Constraints on background torsion field from K physics

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Abstract

We point out that a background torsion field will produce an effective potential to the K and $\bar{K}$ with opposite signs. This allows us to constrain the background torsion field from the K_L and K_S mass difference, CPT violating K° and $\bar{K}^\circ$ mass difference and the CP violating quantities ϵ and η_{+-}. The most stringent bound on the cosmological background torsion $\langle T^0 \rangle < 10^{-25}$ GeV comes from the direct measurement of the CPT violation.

General Theory of relativity has so far succeeded in confronting all experimental tests. However the problem of quantising gravity leads one to believe that Einsteins theory though correct may not be the most general theory which describes the dynamics of the metric tensor and its interactions with matter. The ultimate quantum theory of gravity must also explain the low energy phenomenology. This gave rise to the birth of string theory, which is now considered as the most consistent theory of quantum gravity. In string theory the metric tensor field comes out naturally and gives the response of matter to this metric. However, it also predicts several other fields like the antisymmetric second rank field, which enters via its antisymmetrized derivatives, $T_{\alpha\beta\gamma} = \partial_{[\alpha} A_{\beta\gamma]}$, which are usually referred to as the torsion field. In addition to the string inspired approach to study the torsion dynamics, there are some modifications of the GTR where connection is treated as more fundamental than the metric.

In a metric compatible theory of gravity one generalizes the connection by including the torsion tensor [1, 2]. The symmetric part of the generalized connection are the Christoffel symbols given by the usual formula in terms of the metric, whereas the torsion is the antisymmetric part of the connection which in Einsteins gravity is assumed to be zero. The coupling of the matter fields to torsion arises from the covariant derivative with respect to the generalized connection . The covariant derivative of a fundamental scalar field is just the partial derivative therefore the torsion term does not couple to scalars . The torsion field also does not couple to electro-magnetic fields as the gauge invariant field strength is the antisymmetric partial derivative,$F_{\mu\nu} \equiv (\mathrm{d}A)_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$. This is the general definition for the field strength even in curved space. If one were to generalise the definition of $F_{\mu\nu}$ by replacing the partial derivatives with the covariant derivative (4), the extra terms involving torsion will not be gauge invariant. Coupling of the electro-magnetic field with torsion can only arise in some more generalized versions of torsion theories [3].

It has long been known that fermions can couple with torsion field [1, 2]. The fermions couple as an axial current to the dual of the torsion tensor. Since an axial current is spin dependent in the forward scattering limit, the torsion force on a macroscopic collection of fermions will average to zero unless the fermion spins is polarized (which is difficult to attain in experiments). For this reason one cannot constrain the torsion couplings from the usual fifth force experiments [4].

In this paper we point out that composite scalars like mesons can couple to the torsion field through the constituent quarks. In a pseudoscalar meson the torsion potential being spin dependent couples to the difference of the dipole moments of the quarks . We show that in a background torsion field , the potentials of the K° and $\bar{K}^0$ have opposite signs. This apparent violation of CPT and CP in the $K-\bar{K}$ system can be constrained from the kaon oscillation experiments and these constraints allow us to put bounds on the cosmological torsion background to be $\langle T^0 \rangle < 10^{-25}$ GeV .

We start with a brief review of gravity with torsion and then point out how the torsion field couple to the K system. We then parameterize the effective potential due to this non vanishing torsion field and put bound from various experiments.

In a metric compatible theory of gravity one generalise s the connection by including the torsion tensor [1, 2] ,

$$\Gamma^{\alpha}_{\mu\nu} \equiv \left\{ \begin{array}{c} \alpha \\ \mu\nu \end{array} \right\} + \frac{1}{2} \left(T_{\mu\nu}{}^{\alpha} - T_{\nu}{}^{\alpha}{}_{\mu} + T^{\alpha}{}_{\mu\nu} \right) \tag{1}$$

where the Christoffel symbols $\left\{ \begin{array}{c} \alpha \\ \mu\nu \end{array} \right\}$ are the symmetric part of $\Gamma^{\alpha}_{\mu\nu}$ and are given by the usual formula

$$\left\{ \begin{array}{c} \alpha \\ \mu\nu \end{array} \right\} \equiv \frac{1}{2} g^{\alpha\beta} \left(\partial_\mu g_{\nu\beta} + \partial_\nu g_{\beta\mu} - \partial_\beta g_{\mu\nu} \right). \tag{2}$$

whereas the torsion is the antisymmetric part of $\Gamma^{\alpha}_{\mu\nu}$,

$$T_{\rho\sigma}{}^{\mu} = \Gamma^{\mu}_{\rho\sigma} - \Gamma^{\mu}_{\sigma\rho} \tag{3}$$

The coupling of the matter fields to torsion arises from the covariant derivative with respect to the generalised connection (1),

$$\nabla_\mu X^\alpha \equiv \partial_\mu X^\alpha + \Gamma^\alpha_{\mu\nu} X^\nu \tag{4}$$

The minimal coupling action of the Dirac spinor fields in an external gravitational field with torsion will now become

$$\mathcal{L} = \frac{i}{2}\int d^4x \left(\nabla_\mu \bar{\psi}\gamma^\mu\psi - \bar{\psi}\gamma^\mu\nabla_\mu\psi - 2im\bar{\psi}\psi\right). \tag{5}$$

This contains the usual torsion free part of the lagrangian and the interaction part is given by (we set $g_{\mu\nu} = \eta_{\mu\nu}$),

$$\mathcal{L}_I = \frac{i}{8}\int d^4x T_{\mu\nu\lambda}\bar{\psi}\gamma^{[\mu}\gamma^\nu\gamma^{\lambda]}\psi = \frac{3}{4}T^\sigma(\bar{\psi}\gamma_5\gamma_\sigma\psi). \tag{6}$$

where the pseudo-trace irreducible component of the torsion field is defined as dual to the antisymmetric part of the torsion field,

$$T^\sigma \equiv \frac{1}{3!}\epsilon^{\mu\nu\lambda\sigma}T_{\mu\nu\lambda}. \tag{7}$$

Thus the torsion coupling to matter reduces to a coupling of an axial vector current to a torsion pseudo-vector T_σ. The other components of the torsion, namely, the trace $T_\beta = T^\alpha{}_{\beta\alpha}$ and the tensor $q^\alpha{}_{\beta\lambda}$ couples to this pseudo-vector component but not with matter directly. From the fact that the torsion pseudo-vector T^σ couples to a axial vector current , one can assign the following transformation properties to $T^\sigma = (T^0, \vec{T})$ under transformations of Charge conjugation (C), Parity (P) and Time reversal (T) symmetries :

$$\begin{array}{cccc} & C & P & T \\ (T^0, \vec{T}) \Rightarrow & (T^0, \vec{T}) & (-T^0, \vec{T}) & (T^0, -\vec{T}) \end{array} \tag{8}$$

and under the combined operation of $CPT : T^\sigma \rightarrow -T^\sigma$. We shall now assume that there is a fixed background potential due to a non vanishing value of a background torsion, which breaks CP , T and CPT along with the local Lorentz invariance . However, we would like to preserve the isotropy of the field and assume a non-vanishing value only time-like component of the pseudo-vector $(3/4) < T^0 >= t^0$, where t^0 has a dimension of mass. In this way a non-zero background t breaks CP and CPT, and this should give rise to some observable effects of these symmetry violations.

In the rest of this article we study if it is possible to constrain this background potential due to t from some experiments. We point out that the background potential couples to K and $\bar{K}$ in a different way, and we get an additional CP and CPT violating potential for the K-system. Since the torsion term couples to fermions as a axial vector, in the forward scattering limit the dipole moment of the fermion gives the dominant coupling. This can be seen by writing the Gordon decomposition of the fermion axial current as

$$\bar{\psi}(p')\gamma_\mu\gamma_5\psi(p) = \bar{\psi}(p')\left[\frac{(p'-p)_\mu}{2m} + \frac{i}{2m}\sigma_{\mu\nu}\ (p'+p)^\nu\gamma_5\right]\psi(p) \tag{9}$$

In the forward scattering limit the effective potential of a fermion due to a background torsion is

$$\frac{t^0}{m}\ \bar{\psi}(p)\ \sigma_{0i}\ \gamma_5 p^i\ \psi(p) = \frac{2t^0}{m}\ \vec{s}\cdot\vec{p} = \frac{2t^0\ |\vec{p}|}{m}\ \lambda \tag{10}$$

where $\vec{s}$ is the spin and λ is the helicity of the fermion. At high energies the $(E >> m)$ the helicity is a conserved (upto $O(m^2/E^2)$) and one can assign a fermion in a torsion background t the potential $\pm t^0|\vec{p}|/m$ where the sign depends upon the helicity. Unlike the gravitational potential energy, the torsion potential energy of a macroscopic body is not large as the spins of a macroscopic body are aligned randomly. Due to this reason it is not possible to put bounds on the torsion force with any of the usual fifth force experiments [4]. In a psedoscalar meson like K° or $\bar{K}^\circ$ the quark and the anti-quark have opposite spins and at high energies they can be assigned the helicities $1/2$ and $-1/2$ respectively. The net dipole form factor of mesons will be non-zero if the quark and anti-quark have different masses. We can write the quark model of the K° as

$$|K\rangle = \frac{1}{\sqrt{2}}[s(+)\ \bar{d}(-) - s(-)\ \bar{d}(+)] \tag{11}$$

The signs in the brackets indicate the helicity quantum numbers of the s-quark and the d-anti-quark. The combination appears with a relative negative sign as under Parity the helicity flips sign and the mesons being pseudo-scalars under Parity $K^\circ \rightarrow \bar{K}^\circ$. The net torsion potential of the quark-antiquark combination (11) of the K° meson is

$$t^0\frac{|\vec{p}|}{\sqrt{2}}[(\frac{1}{m_s} - \frac{1}{m_d}) - (\frac{-1}{m_s} - \frac{1}{m_d})) = -t^0|\vec{p}|\sqrt{2}\frac{(m_s - m_d)}{m_s m_d} \tag{12}$$

The quark quantum numbers of the $\bar{K}^\circ$ is given by

$$|\bar{K}\rangle = \frac{1}{\sqrt{2}}[d(+)\ \bar{s}(-) - d(-)\ \bar{s}(+)] \tag{13}$$

This is consistent with (11) and the requirement that under CP : $|K^\circ\rangle \rightarrow -|\bar{K}^\circ\rangle$. The net torsion potential of the $\bar{K}^\circ$ meson is therefore

$$t^0\frac{|\vec{p}|}{\sqrt{2}}[(\frac{1}{m_d} - \frac{1}{m_s}) - (\frac{-1}{m_d} - \frac{1}{m_s})] = t^0|\vec{p}|\sqrt{2}\frac{(m_s - m_d)}{m_s m_d} \tag{14}$$

So we can see from the torsion potentials (12) for K° and (14) for $\bar{K}^\circ$ that the two terms have opposite sign as is expected from the fact that when the background torsion field is non-zero then the potential must be CP and CPT violating.

In the basis $[K^\circ \quad \bar{K}^\circ]$ the background potential due to the non-vanishing torsion field now becomes

$$\mathcal{V} = -\ V \quad \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \tag{15}$$

where $V = \frac{t}{m}|\vec{p}|$; m is the kaon mass and $t \equiv t^0\sqrt{2}(m_s - m_u)m_K/(m_u m_s)$ is the background torsion parameter.

We shall be comparing our results with experiments where the kaons are ultra-relativistic. The total hamiltonian in the basis $[K^\circ \quad \bar{K}^\circ]$ will now become,

$$H \quad = \quad pI + \frac{1}{2p}\begin{pmatrix} m - \frac{i}{2}\Gamma & \frac{1}{2}\left(\delta m - \frac{i}{2}\delta\Gamma\right) \\ \frac{1}{2}\left(\delta m - \frac{i}{2}\delta\Gamma\right) & m - \frac{i}{2}\Gamma \end{pmatrix}^2 + \mathcal{V}$$

$$\equiv \quad pI + \frac{1}{2p}\begin{pmatrix} M_+ - \frac{i}{2}\Gamma_+ & M_{12} - \frac{i}{2}\Gamma_{12} \\ M_{21} - \frac{i}{2}\Gamma_{21} & M_- - \frac{i}{2}\Gamma_- \end{pmatrix}^2 \tag{16}$$

where I is the identity matrix. If CPT is conserved, then $M_+ = M_-$ and $\Gamma_+ = \Gamma_-$. The direct CPT violating quantity $M_+ - M_-$ is then given by,

$$M_+^2 - M_-^2 = 4pV \qquad \text{or} \qquad |M_+ - M_-| = 2\left(\frac{p}{m}\right)^2 t, \tag{17}$$

which depends quadratically on energy. Similar energy dependence in the K system was discussed in the literature [5, 6, 7, 8], which could arise from violation of the weak equivalence principle or the violation of the Local Lorentz invariance.

We shall first constrain the background torsion T from the measurements of the K_L and K_S mass difference, the comparison of the induced CP violation due to the CPT violation, and from the direct measurement of the CPT violating quantity – the mass difference of K° and $\bar{K}^\circ$.

In the basis of the physical states $[K_L \quad K_S]$ the hamiltonian will become,

$$H = \begin{pmatrix} p + \frac{m_L^2}{2p} & 0 \\ 0 & p + \frac{m_S^2}{2p} \end{pmatrix}. \tag{18}$$

Comparing the two we can write down the masses of these physical states K_L and K_S as,

$$\begin{aligned} m_L^2 &= [(m + \frac{\delta m}{2}) - \frac{i}{2}(\Gamma + \frac{\delta\Gamma}{2})]^2 + \frac{2p^2V^2}{(m - \frac{i}{2}\Gamma)(\delta m - \frac{i}{2}\delta\Gamma)} \\ m_S^2 &= [(m - \frac{\delta m}{2}) - \frac{i}{2}(\Gamma - \frac{\delta\Gamma}{2})]^2 - \frac{2p^2V^2}{(m - \frac{i}{2}\Gamma)(\delta m - \frac{i}{2}\delta\Gamma)} \end{aligned} \tag{19}$$

To constrain the torsion parameter we now write down the K_L and K_S mass difference as,

$$m_L - m_S = \left[(\delta m)^2 + 4\left(\frac{p^2}{m^2}t\right)^2\right]^{1/2} \tag{20}$$

For the experimental value of the K_L and K_S mass difference we consider the CDF experiments [9, 10], which were done at energies as high as 160 GeV. Although earlier low energy experiments [11, 12, 13] differ from these CDF values by 2 σ, a recent low energy experiment at CERN [14] gives a value close to the CDF values. Another advantage of CDF value is that it is done at higher energies. So because of the energy dependence of the torsion parameter, the bound will be stronger. Taking the experimental value of the K_L and K_S mass difference to be $(m_L - m_S)_{expt} = (.528 \pm .0030) \times 10^{10} \hbar s^{-1} = 3.49 \times 10^{-15} GeV$ we get an bound on the torsion parameter to be, $t < 1.3 \times 10^{-20}$ GeV.

Usually the bound on the CPT violating parameters is obtained from a direct measurement [15] of the upper bound on $|M_+ - M_-|/m_K$, which is 9×10^{-19} [16]. Although the CPLEAR bound on the direct measurement on the bound of the K° and $\bar{K}^\circ$ mass difference is stronger than the NA31, we use the latter bound because of the energy dependence of the torsion parameter. The bound on the former would constrain strongly the amount of CPT violation arising from the string motivated violation of quantum mechanics [17, 18]. The NA31 experiment was done at energies around 100 GeV [15], which gives the strongest bound on the torsion parameter

$$t = \frac{1}{2}\left(\frac{m}{p}\right)^2 |M_+ - M_-| < 5.6 \times 10^{-24} GeV.$$

Another approach of constraining the CPT violating parameter is by following Kenyon [8]. They introduce the parameter,

$$\Delta = \frac{1}{2}\frac{M_+ - M_-}{\delta M - \delta\Gamma}, \tag{21}$$

where $\delta M = m_L - m_S$ and $\delta\Gamma = \Gamma_L - \Gamma_S$. They relate this parameter to the CPT violating parameter, in our case it will be the torsion parameter, as

$$\left(\frac{p}{m}\right)^2 t = 2\Delta M Im(\Delta).$$

Using the Bell-Steinberger relation they obtain, $Im(\Delta) \leq 2 \times 10^{-4}$. This implies a bound on the torsion parameter, $t < 1.75 \times 10^{-23}$ GeV.

We shall now constrain the torsion parameter from an analysis of the measurements of the CP violating quantities. If we assume that the observed CP violation comes entirely from the torsion parameter, then we are led to immediate contradiction, because of two reasons, as we shall discuss next. However, it is possible to constrain the torsion parameter from the measurement of the CP violating quantities in the K system.

The eigenfunctions whose time evolution is given by $|K_L(t)\rangle = |K_L\rangle \quad exp\{-im_L t\}$ and $|K_S(t)\rangle = |K_S\rangle \quad exp\{-im_S t\}$ are given by the expressions

$$
\begin{aligned}
|K_L\rangle &= \frac{1}{(2(1+|\epsilon|^2)^{1/2}} \Big((1+\epsilon)|K^\circ\rangle - (1-\epsilon)|\bar{K}^\circ\rangle\Big) \\
|K_S\rangle &= \frac{1}{(2(1+|\epsilon|^2)^{1/2}} \Big((1+\epsilon)|K^\circ\rangle + (1-\epsilon)|\bar{K}^\circ\rangle\Big)
\end{aligned} \tag{22}
$$

where the mixing parameter ϵ is given by

$$\epsilon = \frac{2pV}{(m - \frac{i}{2}\Gamma)(\delta m - \frac{i}{2}\delta\Gamma)} \tag{23}$$

From (22) we find that the mixing parameter between $|K_L\rangle$ and $|K_S\rangle$ is given by

$$\langle K_S|K_L\rangle = 2Re\ \epsilon = 4\left(\frac{p}{m}\right)^2\ t\ \ \frac{\delta m}{(\delta m)^2 + (\frac{\delta\Gamma}{2})^2} \tag{24}$$

In this derivation we have followed the possibility that the mixing between K_L and K_S - the source of CP violation is only due to the torsion potential. This assumption implies $\eta_{+-} = \epsilon$ and taking the experimental values [16] of $\frac{\delta\Gamma}{2} = 3.68 \times 10^{-15}$ GeV; it predicts $\phi_{+-} \approx 45^\circ$ as in the super-weak model which is ruled out experimentally. This and the non-observation of the energy dependence of η_{+-} rules out the possibility of explaining the observed CP violation in kaons entirely from the torsion background. However, if we assume that the constant value of η_{+-} till the highest energy of the CDF experiments (160 GeV) is not due to the torsion parameter then we can put bound on this parameter. Taking $\mathrm{Re}\epsilon = 2.27 \times 10^{-3}$, we get,

$$t = \frac{1}{2}(Re\ \epsilon)(\frac{m}{p})^2 \quad \frac{(\delta m)^2 + (\frac{\delta\Gamma}{2})^2}{\delta m} < 8.5 \times 10^{-23} GeV. \tag{25}$$

In other words, if we consider the bound on the torsion parameter from the direct measurement of the CPT violating K° and $\bar{K}^\circ$ mass difference, then the prediction for the CP violation would be less than what has been observed experimentally.

The most stringent bound $t < 5.6 \times 10^{-24} GeV$ therefore arises from the NA31 constraint on $|M_+ - M_-|$ leads to the following bound on the time-like component of the torsion pseudo-vector T^σ,

$$\langle T^0 \rangle = \frac{4}{3} t^0 = \frac{4}{3\sqrt{2}} \frac{m_u m_s}{(m_s - m_u) m_K} t < 10^{-25} \quad GeV \tag{26}$$

To summarize, we pointed out that if there is any background torsion field, it will lead to an apparent violation of CP and CPT in composite pseudo-scalar particles. As a result one can constrain this parameter severely from the K system. The measurements of the direct CPT violating parameter, which is the mass difference of the K° and $\bar{K}^\circ$, gives the most stringent bound on the background torsion to be $\langle T^0 \rangle < 10^{-25}$ GeV.

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