Enhancement of tunneling density of states at a junction of three Luttinger liquid wires

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We study the tunneling density of states (TDOS) for a junction of three Tomonaga-Luttinger liquid wires. We show that there are fixed points which allow for the *enhancement* of the TDOS, which is unusual for Luttinger liquids. The distance from the junction over which this enhancement occurs is of the order of $x = v/(2\omega)$, where v is the plasmon velocity and ω is the bias frequency. Beyond this distance, the TDOS crosses over to the standard bulk value independent of the fixed point describing the junction. This finite range of distances opens up the possibility of experimentally probing the enhancement in each wire individually.

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Introduction.- Junctions of multiple quantum wires (QWs) have attracted considerable attention in recent years since they form the basic building blocks of quantum circuitry. Experimental realizations of three-wire Y-junctions of carbon nanotubes have also given the field a strong impetus [1]. Earlier studies of junctions of Tomonaga-Luttinger liquids (LL) were mostly focused on searching for various low-energy fixed points (FPs) [2, 3, 4, 5, 6, 7, 8, 9] and the corresponding conductance matrices; this includes the recent development involving the inclusion of a finite superconducting gap at the junction [10, 11].

Here we will focus on the local single-particle tunneling density of states (TDOS); this can be found by measuring the differential tunneling conductance of a scanning tunneling microscope (STM) tip at a finite bias. The differential conductance provides a direct measure of the TDOS of the electrons at an energy given by the bias voltage [12], provided the density of states for the STM is energy independent. For a two-wire LL junction, the current measured by the STM tip varies as a power of the bias [13]; the power depends on the Luttinger parameter g and the FP to which the junction has been tuned.

Earlier studies of the TDOS in a LL system with [14] and without [15] impurities revealed that the TDOS vanishes as a power law in the zero bias limit [19]. This is popularly considered to be a hallmark of a LL system. The TDOS for a LL wire with an impurity was studied in Ref. [13] where the tunneling STM current close to the impurity was shown to be power law suppressed in the zero bias limit. An enhancement of the TDOS was found at a single LL -superconductor [16] junction and was explained in terms of the proximity effect. An enhancement of the spectral weight was also found at a junction of multiple LLs tuned to a *fermionic* FP (linear boundary condition (BC) between fermion fields at the junction) [6]. All the earlier studies of the TDOS were focused on either a multiple wire junction tuned to a fermionic FP or a single LL -superconductor junction, but not a junction of multiple LLs tuned to *bosonic* FPs. Bosonic FPs refer to linear BCs connecting incoming and outgoing currents (which are bilinears in the fermion fields) at the junction.

Here we study the TDOS of a three-wire junction of a single channel QW modeled as a LL. We find that close to the junction, the TDOS depends on the details of the current splitting matrix at the junction which relates the incoming and outgoing currents; this matrix describes *bosonic* FPs of the system. We find that for a certain range of repulsive inter-electron interaction ($g < 1$) and certain current splitting matrices, the TDOS close to the junction shows an *enhancement* in the zero bias limit. We show that this is related to reflection of holes off the junction which mimics the Andreev reflection process, even though there is no superconductor in the present scenario. This is in sharp contrast to the case of a two-wire junction where a repulsive electron-electron interaction always results in a suppression of the TDOS near the junction. This is the central result of this Letter. Far away from the junction, the TDOS reduces to that of the bulk LL wire, $\rho(\omega) \sim \omega^{(g+g^{-1}-2)/2}$, independent of the details of the junction, and shows a suppression for both repulsive ($g < 1$) and attractive ($g > 1$) interactions.

Bosonization.- The electron field (taken to be spinless for simplicity) can be bosonized as $\psi(x) = (1/\sqrt{2\pi\alpha})$ $[F_O\, e^{ik_F x + i\phi_O(x)} + F_I e^{-ik_F x + i\phi_I(x)}]$, where $\phi_O(x)$, F_O and $\phi_I(x)$, F_I are the outgoing and incoming chiral bosonic fields and corresponding Klein factors, k_F is the Fermi momentum, and α is a short distance cut-off. We model the wires as spinless LLs on a half-line ($x > 0$), *i.e.*, all the wires are parametrized by a coordinate x running from 0 to ∞. The corresponding Hamiltonian is given by

$$H \;=\; \int_0^\infty dx \;\sum_{i=1}^{N}\; \frac{v}{2\pi}\; \left\{ g\;(\phi_i')^2 + \frac{1}{g}\;(\theta_i')^2 \right\}, \quad (1)$$

where prime (dot) stands for spatial (time) derivative, $\phi_i(x,t) = (\phi_{Oi} + \phi_{Ii})/2$, $\theta_i(x,t) = (\phi_{Oi} - \phi_{Ii})/2$, $\dot{\phi}_i = (v/g)\,\theta_i'$, and $\dot{\theta}_i = (vg)\,\phi_i'$. The total density and current can be expressed in terms of the incoming and outgoing fields as $\rho = \rho_O + \rho_I$ with $\rho_{O/I} = \pm(1/2\pi)\,\phi'_{O/I}$, and $j = j_O - j_I$ with $j_{O/I} = \pm(v_F/2\pi)\,\phi'_{O/I}$. Finally, we need to impose a boundary condition on the fields at $x = 0$. Following Ref. [7], the incoming and outgoing currents, and consequently the bosonic fields, are related at the junction by a current splitting matrix $\mathbb{M}$, *i.e.*, $j_{Oi} = \sum_j \mathbb{M}_{ij}\; j_{Ij}$, which leads to $\phi_{Oi} = \sum_j \mathbb{M}_{ij}\; \phi_{Ij}$. To ensure that the matrix $\mathbb{M}$ represents a FP of the theory, the incoming and outgoing fields must satisfy the bosonic commutation relations; this restricts the matrix $\mathbb{M}$ to be orthogonal. Scale invariance imposes the same constraint of orthogonality on $\mathbb{M}$ as shown in Ref. [11]; orthogonality also implies that there is no dissipation in the system. Current conservation at the junction implies [9] that each row and column of $\mathbb{M}$ adds up to 1.

In general, for a three-wire charge-conserving junction, $\mathbb{M}$ can be parametrized by a single continuous parameter θ, and it falls into one of two classes with (a) $\det \mathbb{M}_1 = 1$, and (b) $\det \mathbb{M}_2 = -1$. These classes can be expressed as

$$\mathbb{M}_1 = \begin{pmatrix} a & b & c \\ c & a & b \\ b & c & a \end{pmatrix}, \quad \mathbb{M}_2 = \begin{pmatrix} b & a & c \\ a & c & b \\ c & b & a \end{pmatrix}. \tag{2}$$

In Eq. (2), $a = (1 + 2\cos\theta)/3$, $b(c) = (1 - \cos\theta + (-)\sqrt{3}\sin\theta)/3$. This provides us with an explicit parametrization of the two one-parameter families of FPs; any FP in the theory can be identified in terms of θ, with the FPs at $\theta = 0$ and $\theta = 2\pi$ being identical. The $\det \mathbb{M}_1 = 1$ class represents a Z_3 symmetric (in the wire index) class of FPs, while $\det \mathbb{M}_2 = -1$ represents an asymmetric class of FPs. In the $\mathbb{M}_1$ class, $\theta = \pi$ corresponds to the $\mathsf{D_P}$ FP, $\theta = 0$ corresponds to the disconnected N FP, and $\theta = 2\pi/3$ and $4\pi/3$ correspond to the chiral cases $\chi_\pm$, following the notation of Ref. [5]. Since ϕ_O and ϕ_I are interacting fields, we must perform a Bogoliubov transformation, $\phi_{O/I} = (1/2\sqrt{g})\{(1+g)\tilde{\phi}_{O/I} + (1-g)\tilde{\phi}_{I/O}\}$, to obtain the corresponding free outgoing (incoming) ($\tilde{\phi}_{O/I}$) chiral fields satisfying the commutation relations, $[\tilde{\phi}_{O/I}(x,t), \tilde{\phi}_{O/I}(x',t)] = \pm i\pi Sgn(x - x')$. Unlike the usual Bogoliubov transformation, here we also need to consider the effect of the junction matrix $\mathbb{M}$ relating the interacting incoming and outgoing fields. Following Ref. [7], we obtain a Bogoliubov transformed matrix $\widetilde{\mathbb{M}}$ which relates $\tilde{\phi}_{Oi}$ to $\tilde{\phi}_{Ii}$. We find that $\tilde{\phi}_{Oi}(x) = \sum_j \widetilde{\mathbb{M}}_{ij}\; \tilde{\phi}_{Ij}(-x)$ where $\widetilde{\mathbb{M}} = [(1+g)\mathbb{I} - (1-g)\mathbb{M}]^{-1}\,[(1+g)\mathbb{M} - (1-g)\mathbb{I}]$. Note that the $\mathbb{M}_2$ class of matrices satisfy $(\mathbb{M}_2)^2 = \mathbb{I}$; hence $\widetilde{\mathbb{M}}_2 = \mathbb{M}_2$, and the interacting and free fields satisfy identical BCs at the junction. This is not true for the $\mathbb{M}_1$ class, but $\widetilde{\mathbb{M}}_1$ still has the same form as $\mathbb{M}_1$ with the corresponding parameters $\tilde{a} = (3g^2 - 1 + (3g^2+1)\cos\theta)/\eta$ and $\tilde{b}(\tilde{c}) = 2(1 - \cos\theta + (-)\sqrt{3}g\sin\theta)/\eta$, where $\eta = 3(1 + g^2 + (g^2-1)\cos\theta)$.

The $\mathbb{M}$ matrix is related to the DC conductance matrix given by $\mathbb{G} = (2e^2/h)(\mathbb{I} - \mathbb{M})$ for Fermi liquid leads [5, 11]. Qualitatively, $\mathbb{M}$ is related to tunnelings between the different wires and backscatterings in each wire. The experimental set-up can be a junction of several edges of a quantum Hall system as in Ref. [7], and $\mathbb{M}$ (or θ) can be tuned by applying gate voltages and a magnetic field at the junction. We note that $\mathbb{M}_2$ is time-reversal invariant, but $\mathbb{M}_1$ is generally not and tuning it will require a magnetic field piercing the junction (see Fig. 1).

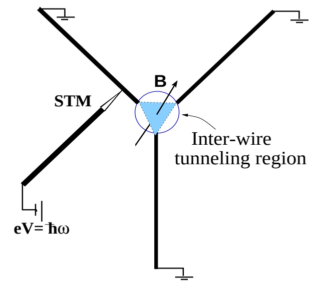

FIG. 1: Schematic picture of STM tip for measuring the TDOS near the junction, the region of inter-wire tunnelings and a magnetic field **B** at the junction.

Tunneling density of states.- We now compute the TDOS for adding an electron with energy $\hbar\omega$ on the $i^{\rm th}$ wire [12],

$$\begin{aligned}\rho_i(\omega) \;&=\; 2\pi \sum_n |\langle 0 \,|\, \psi_i^\dagger(x) \;|n\rangle|^2 \;\delta(E_n - E_0 - \hbar\omega) \\ &=\; 2\mathcal{R}e \int_0^\infty dt\; \langle 0 \,|\, \psi_i(x,t)\,\psi_i^\dagger(x,0) \;|\, 0\rangle\; e^{i\omega t}. \end{aligned}\tag{3}$$

Here $|n\rangle$ (E_n) denotes the $n^{\rm th}$ eigenstate (eigenvalue) of the Hamiltonian in Eq. (1). The Green's function in the $i^{\rm th}$ wire is $\mathcal{G} = \langle \psi_i(x,t)\psi_i^\dagger(x,0)\rangle = \langle \psi_{Ii}(x,t)\psi_{Ii}^\dagger(x,0)\rangle + \langle \psi_{Oi}(x,t)\psi_{Oi}^\dagger(x,0)\rangle + e^{-2ik_F x}\langle \psi_{Ii}(x,t)\psi_{Oi}^\dagger(x,0)\rangle + e^{2ik_F x}\langle \psi_{Oi}(x,t)\psi_{Ii}^\dagger(x,0)\rangle$. The two non-oscillatory terms are

$$\begin{aligned}&\langle \psi_{Ii}(x,t)\,\psi_{Ii}^\dagger(x,0)\rangle = \langle \psi_{Oi}(x,t)\,\psi_{Oi}^\dagger(x,0)\rangle \\ &= \frac{1}{2\pi\alpha}\left[\frac{i\alpha}{-vt + i\alpha}\right]^{\frac{(1+g^2)}{2g}} \left[\frac{-\alpha^2 - 4x^2}{(-vt+i\alpha)^2 - 4x^2}\right]^{\frac{\tilde{d}_i(1-g^2)}{4g}} \end{aligned}\tag{4}$$

The oscillatory part vanishes as $L \to \infty$ and can be dropped in further discussions. For the $\widetilde{\mathbb{M}}_1$ class, $\tilde{d}_i = \tilde{a}$; for the $\widetilde{\mathbb{M}}_2$ class, $\tilde{d}_i = \tilde{a}, \tilde{b}, \tilde{c}$ depending on the wire index.

Treating the tunneling strength γ between the $i^{\rm th}$ wire and the STM tip perturbatively and using Eqs. (3-4), the differential tunneling conductance evaluated to leading order in γ is found to be directly proportional [12] to the TDOS on the $i^{\rm th}$ wire. The TDOS has the same form in the $x \to 0$ and $x \to \infty$ limits and is given by

$$\rho_i(\omega) = \frac{1}{\alpha\hbar\boldsymbol{\Gamma}(\Delta)}\;{\tau_c}^{\Delta}\;\omega^{\Delta-1}\;e^{-|\omega|\alpha/v}\;\Theta(\omega), \tag{5}$$

where $\boldsymbol{\Gamma}(\Delta)$ is the Gamma function, $\Theta(\omega)$ is the Heaviside step function, $\tau_c = \alpha/v$ is the short time cut-off, and $\omega = eV/\hbar$, where e is the electronic charge and V

is the bias voltage between the STM tip and wire system held at a uniform potential. The cut-off frequency scale for the validity of the perturbation theory is given by $\omega_0 = [|\gamma|^{-2/(\Delta_i - 1)} v]/\alpha$. For $x \to 0$, Δ is a function of the $\tilde{d}_i$ which is the corresponding diagonal element of the appropriate $\widetilde{\mathbb{M}}$ matrix. For the $\mathbb{M}_1$ class, $\Delta = \Delta_0(\tilde{a})$, while for $\mathbb{M}_2$, $\Delta = \Delta_i$ is a function of $\tilde{a}, \tilde{b}, \tilde{c}$ depending on the wire index i. For $x \to \infty$, $\Delta = (g + g^{-1})/2$ independent of the $\mathbb{M}$ matrix at the junction. Thus we recover the expression for the bulk TDOS in a LL as $x \to \infty$ irrespective of the details of the junction. For the $\mathbb{M}_1$ class, the power law exponent for $x \to 0$ is the same on all the three wires due to Z_3 symmetry and is given by

$$\Delta_0 = \frac{1}{3g}\;\frac{5g^2 + 1 + (g^2 - 1)\;\cos\theta}{g^2 + 1 + (g^2 - 1)\;\cos\theta}. \qquad (6)$$

Eq. (6) indicates that for $g < 1$ there are values of θ for which $\Delta < 1$. This implies that there are FPs in the theory which show an *enhancement* (see Eq. (5)) of the TDOS in the zero bias limit $\omega \to 0$. This is in sharp contrast to previous studies for various cases of normal (not superconducting) junctions of two-wire systems which always showed a suppression of the TDOS in the zero bias limit for $g < 1$. This is our main result. Note that whenever $g = 1$, $\Delta = 1$; this implies that the TDOS is independent of the bias for $g = 1$. This might look natural since $g = 1$ corresponds to free fermions in the wire for which the TDOS is expected to be bias (energy) independent. However, this is misleading; the BC conditions expressed in terms of the matrix $\mathbb{M}$ at the junction correspond to non-linear relations between the fermions on each wire in the vicinity of the junction, and hence represent non-trivial interaction between the fermions at the junction. Hence, the TDOS being energy independent for $g = 1$ for any FP represented by $\mathbb{M}_1$ is a non-trivial result by itself. To get a clear idea about the FPs which show an enhancement of the TDOS , we present contour plots of Δ in the $g - \theta$ plane in Fig. 2. In the left plot (corresponding to the $\mathbb{M}_1$ class), we can see a dome shaped region for $g < 1$ which corresponds to FPs showing an enhancement, *i.e.*, $\Delta_0 < 1$. It is interesting to note that this region is bounded by the two chiral FPs, $\chi_\pm$ at $\theta = 2\pi/3,\ 4\pi/3$. The $\mathsf{D_P}$ FP at $\theta = \pi$ also falls in this region and shows an enhancement of the TDOS for $1/2 < g < 1$. For the $\mathbb{M}_2$ class, the power law exponents for the three wires are given by

$$\Delta_1 \;=\; \frac{4 + 2g^2 + (\cos\theta - \sqrt{3}\sin\theta)\;(g^2 - 1)}{6g}, \qquad (7)$$

Δ_2 and Δ_3 which are obtained by shifting $\theta \to \theta \mp 2\pi/3$ in Δ_1. For this class also, there are FPs which show an enhancement of the TDOS for $g < 1$; they correspond to the dome shaped region in the right plot in Fig. 2. In contrast to the $\mathbb{M}_1$ class, in this case there can be an enhancement in one wire and suppression in the other wires due to the broken Z_3 symmetry.

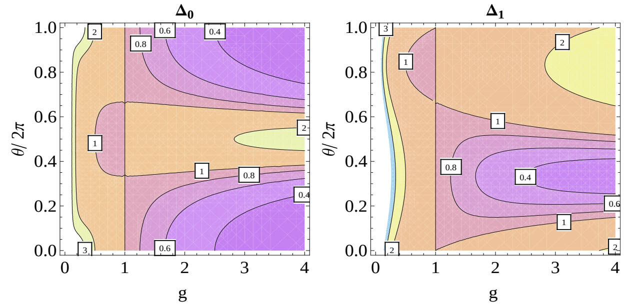

FIG. 2: Δ_0 for all wires in the $\mathbb{M}_1$ class (left), and Δ_1 for wire 1 in the $\mathbb{M}_2$ class (right) in the $g - \theta$ plane.

The observability of the different FPs and the enhancement of the TDOS near the junction crucially depends on the renormalization group (RG) stability of the FP against various perturbations in the form of inter-wire electron tunnelings at the junction (see Fig. 1). To understand the stability of the FP, we present the scaling dimension δ_0 for all possible single-electron tunneling events (quadratic in fermion operators) in Tables I and II; in these tables, $\psi_{I/O}$ are related to $\phi_{I/O}$ by the bosonization formula given earlier. In Fig. 3, we plot the values of $1 - \delta_0$ as functions of θ for $g = 0.9$. A positive value of $1 - \delta_0$ implies that the operator is RG relevant.

Operator	Scaling dimension δ_0
$\psi^\dagger_{iO}\psi_{iI}$	$\frac{4g(1-\cos\theta)}{3(g^2+\cos\theta(g^2-1)+1)}$
$\psi^\dagger_{2O}\psi_{1I}, \psi^\dagger_{3O}\psi_{2I}, \psi^\dagger_{1O}\psi_{3I}$	$\frac{2g(\cos\theta+\sqrt{3}\sin\theta+2)}{3(g^2+\cos\theta(g^2-1)+1)}$
$\psi^\dagger_{1O}\psi_{2I}, \psi^\dagger_{2O}\psi_{3I}, \psi^\dagger_{3O}\psi_{1I}$	$\frac{2g(\cos\theta-\sqrt{3}\sin\theta+2)}{3(g^2+\cos\theta(g^2-1)+1)}$
$\psi^\dagger_{2I}\psi_{1I}, \psi^\dagger_{3I}\psi_{2I}, \psi^\dagger_{1I}\psi_{3I}$	$\frac{2g}{g^2+\cos\theta(g^2-1)+1}$
$\psi^\dagger_{2O}\psi_{1O}, \psi^\dagger_{3O}\psi_{2O}, \psi^\dagger_{1O}\psi_{3O}$	$\frac{2g}{g^2+\cos\theta(g^2-1)+1}$

TABLE I: Table of tunneling operators for the $\mathbb{M}_1$ class.

Operator	Scaling dimension δ_0
$\psi^\dagger_{1O}\psi_{1I}$	$\frac{1}{3}g(2 - 2\cos\theta)$
$\psi^\dagger_{2O}\psi_{2I}$	$\frac{1}{3}g(2 + \cos\theta + \sqrt{3}\sin\theta)$
$\psi^\dagger_{3O}\psi_{3I}$	$\frac{1}{3}g(2 + \cos\theta - \sqrt{3}\sin\theta)$
$\psi^\dagger_{1O}\psi_{2I}, \psi^\dagger_{2O}\psi_{1I}$	$\frac{3+g^2}{12g}(2 - 2\cos\theta)$
$\psi^\dagger_{2O}\psi_{3I}, \psi^\dagger_{3O}\psi_{2I}$	$\frac{3+g^2}{12g}(2 + \cos\theta - \sqrt{3}\sin\theta)$
$\psi^\dagger_{3O}\psi_{1I}, \psi^\dagger_{1O}\psi_{3I}$	$\frac{3+g^2}{12g}(2 + \cos\theta + \sqrt{3}\sin\theta)$
$\psi^\dagger_{1I}\psi_{2I}, \psi^\dagger_{1O}\psi_{2O}$	$\frac{2(g^2+1)+(g^2-1)2\cos\theta}{4g}$
$\psi^\dagger_{2I}\psi_{3I}, \psi^\dagger_{2O}\psi_{3I}$	$\frac{2(g^2+1)-(g^2-1)(\cos\theta-\sqrt{3}\sin\theta)}{4g}$
$\psi^\dagger_{3I}\psi_{1I}, \psi^\dagger_{3O}\psi_{1I}$	$\frac{2(g^2+1)-(g^2-1)(\cos\theta+\sqrt{3}\sin\theta)}{4g}$

TABLE II: Table of tunneling operators for the $\mathbb{M}_2$ class.

It is clear from Fig. 3 that all the FPs showing an enhancement of the TDOS are unstable; however, for the $\mathbb{M}_1$ class close to $\theta = 2\pi/3,\ 4\pi/3$ (the $\chi_\pm$ FPs), only one operator is highly relevant and the rest are almost marginal. Hence this part of the parameter space al-

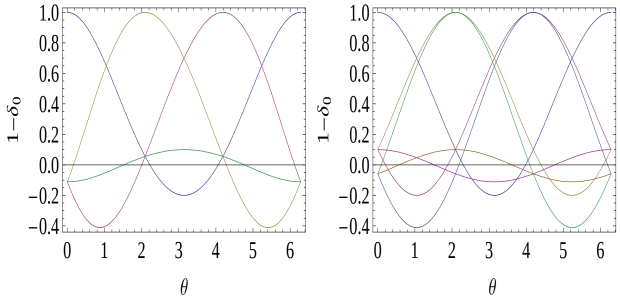

FIG. 3: $1-\delta_0$ as functions of θ for the classes $\mathbb{M}_1$ (left) and $\mathbb{M}_2$ (right) for various tunneling operators, for $g = 0.9$.

lows for a large temperature window for observing an enhancement if one can experimentally suppress the most relevant tunneling by tuning the junction appropriately.

To gain a better understanding of the enhancement, we expand the Δ's for $g \simeq 1$ in the small parameter $1-g$ to obtain $\Delta_j = 1 + (1-g)d_j$, where $j = 0,1,2,3$, and d_j are the diagonal elements of the corresponding $\mathbb{M}$ matrix. This limit corresponds to weakly interacting electrons in the bulk of the wires away from the junction. Whenever $d_j < 0$ and $g < 1$, Δ_j is less then unity which corresponds to an enhancement of the TDOS in the zero bias limit. But $d_j < 0$ corresponds to a hole current being reflected from the junction when an electron current is incident on the junction. Hence we conclude that all the FPs which involve reflection of a hole off the junction lead to an enhancement of the TDOS. As discussed earlier, an enhancement of the TDOS was previously observed in a junction of a LL wire with a superconductor [16]; this can be attributed to the proximity induced Andreev process at the junction which results in the reflection of a hole from the junction in response to an incident electron. It is interesting to note that for our case too, the enhancement is connected to holes being reflected off the junction, even though there is no superconductor in our model. Finally, note from Eqs. (4-5) that the cross-over length scale beyond which the TDOS goes over to its bulk form is given by $x = v/(2\omega)$. For a typical bias voltage of 10 μV and a Fermi velocity $v \approx 10^5$ m/s (typical of a two-dimensional electron gas), we get a cross-over length of about 3 μm; this is readily accessible within present day experimental realizations of a one-dimensional QW in a two-dimensional electron gas.

Discussion.- It is important to note that the interesting prediction of an enhancement of the TDOS for $g < 1$ involves unstable FPs; hence the enhancement can be expected to be observed in experiments at high temperatures only. If the junction is tuned to one of the FPs which show enhancement, a variation of the temperature from high to low will first show an enhancement and then a suppression of the TDOS as the system finally flows to the disconnected stable FP at low temperatures. This non-monotonicity observed via the STM current will be a hallmark of our prediction. Here, high and low temperatures are defined with respect to a cross-over scale called ω_0 after Eq. (5). In other Luttinger liquid systems, such as the one studied experimentally in Ref. [18], the cross-over scale T_B was found to be of the order of $0.5 - 3K$ (corresponding to $\omega_0 \sim k_B T_B/\hbar \sim 60 - 400GHz$); this scale can be varied by tuning the tunneling strength γ. Thus the cross-over scale can be tuned experimentally as was done in Ref. [18]. Hence the temperature window in which the enhancement of the TDOS can be observed is experimentally tunable.

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