

Neutrino Masses and Leptogenesis with Heavy Higgs Triplets

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Abstract

A simple and economical extension of the minimal standard electroweak gauge model (without right-handed neutrinos) by the addition of two heavy Higgs scalar triplets would have two significant advantages. Naturally small Majorana neutrino masses would become possible, as well as leptogenesis in the early universe which gets converted at the electroweak phase transition into the present observed baryon asymmetry.

In the minimal standard electroweak gauge model, neutrinos are massless because there are no right-handed neutrino singlets and there is no Higgs scalar triplet. The latter may be added[1] so that the interaction

$$f_{ij}[\xi^0\nu_i\nu_j + \xi^+(\nu_i l_j + l_i\nu_j)/\sqrt{2} + \xi^{++}l_i l_j] + h.c. \quad (1)$$

would induce a Majorana mass matrix of the neutrinos if ξ^0 has a small nonzero vacuum expectation value. However, since the triplet ξ carries lepton number, a massless Goldstone boson (the triplet Majoron) would appear if the model conserves lepton number before spontaneous symmetry breaking. This would have counted as the equivalent of two extra neutrino flavors in the invisible decay of the Z boson. Hence it is already ruled out experimentally[2]. On the other hand, if lepton number is explicitly broken, this problem may not arise. In particular, we will show in the following that if the scalar triplet is very heavy, *i.e.* of order 10^{13} GeV, then it is in fact natural for neutrinos to be of order 1 eV or less, and if there are two such triplets, their decays could generate a lepton asymmetry[3] in the early universe which would get converted at the electroweak phase transition[4] into the present observed baryon asymmetry.

Consider the most general Higgs potential of one doublet $\Phi = (\phi^+, \phi^0)$ and one triplet $\xi = (\xi^{++}, \xi^+, \xi^0)$:

$$\begin{aligned} V &= m^2\Phi^\dagger\Phi + M^2\xi^\dagger\xi \\ &+ \frac{1}{2}\lambda_1(\Phi^\dagger\Phi)^2 + \frac{1}{2}\lambda_2(\xi^\dagger\xi)^2 + \lambda_3(\Phi^\dagger\Phi)(\xi^\dagger\xi) \\ &+ \mu(\bar{\xi}^0\phi^0\phi^0 + \sqrt{2}\xi^-\phi^+\phi^0 + \xi^{--}\phi^+\phi^+) + h.c. \end{aligned} \quad (2)$$

Let $\langle\phi^0\rangle = v$ and $\langle\xi^0\rangle = u$, then the conditions for the minimum of V are given by

$$m^2 + \lambda_1 v^2 + \lambda_3 u^2 + 2\mu u = 0, \quad (3)$$

$$u(M^2 + \lambda_2 u^2 + \lambda_3 v^2) + \mu v^2 = 0. \quad (4)$$

In the triplet Majoron model[1], $\mu = 0$, hence for $u \neq 0$, we have

$$v^2 = \frac{-\lambda_2 m^2 + \lambda_3 M^2}{\lambda_1 \lambda_2 - \lambda_3^2}, \quad u^2 = \frac{\lambda_3 m^2 - \lambda_1 M^2}{\lambda_1 \lambda_2 - \lambda_3^2}. \quad (5)$$

Since $v = 174$ GeV and u should not be greater than a few keV (to suppress the $\gamma + e \rightarrow e + \text{Majoron}$ cross section[5]), the parameter M^2 must be fine-tuned to equal $(\lambda_3/\lambda_1)m^2$ to extreme accuracy. This model is thus rather unnatural, but of course it is also experimentally ruled out. To see this, we consider the mass matrix spanned by the neutral scalar fields $\sqrt{2}Re\phi^0$ and $\sqrt{2}Re\xi^0$, *i.e.*

$$\mathcal{M}^2 = \begin{bmatrix} 2\lambda_1 v^2 & 2\lambda_3 uv + 2\mu v \\ 2\lambda_3 uv + 2\mu v & 2\lambda_2 u^2 - \mu v^2/u \end{bmatrix}. \quad (6)$$

If $\mu = 0$, then the eigenvalues of the above are $2\lambda_1 v^2$ and $2(\lambda_2 - \lambda_3^2/\lambda_1)u^2$. The latter is the square of the mass of the partner of the Majoron and it is necessarily small. Hence the Z boson must decay into them if the model is correct.

If $\mu \neq 0$, then lepton number is explicitly violated and the mass matrix spanned by the neutral scalar fields $\sqrt{2}Im\phi^0$ and $\sqrt{2}Im\xi^0$ is given by

$$\mathcal{M}^2 = \begin{bmatrix} -4\mu u & 2\mu v \\ 2\mu v & -\mu v^2/u \end{bmatrix}. \quad (7)$$

The above contains one zero eigenvalue for the longitudinal component of the Z boson, but the would-be Majoron is now massive with mass-squared given by

$$-\frac{\mu}{u}(v^2 + 4u^2) = (M^2 + \lambda_3 v^2) \left[1 + \mathcal{O}\left(\frac{u^2}{v^2}\right) \right], \quad (8)$$

which is also approximately the mass-squared of its partner. In the above, we have used Eq. (4) and the fact that $u^2 \ll v^2$. Hence $M^2 + \lambda_3 v^2$ must be positive and if it is also large enough, present experiments cannot rule out this model.

Let us in fact make M very large. In that case, we have a natural understanding of why u can be so small because

$$u \simeq \frac{-\mu v^2}{M^2}, \quad (9)$$

which is analogous to the usual seesaw mechanism for obtaining small Majorana neutrino masses, except that here we do not have any right-handed neutrinos. [In a left-right gauge model, where there is already a right-handed neutrino, the left-handed neutrino also gets a mass from a Higgs triplet[6] in addition to the canonical seesaw mechanism.] Substituting the above into Eq. (3), we find

$$v^2 \simeq \frac{-m^2}{\lambda_1 - 2\mu^2/M^2}. \quad (10)$$

Note that we have no fine tuning (*i.e.* the cancellation of two large quantities to obtain a small one) in this model.

It may appear strange that the heavy triplet ξ gets a tiny vacuum expectation value u . However, this actually already occurs in the well-known case of the spontaneous breaking of $SU(5)$ using the **24** scalar representation. Under $SU(3)_C \times SU(2)_L \times U(1)_Y$,

$$\mathbf{24} = (1, 1, 0) + (8, 1, 0) + (1, 3, 0) + (3, 2, -5/6) + (3^*, 2, 5/6). \quad (11)$$

What everyone knows is that a large vacuum expectation value v_1 for the $(1, 1, 0)$ component breaks $SU(5)$ down to the standard-model gauge group. What many people do not realize is that the further breaking of $SU(2)_L \times U(1)_Y$ down to $U(1)_Q$ using the fundamental **5** scalar representation necessarily induces a small vacuum expectation value v_3 for the heavy $(1, 3, 0)$ component. It has been shown recently[7] that $v_3 \sim v_2^2/v_1$, where v_2 is the electroweak vacuum expectation value. Again the seesaw structure appears automatically.

Another way of handling the heavy Higgs triplet is to integrate it out. From Eqs. (1) and (2), we obtain the effective nonrenormalizable term

$$\frac{-f_{ij}\mu}{M^2}[\phi^0\phi^0\nu_i\nu_j - \phi^+\phi^0(\nu_i l_j + l_i\nu_j) + \phi^+\phi^+ l_i l_j] + h.c. \quad (12)$$

From Eq. (2) itself, the reduced Higgs potential involving only Φ is

$$V = m^2\Phi^\dagger\Phi + \frac{1}{2}\left(\lambda_1 - \frac{2\mu^2}{M^2}\right)(\Phi^\dagger\Phi)^2, \quad (13)$$

the last term coming[7] from the exchange of ξ . As ϕ^0 acquires a nonzero vacuum expectation value v , we obtain Eq. (10) as we should, and the neutrino mass matrix becomes $-2f_{ij}\mu v^2/M^2 = 2f_{ij}u$ as expected.

Armed with Eq. (9) and making the reasonable assumption that $|\mu|$ is of order M or less, we find, for $M \sim 10^{13}$ GeV, that u is less than a few eV. This is then a suitable natural mechanism for small Majorana neutrino masses. Furthermore, a mass of 10^{13} GeV or so is very evocative of the natural energy scale for leptogenesis[3]. For this, we would need (at least) two such heavy Higgs triplets to have the proper CP violation which distinguishes states of different lepton number.

We now write down the mass terms and the Yukawa couplings of the heavy Higgs triplets (ξ_a , $a = 1, 2$), which are relevant for the study of leptogenesis in this scenario:

$$\begin{aligned}
-\mathcal{L} = & \sum_{a=1,2} \left\{ M_a^2 \xi_a^\dagger \xi_a + \left(f_{aij} [\xi_a^0 \nu_i \nu_j + \xi_a^+ (\nu_i l_j + l_i \nu_j) / \sqrt{2} + \xi_a^{++} l_i l_j] \right. \right. \\
& \left. \left. + \mu_a [\bar{\xi}_a^0 \phi^0 \phi^0 + \sqrt{2} \xi_a^- \phi^+ \phi^0 + \xi_a^{--} \phi^+ \phi^+] + h.c. \right) \right\}. \tag{14}
\end{aligned}$$

At an energy scale far above that of electroweak symmetry breaking, $SU(2)_L \times U(1)_Y$ gauge invariance means that we can pick any one of the three components of the triplet for consideration and the results are guaranteed to be valid for the other two. Let us choose ξ_a^{++} , then their decays are:

$$\xi_a^{++} \rightarrow \begin{cases} l_i^+ l_j^+ & (L = -2) \\ \phi^+ \phi^+ & (L = 0) \end{cases} \tag{15}$$

The coexistence of the above two types of final states indicates the nonconservation of lepton number. On the other hand, any lepton asymmetry generated by ξ_a^{++} would be neutralized by the decays of ξ_a^{--} , unless CP conservation is also violated and the decays are out of thermal equilibrium[8] in the early universe.

We will use the effective mass-matrix formalism[9] to discuss the generation of lepton asymmetry in this model. We note that in the often studied case of the decays of heavy

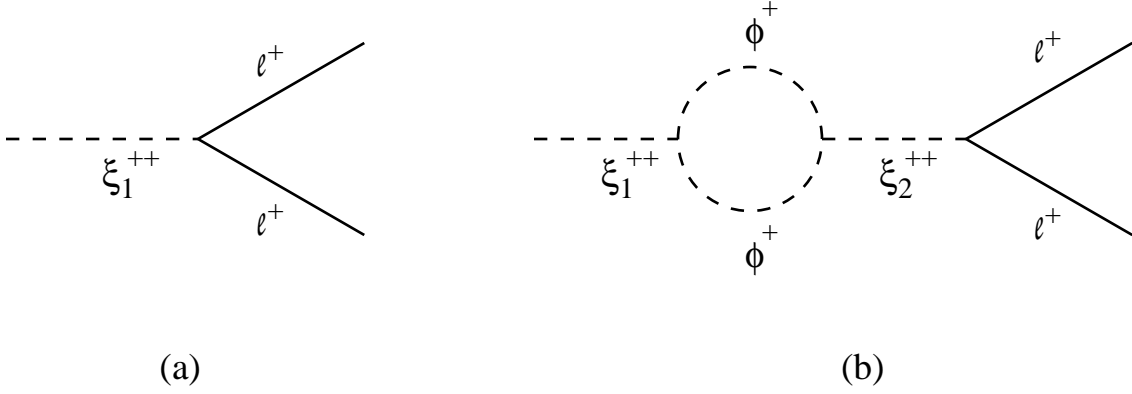


Figure 1: The decay of $\xi_1^{++} \rightarrow l^+l^+$ at tree level (a) and in one-loop order (b). A lepton asymmetry is generated by their interference.

singlet neutrinos, there is always $\nu_R - \bar{\nu}_R$ mixing, whereas $\xi_a^0 - \bar{\xi}_b^0$ mixing is strictly forbidden here before $SU(2)_L \times U(1)_Y$ symmetry breaking. Without loss of generality, we can choose the tree-level mass matrix for the triplets ξ_a to be diagonal and real, as already assumed in Eq. (14). Hence CP is conserved at this level. However, CP nonconservation may still occur at the one-loop level due to the interference between tree and one-loop diagrams, as shown in Fig. 1. We note also that in all other previous models of baryogenesis (or leptogenesis), where the decays of heavy particles generate the asymmetry, there is always a one-loop vertex correction, whereas in this model, there is none. However, leptogenesis is still possible if there are two triplets because off-diagonal one-loop self-energy terms will have absorptive parts.

Before we present the details of our calculation, let us consider how CP nonconservation appears in Eq. (14). If there is only one ξ , then the relative phase between any f_{ij} and μ can be chosen real. Hence a lepton asymmetry cannot be generated. With two ξ 's, even if there is only one lepton family, one relative phase must remain among the quantities $f_1, f_2, \mu_1,$ and μ_2 . As for the possible relative phases among the f_{aij} 's, they cannot generate a lepton asymmetry because they all refer to final states of the same lepton number.

In the presence of interactions, the diagonal tree-level mass matrix M_a^2 is replaced by

$$\frac{1}{2}\xi_a^\dagger(\mathcal{M}_+^2)_{ab}\xi_b + \frac{1}{2}(\xi_a^*)^\dagger(\mathcal{M}_-^2)_{ab}\xi_b^*, \quad (16)$$

where

$$\mathcal{M}_\pm^2 = \begin{pmatrix} M_1^2 - i\mathcal{G}_{11} & -i\mathcal{G}_{12}^\pm \\ -i\mathcal{G}_{21}^\pm & M_2^2 - i\mathcal{G}_{22} \end{pmatrix}, \quad (17)$$

where $\mathcal{G}_{ab}^+ = \Gamma_{ab}M_b$, $\mathcal{G}_{ab}^- = \Gamma_{ab}^*M_b$, and $\mathcal{G}_{aa} = \Gamma_{aa}M_a$. Now

$$\Gamma_{ab}M_b = \frac{1}{8\pi} \left(\mu_a\mu_b^* + M_aM_b \sum_{k,l} f_{akl}^* f_{bkl} \right), \quad (18)$$

and it comes from the absorptive part of the one-loop self-energy diagram for $\xi_b \rightarrow \xi_a$ which is of course equal to that for $\xi_a^* \rightarrow \xi_b^*$. Hence $\Gamma_{12}M_2 = \Gamma_{21}^*M_1$ as expected. If there is no phase convention which allows us to choose Γ_{ab} to be real, then CP conservation is violated.

Assuming that $\Gamma_a \equiv \Gamma_{aa} \ll M_a$, we solve for the eigenvalues of \mathcal{M}_\pm^2 :

$$\lambda_{1,2} = \frac{1}{2}(M_1^2 + M_2^2 \pm \sqrt{\mathcal{S}}), \quad (19)$$

where $\mathcal{S} = (M_1^2 - M_2^2)^2 - 4|\Gamma_{12}M_2|^2$, and $M_1 > M_2$ is assumed. The corresponding physical states are

$$\psi_{1,2}^+ = a_{1,2}^+\xi_1 + b_{1,2}^+\xi_2, \quad \psi_{1,2}^- = a_{1,2}^-\xi_1^* + b_{1,2}^-\xi_2^*, \quad (20)$$

where $a_1^\pm = b_2^\pm = 1/\mathcal{N}$, $b_1^\pm = \mathcal{C}_1^\pm/\mathcal{N}$, $a_2^\pm = \mathcal{C}_2^\pm/\mathcal{N}$, with

$$\mathcal{C}_1^+ = -\mathcal{C}_2^- = \frac{-2i\Gamma_{12}^*M_2}{M_1^2 - M_2^2 + \sqrt{\mathcal{S}}}, \quad \mathcal{C}_1^- = -\mathcal{C}_2^+ = \frac{-2i\Gamma_{12}M_2}{M_1^2 - M_2^2 + \sqrt{\mathcal{S}}}, \quad (21)$$

and $\mathcal{N} = \sqrt{1 + |\mathcal{C}_i^\pm|^2}$. Note that whereas ξ_a and ξ_a^* are CP conjugates, ψ_i^+ and ψ_i^- are not, even though they have the same mass. This is analogous to having physical Majorana neutrinos which are not CP eigenstates, as discussed in Ref.[9].

The physical states $\psi_{1,2}^\pm$ will now evolve with time and decay into leptons and antileptons. The resulting lepton asymmetries $\delta_i = n_l/n_i$ will be given by

$$\delta_i = 2 \left[B(\psi_i^- \rightarrow ll) - B(\psi_i^+ \rightarrow l^c l^c) \right], \quad (22)$$

where n_i is the number density of ψ_i^\pm , and

$$B(\psi_i^- \rightarrow ll) = \frac{\sum_{k,l} |a_i^- f_{1kl}^* + b_i^- f_{2kl}^*|^2}{|a_i^- \mu_1 + b_i^- \mu_2|^2 / M_i^2 + \sum_{k,l} |a_i^- f_{1kl}^* + b_i^- f_{2kl}^*|^2}, \quad (23)$$

$$B(\psi_i^+ \rightarrow l^c l^c) = \frac{\sum_{k,l} |a_i^+ f_{1kl} + b_i^+ f_{2kl}|^2}{|a_i^+ \mu_1^* + b_i^+ \mu_2^*|^2 / M_i^2 + \sum_{k,l} |a_i^+ f_{1kl} + b_i^+ f_{2kl}|^2}. \quad (24)$$

Assuming $(M_1^2 - M_2^2)^2 \gg 4|\Gamma_{12} M_2|^2$, so that $\sqrt{\mathcal{S}} \simeq M_1^2 - M_2^2$, we get

$$\delta_i \simeq \frac{\text{Im} \left[\mu_1 \mu_2^* \sum_{k,l} f_{1kl} f_{2kl}^* \right]}{8\pi^2 (M_1^2 - M_2^2)} \left[\frac{M_i}{\Gamma_i} \right]. \quad (25)$$

Note that there is no contribution from the purely leptonic term because it is identically zero as expected.

In calculating the lepton asymmetry δ_i , we have assumed that when the temperature was much higher than the masses of the ψ 's, there was no lepton asymmetry. Only around the time when the ψ 's started decaying was a lepton asymmetry created. At that time, these scalars also became nonrelativistic. In the case $M_1 > M_2$ as we have assumed, when the universe cooled down to below M_1 , most of ψ_1 would decay away. However, the asymmetry so created would be erased by the lepton-number nonconserving interactions of ψ_2 . Hence only the subsequent decay of ψ_2 at $T < M_2$ would generate a lepton asymmetry to remain until the onset of the electroweak phase transition. This asymmetry would evolve with time following the Boltzmann equation,

$$\frac{dn_l}{dt} + 3Hn_l = \delta_2 \Gamma_2 [n_2 - n_2^{eq}] - \left(\frac{n_l}{n_\gamma} \right) n_2^{eq} \Gamma_2 - 2n_\gamma n_l \langle \sigma |v| \rangle \quad (26)$$

The second term on the left side comes from the expansion of the universe, where $H = 1.66g_*^{1/2}(T^2/M_{Pl})$ is the Hubble constant with g_* the effective number of massless particles. Γ_2 is the thermally averaged decay rate of ψ_2 , n_γ is the photon density and the term $\langle \sigma |v| \rangle$ describes the thermally averaged cross section of $l + l \leftrightarrow \phi + \phi$ scattering. The density of ψ_2

satisfies the Boltzmann equation,

$$\frac{dn_2}{dt} + 3Hn_2 = -\Gamma_2(n_2 - n_2^{eq}) \quad (27)$$

It is now convenient to use the dimensionless variable $x = M_2/T$ as well as the particle density per entropy density $Y_i = n_i/s$, and the relation $t = x^2/2H(x = 1)$. We also define the parameter $K \equiv \Gamma_2(x = 1)/H(x = 1)$ as a measure of the deviation from equilibrium. For $K \ll 1$ at $T \sim M_2$, the system is far from equilibrium; hence the last two terms responsible for the depletion of n_l would be negligible. With these simplifications and the above redefinitions, the Boltzmann equations effectively read:

$$\frac{dY_l}{dx} = (Y_2 - Y_2^{eq})\delta_2 Kx, \quad \frac{dY_2}{dx} = -(Y_2 - Y_2^{eq})Kx. \quad (28)$$

In this limit $K \ll 1$, it is not difficult to obtain an asymptotic solution for n_l . Although the decay rate of ψ_2 is not fast enough to bring the number density n_2 to its equilibrium density, it is a good approximation to assume that the universe never goes far away from equilibrium. In other words, we can assume $d(Y_2 - Y_2^{eq})/dx = 0$ to get an asymptotic value for Y_l , given by $Y_l = n_l/s = \delta_2/g_*$. However, if $K > 1$, the terms which deplete n_l dominate for some time and the lepton number density reaches its new asymptotic value, which is lower than the value it reaches in the out-of-equilibrium case. In this case although it is difficult to get an analytic solution of the Boltzmann equations, it is possible[10] to get an approximate suppression factor given by $1/K(\ln K)^{0.6}$.

The lepton asymmetry thus generated after the Higgs triplets decayed away would be the same as the $(B - L)$ asymmetry before the electroweak phase transition. During the electroweak phase transition, the presence of sphaleron fields would relate this $(B - L)$ asymmetry to the baryon asymmetry of the universe[11]. The final baryon asymmetry thus generated can then be given by the approximate relation

$$\frac{n_B}{s} \sim \frac{\delta_2}{3g_*K(\ln K)^{0.6}} \quad (29)$$

It is clear from Eq. (25) that we must have two Higgs triplets for n_l to be nonzero. Hence the neutrino mass matrix is now given by $(m_\nu)_{ij} = -2v^2(f_{1ij}\mu_1/M_1^2 + f_{2ij}\mu_2/M_2^2)$. Similarly, the effective quartic coupling of the Higgs doublet Φ is now modified in Eqs. (10) and (13) to read $\lambda_1 - 2\mu_1^2/M_1^2 - 2\mu_2^2/M_2^2$. This means that we have the flexibility to choose M_1 and M_2 somewhat differently but within an order of magnitude to obtain a neutrino mass of order 1 eV, as well as the observed baryon asymmetry of the universe. For example, let $M_2 = 10^{13}$ GeV, $\mu_2 = 2 \times 10^{12}$ GeV, and $f_{233} = 1$, then $m_{\nu_\tau} = 1.2$ eV, assuming that the M_1 contribution is negligible. Now let $M_1 = 3 \times 10^{13}$ GeV, $\mu_1 = 10^{13}$ GeV, and $f_{1kl} \sim 0.1$, then the decay of ψ_2^\pm generates a lepton asymmetry δ_2 of about 8×10^{-4} if the CP phase is maximum. Using $M_{Pl} \sim 10^{19}$ GeV and $g_* \sim 10^2$, we find $K \sim 2.4 \times 10^3$. Hence $n_B/s \sim 10^{-10}$ as desired.

In conclusion, we have presented in this paper a simple and economical extension of the minimal standard model to obtain naturally small Majorana neutrino masses and explain the observed baryon asymmetry of the universe. This is achieved by the addition of two heavy Higgs triplets. We show that it is in fact natural for them to have very small nonzero vacuum expectation values. For neutrino masses of less than a few eV, the mass scale of these triplets is of order 10^{13} GeV, which is very suitable for leptogenesis. We then calculate the lepton asymmetry, using a newly developed effective mass-matrix formalism[9]. Our proposal is an equally viable and attractive alternative to the canonical scenario where there are three additional right-handed singlet neutrinos with large Majorana masses. They are distinguished in principle by the fact that the seesaw mechanism in the latter case decreases very slightly the coupling of the left-handed neutrinos, whereas there is no such deviation at all in our case. Otherwise, they are identical in their two significant advantages over the minimal standard model, *i.e.* naturally small neutrino masses and leptogenesis.

Note added: Equation (2) is missing the term $\Phi_i^\dagger \tau_{ij}^a \Phi_j \xi_b^\dagger t_{bc}^a \xi_c$. Hence λ_3 in all subsequent

equations should include the coupling of this term as well.

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