere has the form

$$T(\theta, \phi) = T_0 \left[1 + \frac{\delta T}{T}(\theta, \phi)\right], \tag{5}$$

T_0 is the blackbody temperature, $T_0 = 2.728 \pm 0.004°$K. $\delta T/T$ can be written as a multipole expansion

$$\frac{\delta T}{T}(\theta, \phi) = \sum_{l=2}^{\infty} \sum_{m=-l}^{l} a_{lm} Y_l^m(\theta, \phi), \tag{6}$$

where the coefficients a_{lm} are statistically independent and distributed in the manner of a Gaussian random field with zero mean and variance given by the angular power spectrum

$$C_l \equiv \langle |a_{lm}|^2 \rangle. \tag{7}$$

The angle brackets indicate averaging over an ensemble of possible universes. The location of the first acoustic peak in C_l is determined by the angle subtended by the sound horizon at the time of decoupling which depends upon $\Omega_{\rm baryon}, \Omega_m$ and Ω_Λ. Thus the amplitude and location of the peak depends sensitively upon both the curvature of the universe as well as its matter content. Observations made by the 1997 test flight of the BOOMERANG experiment appear to have measured a doppler peak at $l \simeq 200$ [21]. In the standard gravitational clustering scenario a peak in C_l is predicted at $l_{\rm peak} \sim 200\Omega_{\rm total}^{-1/2}$ which, when combined with the BOOMERANG measurements suggests $0.85 \leq \Omega_{\rm total} \leq 1.25$ at the 68% confidence level.

It is extremely fortuitous that CMB and supernovae (Sn) measurements display orthogonal degeneracies in the parameters Ω_m, Ω_Λ (see [32] and references therein). Thus by combining CMB and Sn observations one gets much better constraints on Ω_m and Ω_Λ than by using each set of measurements separately. For instance combining BOOMERANG + supernovae gives [21]

$$0.2 \leq \Omega_m \leq 0.45, \;\; 0.6 \leq \Omega_\Lambda \leq 0.85. \tag{8}$$

Current observations therefore appear to favour a flat universe $\Omega_m + \Omega_\Lambda \simeq 1$ in excellent agreement with predictions of inflationary models made almost two decades ago.

A flat matter dominated universe is constantly *decelerating*, its expansion described by $a(t) \propto t^{2/3}$. A cosmological constant dominated universe on the other hand accelerates, approaching the asymptotic expansion law $a(t) \propto \exp\sqrt{\Lambda/3}t$ as $t \to \infty$. The redshift z_* at which deceleration is succeeded by acceleration and the redshift $z_\star$ when the energy density in the cosmological constant exceeds the density of matter occur respectively at [26,32]

$$(1+z_*)^3 = 2\frac{\Omega_\Lambda}{\Omega_m}, \;\; (1+z_\star)^3 = \frac{\Omega_\Lambda}{\Omega_m}. \tag{9}$$

Substituting the best-fit values $\Omega_m \simeq 0.3, \Omega_\Lambda \simeq 0.7$ one obtaines $z_* \simeq 0.73$, $z_\star \simeq 0.37$. Thus the acceleration of the universe is a relatively recent phenomenon!

The presence of a cosmological Λ-term has several important astrophysical consequences [32]:

1. $\Lambda > 0$ leads to a longer age for the universe and could resolve the 'age problem' which has proved problematic for matter dominated cosmologies with a large value of the Hubble parameter.

2. Since the distance to high redshift objects increases, so does the probability that they will be lensed by massive dense foreground objects such as galaxy clusters. Gravitational lensing combined with CMB and supernovae is proving to be a very useful test of the cosmic equation of state [35,36].
3. The presence of Λ slows down the growth of long wavelength density perturbations which are still in the linear regime. An associated result is that most clusters and superclusters of galaxies observed today are expected to have formed by a redshift of unity in a ΛCDM universe in sharp contrast to structure formation in standard CDM cosmology which is expected to be more recent. Observations of clusters at high redshifts (made, for instance, using the Chandra satellite) are therefore expected to provide a sensitive probe of Λ.

3. Theoretical issues associated with a small Λ-term

The cosmological constant has had a chequered history. Conceived by Einstein in 1917 and discarded by him soon after, it has made several 'come-backs' of which perhaps the most significant was the recognition by Zeldovich (1968) that the zero-point vacuum energy-momentum tensor had precisely the form of a cosmological constant i.e. $\langle T_{ik}\rangle_{\rm vac} \propto \Lambda g_{ik}$. Unfortunately the energy density of the vacuum diverges as $\langle T_{00}\rangle \sim k^4$. Invoking an ultra-violet cutoff at the Planck scale results in a very large vacuum density $\langle T_{00}\rangle \sim 10^{76}$ GeV^4, which is 123 orders of magnitude larger than the currently observed value $\rho_{\rm vac} \sim 10^{-47}$ GeV^4. This discrepancy is well known as the *cosmological constant problem* [37]. Attempts to resolve it include invoking supersymmetry [41] since bosons and fermions contribute towards $\langle T_{00}\rangle$ with opposite signs. However since supersymmetry is broken at $T_{\rm CMB} \sim 3^\circ$ K, the cosmological constant is likely to reappear at low temperaturs even though it might be made to vanish in the early universe. As an example the QCD vacuum is expected to generate a cosmological constant of the order of $\Lambda^4_{\rm QCD} \sim 10^{-3}$ GeV^4 which, though smaller than $\rho_{\rm Pl}$, is still several orders of magnitude larger than the observed value of $\rho_{\rm vac}$. It is interesting that in some models the scale of supersymmetry breaking is rather low and occurs near the electroweak scale $M_{\rm SUSY} \simeq 10^3$ GeV [2]. The corresponding value of the vacuum density $\rho_{\rm SUSY} \sim M^4_{\rm SUSY} \simeq 10^{12}$ GeV^4 lies about midway between $\rho_{\rm vac} \sim 10^{-47}$ GeV^4 and $\rho_{\rm Pl} \sim 10^{76}$ GeV^4. Therefore a theory in which the effective energy scale of the vacuum is given by $\rho_{\rm vac} \sim M_X^4$ where $M_X = M^2_{\rm SUSY}/M_{\rm Pl} \simeq 10^{-3}$ eV might 'explain' the small value of $\rho_{\rm vac}$ observed today.

3.1 *Quantum effects and the value of* Λ

Attempts to generate a small value of Λ at the present epoch from one-loop quantum effects in a curved space-time have been considered in [30,23]. For instance Sahni and Habib (1998) show that vacuum polarization and particle production of non-minimally coupled massive fields in an expanding universe give rise to a vacuum energy-momentum tensor $\langle T_{ik}\rangle_{\rm vac} \propto \Lambda g_{ik}$ and the corresponding dimensionless vacuum energy density is $\Omega_\Lambda \simeq 1/(6|\xi|)(m/H)^2$ (the coupling to gravity is assumed to be weak $|\xi| \ll 1$). This treatment together with [23] suggests that ultra-light fields with $m \sim H \sim 10^{-33}$ eV could give rise to a small cosmological constant at the present epoch. Such ultra-light fields have been discussed in the context of pseudo–Nambu–Goldstone bosons in [16]. (Pseudo-Nambu-Goldstone bosons can rotate polarized radiation from distant radio sources and therefore provide the possibility of detection [11].) Hypothetically a small mass m may also be associated with the mass difference implied by solar neutrino oscillations and the Planck (or GUT) scale: $m = \Delta m_\nu^2/M_P \simeq 10^{-33}$ eV where $\Delta m_\nu^2 \simeq 10^{-5}$ eV^2.

3.2 *A time dependent* Λ*-term*

As pointed out earlier, the presence of a small cosmological *constant* at the present epoch raises a number of problematic issues for cosmology including: (i) a degree of fine tuning of parameters may be required to explain the exceedingly small numbers $\rho_\Lambda/\rho_{\rm Pl} \sim 10^{-123}$ or even $\rho_\Lambda/\rho_{\rm EW} \sim 10^{-53}$; (ii) for $\Omega_m \simeq 0.3, \Omega_\Lambda \simeq 0.7$, $\rho_m/\rho_\Lambda \simeq 1$ at a redshift $z_\star \simeq 0.37$, which could be interpreted to mean that we are living at a preferred epoch when the energy densities in matter and Λ are almost equal. This is sometimes referred to as the 'coincidence problem' [40].

Some of these problems can be alleviated if we consider the energy density in Λ to be a function of time, such a model may be called a Λ-field or 'quintessence'. (A time dependent Λ-term can also arise if the the actual vacuum energy is zero, but the universe takes a long time in relaxing to that state.) The simplest Λ-field models borrow heavily from inflationary model building. It is well known that for a homogeneous, minimally coupled massive scalar field the energy density and pressure are given by

$$\rho = \frac{1}{2}\dot\phi^2 + V(\phi),$$
$$P = \frac{1}{2}\dot\phi^2 - V(\phi), \tag{10}$$

and the evolution of the scalar field and expansion of the universe are governed by

$$H^2 = \frac{8\pi G}{3}(\rho_m + \frac{\dot\phi^2}{2} + V),\ \rho_m = \frac{3\Omega_0 H_0^2}{8\pi G}(\frac{a_0}{a})^3, \tag{11}$$

$$\ddot\phi + 3H\dot\phi + \frac{{\rm d}V}{{\rm d}\phi} = 0, \tag{12}$$

$$\dot H = -4\pi G(\rho_m + \dot\phi^2). \tag{13}$$

From (10) we find that the Λ-term equation of state $P \simeq -\rho$ arises if $\dot\phi^2 \ll V(\phi)$. This model works very well for potentials encountered in 'chaotic inflation' such as $V \propto \lambda\phi^4$ for which the scalar field rolls down its potential very slowly if $\phi \gg m_{\rm Pl}$ leading to $P \simeq -\rho$ and $T_{ik} \simeq V(\phi)g_{ik}$. However, precisely for this reason this class of potentials also runs into problems similar to those encountered by a cosmological constant. The scalar field eq. (12) is overdamped during radiation and matter dominated epochs causing $V(\phi) \propto \Lambda$ to remain unchanged virtually from the Planck epoch $z_{\rm Pl} \sim 10^{19}$ to $z \sim 2$ [16]. This leads to an enormous difference in the scalar field energy density and that of matter/radiation at early times and one runs into the fine tuning problem: the relative values of ρ_ϕ and ρ_m must be adjusted to very high levels of accuracy in order to ensure $\rho_\phi/\rho_m \sim 1$ at precisely the present epoch [31].

Luckily the fine tuning problem can be substantially reduced for the following class of potentials:

1. $V_1(\phi) = k/\phi^\alpha$, $\alpha > 1$, [22];
2. $V_2(\phi) = V_0[\cosh \lambda\phi/M_P - 1]^p$, $p \leq 1$, [31];
3. $V_3(\phi) = V_0[\mathrm{e}^{M_P/\phi} - 1]$ [40].

In all three models the density in the ϕ-field decreases *rapidly* at early times and *slowly* at late times. Therefore the early-time value of ρ_ϕ can be comparable to the density in radiation and one can conceive of the ϕ-field being produced by mechanisms similar to those giving rise to $\rho_{\rm rad}$ (preheating etc.). An interesting feature of potential (ii) is that for large values $|\lambda\phi| \gg 1$, $V(\phi) \propto \exp(-p\lambda\phi)$. As discussed in [22,38,15] the exponential potential has an interesting property: the energy density of a scalar field rolling down $V(\phi)$ scales like the background density of matter driving the expansion of the universe, as a result the ratio of the scalar field density to the total matter density approaches a constant value

$$\frac{\rho_\phi}{\rho_B + \rho_\phi} = \frac{3(1+w_B)}{p^2\lambda^2} \tag{14}$$

($w_B = 0,\ 1/3$ is the equation of state for dust, radiation).

Thus at early times when $\lambda|\phi| \gg 1$, ρ_ϕ 'tracks' the density in radiation $\rho_{\rm rad}$. At late times when $|\lambda\phi| \ll 1$ the potential approaches a power law form $V_2(\phi) \propto (\lambda\phi)^{2p}$. Rapid oscillations of the scalar field during this stage result in an averaged equation of state $\langle w_\phi \rangle = \langle P_\phi \rangle / \langle \rho_\phi \rangle = p - 1/p + 1$. We therefore find that, depending upon the value of the exponent p, scalar field models based on $V_2(\phi)$ can play the role both of cold dark matter ($p = 1$) as well as a negative pressure Λ-field ($p \leq 1/2$). The evolution of the dimensionless density parameter Ω_ϕ is shown as a function of expansion factor $a(t)$ for a universe consisting of radiation, matter and a ϕ-field in figure 2. It is interesting to see that Ω_ϕ remains virtually unchanged even though both Ω_r and Ω_m change substantially as the universe expands.

Λ-field (quintessence) models have been discussed in the context of supersymmetric theories and supergravity [7,20,9,10], extra dimensions [3,5,6], string/M-theory [1,13], spinodal instabilities [14] and as solutions to the non-perturbative renormalization group equations [8].

An accelerating universe can also arise in topological defect models. For instance a tangled network of random non-intercommuting cosmic strings possesses an averaged equation of state $P = -\rho/3$, the mean energy density of a string network dominated by straight strings decreases as $\rho \propto a^{-2}$ leading to the linear expansion law $a \propto t$ (in the absence of other forms of matter). Similarly $P = -2\rho/3$ for a network of walls resulting in $\rho \propto a^{-1}$ and an accelerating expansion rate $a \propto t^2$. Cosmological defect models and their observational consequences have been extensively discussed in [34,4].

3.3 *Reconstructing Λ from observations*

It is perhaps fair to say that although several promising models of a time dependent Λ-term exist, none is singled out uniquely on the basis of a fundamental theory of particle physics such as supergravity or M-theory and the situation in many ways resembles that faced by the inflationary scenario. It is therefore important that one can *reconstruct* the Λ-field

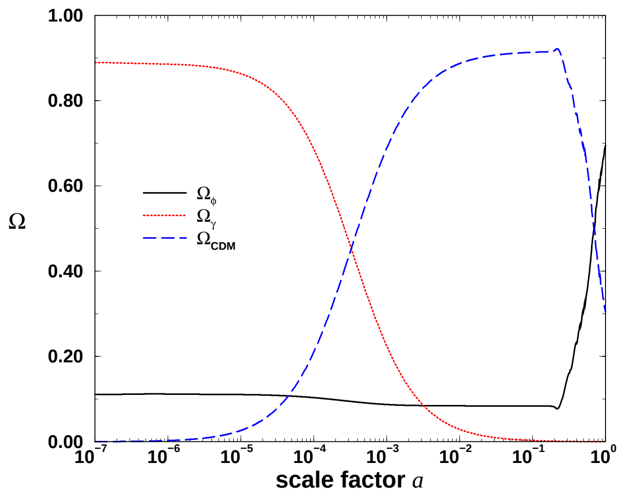

Figure 2. The evolution of the dimensionless density parameter for the quintessence field Ω_ϕ (solid line) is shown for the potential $V(\phi) = V_0[\cosh \lambda\phi/M_P - 1]^{0.2}$. Matter (dashed line) and radiation densities (dotted line) are also shown. At early times the energy in the ϕ-field is subdominant, but at later times $z \lesssim 2$ scalar field oscillations commence and the density in the ϕ-field rapidly dominates the mass density of the universe leading to $\Omega_\phi \sim 0.7$ today (from Sahni and Wang (1999)).

potential solely on the basis of observational data and hence test the 'quintessence' hypothesis in a model independent manner as demonstrated in [33,17,29]. (A similar exercise for the inflationary potential is reviewed in [19].)

One begins by noting that in a flat universe the Hubble parameter can be uniquely defined through the luminosity distance by the relation (2) which can be rewritten as

$$H(z) \equiv \frac{\dot{a}}{a} = \left[\frac{\mathrm{d}}{\mathrm{d}z}\left(\frac{d_L(z)}{1+z}\right)\right]^{-1}. \tag{15}$$

In order to determine the Λ-field potential we simply rewrite the Einstein equations as

$$\frac{8\pi G}{3H_0^2}V(x) = \frac{H^2}{H_0^2} - \frac{x}{6H_0^2}\frac{\mathrm{d}H^2}{\mathrm{d}x} - \frac{1}{2}\Omega_m x^3, \tag{16}$$

$$\frac{8\pi G}{3H_0^2}\left(\frac{\mathrm{d}\phi}{\mathrm{d}x}\right)^2 = \frac{2}{3H_0^2 x}\frac{\mathrm{d}\ln H}{\mathrm{d}x} - \frac{\Omega_m x}{H^2}, \tag{17}$$

where $x \equiv 1+z$. The corresponding equation of state of the Λ-field is

$$w_\phi(x) \equiv \frac{p}{\rho} = \frac{(2x/3)\mathrm{d}\ln H/\mathrm{d}x - 1}{1 - (H_0^2/H^2)\,\Omega_m x^3}. \tag{18}$$

Thus knowing d_L we can determine $H(x)$ and $\mathrm{d}H/\mathrm{d}x$ and hence reconstruct both the form of $V(\phi)$ and the cosmic equation of state $w_\phi(x)$. This reconstruction method depends crucially upon the ansatz used for determining the luminosity distance d_L. Saini *et al* (1999)

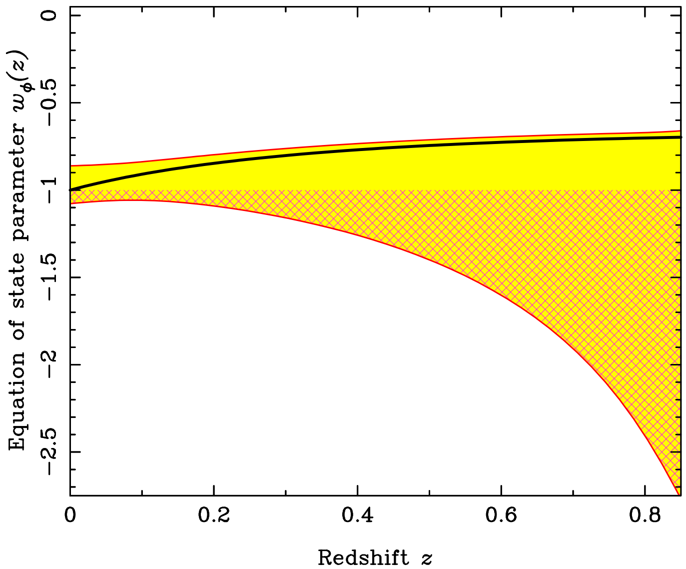

Figure 3. The reconstructed *model independent* equation of state of the Λ-field is shown as a function of cosmological redshift. The solid line corresponds to the best-fit values of parameters in the ansatz for d_L. The shaded area covers the range of 68% errors. A small evolution in w_ϕ is supported by observations, however a cosmological constant also agrees with the data. The hatched region corresponds to $w_\phi < -1$ which is unphysical for a minimally coupled scalar field (from Saini *et al* (1999)).

suggest a three parameter ansatz which has been shown to be extremely accurate. The resulting form of the cosmic equation of state is shown in figure 3. We find that although there is evidence for some evolution in the equation of state between the present epoch and the redshift $z = 0.83$ (the redshift of the most distant supernova in the sample) an unevolving cosmological constant with $w = -1$ is also consistent with current supernovae data.

4. Conclusions

The supernova inventory is growing rapidly with up to ~ 50 new Type Ia events being added every year. Coupled to this is the expectation that the MAP and PLANCK missions will pinpoint the location and amplitude of the first Doppler peak in the CMB to within an accuracy of a few per cent. Thus, provided systematics is properly understood, the issue of whether or not we live in a Λ-dominated universe should be resolved within the next few years. It will then be up to theorists to explain the 'riddle of Λ': if it exists then why is it so small, and if it does not then which physical principles set the value of Λ to precisely zero!

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