Landau-Zener problem with waiting at the minimum gap and related quench dynamics of a many-body system

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(Dated: February 22, 2010)

We discuss a technique for solving the Landau-Zener (LZ) problem of finding the probability of excitation in a two-level system. The idea of time reversal for the Schrödinger equation is employed to obtain the state reached at the final time and hence the excitation probability. Using this method, which can reproduce the well-known expression for the LZ transition probability, we solve a variant of the LZ problem which involves waiting at the minimum gap for a time t_w; we find an exact expression for the excitation probability as a function of t_w. We provide numerical results to support our analytical expressions. We then discuss the problem of waiting at the quantum critical point of a many-body system and calculate the residual energy generated by the time-dependent Hamiltonian. Finally we discuss possible experimental realizations of this work.

I. INTRODUCTION

Introduced in the 1930s independently by Landau and Zener [1, 2], the Landau-Zener (LZ) transition formula provides an exact expression for the excitation probability in the final state when two levels approach each other due to a linear variation of the diagonal terms of a two-level Hamiltonian with an avoided level crossing at the point of minimum gap. Even after seventy years, the LZ formula is being applied extensively in problems over a wide range of modern physics, including neutrino oscillations, atomic physics, quantum optics, and mesoscopic systems [3]. Recently, it has found several applications in quantum computations [4] and adiabatic quantum dynamics of many-body systems [5]. In particular, the general Kibble-Zurek (KZ) scaling [6–9] of the residual energy or the defect density produced in the final state of a many-body Hamiltonian following a slow passage across a quantum critical point (QCP) [10] has been established for a number of exactly solvable low-dimensional non-random spin models using the LZ formula [11–14]. All these systems factorize into decoupled 2×2 matrices in momentum space, and the dynamics gets mapped to a set of decoupled LZ problems. We note that there are several other studies of quenching dynamics of different critical systems [15–17]. Quenching through a multicritical point [18], repeated passage through a QCP [19] and periodic variation of a parameter [20], quenching dynamics of a disordered spin chain [21], and the dynamics of an open system coupled to a heat bath [22] have also been studied.

In this paper, we address the following question: how does the transition probability of a LZ problem get altered if we introduce a waiting time t_w at the minimum gap? Here, starting from the ground state at $t = -\infty$, the system is brought to the point of minimum gap (2Δ) at time $t = 0$ by linearly varying the diagonal terms of the Hamiltonian at a rate $1/(2\tau)$. At the minimum gap,

the system is allowed to evolve without any external driving for a time t_w, after which the linear variation is again resumed up to $t = \infty$. We study how the probability of finding the system in the excited state at $t = \infty$, given exactly by $\exp(-2\pi\Delta^2\tau)$ for the conventional LZ problem [1], gets modified due to the additional time scale t_w. We then extend our results for the Landau-Zener problem with waiting to a many-body system whose quench dynamics through a QCP can be viewed as a set of decoupled LZ problems; the system is brought from its initial ground state at $t = -\infty$ to the QCP at $t = 0$, where the Hamiltonian is not changed for a time t_w, before it is again varied in time up to $t = \infty$. As the relaxation time of the system diverges at the critical point, the response of the system to any perturbation becomes infinitely slow; hence the system is no longer able to follow the ground state and therefore excitations are produced. As mentioned already, there have been several studies in this field recently, but the effect of waiting for a time t_w at the critical point of a many-body system has not been studied. In a many-body system, it is more meaningful to look at quantities like the residual energy e_r defined as the difference in energy between the actual state reached and the true ground state at the final time, or the density of defects (wrongly oriented spins) n which is obtained by integrating the probability of excitations over all modes.

We will study the effect of waiting on the residual energy e_r in the final state of the one-dimensional Kitaev model following a quench through the QCP along with waiting. We study the possible correction to the KZ power law scaling $n \sim 1/\tau^{d\nu/(\nu z+1)}$, where d is the spatial dimension and ν and z are the correlation length and dynamical exponents associated with the QCP [10] across which the system is swept. To the best of our knowledge, these questions have not been addressed before from a theoretical point of view, although experimental results with waiting for single molecular magnets $Mn_{12}Ac$ are available [23]. The possibility of experimental realizations of quenching dynamics with waiting in

optical lattices serves as another motivation. It is interesting to note that a similar concept is used in Ramsey spectroscopy where a molecule is subjected to an oscillating perturbation for a time T_1 which induces transitions between two specific levels of the molecule. The perturbation is then switched off for a time T_2 after which it is again switched on for a time T_1 and the probability of excitations is found [24]. We note that LZ sweeps have been used to generate coherent superpositions in quantum optical experiments [25].

Our results can be summarized as follows. The effect of a waiting time t_w can be understood by visualizing the dynamics in two parts: from $t = -\infty$ to 0 and from $t = 0$ to ∞. If the solution of the first part is known, the second part can be solved by applying the idea of time reversal on the first part; in this way, given the amplitudes of the two basis states at $t = 0$, one can find the values of the same at $t = \infty$. This reproduces the exact result for the conventional LZ problem. We then apply the method to a problem with waiting at the minimum gap to obtain an exact expression for the excitation probability in the final state; this gives simple forms for both the diabatic ($\Delta^2\tau \to 0$) and the adiabatic ($\Delta^2\tau \to \infty$) limit. The probability of excitations exhibits a sinusoidal behavior in both cases but with different prefactors and phases. The method is then used for a many-body system, namely, the one-dimensional Kitaev model [26] to obtain the residual energy. We quench this system through a QCP by linearly varying the anisotropy in the interaction, $dJ_-/dt = 1/\tau$ [13, 27], with a waiting time t_w at the QCP. We show that for $t_w/\sqrt{\pi\tau} \ll 1$, the residual energy shows an exponential decay with t_w given by $e_r \sim (a/\sqrt{\tau})[1 + b\exp(-ct_w/\sqrt{\tau})]$. We find that the parameters a, b and c obtained by an approximate analytical calculation give a good fit with the results obtained by numerically solving the Schrödinger equation.

The outline of this paper is as follows. We describe our method of solving the Landau-Zener method in Sec. II A and obtain an expression for the excitation probability in the presence of waiting in Sec. II B. In Sec. III, we study the waiting problem for a many-body system taking the example of the one-dimensional Kitaev model.

II. LANDAU-ZENER PROBLEM

A. Landau-Zener revisited

To illustrate our method, let us revisit the conventional two-level LZ problem. Although the model has been exactly solved and the evolution matrix is known exactly for all times [28, 29], and it has also been studied within a rotating wave approximation [30], we describe a method below which can be easily generalized to study LZ dynamics with waiting.

The state $\psi(t) = C_1(t)|1\rangle + C_2(t)|2\rangle$, where $|1\rangle$ and $|2\rangle$ are the basis states (the initial and final ground states, respectively), evolves according to the Schrödinger equation

$$i\frac{d}{dt}\psi(t) = \begin{bmatrix} t/(2\tau) & \Delta \\ \Delta & -t/(2\tau) \end{bmatrix}\psi(t) = H\psi(t), \qquad (1)$$

where Δ is chosen to be real without any loss of generality. With the initial condition $|C_1(-\infty)|^2 = 1$, the wave function at $t = 0$ is given by $\psi(t = 0) = \alpha|1\rangle + \beta|2\rangle$, where

$$\alpha = C_1(0) = e^{-\frac{\pi}{4}\Delta^2\tau}\, e^{i\frac{3\pi}{4}}\, \sqrt{\pi}\, \frac{2^{-iy}}{\Gamma(1/2 + iy)},$$

$$\beta = C_2(0) = \Delta\sqrt{\tau}\, e^{-\frac{\pi}{4}\Delta^2\tau}\sqrt{\frac{\pi}{2}}\, \frac{2^{-iy}}{\Gamma(1 + iy)}, \qquad (2)$$

with $y = \Delta^2\tau/2$ [28, 31]. Henceforth, $(\cdot, \cdot)^T$ will denote the transpose of a given row vector. Since $\psi(-\infty) = (1, 0)^T$ evolves to $\psi(0) = (\alpha, \beta)^T$, orthogonality implies that $\psi(-\infty) = (0, 1)^T$ must evolve to $\psi(0) = (\beta^*, -\alpha^*)^T$, up to a phase. Using properties of the Gamma functions [32], one can show that $|\alpha|^2 + |\beta|^2 = 1$ as desired, and $|\alpha|^2 - |\beta|^2 = e^{-\pi\Delta^2\tau}$.

Let us now ask: what wave functions at $t = 0$ will evolve to $(1, 0)^T$ and $(0, 1)^T$ at $t = \infty$? To answer this question, let us multiply the Hamiltonian in Eq. (1) by σ^z on both sides, which gives

$$-i\frac{d}{dt}\sigma^z\psi(t) = \begin{bmatrix} -t/(2\tau) & \Delta \\ \Delta & t/(2\tau) \end{bmatrix}\sigma^z\psi(t). \qquad (3)$$

Clearly, by substituting $t' = -t$ and $\psi'(t') = \sigma^z\psi(t)$ in Eq. (3), we recover Eq. (1). Thus, the dynamics occurring in Eq. (1) from $t = 0$ to $-\infty$ is the same as in Eq. (3) with t' going from 0 to ∞ and ψ replaced by $\sigma^z\psi$. Since $\psi(-\infty) = (1, 0)^T$ evolves to $\psi(0) = (\alpha, \beta)^T$, the above argument shows that $\psi(0) = \sigma^z(\alpha, \beta)^T = (\alpha, -\beta)^T$ evolves to $\psi(\infty) = (1, 0)^T$ through the Hamiltonian given in Eq. (1). By similar arguments, or by orthogonality, we see that $\psi(0) = \sigma^z(\beta^*, -\alpha^*)^T = (\beta^*, \alpha^*)^T$ evolves to $\psi(\infty) = (0, 1)^T$, again up to a phase. We can now find the probability of ending in the excited state $|1\rangle$ at $t = \infty$ as follows. We can write $\psi(0) = (\alpha, \beta)^T$ as

$$\psi(0) = \left[\begin{pmatrix} \alpha \\ -\beta \end{pmatrix}(\alpha^*, -\beta^*) + \begin{pmatrix} \beta^* \\ \alpha^* \end{pmatrix}(\beta, \alpha)\right]\begin{pmatrix} \alpha \\ \beta \end{pmatrix}$$

$$= (|\alpha|^2 - |\beta|^2)\begin{pmatrix} \alpha \\ -\beta \end{pmatrix} + 2\alpha\beta\begin{pmatrix} \beta^* \\ \alpha^* \end{pmatrix} \qquad (4)$$

where, in the first line, we introduced an identity operator using the orthonormal basis $(\alpha, -\beta)^T$ and $(\beta^*, \alpha^*)^T$. Since we now know the evolution of $(\alpha, -\beta)^T$ and $(\beta^*, \alpha^*)^T$ from $t = 0$ to ∞, we see that

$$\psi(\infty) = (|\alpha|^2 - |\beta|^2)\begin{pmatrix} 1 \\ 0 \end{pmatrix} + 2\alpha\beta\begin{pmatrix} 0 \\ 1 \end{pmatrix}, \qquad (5)$$

where there may be an unimportant phase difference between the two terms. The probability of excitations at the final time is the probability to be in state $|1\rangle$ and is thus given by $p = (|\alpha|^2 - |\beta|^2)^2 = e^{-2\pi\Delta^2\tau}$ which is the exact expression for the LZ transition probability [1].

B. Landau-Zener with waiting

We now apply our method to the LZ problem with waiting. Within the time interval $[0, t_w]$, the eigenvectors of H are given by $(1, \pm 1)^T/\sqrt{2}$ with eigenvalues $\pm\Delta$. So the wave function changes from $\psi(0) = (\alpha, \beta)^T$ to

$$\psi(t_w) = \frac{\alpha+\beta}{2} \, e^{-i\Delta t_w} \begin{pmatrix} 1 \\ 1 \end{pmatrix} + \frac{\alpha-\beta}{2} \, e^{i\Delta t_w} \begin{pmatrix} 1 \\ -1 \end{pmatrix}$$

Inserting the identity operator used in Eq. (4), we get

$$\psi(t_w) = \left[\begin{pmatrix} \alpha \\ -\beta \end{pmatrix} (\alpha^*, -\beta^*) + \begin{pmatrix} \beta^* \\ \alpha^* \end{pmatrix} (\beta, \alpha) \right] \psi(t_w).$$

The state at $t = \infty$ can again be obtained by using the information about the evolution of $(\alpha, -\beta)^T$ and $(\beta^*, \alpha^*)^T$:

$$\psi(\infty) = (\alpha^*, -\beta^*)\psi(t_w) \begin{pmatrix} 1 \\ 0 \end{pmatrix} + (\beta, \alpha)\psi(t_w) \begin{pmatrix} 0 \\ 1 \end{pmatrix}.$$

Hence the excitation probability is given by

$$\begin{aligned} p_{t_w} &= |(\alpha^*, -\beta^*)\psi(t_w)|^2 \\ &= \left[(|\alpha|^2 - |\beta|^2) \cos(\Delta t_w) - i(\alpha^*\beta - \alpha\beta^*)\sin(\Delta t_w) \right]^2. \end{aligned} \quad (6)$$

This expression simplifies in the limits $\Delta^2\tau \to 0$ and ∞. For $\Delta^2\tau = 0$, we have $p_{t_w} = \cos^2(\Delta t_w)$; this is expected as the system does not get any time to evolve and therefore remains in the state $|1\rangle$ up to $t = 0$, then oscillates between the states $|1\rangle$ and $|2\rangle$ from $t = 0$ to t_w, and then remains in the superposed state reached at t_w for all $t > t_w$. If $\Delta^2\tau \ll 1$, one gets $i(\alpha^*\beta - \beta^*\alpha) \simeq \Delta\sqrt{\pi\tau}$ which leads to an approximate expression

$$p_{t_w} \simeq e^{-\pi\Delta^2\tau} \cos^2[\Delta(t_w + \sqrt{\pi\tau})]. \quad (7)$$

Comparison with the case $\Delta^2\tau = 0$ shows a decrease in amplitude from 1 to $e^{-\pi\Delta^2\tau}$ and a phase shift of $\sqrt{\pi\tau}$. In the adiabatic limit $\Delta^2\tau \to \infty$, $|\alpha|^2 - |\beta|^2 = 0$, and one can use the asymptotic expansions of the Gamma function [32] to obtain the excitation probability

$$p_{t_w} \simeq \frac{1}{16\Delta^4\tau^2} \sin^2(\Delta t_w). \quad (8)$$

In the next section, we consider waiting in a many-body problem, namely, the one-dimensional Kitaev model.

III. WAITING IN KITAEV MODEL

We now use the above results to study the quenching dynamics with waiting at the QCP of the one-dimensional Kitaev model given by the Hamiltonian [26, 27]

$$H = \sum_j (J_1\sigma_{2j}^x\sigma_{2j+1}^x + J_2\sigma_{2j-1}^y\sigma_{2j}^y), \quad (9)$$

where j refers to the site index and $\sigma^{x,y}$ are the Pauli matrices. The model can be exactly solved in terms of Jordan-Wigner fermions [33] defined as

$$a_j = (\prod_{i=-\infty}^{2j-1} \sigma_i^z) \sigma_{2j}^y, \quad b_j = (\prod_{i=-\infty}^{2j} \sigma_i^z) \sigma_{2j+1}^x. \quad (10)$$

Going to momentum space with $\psi_k \equiv (a_k, b_k)^T$, where a_k and b_k are the Fourier transform of a_j and b_j, and performing an appropriate unitary transformation, the Hamiltonian decouples into a 2×2 form given by [27]

$$H = \sum_{k=0}^{\pi/2} \psi_k^\dagger \, H_k \, \psi_k,$$

where $H_k = 2 \begin{bmatrix} J_- \sin(k) & J_+ \cos(k) \\ J_+ \cos(k) & -J_- \sin(k) \end{bmatrix}$, $\quad (11)$

$J_\pm = J_1 \pm J_2$ and k ranges from 0 to $\pi/2$. The vanishing of the gap $(= 4\sqrt{J_-^2 \sin^2 k + J_+^2 \cos^2 k})$ for the mode $k = \pi/2$ at $J_- = 0$ signals a quantum phase transition of topological nature [34], with $\nu = z = 1$.

Setting $J_+ = 1$, we now apply the quenching scheme

$$\begin{aligned} J_-(t) &= t/\tau \quad \text{for} \quad -\infty < t \le 0 \\ &= 0 \quad \text{for} \quad 0 \le t \le t_w \\ &= (t - t_w)/\tau \quad \text{for} \quad t_w \le t < \infty, \end{aligned} \quad (12)$$

which incorporates waiting for a time t_w at the QCP. The Schrödinger equation for each mode is

$$i\frac{d}{dt}\psi_k(t) = 2\begin{bmatrix} J_-(t)\sin k & \cos k \\ \cos k & -J_-(t)\sin k \end{bmatrix}\psi_k(t), \quad (13)$$

so that the excitation probability for each mode is given by the modified LZ formula in Eq. (6). In Fig. 1, we compare the exact analytical expression given in Eq. (6) with its corresponding approximate form for small $\Delta^2\tau$ given in Eq. (7); the picture in the figure is reminiscent of Ramsey fringes. Noting that $\Delta^2\tau = \tau \cos^2 k / \sin k \simeq 0.00012$ for the mode shown in Fig. 1, we can use the diabatic limit result in Eq. (7),

$$p_{k,t_w} \simeq e^{-\pi\tau \cos^2 k / \sin k} \cos^2[2\cos k(t_w + \frac{1}{2}\sqrt{\frac{\pi\tau}{\sin k}})]. \quad (14)$$

This gives a good understanding of the peak heights and phase shift in Fig. 1.

The variation of p_{k,t_w} vs k for different t_w (shown in Fig. 2 with $\tau = 10$) shows secondary maxima, the peak heights of which increase as t_w increases, in contrast to the conventional quenching case [27]. The increase in the number of maxima with increasing t_w can be explained by the expression in Eq. (14) which has maxima at

$$\cos k = \frac{m\pi}{2(t_w + \sqrt{\pi\tau/2})}, \quad m = 0, 1, 2, \cdots \quad (15)$$

in the limit of small k. With increasing t_w, Eq. (15) is satisfied by more and more values of k which still satisfy
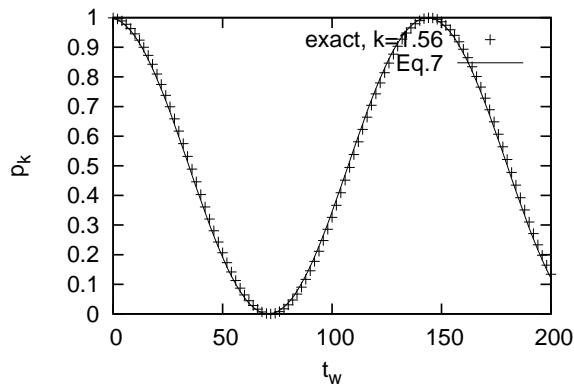
FIG. 1: Variation of probability of excitations p_k with waiting time t_w, for $\tau = 1$ and $k = 1.56$. The dots correspond to the exact solution given in Eq. (6), whereas the line corresponds to the approximate analytical expression given in Eq. (7).
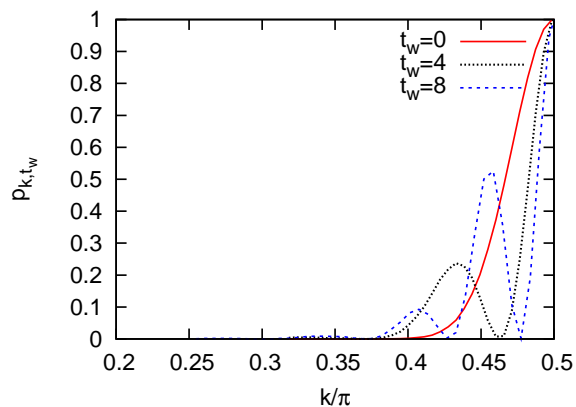

FIG. 2: (Color online) Variation of probability of excitations p_{k,t_w} with k for $\tau = 10$ obtained by numerically solving the Schrödinger equation, for $t_w = 0$ (solid red line), 4 (dotted black line) and 8 (dashed blue line).

the condition that $\cos k$ is small enough so that the peak height given by the exponential pre-factor in Eq. (14) is not very small. Further, for a given value of m in Eq. (15), $\cos k$ decreases as t_w increases which justifies the increase in the peak height given by Eq. (14).

The residual energy per site, e_r, is given by the difference of the expectation value of the operator

$$\mathcal{O} = \frac{1}{N} \sum_m (\sigma_{2m}^x \sigma_{2m+1}^x - \sigma_{2m-1}^y \sigma_{2m}^y) \qquad (16)$$

between the many-body state that is actually reached and the true ground state of H at $t = \infty$ (this is also the ground state of $\mathcal{O}$); N denotes the number of sites. We find that e_r is given by

$$e_r = \int_0^{\pi/2} \frac{dk}{2\pi} \, 8 \sin k \, p_{k,t_w}. \qquad (17)$$

Although this expression cannot be evaluated analytically in general, one can obtain an approximate expres-
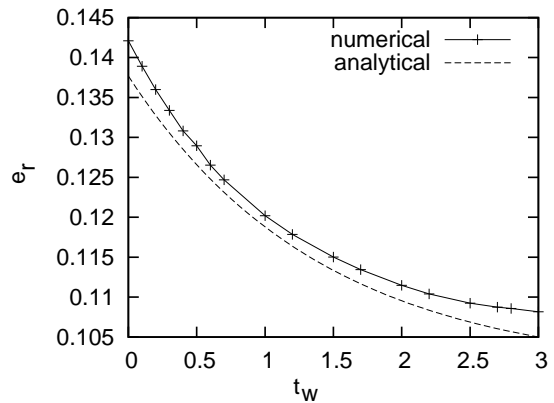

FIG. 3: Variation of residual energy e_r with waiting time t_w, for $\tau = 10$. The line corresponds to the integral given in Eq. (17), while the dotted line corresponds to Eq. (18).

sion when $\tau \gg 1$ and $t_w/\sqrt{\pi\tau} \ll 1$. Most of the contribution to the integral in Eq. (17) then comes from k close to $\pi/2$ in Eq. (14). Approximating $\cos k \simeq \pi/2 - k$ and $\sin k \simeq 1$, redefining $k = \pi/2 - k$ and finally extending the limits of integration from $[0, \pi/2]$ to $[0, \infty]$, we obtain

$$
\begin{aligned}
e_r &= \frac{2}{\pi} \int_0^\infty dk \, e^{-\pi\tau k^2} \left[1 + \cos[4k(t_w + \frac{1}{2}\sqrt{\pi\tau})] \right] \\
&= \frac{1}{\pi\sqrt{\tau}} \left[1 + e^{-4(t_w + \frac{1}{2}\sqrt{\pi\tau})^2/(\pi\tau)} \right] \\
&\simeq \frac{0.32}{\sqrt{\tau}} \left[1 + 0.37 \, e^{-2.3 t_w/\sqrt{\tau}} \right], \qquad (18)
\end{aligned}
$$

where we have used the approximation $t_w/\sqrt{\pi\tau} \ll 1$ in the third line. In Fig. 3, the numerical results obtained by solving the Schrödinger equation are compared with the approximate analytical expression given in Eq. (18). This comparison shows that the numerical and analytical expressions are in good agreement.

The decrease in the residual energy as a consequence of waiting can be understood as follows. For slow driving, only modes close to the critical modes contribute. For a mode with momentum k, the frequency of oscillations of $|C_1(t)|^2$ between the two levels during waiting is proportional to $\Delta_k = 2\cos k$; this vanishes as k approaches the critical value $\pi/2$, leading to a diverging time period T. Further, the vanishing of the off-diagonal element in the Hamiltonian in Eq. (13) implies that the modes close to $k = \pi/2$ will remain close to the excited state under time evolution, i.e., $|C_1|$ will remain close to 1. Now consider the variation of the diabatic excitation probability $|C_1|^2$ between $t = 0$ and $t = t_w$. The waiting gives $|C_1|$ the time to oscillate to $|C_2|$. Since we are considering only values of t_w much smaller than the time period T, we encounter only the decreasing part of the oscillating $|C_1|^2$ such that when the variation of the parameter is again started at t_w, $|C_1(t_w)|^2 < |C_1(t = 0)|^2$. This reduces the probability of excitations at the final time.

The advantage of studying the waiting problem in the Kitaev model is that the minimum gap for all the modes

occurs at the same time ($t = 0$) which makes the analytical calculations easier. In many other models, the minimum gaps for the different modes, given by the vanishing of the diagonal term of the 2×2 Hamiltonians, do not occur at the same time. This makes it difficult to specify the precise time at which the waiting should be initiated so that our analytical results can be applied. But one can prove that in the limit of large τ, the minima for all the modes approach the time at which the minima of the critical mode occurs. For large τ, we therefore expect our analysis to go through with small corrections to the probability of excitations given in Eq. (14). To understand why this is so, consider the quenching of the transverse magnetic field h in the transverse field XY model [12], where the diagonal term of the equivalent 2×2 matrix is $h + J \cos k$ and the off-diagonal term is $\gamma \sin k$, with $J = J_x + J_y$ and $\gamma = J_x - J_y$. By expanding the diagonal term about the critical mode $k = 0$, multiplying the corresponding Schrödinger equation with $\sqrt{\tau}$ and redefining $t' = t/\sqrt{\tau}$, the diagonal term can be rewritten as $t' + 1/(2\gamma\sqrt{\tau})$, where the characteristic momentum scale is given by $1/(\gamma\sqrt{\tau})$ by the LZ tunneling formula. Clearly, in the limit $\tau \to \infty$, the minima for the mode occurs approximately at $t' = 0$. These arguments are applicable to the modes close to the critical mode; the modes far away from the critical mode anyway do not contribute to the residual energy or defect density as the excitation gap is very large.

The waiting problem is also interesting from an experimental point of view. In Ref. 23, the relaxation dynamics with waiting at a resonance was studied for single molecular magnets (SMM) called $Mn_{12}Ac$, where each molecule has total spin $S = 10$. As the magnetic field in the $\hat{z}$-direction is varied from a large negative to a large positive value, S^z changes from 10 to -10. In Ref. [23], the magnetic field is swept to a resonance value, starting from a field of -6 T, where it is held for different waiting times causing tunneling of the spins; eventually the field is brought back to its initial value. The number of molecules which have tunneled through the barrier, measured using electron paramagnetic resonance techniques, shows nearly a stretched exponential decay with t_w, where the decay constant is governed by the relaxation time of the system. Though there are limitations in mapping SMM to a LZ problem for all quenching rates [35], it is worth noting that there have been experimental studies on the effect of waiting at the resonance, and important quantities like the relaxation time of the system can be obtained from the waiting problem. Although, in our case, J_- is driven to a final value of $+\infty$ at $t = +\infty$, we do observe a qualitatively similar behavior, i.e., an increase in the tunneling probability to the second state

with increasing waiting time. We believe that similar experiments with waiting and forward driving of the magnetic field can be performed and it would be interesting to compare the results with our predictions. However, if the non-linear term of a SMM Hamiltonian [35] dominates, one may need to look at non-linear LZ problems [36] with waiting.

It may be mentioned here that quenching with waiting can be studied if the Kitaev model can be experimentally realized using cold atoms and molecules trapped in an optical lattice as proposed in Ref. 37. In this proposal, each of the couplings can be independently tuned using different microwave radiations. It is possible to investigate the evolution of the spatial correlation function of the operator $ib_n a_{n+r}$, defined in Ref. 13 as a function of various parameters, where a_n and b_n denote some Majorana fermions operators. This spatial correlation function depends on p_k which we have already obtained for the waiting case. Then the evolution of defect correlations can be detected by spatial noise correlation measurements as discussed in Ref. 38.

IV. CONCLUSION

To summarize, the technique proposed here not only provides an exact result for the standard LZ problem but also enables us to estimate the excitation probability for the dynamics with waiting at the minimum gap and the residual energy for the related quenching dynamics of some many-body systems like the one-dimensional Kitaev model. We can derive simple expressions in some limiting situations and the approximate analytical results are in excellent agreement with numerical results when $t_w/\sqrt{\pi\tau} << 1$. The arguments leading up to Eq. (18) indicate that the KZ scaling law $1/\tau^{d\nu/(\nu z+1)}$ will generally remain valid in the presence of waiting, except that the function multiplying the scaling term has a piece which decays with increasing t_w. Finally, we have discussed some possibilities for experimentally testing our results.

V. ACKNOWLEDGMENTS

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