

Coulomb gap in one-dimensional disordered electronic systems

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We study a one-dimensional system of spinless electrons in the presence of a long-range Coulomb interaction (LRCI) and a random chemical potential at each site. We first present a Tomonaga-Luttinger liquid (TLL) description of the system. We use the bosonization technique followed by the replica trick to average over the quenched randomness. An expression for the localization length of the system is then obtained using the renormalization group method and also a physical argument. We then find the density of states for different values of the energy; we get different expressions depending on whether the energy is larger than or smaller than the inverse of the localization length. We work in the limit of weak disorder where the localization length is very large; at that length scale, the LRCI has the effect of reducing the interaction parameter K of the TLL to a value much smaller than the noninteracting value of unity.

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I. INTRODUCTION

In a strongly localized electron system with an un-screened long-range Coulomb interaction (LRCI) between the electrons, the density of states (DOS) is expected to exhibit a gap at the Fermi energy. This is called the “Coulomb gap” [1] as it arises due to the LRCI. A phenomenological argument [1] suggests that the DOS should obey a power-law of the form

$$D(\omega) \sim |\omega|^{d-1}, \quad (1)$$

where d is the spatial dimensionality of the system, and ω is measured from the Fermi energy. We shall consider here the one-dimensional case where the DOS is expected to show a logarithmic behavior in the extremely localized limit [2–5].

Following the recent developments in carbon nanotube technology [6], there has been an upsurge in the studies of one and quasi-one dimensional electron systems. In low dimensional systems, the electron-electron interactions play a dominant role leading to a behavior significantly different from that of conventional Fermi liquids. Short-range repulsive interactions between the electrons lead to the Tomonaga-Luttinger liquid (TLL) behavior [7–9], whereas the LRCI is believed to lead to a Wigner crystal [10].

It is well known that for a one-dimensional noninteracting electron system with random disorder, all the states are localized due to repeated back-scatterings of the electrons [11]. However, the DOS of noninteracting electrons is finite at the Fermi energy [12]. The presence of LRCI together with random impurities modifies the DOS in a drastic way and supposedly leads to the Coulomb gap behavior as discussed above. In an earlier study, Vojta and John [5] studied a one-dimensional electron system, with LRCI and randomness, in the extremely localized

limit where the overlap of electronic wave functions can be neglected. They find the form of the DOS to be [5]

$$D(\omega) \sim \left(\ln \frac{E_c}{|\omega|} \right)^{-1}, \quad (2)$$

where E_c is a cut-off energy [3,4]. In a recent paper, Lee [13] derived the expression of the DOS of a one-dimensional system of spinless electrons with LRCI and impurities at random positions in the opposite limit where the quantum effects (electron hopping) play a dominant role, and the pinning is weak, i.e., the localization length is much larger than the inter-impurity distance. Following studies of a pinned Wigner crystal [14,15] and using a semiclassical approximation, Lee [13] finds the DOS at low energy to be

$$D(\omega) \sim |\omega|^{\sqrt{1+\eta}/2}, \quad (3)$$

where the exponent η (defined in Eq. (19) below) is determined by the localization length of the system. This power-law behavior is consistent with the existing numerical studies [16].

In this paper, we study a model of spinless electrons with LRCI as in Ref. [13] (with quantum effects, i.e., with the electron hopping term) in the presence of a random chemical potential at each site. In our approach, we find that a TLL picture [9], rather than a semiclassical approach [13], provides a convenient description of the system. The localization length L_0 is derived using a renormalization group (RG) study of the effective disorder-averaged bosonized action [17], and is found to be very large in the limit of weak disorder. (We also present a simple physical understanding of the expression for L_0). Due to the LRCI, the interaction parameter K of the TLL is found to be effectively a function of the length scale. At the length scale L_0 , the value of K is

given by a value much smaller than unity (which is the value of K for the noninteracting system). We should remark here that the present one-dimensional disordered electron system with LRCI appears to be a unique example of a TLL with $K \ll 1$.

Once we make use of a TLL description, the DOS can be found in a standard way [9,18]. We will show that for $\omega \gtrsim v_F/L_0$, the DOS is given by

$$D(\omega) \sim |\omega|^\beta, \quad \beta = \frac{\sqrt{1+\eta}}{2} + \frac{1}{2\sqrt{1+\eta}} - 1, \quad (4)$$

where, *unlike* a standard TLL, η itself depends on ω as discussed below. Eq. (4) resembles the result of Lee [13] in the limit $\eta \gg 1$; however, η is not taken to be a function of ω in Ref. [13]. On the other hand, for $\omega \lesssim v_F/L_0$, we will argue that the DOS is given by the expression in Eq. (2).

In Sec. II, we will consider a system of spinless electrons with LRCI and disorder. After bosonizing the Hamiltonian, we will use the RG equations to obtain the localization length L_0 in terms of the disorder strength. We will also present a physical understanding of the expression for L_0 . In Sec. III, we will obtain expressions for the DOS for different ranges of values of ω . In Sec. IV, we will briefly discuss the case of a screened Coulomb interaction.

II. BOSONIZATION AND THE LOCALIZATION LENGTH

The Hamiltonian of a disordered system of spinless electrons with LRCI consists of three parts,

$$H = H_0 + H_C + H_{\text{random}}. \quad (5)$$

The noninteracting part H_0 and the Coulomb interaction part H_C are written in terms of the continuum chiral electron fields as follows,

$$H_0 = v_F \int_{-\infty}^{\infty} dx \left(-i\psi_R^\dagger \partial_x \psi_R + i\psi_L^\dagger \partial_x \psi_L \right), \quad (6)$$

$$H_C = \frac{1}{2} \int_{-\infty}^{\infty} dx dy U(x-y) \rho(x) \rho(y), \quad (7)$$

where the form of $U(x-y)$ will be specified later. The fields $\psi_R(\psi_L)$ are the right-moving (left-moving) electron operators; they are related to the lattice electron operators by

$$c(x) = \sqrt{a} \left[e^{ik_F x} \psi_R(x) + e^{-ik_F x} \psi_L(x) \right], \quad (8)$$

where a is the lattice spacing. The density $\rho = \rho_R + \rho_L$, where ρ_R (ρ_L) is the normal-ordered density of the right (left) moving electrons.

In our model, the random part of the Hamiltonian consists of two parts in the continuum limit, (i) the forward scattering part, H_f , where the scattered electrons remain in the vicinity of the same Fermi point, and (ii) the backward scattering part, H_b , where an electron is scattered from $-k_F$ to k_F , or vice versa.

Using the rules of bosonization [9,18], we rewrite the full Hamiltonian in the bosonic language. The low-energy and long-wavelength excitations of the noninteracting part can be written as

$$H_0 = \frac{v_F}{2\pi} \int_{-\infty}^{\infty} dx \left[(\partial_x \theta(x))^2 + (\partial_x \phi(x))^2 \right], \quad (9)$$

while the Coulomb part is given by

$$H_C = \frac{1}{2\pi^2} \int_{-\infty}^{\infty} dx dy U(x-y) \partial_x \phi(x) \partial_y \phi(y). \quad (10)$$

The forward scattering part is given by

$$\begin{aligned} H_f &= \int_{-\infty}^{\infty} dx h(x) \left[\psi_R^\dagger(x) \psi_R(x) + \psi_L^\dagger(x) \psi_L(x) \right] \\ &= -\frac{1}{\pi} \int_{-\infty}^{\infty} dx h(x) \partial_x \phi(x). \end{aligned} \quad (11)$$

The forward scattering is due to the real random field $h(x)$, and its effect can be taken care of by a rescaling of the bosonic field ϕ [17]; henceforth, we shall ignore the forward scattering part. The back-scattering part can be written as

$$\begin{aligned} H_b &= \int_{-\infty}^{\infty} dx \left[\xi(x) \psi_R^\dagger(x) \psi_L(x) + \xi^*(x) \psi_L^\dagger(x) \psi_R(x) \right] \\ &= \frac{1}{\pi\alpha} \int_{-\infty}^{\infty} dx \left[\xi(x) e^{i(2\phi(x)+2k_F x)} + \text{h.c.} \right]. \end{aligned} \quad (12)$$

where h.c. denotes the hermitian conjugate. The back-scattering is due to a complex random field $\xi(x)$ with the probability distribution

$$P[\xi(x)] = \exp \left[-D_\xi^{-1} \int dx \xi^*(x) \xi(x) \right], \quad (13)$$

so that $\langle \xi(x) \rangle = 0$ and $\langle \xi^*(x) \xi(x') \rangle = D_\xi \delta(x-x')$.

The Hamiltonians given in Eqs. (9), (10) and (12), along with the probability distribution (13) of the random field ξ , constitute the complete low-energy and long-wavelength description of the model in the bosonic language. We now rewrite the Coulomb interaction in Eq. (10) in momentum space as

$$\begin{aligned} &\frac{1}{2\pi^2} \int dx dy U(x-y) \partial_x \phi(x) \partial_y \phi(y) \\ &= \frac{1}{4\pi^3} \int dk k^2 \hat{U}(k) \hat{\phi}(k) \hat{\phi}(-k), \end{aligned} \quad (14)$$

where $\hat{U}(k)$ is the Fourier transform of the Coulomb interaction. We can expand $\hat{U}(k)$ in powers of k as

$$\hat{U}(k) = \hat{U}_0 + \hat{U}_1 k + \hat{U}_2 k^2 + \dots . \quad (15)$$

From the unperturbed Hamiltonian in Eq. (9), we find the naive scaling dimensions of various quantities to be

$$[\omega] = [k] = (\text{Length})^{-1}, \quad [\hat{\phi}] = (\text{Length})^1. \quad (16)$$

Considering the scaling dimensions of the various terms in Eq. (15), we conclude that only the constant term is marginal under renormalization, whereas the terms involving powers of k are irrelevant. Hence, the important contribution arising from the Coulomb interaction term in Eq. (14) can be written as

$$\begin{aligned} H_C &= \frac{\hat{U}_0}{4\pi^3} \int dk k^2 \hat{\phi}(k) \hat{\phi}(-k) \\ &= \frac{\hat{U}_0}{2\pi^2} \int dx (\partial_x \phi(x))^2. \end{aligned} \quad (17)$$

The constant \hat{U}_0 is given by

$$\hat{U}_0 = \int_{-L/2}^{L/2} dx \frac{q^2}{\sqrt{x^2 + d^2}} = 2q^2 \ln \frac{L}{d}, \quad (18)$$

where q is the charge of the electron, and d is the width of the wire which is of the order of the lattice spacing a ; it ensures convergence of the integral in Eq. (18) near $x = 0$. The meaning of the cut-off length L will be discussed below. (We have assumed that $L \gg a, d$; this will be justified later for various cases of interest). We now define a dimensionless quantity

$$\eta = \frac{2q^2}{\pi v_F} \ln \frac{L}{a}. \quad (19)$$

Then the Hamiltonian can be written in the form $H = H_0 + H_C + H_b$, where

$$H_0 + H_C = \frac{v_F}{2\pi} \int dx [(\partial_x \theta(x))^2 + (1 + \eta)(\partial_x \phi(x))^2]. \quad (20)$$

Eq. (20) can be brought into a standard form by defining

$$K = \frac{1}{\sqrt{1 + \eta}}, \quad \text{and} \quad u = v_F \sqrt{1 + \eta}. \quad (21)$$

We then arrive at the expression

$$H_0 + H_C = \frac{u}{2\pi} \int dx \left[K (\partial_x \theta(x))^2 + \frac{1}{K} (\partial_x \phi(x))^2 \right]. \quad (22)$$

In the imaginary time representation, the quadratic and random parts of the action are respectively given by

$$\begin{aligned} S_0 + S_C &= \frac{u}{2\pi} \int dx d\tau \left[K (\partial_\tau \phi(x, \tau))^2 + \frac{1}{K} (\partial_x \phi(x, \tau))^2 \right], \\ S_b &= \frac{1}{\pi\alpha} \int dx d\tau \left[\xi(x) e^{i(2\phi(x, \tau) + 2k_F x)} + h.c. \right]. \end{aligned} \quad (23)$$

The randomness in the back-scattering term is dealt with using the standard replica method and averaging over the randomness using the distribution in Eq. (13). The final form of the disorder-averaged n -replicated action is found to be [17]

$$\begin{aligned} S_n &= \frac{u}{2\pi} \sum_a \int dx d\tau \left[K (\partial_\tau \phi^a(x, \tau))^2 + \frac{1}{K} (\partial_x \phi^a(x, \tau))^2 \right] \\ &\quad - \frac{D_\xi}{(\pi\alpha)^2} \sum_{a,b} \int dx d\tau d\tau' \cos(2\phi^a(x, \tau) - 2\phi^b(x, \tau')), \end{aligned} \quad (24)$$

where a, b are the replica indices running from 1 to n . Eq. (24) is the same as the replicated action of a one-dimensional disordered electronic system given in Ref. [17]. However, the back-scattering term arising due to the spin of the electrons is absent in the present case of spinless electrons. The RG equations of the different parameters are given by [17]

$$\begin{aligned} \frac{dD}{dl} &= (3 - 2K)D, \\ \frac{dK}{dl} &= -\frac{1}{2}K^2D, \\ \frac{du}{dl} &= -\frac{uK}{2}D. \end{aligned} \quad (25)$$

where $D = (2D_\xi\alpha)/\pi u^2$, and $l = \ln(L/a)$ is the logarithm of the length scale. The RG equations given above show a quantum phase transition at $K = 3/2$. For $K > 3/2$, the disorder is irrelevant. For $K < 3/2$, the disorder grows under RG and leads to localization. Thus the system undergoes a zero-temperature localized-delocalized transition at $K = 3/2$.

The localization length, denoted by L_0 , can be obtained by integrating the RG equations in (25). At the microscopic length scale a , we begin with some values of D (much smaller than 1), and u, K given in Eq. (21). (The latter requires a knowledge of the length scale L through Eq. (19); this will be determined self-consistently by the solution of the RG equations). Since we are assuming that $K \ll 1$, we find (as can be verified numerically using Eqs. (25)) that the quantities K and u flow very little while D flows from a small number of order D_ξ to a number of order 1. Thus the length scale at which the disorder strength becomes of order 1 is given by

$$\frac{L_0}{a} \sim \left(\frac{1}{D_\xi} \right)^{1/(3-2K)}. \quad (26)$$

We identify this as the localization length whose physical meaning will be discussed below.

In the limit of weak disorder ($D_\xi \rightarrow 0$), the form of the localization length in Eq. (26) implies, in a self-consistent manner, that (i) $L_0 \gg a$, (ii) the parameter $\eta \gg 1$ (replacing L by L_0 in Eq. (19)), and (iii) the interaction parameter $K \ll 1$ due to Eq. (21). Hence the localization length in the $K \rightarrow 0$ limit assumes the classical value

$$\frac{L_0}{a} \sim \left(\frac{1}{D_\xi} \right)^{\frac{1}{3}}. \quad (27)$$

[Note that the bosonization approach cannot be used in the extremely localized limit ($L_0 \sim a$) studied in Refs. [3–5]. In that limit, one cannot use a continuum description, and thus a bosonized description of the system is not possible].

The significance of the localization length is as follows. Although two electrons which are separated by more than the distance L_0 do interact with each other through the Coulomb potential, the overlap of their wave functions is exponentially small, and hence their positions are uncorrelated. Such interactions will therefore only contribute to a uniform and static chemical potential which is the same for all electrons. Namely, the Coulomb potential felt by an electron from other electrons which are separated from it by a distance larger than L_0 is described by a part of Eq. (7) given by

$$H_C = \frac{1}{2} \int_{-\infty}^{\infty} dx \rho(x) \rho_0 \left[\int_{x+L_0}^{\infty} dy U(x-y) + \int_{-\infty}^{x-L_0} dy U(x-y) \right], \quad (28)$$

where ρ_0 is the average density. Eq. (28) represents a uniform one-body potential. On the other hand, when two electrons are closer to each other than L_0 that their positions are correlated; then their Coulomb interaction has to be included in the quadratic part of the bosonic Hamiltonian (Eq. (10) to be specific) which governs the density fluctuations. In short, portions of the system which are separated by distances larger than L_0 are uncorrelated, whereas, within a distance L_0 , the system can be described in terms of a TLL with $K \ll 1$.

Before ending this section, we would like to present a simple physical understanding of the important result in Eq. (26) which does not make use of the replica idea. First, let us consider a weak δ -function impurity of strength V_0 at one point in a TLL. According to an RG equation derived by Kane and Fisher [19], the impurity strength V flows according to the equation $dV/dl = (1-K)V$; hence, at a distance scale L , we have $V(L) \sim V_0 (L/a)^{1-K}$. From quantum mechanics, we know that the reflection amplitude for an electron scattering from a single impurity is proportional to V if

V/v_F is small. Now suppose that each site in the lattice has a random impurity of strength $V(x)$ which satisfies $\langle V(x)V(y) \rangle = D_\xi \delta(x-y)$. This means that at each site, $V(x)$ is of the order of $\sqrt{D_\xi}$, and its sign is random. Over a length L , there are L/a reflections since there is a reflection at each site. In order that these L/a reflections should add up to a total reflection amplitude of order 1 (which would localize the electron), we must have $\sqrt{L/a} V(L) \sim 1$ (assuming that L/a is large, and using the well-known result for the sum of a large number of random terms in a Brownian motion). This gives us the estimate $L_0/a \sim 1/(V(L_0))^2$, i.e., $L_0/a \sim 1/(V_0(L_0/a)^{1-K})^2$. Since $V_0 \sim \sqrt{D_\xi}$, we obtain $L_0/a \sim (1/D_\xi)^{1/(3-2K)}$.

Our approach shows that the two ways of introducing disorder, (i) at random positions with a density n_i but with equal strengths V_0 as in Ref. [13], and (ii) at all sites but with random strengths $\xi(x)$ as in the present case, have the same effect. This is because the only thing which matters finally is the disorder parameter D_ξ which appears in the expression in Eq. (26) for the localization length [17]. The exact nature of the randomness is not relevant for determining the form of L_0 . The relation between the two sets of disorder parameters is given by $D_\xi \sim V_0^2 n_i$. Using this relation, one can write the expression for L_0 given in Ref. [13] directly in terms of D_ξ and check that L_0 is proportional to $(1/D_\xi)^{1/3}$ (except for a logarithmic correction arising due to the LRCI [14]). This is also what the RG equation yields for $\eta \gg 1$. Hence, our approach provides an independent (RG) derivation of L_0 which is an alternative to the derivation using considerations of energy minimization [14,15]. We have also presented above an alternative understanding of the expression in Eq. (26) which does not use the replica idea and is based only on the RG equation for a single impurity.

III. DENSITY OF STATES

We are now ready to discuss the density of states. For energies satisfying $\omega \gtrsim v_F/L_0$, the disorder parameter D is small, and our system can be described by a clean TLL. The DOS of a TLL is given by the relation [9]

$$D(\omega) \sim |\omega|^\beta, \quad \beta = \frac{(1-K)^2}{2K} = \frac{\sqrt{1+\eta}}{2} + \frac{1}{2\sqrt{1+\eta}} - 1, \quad (29)$$

where we have used the relation $K = 1/\sqrt{1+\eta}$. This is the result quoted in Eq. (4). However, we now have to determine what value of L one should take in Eq. (19) in order to determine the value of η to be used in Eq. (29).

It turns out that the value of ω itself determines the appropriate value of η to use in Eq. (29). If $\omega = v_F/L_0$, then the length scale of interest is v_F/ω which is equal

to L_0 . The value of η to use in Eq. (29) is then given self-consistently by Eqs. (19) (with L replaced by L_0), (21) and (26). However, if $\omega > v_F/L_0$, then the length scale of interest is $L = v_F/\omega$ which is smaller than L_0 . Then the Coulomb interaction should be cut-off at the length scale L as indicated in Eq. (18). The reason for this lies in the basic idea behind the RG method, namely, that the properties of a system at a length scale L (or, equivalently, at an energy scale v_F/L) are governed essentially by the modes whose wavelengths are smaller than L . (The modes whose wavelengths are larger than L only contribute to a uniform and static chemical potential in the sense described in Eq. (28)). If the Coulomb interaction is cut-off at the distance L , the value of η to be used in Eq. (29) is simply given by Eq. (19). We thus conclude that as long as $\omega \gtrsim v_F/L_0$, the DOS of states is given by Eq. (29) where η depends on ω through Eq. (19) with $L = v_F/\omega$.

We note that the expression in Eq. (4) agrees with the semiclassical result in Eq. (3) in the limit $\eta \gg 1$. Moreover, Eq. (4) correctly reproduces the expression of the DOS of a noninteracting disordered electronic system (for which $\eta = 0$ and $K = 1$), where there is a finite DOS at the Fermi energy [12].

We now consider the case of low energies satisfying $\omega < v_F/L_0$. In this case, the RG equations in (25) imply that the disorder is strong; hence the system cannot be described by a clean TLL. However, this is precisely the regime described by the localized limit discussed in Refs. [3–5], where the localization length L_0 is smaller than the length scale of interest, namely, $L = v_F/\omega$. We therefore expect the DOS in this regime to be described by Eq. (2).

We have thus found expressions for the DOS in the two regimes $\omega \gtrsim v_F/L_0$ and $\omega \lesssim v_F/L_0$ (strictly speaking, $\omega \gg v_F/L_0$ and $\omega \ll v_F/L_0$ respectively). The DOS must of course cross over smoothly from one regime to the other, but this cross over is difficult to determine analytically.

Let us summarize our key results. In the limit of weak disorder, the system is power-law correlated only over a large distance of order L_0 , called the localization or pinning length. At that length scale, the system is described by a TLL in which the LRCI drastically reduces the interaction parameter K to a value much smaller than unity; such a value of K is quite uncommon in the literature. (Thus the Coulomb interaction has the same effect as a very strong short-range interaction)! We note that in the absence of disorder, a system with a LRCI is *not* equivalent to one with a short-range interaction. For a clean system, the LRCI leads to a Wigner crystal [10] for which a TLL description is not valid. In our formalism, we can see this by noting that if the disorder was absent (i.e., $D_\xi = 0$), L_0 would be infinite; then η and K would respectively be ∞ and zero, and the TLL description would no longer be valid. On the other hand, for the disordered

but noninteracting system (i.e., $q^2 = 0$), we have $\eta = 0$ and $K = 1$ even if L_0 is very large.

The TLL description enables us to derive the DOS easily using well-known relations without having to make use of a semiclassical approach [13]. Even though the final expressions for the DOS in Eqs. (3) and (4) match in the limit of $\eta \gg 1$, the expression in Eq. (3) derived in Ref. [13] shows a gap at the Fermi energy of the form $\omega^{1/2}$, and thus fails to reproduce the result for the noninteracting system. Our TLL approach correctly captures the physics of both the interacting and noninteracting limits by using the appropriate values of the interaction parameter K .

Finally, beyond the length scale L_0 , the TLL description is no longer valid since the disorder strength is of order 1. In this regime, the DOS is given by Eq. (2); thus as $\omega \rightarrow 0$, the DOS goes to zero logarithmically rather than as a power-law.

IV. DISCUSSION

It is interesting to consider what would happen if the Coulomb interaction was screened. We then have

$$\hat{U}_0 = \int_{-L/2}^{L/2} dx \frac{q^2 e^{-x/L_s}}{\sqrt{x^2 + d^2}},$$

$$\eta = \frac{\hat{U}_0}{\pi v_F}, \quad \text{and} \quad K = \frac{1}{\sqrt{1 + \eta}}, \quad (30)$$

where L_s is the screening length. It is clear that \hat{U}_0 now has a finite limit, and that η and K no longer approach ∞ and zero respectively, as $L \rightarrow \infty$. For the case of weak disorder, the localization length L_0 is again given by the expression in Eq. (26), where K is self-consistently determined by Eq. (30) with L replaced by L_0 .

Let us consider the case in which the screening length L_s is much smaller than L_0 . Now there are three regimes of energy to consider for the DOS, (i) $\omega \gtrsim v_F/L_s$, (ii) $v_F/L_s \gtrsim \omega \gtrsim v_F/L_0$, and (iii) $v_F/L_0 \gtrsim \omega$. In regimes (i) and (ii), arguments similar to those presented in Sec. III will show that the system can again be described by a clean TLL in which the DOS is given by Eq. (29), with η being determined by ω through Eq. (30) with $L = v_F/\omega$. Clearly, η will increase as ω decreases in regime (i). However, η will stop changing appreciably once L becomes much bigger than L_s ; hence, η will be almost constant in regime (ii). Finally, in regime (iii), the disorder is of order 1, the system is in the localized limit (the length scale of interest is larger than L_0), and the DOS is given by Eq. (2).

Before ending, we would like to mention that the results obtained here for the localization length and the DOS can be used to study disordered antiferromagnetic spin chains. In particular, the spin-1/2 chain with random magnetic field and random exchanges of various

kinds has been studied extensively in recent years [20–22]. Our analysis can be applied to such systems after mapping the spins to Jordan-Wigner fermions [20].

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