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The cosmological constant revisited

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Abstract. I briefly review the observational evidence for a small cosmological constant at the present epoch. This evidence mainly comes from high redshift observations of Type 1a supernovae, which, when combined with CMB observations strongly support a flat Universe with $\Omega_m + \Omega_\Lambda \simeq 1$. Theoretically a cosmological constant can arise from zero point vacuum fluctuations. In addition ultra-light scalar fields could also give rise to a Universe which is accelerating driven by a time dependent Λ-term induced by the scalar field potential. Finally a Λ dominated Universe also finds support from observations of galaxy clustering and the age of the Universe.

Keywords. Cosmology; supernovae; cosmic microwave background; cosmological constant; vacuum fluctuations.

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1. Introduction

The cosmological constant Λ was introduced into cosmology by Einstein in 1917. Einstein, under the influence of Mach's principle, constructed a Universe which was static and closed – a configuration that could only arise under the joint influence of both matter and a cosmological constant. Subsequently Friedmann derived expanding solutions to the Einstein equations and Einstein acknowledged that the introduction of Λ was probably unnecessary, particularly in view of Hubble's discovery that the Universe was expanding. However, interest in the cosmological constant remained, partly due to the rich variety of new solutions which arise in the Einstein equations with a cosmological constant including: the static Einstein Universe, singularity free 'bouncing' models, quasi-static 'loitering' models etc. Interest in Λ was reignited in the late 1960's, when it was felt than an excess of QSO's was being observed at redshift 1.95. This observation was difficult to explain in the framework of standard FRW cosmology, but easier to account for if the Universe loitered at that redshift. More recently a large cosmological constant at an early epoch is the basis of the inflationary model, and a much smaller cosmological constant at a much later epoch is suggested by current observations.

2. Observational issues

Recent observations using Type 1a supernovae (Sn) as standard candles have shown that the Universe may be accelerating driven by the negative pressure of a cosmological constant.

The small dispersion of Type 1a supernova at maximum light ($\lesssim 0.3$ mag), combined with their high absolute luminosity ($M_B \simeq -19.5$mag) makes these objects ideal standard candles with which to try and determine cosmological parameters such as q_0, H_0, Ω_Λ, Ω_m etc. The luminosity flux reaching an observer at redshift $z = 0$ from an object of absolute luminosity L located at a redshift z is

$$F = \frac{L}{4\pi d_L^2}, \tag{1}$$

where $d_L(z)$ – the *luminosity distance* to a source at redshift z – depends crucially upon both the curvature of space and the matter content of the Universe

$$d_L(z) = \frac{(1+z)cH_0^{-1}}{|\Omega_{\rm total} - 1|^{1/2}} S(\eta_0 - \eta), \tag{2}$$

where

$$\eta_0 - \eta = |\Omega_{\rm total} - 1|^{1/2} \int_0^z \frac{dz'}{h(z')}, \tag{3}$$

and $S(x)$ is defined as follows, $S(x) = \sin(x)$ if $\kappa = 1$ ($\Omega_{\rm total} > 1$), $S(x) = \sinh(x)$ if $\kappa = -1$ ($\Omega_{\rm total} < 1$), $S(x) = x$ if $\kappa = 0$ ($\Omega_{\rm total} = 1$). $h(z) = H(z)/H_0$ is the dimensionless Hubble parameter

$$h(z) = (1+z)\left[1 - \Omega_{\rm total} + \sum_\alpha \Omega_\alpha (1+z)^{\gamma_\alpha}\right]^{\frac{1}{2}} \tag{4}$$

where $\Omega_{\rm total} = \sum_\alpha \Omega_\alpha$, $\gamma_\alpha = 1 + 3w_\alpha$ where $w_\alpha = P_\alpha/\rho_\alpha$ is the equation of state ($p = -\rho$ for a cosmological constant) and $1 + z = a_0/a(t)$ is the cosmological redshift parameter. Expressions (2) and (3) demonstrate that, since for a given value of $\Omega_{\rm total}$ the luminosity distance increases in the presence of Λ, an object at a fixed redshift will appear brighter in a Universe with $\Lambda = 0$ than it will in a Universe with $\Lambda > 0$. Therefore by observing a sufficiently large number of Type 1a supernovae at a high redshift it should, in principle, be possible to distinguish the cosmological effects of a Λ-term if the latter exists. In practice (1) translates into the magnitude-redshift relation between the apparent magnitude m of an object and its absolute magnitude M

$$\mu \equiv m - M = 5\log_{10}\frac{d_L}{\rm Mpc} + 25 \tag{5}$$

where μ is known as the distance modulus. If both m and M are known then the parameter pair $\{\Omega_m, \Omega_\Lambda\}$ can be estimated by minimising the χ^2 statistic

$$\chi^2(H_0, \Omega_m, \Omega_\Lambda) = \sum_i \frac{\{\mu_{p,i}(z_i; H_0, \Omega_m, \Omega_\Lambda) - \mu_{0,i}\}^2}{\sigma_{\mu 0,i}^2} \tag{6}$$

where $\mu_o(z_i)$ are the observed values of the distance modulus and $\mu_p(z_i)$ are the theoretically predicted values, $\Omega_\Lambda = \Lambda/3H^2$. The resulting best-confidence regions obtained by Perlmutter *et al* [1] from the analysis of 42 Type 1a high redshift supernovae are shown in figure 1. A positive value of the cosmological constant is clearly suggested by these measurements.

The parameter pair $\{\Omega_m, \Omega_\Lambda\}$ can be further constrained by combining Sn observations with those of the cosmic microwave background (CMB) anisotropy on intermediate angular scales $\theta \lesssim 1°$. The CMB anisotropy when expanded on the celestial sphere has the form

$$\frac{\delta T}{T}(\theta, \phi) = \sum_{l=2}^{\infty} \sum_{m=-l}^{l} a_{lm} Y_l^m(\theta, \phi), \tag{7}$$

where the coefficients a_{lm} are statistically independent and distributed in the manner of a Gaussian random field with zero mean and variance

$$C_l \equiv \langle |a_{lm}|^2 \rangle. \tag{8}$$

The quantity C_l defines the *angular power spectrum* of the CMB. On large angular scales (small l) the form of C_l is determined by the spectrum of primordial density perturbations generated during inflation. On smaller scales ($\theta \lesssim 1°, l \gtrsim 60$) C_l has sharp peaks reflecting accoustic oscillations in the photon-baryon plasma prior to recombination. The location and amplitude of these so-called 'Doppler peaks' depends sensitively upon both the curvature of the Universe and its matter content. From figure 2 we see that a spatially

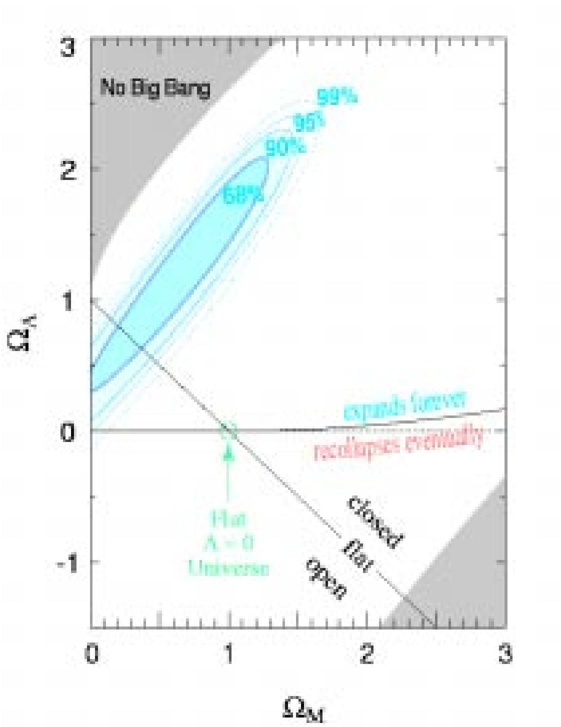

Figure 1. Constraints on $\{\Omega_m, \Omega_\Lambda\}$ from the analysis of Type 1a high redshift supernovae by Perlmutter *et al* [1].

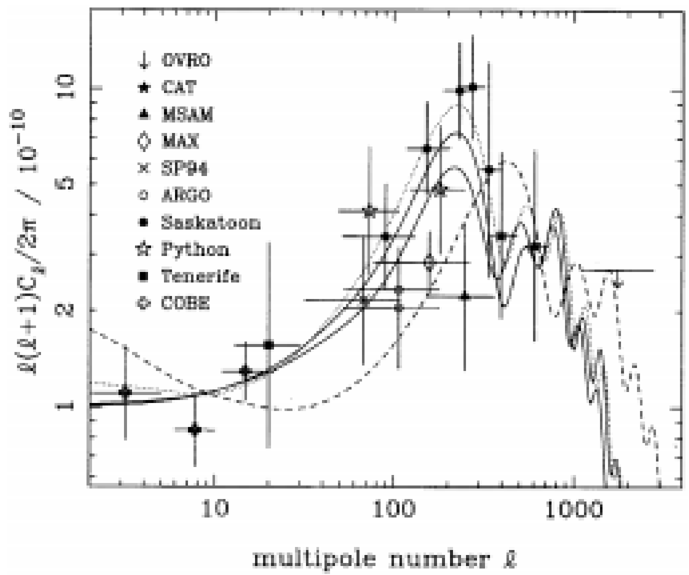

Figure 2. The angular spectrum of CMB fluctuations as measured by different CMB experiments. Also shown are predictions from the following theoretical models: (i) Flat ΛCDM model with parameters $(\Omega_\Lambda, \Omega_m, \Omega_b, h) = (0.7, 0.3, 0.05, 0.65)$ (dotted line). (ii) Flat CDM models with $(\Omega_m, \Omega_b, h) = (1, 0.1, 0.5)$ and $(\Omega_m, \Omega_b, h) = (1, 0.0.5, 0.5)$ (solid lines). The larger Ω_b model shows a higher Doppler peak. (iii) Open CDM model with $(\Omega_m, \Omega_b, h) = (0.3, 0.05, 0.65)$ (broken line). For more details, see Peacock [13].

flat Λ-dominated cold dark matter model (ΛCDM) agrees well with observations. A major advantage of combining low redshift supernovae and high redshift CMB observations arises because the degeneracy in the parameter pair $\{\Omega_m, \Omega_\Lambda\}$ from Sn measurements is orthogonal to the corresponding degeneracy in CMB experiments. A combined likelyhood analysis of CMB anisotropy and Type 1a Supernovae data gives the best fit values [2]

$$\Omega_m = 0.25^{+0.18}_{-0.12},\ \Omega_\Lambda = 0.63^{+0.17}_{-0.23}. \tag{9}$$

which is strongly supportive of a flat Universe with $\Omega_m + \Omega_\Lambda \simeq 1$. The corresponding value of the deceleration parameter turns out to be

$$q_0 \equiv -H_0^{-2}(\ddot{a}/a)_0 = \frac{\Omega_m}{2} - \Omega_\Lambda \simeq -0.5 \tag{10}$$

which shows that the Universe is currently *accelerating*.

Other features of a ΛCDM model include:

1. A longer age.
2. Greater long range power in the perturbation spectrum. (The latter is quantified by means of the dimensionless parameter $\Gamma = \Omega_m h$, the value $\Gamma \simeq 0.25$ appears to agree with most current observations of galaxy clustering.)

3. The growth of long wavelength perturbations still in the linear regime is slower in ΛCDM, affecting the abundance of rich galaxy clusters at high redshift.

4. A closed Λ-dominated Universe can expand forever, while open/flat universes with $\Lambda < 0$ will eventually recollapse. Thus the late time behaviour of a Λ dominated Universe is radically different from standard FRW.

3. Theoretical issues

In 1968 Zeldovich [3] having being drawn to the debate surrounding Λ, showed that zero-point vacuum fluctuations possessed the Lorentz invariant equation of state $p_{\rm vac} = -\rho_{\rm vac}$ and therefore described matter with an energy-momentum tensor $\langle T_{ik}\rangle = \Lambda g_{ik}$. (The presence of zero-point fluctuations had by then been convincingly demonstrated by the experimental verification of the Casimir effect.) However it turns out that for bosons and fermions the vacuum energy is an infinite quantity, having the form

$$\langle 0|H_{b,f}|0\rangle = \pm\frac{1}{2}\sum_{\mathbf{k}}\omega_{\mathbf{k}}. \qquad (11)$$

The fact that the 'infinite' contribution from fermions is equal and opposite to the corresponding 'infinite' contribution from bosons (having identical mass) led to the hope that bosonic and fermionic infinities might cancel each other within the framework of a supersymmetric theory [4]. However this would only occur at high temperatures prevailing in the early Universe when supersymmetry is restored, at lower temperatures supersymmetry would be broken and a large cosmological constant would therefore reappear during the present epoch. (In fact the exact opposite of this scenario is good for cosmology: a large Λ during an early epoch will drive inflation and resolve the horizon and flatness problems and seed structure formation while a small value of Λ today is indicated by observations.)

Zeldovich also suggested that after the removal of infinites (associated with one loop effects) a small residual Λ may be generated at the two loop level due to the gravitational interaction between particle-antiparticle participating in a virtual loop. This would lead to a vacuum energy density $\epsilon_{\rm vac} \equiv \rho_{\rm vac}c^2 \sim (Gm^2/\lambda)/\lambda^3 = Gm^6c^4/\hbar^4$, where $\lambda = \hbar/mc$ is the typical separation between pairs. Substituting for the mass m we find that whereas the electron and proton give too small/large a value for $\rho_{\rm vac}$, the pion mass gives just the right value to be in agreement with observations

$$\rho_\Lambda = \frac{1}{(2\pi)^4}\,\rho_{\rm Pl}\,\left(\frac{m_\pi}{m_{\rm pl}}\right)^6 \simeq 1.45\rho_{\rm Pl}\times 10^{-123}, \qquad (12)$$

$\rho_{\rm Pl}$ is the Planck density: $\rho_{\rm Pl} = c^5/G^2\hbar \simeq 5\times 10^{93}$ g cm $^{-3}$. Expression (12) illustrates the enormous energy difference between the Planck energy and the current value of the vacuum energy. Thus if the value of Λ was set during the Planck epoch of $t_{\rm Pl} \sim 10^{-43}$ sec. it would involve a fine tuning of one part in 10^{123}!

Vacuum effects also play an important role if the Universe is expanding. Consider for instance the Klein Gordon equation describing a massive scalar field which couples non-minimally to gravity

$$[\Box + \xi R + m^2]\Phi = 0. \qquad (13)$$

In a spatially flat FRW Universe field variables separate and

$$\Phi_k = (2\pi)^{-3/2}\frac{\chi_k}{a}(\eta)\,\mathrm{e}^{-i\mathbf{k}\cdot\mathbf{x}}$$

for each wave mode. Substituting in (13) leads to

$$\ddot{\chi}_k + [k^2 - V(\eta)]\chi_k = 0, \tag{14}$$

where $V(\eta) = -m^2a^2 + (1-6\xi)\ddot{a}/a$ and differentiation is carried out with respect to the conformal time $\eta = \int \mathrm{d}t/a$. Comparing (14) with the one dimensional Schrödinger equation in quantum mechanics

$$\frac{\hbar^2}{2m}\frac{\mathrm{d}^2\Psi}{\mathrm{d}x^2} + [E - V(x)]\Psi = 0, \tag{15}$$

we find that $V(\eta)$ in (14) is complementary to $V(x)$ in (15): the latter is a potential barrier in space whereas the former is a *potential barrier in time*. Thus in analogy with quantum mechanics one should expect a particle moving forward in time to be both reflected and transmitted by the barrier $V(\eta)$. The resulting wave function at late times is described by a superposition of positive and negative frequency states

$$\phi_{\mathrm{out}}(k,\eta) = \alpha\phi_k^+ + \beta\phi_k^-, \tag{16}$$

where $\lim_{k\to\infty}\phi_k^\pm \simeq 1/\sqrt{2k}a\,\exp(\mp ik\eta)$. The Bogoliubov coefficients α, β quantify the extent of 'particle production' and 'vacuum polarization' in the space-time brought about by its changing geometry.

The result of a detailed calculation of particle production from an inflationary Universe by Sahni and Habib [5] showed that for an ultra-light scalar field, the vacuum expectation value of the energy momentum tensor had the form of a cosmological constant

$$\Lambda_{\mathrm{eff}} \equiv 8\pi\bar{G}\langle T_{00}\rangle \simeq m^2/2|\xi|,$$
$$\Omega_\Lambda \equiv \Lambda_{\mathrm{eff}}/3H^2 \simeq \frac{1}{6|\xi|}(m/H)^2. \tag{17}$$

Thus quantum effects in an expanding Universe can give rise to a cosmological constant having just the right value to be in agreement with observations!

3.1 *Late time inflation and* Λ

An altogether different means of generating a small Λ at the present epoch is suggested by scenarios involving 'late time inflation'. In this case a scalar field rolls down a potential having the chaotic form $V \propto \phi^p$, where p can be either positive or negative. Consider for instance the case of a massive scalar field $V = 1/2m^2\phi^2$, the inflationary condition $p_\phi \simeq -\rho_\phi$ is realised if $m/H \lesssim 1$, which suggests a very small mass for the scalar $m \sim 10^{-33}$ eV [3,4]. A different possibility arises for the potential $V \propto \phi^{-p}$, in this case it can be shown that the ratio of the scalar field density to that of background matter ρ_B grows as [6,7]

$$\frac{\rho_\phi}{\rho_B} \propto t^{\frac{4}{2+p}} \tag{18}$$

as a result for $p > 0$ the scalar field density can dominate the background matter/radiation density at late times, even if $\rho_\phi < \rho_B$ initially. In both scenario's the Universe accelerates at late times under the action of a scalar field whose potential mimicks a *time dependent* Λ-term. For models with $V \propto \phi^{-p}$ the fine tuning problem is significantly less acute since the initial value of ρ_ϕ can be much larger than its present value of $\sim 10^{-29}$g cm^{-3}. (see [8] for a recent review of the cosmological constant and [9,10,3] for earlier reviews.)

4. Conclusions

We briefly discuss the issue of whether a small cosmological constant can dominate the mass density of the Universe as implied by current observational data. Although the data indicates that the Universe is accelerating, the precise mechanism which generates either a time independent cosmological constant or else a time dependent cosmological Λ-term remains unsettled. Several possibilities exist: the cosmological constant may be induced by quantum effects at either one loop or higher order. Ultra-light scalar fields can give rise to a time dependent Λ-term by virtue of a scenario involving late-time inflation, although it is not clear whether such models arise naturally in particle physics theories of the fundamental interactions. Finally a small cosmological constant may also be motivated by the anthropic principle [11]. Observational evidence for Λ has come from the results of Type 1a supernovae observations made by members of the Supernova Cosmology Project [12,1] and the High-Z Supernova Search Team [14]. Since several dozen supernovae events are likely to be added to the Sn inventory every year, the statistical evidence for (or against) Λ can only get better with time (provided of course systematics is well understood). The results from Sn observations are complemented by CMB analysis. Here too one expects significant improvement in the data from the MAP and PLANCK satellite missions which are expected to be launched in the near future. Thus the ongoing quest for Λ promises to bear rich fruit and could lead to deep theoretical consequences of a radical nature.

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