Test of Special Relativity and Equivalence principle from K Physics

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Abstract

A violation of Local Lorentz Invariance (VLI) and hence the special theory of relativity or a violation of equivalence principle (VEP) in the Kaon system can, in principle, induce oscillations between K^0 and $\bar{K}^0$. We construct a general formulation in which simultaneous pairwise diagonalization of mass, momemtum, weak or gravitational eigenstates is not assumed. We discuss this problem in a general way and point out that, as expected, the VEP and VLI contributions are indistinguishable. We then insist on the fact that VEP or VLI can occur even when CPT is conserved. A possible CP violation of the superweak type induced by VEP or VLI is introduced and discussed. We show that the general VEP mechanism (or the VLI mechanism, but not both simultaneously), with or without conserved CPT, could be clearly tested experimentally through the energy dependence of the $K_L - K_S$ mass difference and of η_{+-}, η_{00}, δ. Constraints imposed by present experiments are calculated.

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1 Introduction.

A few of the basic building blocks of particle physics are the assumptions that nature preserves local Lorentz invariance and hence the special theory of relativity, the product of the discrete symmetries CPT and the equivalence principle. It is also true that to date we have not seen any violation of any of these laws. In recent times many new attempts have been made to obtain new and quantifiable information on the degree of validity of these basic laws. It is in this connection that we plan to investigate the Kaon–system.

Many experiments have tested the special theory of relativity to a high degree of precision [1]. These experiments probe for any dependence of the (non-gravitational) laws of physics on a laboratory's position, orientation or velocity relative to some preferred frame of reference, such as the frame in which the cosmic microwave background is isotropic. Failure to observe such dependence further enhances the validity of (respectively) Local Position Invariance and Local Lorentz Invariance (LLI), and hence of the Einstein Equivalence Principle (EEP) [2]. However, these empirical results have been obtained primarily in the baryon-photon sector of the standard model. There is no logically necessary reason to conclude from these results that the special theory of relativity must be valid in all sectors of the standard model of elementary particle physics. Its validity must be empirically checked for each sector (gauge boson, neutrino, massive lepton, *etc.*) separately [3].

A characteristic feature of LLI violation (VLI) is that every species of matter has its own maximum attainable speed. This yields several novel effects in various sectors of the standard model [3], including vacuum Cerenkov radiation [4], photon decay [5] and neutrino oscillations [6, 7, 8]. Recently we extended these arguments and pointed out that violation of special relativity will in general induce an energy dependent $K_L - K_S$ mass difference [9]; an empirical search for such effects can therefore be used to obtain bounds on VLI in the Kaon sector of the standard model. As we shall discuss later VLI in the kaon sector can occur in a manner that may or may not violate CPT.

The EEP implies universality of gravitational coupling for all forms of mass-energy, thereby ensuring that spacetime is described by a unique operational geometry. An extreme converse of this principle is that every form of stress-energy couples to its own metric, so that the Lagrangian for the standard model is modified to be one of the form

$$\mathcal{L} = \mathcal{L}_G(g^I) + \sum_I \mathcal{L}_M(g^I, \Phi_I) + \mathcal{L}_C \tag{1}$$

where each matter field Φ_I couples to its own metric $g^I_{\mu\nu}$. The gravitational Lagrangian density $\mathcal{L}_G$ describes the behaviour of all of these metrics in the absence of any matter fields. The Lagrangian density $\mathcal{L}_C$ describes the interaction between the different matter fields; it will in general include at least some subset of the metric fields $g^I_{\mu\nu}$. Although such a Lagrangian is generally covariant, spacetime no longer has a unique operational geometry, since clocks and measuring rods constructed out of different types of matter fields will in general yield different results for a given set of experiments that depend on the choice of coordinate frame. Furthermore, while it is possible for any given metric $g^I_{\mu\nu}$ to interpret a diffeomorphism of the manifold as a gauge transformation of the linearized tensor $h^I_{\mu\nu} = g^I_{\mu\nu} - \bar{g}^I_{\mu\nu}$ where $\bar{g}^I_{\mu\nu}$ is some reference metric (typically chosen to be a flat metric), this cannot be done simultaneously for all the metrics (unless they are all the same). This means

that the spin modes of all the other metrics will in general be excited. It is then a theoretical challenge to ensure that the excitations of the additional degrees of freedom of the other metric do not yield unacceptable pathologies such as runaway negative energy solutions, tachyons, etc. One might imagine doing this by giving, say, the gravitons associated with the metrics a tiny mass, save for the metric associated with ordinary stable matter. More general theoretical mechanisms than that given in (1) can also be considered: for example some of the metrics may not be describable by second rank tensor fields, or some sectors of the theory may not even be Lagrangian-based. For an overview and further discussion of the different possibilities, see ref. [2].

From an empirical perspective, the validity of the EEP must therefore be checked sector-by-sector in the standard model, since it cannot be imposed on grounds of logic. Although the EEP has been tested to impressive levels of precision, virtually all such tests have been carried out with matter fields. The possibility that matter and antimatter may have different gravitational couplings remains a fascinating open question. The strongest bound on matter-antimatter gravitational universality comes from the $K^0 - \bar{K}^0$ system. Recent studies of this system have considered a straightforward violation of the weak equivalence principle (WEP) in which it is assumed that $K^0 - \bar{K}^0$ mass and gravitational eigenstates can be simultaneously diagonalised but with differing eigenvalues (*i.e.* differing K^0 and $\bar{K}^0$ masses) [10, 11, 12], in which case violation of gravitational universality also means violation of CPT.

However, more generally, a violation of the EEP (VEP) in the Kaon system will not assume simultaneous pairwise diagonalization of mass, gravitational or weak eigenstates. We shall consider in this article the consequences of such a general VEP mechanism, showing that it can provide a source of CP violation whilst conserving CPT. In this context, previously investigated mechanisms of EEP violation in the Kaon system may be considered either as special cases of maximal CPT violation in the gravitational sector [10, 11, 12] or else CPT conserved VEP, which is the other extreme case [13]. Our analysis is more general, including all the earlier analyses as special cases and in addition allows us to compare with the VLI bounds. We consider constraints imposed on this general VEP mechanism by present experiments.

In section 2 and 3 we derive the general mass matrix including VLI and VEP effects respectively. In both sections we point out that VLI (VEP) allows for a phase α_v (α_G) responsible for CP violation whilst conserving CPT. At the end of section 3 we compare both general mass matrices noticing that VEP and VLI effects are indistinguishable as expected [2] (for the neutrino sector this similarity was pointed out in ref.[6]). In section 4 we discuss the general case where these phases are taken to be 0. We consider first the CPT-conserving case and examine the energy dependence VLI and VEP induce in the mass difference $m_L - m_S$. Constraints on VEP parameters (and hence on VLI parameters) from experiments on $m_L - m_S$ are discussed. We also give constraints on the interesting maximally $CPT-$violating case where matter or antimatter states are the velocity or gravitational eigenstates with differing eigenvalues. Then in section 5 we discuss the effect of the phases α_v and α_G and constraints on VEP and VLI parameters from CP violation experiments. We point out that the CP violation induced by VEP or VLI is of the superweak type and has an inherent energy dependence. Consequently, although this mechanism cannot fully account for observed CP violation in the Kaon system, it yields a definite testable prediction for the

energy dependence of CP violation parameters. This can then be used to put a qualitatively new bound on VEP or VLI. We summarise our results in section 6.

2 Violation of LLI.

The maximum attainable velocities of particles and antiparticles can differ if there is violation of LLI [5]. Here we take a phenomenological approach to this problem and assume that neither the mass nor the weak eigenstates are a priori simultaneously diagonalisable with the momemtum eigenstates.

Then the general form of the effective Hamiltonian associated with the Lagrangian in the $(K^0 \;\; \bar{K}^0)$ basis will be

$$H = U_W H_{SEW} U_W^{-1} + U_v H_v U_v^{-1} \tag{2}$$

with,

$$H_{SEW} = \frac{(M_{SEW})^2}{2p} = \frac{1}{2p}\begin{pmatrix} m_1 - i\frac{\Gamma_1}{2} & 0 \\ 0 & m_2 - i\frac{\Gamma_2}{2} \end{pmatrix}^2 \tag{3}$$

and

$$H_v = \begin{pmatrix} v_1 & 0 \\ 0 & v_2 \end{pmatrix} p \tag{4}$$

to leading order in $\bar{m}^2/p^2$ with p the momentum and $\bar{m} = (m_1 + m_2)/2$ the average mass. From now on we define $\delta X \equiv (X_1 - X_2)$, $\bar{X} \equiv (X_1 + X_2)/2$ for any quantity X. H_{SEW} refers to the strong and electroweak part of the hamiltonian. The constants v_1 and v_2 correspond to the maximum attainable speeds of each eigenstate. If special relativity is valid within the Kaon sector these are both equal to their average $\bar{v} = (v_1 + v_2)/2$, which we normalize to unity. Hence $v_1 - v_2 = \delta v$ is a measure of VLI in the Kaon sector. If $\bar{v}$ corresponds to the speed of electromagnetic radiation then special relativity is valid within the Kaon–photon sector of the standard model. In the limit $v_1 = v_2$, $m_{1,2}$ and $\Gamma_{1,2}$ are interpreted as the masses and the decay widths of the physical states $\tilde{K}_{1,2}$. These states are usually denoted as $K_{L,S}$, but since we shall be representing the physical states including VLI effects with the same notation we shall refer to them as $\tilde{K}_{1,2}$. The transformation matrix U_W which relates the states $\tilde{K}_{1,2}$ to the states $K^0, \bar{K}^0$ can be written as

$$U_W = \frac{1}{\sqrt{2(1+|\tilde{\varepsilon}|^2)}} e^{i\chi_W} \begin{pmatrix} (1+\tilde{\varepsilon}) & (1+\tilde{\varepsilon}) \\ -(1-\tilde{\varepsilon}) & (1-\tilde{\varepsilon}) \end{pmatrix} \begin{pmatrix} e^{-i\beta_W} & 0 \\ 0 & e^{i\beta_W} \end{pmatrix}. \tag{5}$$

We have assumed that there is no CPT violation in the non-VLI part of the Hamiltonian, but only that CP is violated, parametrized by $\tilde{\varepsilon}$. The phases χ_W and β_W can be eliminated by a redefinition of the $\tilde{K}_{1,2}$ states in such a way that we have the usual formula:

$$\tilde{K}_{\substack{1\\2}} = \frac{1}{\sqrt{2(1+|\tilde{\varepsilon}|^2)}} \left[(1+\tilde{\varepsilon})K^0 \mp (1-\tilde{\varepsilon})\bar{K}^0\right] \tag{6}$$

For the VLI part if we assume that the velocity eigenstates are orthogonal they are related to the $K^0, \bar{K}^0$ by a unitary matrix U_v which can be written in the general form

$$U_v = e^{i\chi_v} \begin{pmatrix} e^{-i\alpha_v} & 0 \\ 0 & e^{i\alpha_v} \end{pmatrix} \begin{pmatrix} \cos\theta_v & \sin\theta_v \\ -\sin\theta_v & \cos\theta_v \end{pmatrix} \begin{pmatrix} e^{-i\beta_v} & 0 \\ 0 & e^{i\beta_v} \end{pmatrix}. \tag{7}$$

The phases χ_v and β_v can be absorbed in a redefinition of the velocity eigenstates. The phase α_v (which is similar to Im$\tilde{\varepsilon}$ in Eq.(5) which to this order in $\tilde{\varepsilon}$ can be written in the form of such a phase) cannot be absorbed because K^0-$\bar{K}^0$ are by definition charge conjuguate states. The phase α_v is a new source of CP violation which can be present even though the velocity states are still orthogonal.

From the form of the transformation matrix U_W and U_v, the total hamiltonian in the $K^0 - \bar{K}^0$ basis is

$$H = pI + \frac{1}{2p}\begin{pmatrix} M_+ & M_{12} \\ M_{21} & M_- \end{pmatrix}^2$$

with

$$\begin{aligned} M_\pm &= \bar{m} - i\frac{\bar{\Gamma}}{2} \pm \frac{p^2}{\bar{m}}\frac{\cos 2\theta_v}{2}\delta v \\ M_{12} &= -\frac{1}{2}\frac{1+\tilde{\varepsilon}}{1-\tilde{\varepsilon}}(\delta m - i\frac{\delta\Gamma}{2}) - e^{-2i\alpha_v}\frac{p^2}{\bar{m}}\frac{\sin 2\theta_v}{2}\delta v \\ M_{21} &= -\frac{1}{2}\frac{1-\tilde{\varepsilon}}{1+\tilde{\varepsilon}}(\delta m - i\frac{\delta\Gamma}{2}) - e^{2i\alpha_v}\frac{p^2}{\bar{m}}\frac{\sin 2\theta_v}{2}\delta v \end{aligned} \tag{8}$$

The mass matrix above is the general formula from which we will discuss different special cases.

3 General mass matrix for VEP.

To formulate the VEP mechanism in the Kaon system, we first study the energy of the particles under consideration, taking the kaons to be relativistic. The gravitational part of the Lagrangian to first order (linearized theory) in a weak gravitational field $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$ (where $h_{\mu\nu} = 2\frac{\phi}{c^2}diag(1,1,1,1)$) can be written as $\mathcal{L} = -\frac{1}{2}(1+g_i)h_{\mu\nu}T^{\mu\nu}$ where $T^{\mu\nu}$ is the stress-energy in the gravitational eigenbasis. The principle of equivalence says that the gravitational couplings g_i are equal.

We can now write down the effective Hamiltonian including the strong, electromagnetic, weak, and gravitational interactions in the $(K^0\ , \bar{K}^0)$ basis :

$$H = pI + U_W H_{SEW} U_W^{-1} + U_G H_G U_G^{-1} \tag{9}$$

with I the identity matrix,

$$H_{SEW} = \frac{(M_{SEW})^2}{2p} = \frac{1}{2p}\begin{pmatrix} m_1 - i\frac{\Gamma_1}{2} & 0 \\ 0 & m_2 - i\frac{\Gamma_2}{2} \end{pmatrix}^2 \tag{10}$$

and

$$H_G = \begin{pmatrix} G_1 & 0 \\ 0 & G_2 \end{pmatrix} = \begin{pmatrix} -2(1+g_1)\phi(p + \frac{\bar{m}^2}{2p}) & 0 \\ 0 & -2(1+g_2)\phi(p + \frac{\bar{m}^2}{2p}) \end{pmatrix} \tag{11}$$

in physical time and length units [12] to first order in $\bar{m}^2/p^2$ with p the momentum. In formalisms where the weak equivalence principle is assumed [11, 12], one starts with U_G proportional to U_W (in the case considered where CP-violating effects in U_W are taken to

be 0), which leads to a violation of CPT if VEP is operative, that is to say if $g_1 \neq g_2$. More generally when VEP is operative, U_G is not necessarily proportional to U_W. Note that in the gravitational Hamiltonian H_G we have neglected terms proportional to δm, and ϕ is the gravitational potential on the surface of earth, which is constant over the range of terrestrial experiments.

In the absence of gravity, $m_{1,2}$ and $\Gamma_{1,2}$ are interpreted as the masses and the decay widths of the physical states $\tilde{K}_{1,2}$ defined by Eq.(6) as in section 2. For the gravitational part if we assume that the gravitational states are orthogonal they are related to the $K^0, \bar{K}^0$ by a unitary matrix U_G which can be written in the general form

$$U_G = e^{i\chi_G} \begin{pmatrix} e^{-i\alpha_G} & 0 \\ 0 & e^{i\alpha_G} \end{pmatrix} \begin{pmatrix} \cos\theta_G & \sin\theta_G \\ -\sin\theta_G & \cos\theta_G \end{pmatrix} \begin{pmatrix} e^{-i\beta_G} & 0 \\ 0 & e^{i\beta_G} \end{pmatrix}. \tag{12}$$

The phases χ_G and β_G can be absorbed in a redefinition of the gravitational states but the phase α_G cannot like in the VLI case [4] and is a new source of CP violation like α_v.

From U_W and U_G we then get

$$\begin{aligned} M_{\pm} &= \bar{m} - i\frac{\bar{\Gamma}}{2} + \frac{p}{\bar{m}}\bar{G} \pm \frac{p}{\bar{m}}\frac{\cos 2\theta_G}{2}\delta G \\ M_{12} &= -\frac{1}{2}\frac{1+\tilde{\varepsilon}}{1-\tilde{\varepsilon}}(\delta m - i\frac{\delta\Gamma}{2}) - e^{-2i\alpha_G}\frac{p}{\bar{m}}\frac{\sin 2\theta_G}{2}\delta G \\ M_{21} &= -\frac{1}{2}\frac{1-\tilde{\varepsilon}}{1+\tilde{\varepsilon}}(\delta m - i\frac{\delta\Gamma}{2}) - e^{2i\alpha_G}\frac{p}{\bar{m}}\frac{\sin 2\theta_G}{2}\delta G \end{aligned} \tag{13}$$

The mass matrix above is the general formula from which we will discuss different cases like Eq.(8) in the VLI case.

Comparing Eq.(8) and Eq.(13) we see that both the VLI and VEP mass matrices are similar. Puting $\theta_v = \theta_G$ and $\alpha_v = \alpha_G$, VLI and VEP effects are indistinguishable to lowest order in $\bar{m}^2/p^2$ providing one identifies the VLI parameter δv with the VEP paramter $-2\phi\delta g$. From now on we will discuss the VEP case knowing that any corresponding VLI formula can be obtained straightforwatdly from this identification.

4 Testing the equivalence principle in the case $\alpha = 0$.

In this section we restrict ourselves to tests of the equivalence principle from $m_L - m_S$ data in the case where gravitational states are related to the states $K^0 - \bar{K}^0$ by a simple orthogonal matrix (i.e. $\alpha_G = 0$). In a first step we neglect the decay widths in Eq.(10)-(13). In the basis of the physical states K_L and K_S, the hamiltonian of Eq.(13) becomes

$$H = \begin{pmatrix} p + \frac{m_L^2}{2p} & 0 \\ 0 & p + \frac{m_S^2}{2p} \end{pmatrix} \tag{14}$$

[4] This is in contrast to the VEP mechanism for neutrinos [7, 15, 16] where charge conjugation plays no role. Relative phases α between neutrino flavour eigenstates can be absorbed in at least one sector, e.g. the weak sector [16].

with $(m_L + m_S)/2 = \bar{m} + \frac{p}{m}\bar{G}$ and

$$m_L - m_S = \left[(\delta m)^2 + \left(2\phi\delta g \frac{p}{\bar{m}}(p + \frac{\bar{m}^2}{2p})\right)^2 - 4\delta m\phi\delta g \frac{p}{\bar{m}}(p + \frac{\bar{m}^2}{2p})\cos(\frac{\pi}{2} - 2\theta_G)\right]^{1/2} \quad (15)$$

where m_L and m_S are the experimentally measured masses of K_L and K_S respectively. From this expression it is clear that the mass difference $m_L - m_S$ is energy dependent. (The possibility of energy dependence of the various parameters in the Kaon system has been previously considered in different contexts [3,9-13]).

From Eq.(13) we can define the amount of CPT violation induced by VEP as follows

$$\Delta_{CPT} = M_+ - M_- = -\cos(2\theta_G)2\phi\delta g \frac{p}{\bar{m}}(p + \frac{\bar{m}^2}{2p}) \quad (16)$$

Recent studies of VEP in the Kaon system [11]-[12] assumed CPT violation in the gravitational sector, from which it was argued that empirical bounds can be placed on the difference between the gravitational couplings to $|K^0>$ and $|\bar{K}^0>$. The difference in gravitational eigenvalues then corresponds to a difference (ΔM_g) in the masses of $|K^0>$ and $|\bar{K}^0>$:

$$|M_+ - M_-| = \phi\Delta M_g = 2\phi|\delta g|\frac{p}{\bar{m}}(p + \frac{\bar{m}^2}{2p}) \quad (17)$$

and is entirely attributable to the amount of CPT violation. The first equality in Eq.(17) was given by Kenyon [11] and the second by Hughes [12], who specified the energy dependence of ΔM_g. From the experimental upper bound on $M_+ - M_-$ [17] the bound $|\ \delta g\ | < 2.5 \times 10^{-18}$ may be obtained, where the potential ϕ is taken to be that due to the local supercluster ($\phi \simeq 3 \times 10^{-5}$) and $p \simeq 100\ GeV$ [17]. In this approach CPT conservation implies no gravitational mass difference and hence no VEP. However it is clear from the expression (16) for Δ_{CPT} that the bound obtained on ΔM_g is actually on some combination of VEP parameters and not on δg and $\cos(2\theta_G)$ separately. When $\theta_G = 0$, Eq.(17) agrees with Eq.(16). More recent experiments [18] find $|M_+ - M_-|/m_K < 9 \times 10^{-19}$, yielding the bound $|\ \delta g\ | < 3.8 \times 10^{-19}$ for the same values of ϕ and p.

In the case of VLI with $\theta_v = 0$, the amount of CPT violation associated with VLI is given by,

$$|M_+ - M_-| = |\delta v|\frac{p^2}{\bar{m}} \quad (18)$$

The same experimental results can be used to constrain the VLI parameter: $|\delta v| < 2.3 \times 10^{-23}$. To leading order this has exactly the same energy dependence as the VEP mechanism.

Next we shall consider a scenario in which CPT is conserved, so that $\Delta_{CPT} = 0$. From the above it is clear that, even if CPT is conserved, there is still a VEP-induced difference between the masses of the physical states. As a result bounds can be placed on the VEP parameter $\phi\delta g$ without the assumption that locality in quantum field theory is violated.

From the expression of Δ_{CPT} it is clear that it is possible to conserve CPT for all momentum taking $\theta_G = \frac{\pi}{4}$ (modulo $\frac{\pi}{2}$). In this case the mass difference is

$$m_L - m_S = \delta m - 2\phi\delta g \frac{p}{\bar{m}}(p + \frac{\bar{m}^2}{2p}) \quad (19)$$

which as noted above is energy dependent and to the leading order similar to the VLI expression in the case $\theta_v = \pi/4$: $m_L - m_S = \delta m + \delta v p^2/m$. It is possible to put a bound on the VEP parameter δg if we know the value of ϕ and the mass difference at various given energies. Alternatively, if mass measurements at two different energies were different, the differing values for $m_L - m_S$ could be used to extract a value for the VEP parameter δg.

We now proceed to find out constraints on the parameters δm and δg (or δv). In the review of particle properties [18] six experiments were taken into account. Two of them [19, 20] are with the kaon momentum p_K between 20 GeV and 160 GeV. The weighted average of these two experiments is [20]: $\Delta m_{LS} = m_L - m_S = (0.5282 \pm 0.0030)10^{10}\hbar s^{-1}$. The four other experiments [21, 22, 23, 24] are at lower energy, with $p_K \approx 5$ GeV, or less with a weighted average $\Delta m_{LS} = (0.5322 \pm 0.0018)10^{10}\hbar s^{-1}$. A fit of equation (19) with the high and low energy value of Δm_{LS} gives : $\delta m = (3.503 \pm 0.012) \times 10^{-12} MeV$ and $\phi\delta g = (8.0 \pm 7.0) \times 10^{-22} \times \left(\frac{90}{E_{av}}\right)^2$, (where E_{av} is the average energy for the high energy experiment). All these bounds on the VEP parameter $\phi\delta g$ are also bounds of the VLI parameter $-\delta v/2$ with the same energy dependence. We shall not explicitly present the VLI bounds.

Taking ϕ to be the earth's potential ($\phi \simeq 0.69 \times 10^{-9}$), we find $\delta g = (1.2 \pm 1.0) \times 10^{-12}$ whereas if ϕ is due to the local supercluster then $\delta g = (2.7 \pm 2.3) \times 10^{-17}$. These values differ from zero by 1.15 standard deviations. A precise fit of mass difference per energy bin in present and future high energy experiments would be extremely useful in constraining the energy dependent VEP or VLI parameters. Improvement on the low energy experiments can also change the bounds. One of the low energy experiments published last year found $\Delta m_{LS} = (0.5274 \pm 0.0029 \pm 0.0005) \times 10^{10}\hbar s^{-1}$ [24]; when fitted with the high energy experiments, a value of δg consistent with 0 at less than 1 standard deviation is obtained. On the other hand, without this new experiment, a similar fit of the other five experiments yields $\phi\delta g = (1.38 \pm 0.77) \times 10^{-21}(90/E_{av})^2$. In this case δg is different from 0 by 1.8 standard deviations.

In the above analysis we have not included the effect of the absorptive part of the Hamiltonian, i.e of the decay widths in Eqs.(10-13). Including them we now obtain

$$m_L - m_S = \frac{1}{\sqrt{2}}\left[\sqrt{F^2 + G^2} + F\right]^{1/2} \tag{20}$$

$$\Gamma_L - \Gamma_S = \sqrt{2}\left[\sqrt{F^2 + G^2} - F\right]^{1/2} \tag{21}$$

$$F = (\delta m)^2 + (2\phi\delta g \frac{p}{\bar{m}}(p + \frac{\bar{m}^2}{2p}))^2 - 4\delta m \phi \delta g \frac{p}{\bar{m}}(p + \frac{\bar{m}^2}{2p}) \cos(\frac{\pi}{2} - 2\theta_G) - (\frac{\delta\Gamma}{2})^2$$

$$G = -(\delta m \delta\Gamma) + 2\cos(\frac{\pi}{2} - 2\theta_G)[\delta\Gamma\phi\delta g \frac{p}{\bar{m}}(p + \frac{\bar{m}^2}{2p})]$$

In deriving these equations we neglected terms in $\delta m\Gamma$, $\delta m\delta\Gamma$ and Γ^2 with respect to the terms in $m\delta m$ or $m\delta\Gamma$. It can be shown that in the CPT-conserving case the mass difference given in Eq. (20) reduces to Eq. (19). So in the $CPT-$conserving case the results above are not affected by inclusion of the widths. In this case the difference $\Gamma_S - \Gamma_L = \delta\Gamma$ is independent of energy. This is consistent with experiment, which indicates that the low

and high energy measurements of $\Gamma_S - \Gamma_L$ are fully compatible [18]. For $\theta_G \neq \pi/4$, an examination of (21) indicates that $\Gamma_S - \Gamma_L$ is energy dependent; however this is small and measurements of $\Gamma_S - \Gamma_L$ do not constrain δg more than measurements of Δm_{LS} even though they are relatively more precise. We note that measurements of $\Gamma_S - \Gamma_L$ would more strongly constrain a possible absorptive part coming from the gravitational sector which presumably would induce a larger energy dependence. We shall not consider this possibility here. For $\theta_G \neq \pi/4$, width effects in Eq.(20) are small and (15) remains valid to within a few percent.

In Fig.1, for completeness, we plot as a function of $\cos(2\theta_G)$ the upper bounds we get on $|\phi\delta g|$ by fixing δm to the central value of the world average [18], $\Delta m_{LS} = (0.5310 \pm 0.0019) \times 10^{10}\hbar s^{-1}$ and requiring that $m_L - m_S$ in (15) does not differ from δm by more that ± 2 standard deviations. Note that in the case of maximal CPT violation ($\theta_G = 0$), $m_L - m_S$ can only increase with energy, as is clear from Eq.(15) or Eq. (20). The actual difference between low and high energy experiments, if valid, could not be explained in this case except for complex values of δg (and similarly for values of θ_G very close to 0).

In Fig.1 we also show the bound coming from Eq.(16) requiring that at high experimental energy ($p \simeq 100GeV$) the experimental upper bound $|M_+ - M_-|/m_K < 9 \times 10^{-19}$ [18] on CPT violation is satisfied.

5 Testing the equivalence principle from CP violation experiments.

Now we consider the effect of the CP-violating phase α_G. Let us note first that in the maximally $CPT-$violating case, $\theta_G = 0$, there is no effect of the phase α_G in the mass matrix Eq.(13) and consequently no $CP-$violating effect coming from this phase (similarly in the VLI case when matter and antimatter states are the velocity eigenstates). In the following we will restrict ourself to the most interesting $CPT-$conserving case with $\theta_G = \pi/4$. The total Hamiltonian can then be diagonalised

$$H = \begin{pmatrix} p + \frac{1}{2p}(m_L - i\frac{\Gamma_L}{2})^2 & 0 \\ 0 & p + \frac{1}{2p}(m_S - i\frac{\Gamma_S}{2})^2 \end{pmatrix} \tag{22}$$

with the physical eigenstates K_L and K_S being given by

$$K_{\substack{L \\ S}} = \frac{1}{\sqrt{2(1+|\varepsilon|^2)}} \left[(1+\varepsilon)K^0 \mp (1-\varepsilon)\bar{K}^0 \right] \tag{23}$$

Defining

$$\begin{aligned} G_c &= \frac{p}{\bar{m}}\delta G \cos 2\alpha_G = -2g_c(\frac{p^2}{\bar{m}} + \frac{\bar{m}}{2}); \qquad g_c = \phi\delta g \cos 2\alpha_G \\ G_s &= \frac{p}{\bar{m}}\delta G \sin 2\alpha_G = -2g_s(\frac{p^2}{\bar{m}} + \frac{\bar{m}}{2}); \qquad g_s = \phi\delta g \sin 2\alpha_G \end{aligned}$$

the new CP-violating parameter ε is now defined in terms of the $CP-$violating parameters $\tilde{\varepsilon}$ and G_s via

$$\varepsilon = \frac{\tilde{\varepsilon}(\delta m - i\frac{\delta\Gamma}{2}) - \frac{i}{2}G_s}{(\delta m + G_c) - i\frac{\delta\Gamma}{2}} \tag{24}$$

to first order in $\tilde{\varepsilon}$ and G_s. Similarly we have to first order in $\tilde{\varepsilon}$ and G_s

$$\Delta m \equiv m_L - m_S = (\delta m + G_c) \quad (25)$$

$$\Delta\Gamma \equiv \Gamma_L - \Gamma_S = \delta\Gamma. \quad (26)$$

Since in the mechanism considered here there is no ε' type CP violation coming from VEP we will neglect other possible ε' effects and the relevant CP-violating quantities are

$$\delta = \frac{\Gamma(K_L \to \pi^- l^+ \nu) - \Gamma(K_L \to \pi^+ l^- \nu)}{\Gamma(K_L \to \pi^- l^+ \nu) + \Gamma(K_L \to \pi^+ l^- \nu)} = 2\mathrm{Re}\varepsilon$$

$$\eta_{+-} = \frac{A(K_L \to \pi^+\pi^-)}{A(K_S \to \pi^+\pi^-)} = |\eta_{+-}|e^{i\phi_{+-}} = \varepsilon$$

$$\eta_{00} = \frac{A(K_L \to \pi^0\pi^0)}{A(K_S \to \pi^0\pi^0)} = |\eta_{00}|e^{i\phi_{00}} = \varepsilon$$

.

Consider first the case $\tilde{\varepsilon} = 0$, *i.e.* there is no CP violation induced from the weak interaction. Can we interpret the observed CP violation parameters above as originating purely due to the relative phase α_G? In other words does the superweak mechanism have a gravitational origin? Eq. (24), with $\tilde{\varepsilon} = 0$, can be written as

$$\varepsilon = -\frac{i}{2}\frac{G_s}{(\Delta m - i\frac{\Delta\Gamma}{2})}. \quad (27)$$

Equating the real and the imaginary parts we get

$$\mathrm{Re}\varepsilon = \frac{G_s}{2}\frac{\Delta\Gamma}{2}[(\Delta m)^2 + (\frac{\Delta\Gamma}{2})^2]^{-1} \quad ; \quad \mathrm{Im}\varepsilon = -\frac{G_s}{2}\Delta m[(\Delta m)^2 + (\frac{\Delta\Gamma}{2})^2]^{-1}.$$

The above equations reproduce the results of the superweak theory: $\phi_{+-} = \phi_{00} = -2\Delta m/\Delta\Gamma \simeq 43.5^0$ and also consequently $|\eta_{+-}| = |\eta_{00}| \simeq \frac{\delta}{\sqrt{2}}$. The fact that the superweak phase is obtained is due to the fact that the VEP mechanism considered here respects the hermiticity of the interaction between the $CP-$eigenstates K_1 and K_2 (i.e. the numerator of Eq.(24) is purely imaginary [5]). Interestingly, we see that by assuming the gravitational, weak and mass eigenstates are all related by unitary transformations, the physical states are still of the superweak type; in particular they are no longer related to the other states by a unitary transformation and are no longer orthogonal.

Taking the experimental value $\delta = (0.327 \pm 0.012)\%$ [18] as input we obtain $G_s \simeq -2 \times 10^{-14}$. This value of G_s yields a consistent fit to all the CP violation parameters above, as with any superweak mechanism. However this does not provide us with positive evidence

[5]Note that insofar as there is no gravitational coupling to the imaginary part of the energy (as we have assumed here), a CP violation of the Reε type coming from gravitation (which would render U_G nonunitary) does not respect the hermiticity of the K_1-K_2 interaction and is consequently experimentally suppressed. We do not consider the possibility of such an effect here.

for VEP-induced CP violation because $|\eta_{+-}|$ and $|\eta_{00}|$ have been observed experimentally with good accuracy to be constant over a large energy range [18]. Hence is not possible to reproduce the data for all energies with $\tilde{\varepsilon} = 0$ since G_s is proportional to p^2. An observed energy dependence in these parameters that is consistent with (24) would be a definitive signature of a VEP-mechanism operative in this sector.

We now demonstrate how a bound on the VEP parameter $\phi\delta g$ can be obtained independently of the phase α_G. We can extract a bound on g_c from the experimental constraint on the energy dependence of Δm, Eq. (25) in the same way that for $\phi\delta g$ above in Eq.(19) substituting δg by $\delta g \cos 2\alpha_G$. The bound obtained on $\phi\delta g$ in Fig.1 is now a bound on g_c:

$$|g_c| < 9 \times 10^{-22}. \tag{28}$$

From CP violation parameter data we can also obtain a bound on g_s. Defining

$$A = \frac{G_c}{\delta m} \qquad B = \frac{\mathrm{Re}\varepsilon}{\mathrm{Im}\varepsilon} - 1 \qquad C = -\frac{\delta\Gamma}{2\delta m} - 1$$

where the magnitudes of A, B and C are all much smaller than unity, implies from (24)

$$\mathrm{Re}\varepsilon = \mathrm{Re}\tilde{\varepsilon}(1 - A) + \frac{1}{2} G_s \frac{\frac{\delta\Gamma}{2}}{(\delta m)^2 + (\frac{\delta\Gamma}{2})^2} \tag{29}$$

$$\mathrm{Im}\varepsilon = \mathrm{Im}\tilde{\varepsilon} - \frac{1}{2} G_s \frac{\delta m}{(\delta m)^2 + (\frac{\delta\Gamma}{2})^2} \tag{30}$$

where only terms linear in A, B and C have been retained. We observe that g_s changes the value of $|\varepsilon|$ but not the phase of ε. In addition, $\mathrm{Im}\varepsilon$ depends on g_s but not on g_c. From the experimental values of $\phi_{+-} = (43.7 \pm 0.6)^\circ$ and $|\eta_{+-}| = (2.284 \pm 0.018)10^{-3}$ [18] we obtain $\mathrm{Im}\varepsilon = (1.58 \pm 0.02).10^{-3}$. Fixing $\mathrm{Im}\tilde{\varepsilon}$ to the central value of $\mathrm{Im}\varepsilon$ and requiring that $\mathrm{Im}\varepsilon$ in Eq.(30) differs from the central value by no more than 2 standard deviations at high experimental energies ($p \simeq 70$ GeV[18]) yields

$$|g_s| < 3 \times 10^{-23}$$

This value hardly varies when we calculate $\mathrm{Im}\varepsilon$ from the experimental value of $|\eta_{00}|$ and ϕ_{00} instead of $|\eta_{+-}|$ and ϕ_{+-}. From the bounds on g_c and g_s we then get

$$|\phi\delta g| < 9 \times 10^{-22}.$$

g_c can also be bounded from CP violation. Indeed the CP-conserving parameter g_c is present in $\mathrm{Re}\varepsilon$ and a bound on it can be obtained by considering the phase of ε

$$\tan\phi_{+-} = \frac{\mathrm{Im}\varepsilon}{\mathrm{Re}\varepsilon} = \frac{\mathrm{Im}\tilde{\varepsilon}}{\mathrm{Re}\tilde{\varepsilon}}(1 + A) \tag{31}$$

which doesn't depend on g_s. Taking the low energy value of ϕ_{+-} to equal its central value above we similarly obtain (requiring that ϕ_{+-} not differ at high energy by more than two standard deviations from its low energy value):

$$|g_c| < 7 \times 10^{-21}.$$

This value is of the same order of magnitude as the upper bound obtained above by looking for energy dependence in Δm. CP violation measurements consequently are a useful means for searching both for $CP-$conserving VEP effects (through the parameter g_c) and $CP-$violating VEP effects (through g_s).

We shall not consider the case where CPT is violated ($\theta_G \neq \pi/4$) with $\alpha_G \neq 0$. Equations similar to Eqs.(24)-(26) can be straightforwardly obtained from Eq.(13) but they are lengthy and do not provide any new interesting physical results which have not already been discussed above.

In ref.[13], the effect of a tensorial field $f_{ij}^{\mu\nu}$ whose $CPT-$violating interactions with the kaons is given by the lagrangian L=$f_{ij}^{\mu\nu} d_\mu \phi_i d_\nu \phi_j$ has been considered with $\phi_{1(2)}$ the CP-eigenstates and $f_{11}^{\mu\nu}$, $f_{22}^{\mu\nu}$, $f_{12}^{\mu\nu} = f_{21}^{\mu\nu}$ real parameters. Writing $f_{ii}^{\mu\nu}$ as $\frac{f_i}{f}\phi\eta^{\mu\nu}$ and $f_{12}^{\mu\nu}$ as $\frac{f_T}{f}\phi\eta^{\mu\nu}$, constraints on $(f_1 - f_2)/f$ and f_T/f have been obtained from experimental energy constraints on the energy dependence of Δm_{LS} and η_{+-} respectively. We observe that, except for a different experimental situation, the constraints obtained on $|(f_1 - f_2)/f|$ and $|f_t/f|$ are similar to ours on the corresponding quantities, $|4\delta g \cos 2\alpha_G|$ and $|\delta g \sin 2\alpha_G|$ respectively with $\theta_G = \pi/4$. From our treatment, we see consequently that provided the tensorial interaction is of gravitational origin the bounds on the parameters $|(f_1 - f_2)/f|$ and $|f_t/f|$ are in fact bounds on a combination of the VEP parameters, the difference δg in the gravitational couplings and the phase α_G.

We close this section by noting that the bounds on $|g_c|$, $|g_s|$ and $|\phi\delta g|$ above are also bounds on the corresponding VLI parameters $|\frac{\delta v}{2}\cos 2\alpha_v|$, $|\frac{\delta v}{2}\sin 2\alpha_v|$ and $|\frac{\delta v}{2}|$ respectively.

6 Summary and Conclusion.

The Kaon system provides us with an interesting physical situation in which we can empirically check the validity of special relativity and/or the equivalence principle in a matter/antimatter sector of the standard model that includes 2nd generation matter. A variety of interesting combinations of VLI/VEP effects exist which can be associated with CP violation and/or CPT violation. Violations of the equivalence principle in the Kaon system need not violate CPT (which in turn implies a loss of locality in quantum field theory) as considered in recent studies.

A general feature of the VLI/VEP mechanisms is that they predict an energy dependence in $m_L - m_S$ and in the CP violation parameters which can be empirically tested to obtain bounds on the relevant parameters (such as $\phi\delta g$ for VEP or δv for VLI). Since both VEP and VLI have the same energy dependence, although we can obtain bounds for both, it will not be possible to experimentally distinguish between the two mechanisms. Under the assumption that all such parameters are within two standard deviations over the energy scales at which they have been measured, present experiments provide rather stringent bounds on the VEP (or VLI) mechanism. A more systematic search for energy dependence in $m_L - m_S$ and in the CP-violating parameters (such as ε) will provide us with more definitive information about the VEP (or VLI) mechanism in this sector. In addition in our formalism, we observe that the CPT-conserving case [13] and the CPT-violating case of recent VEP studies [11, 12] are special cases of the same general mechanism.

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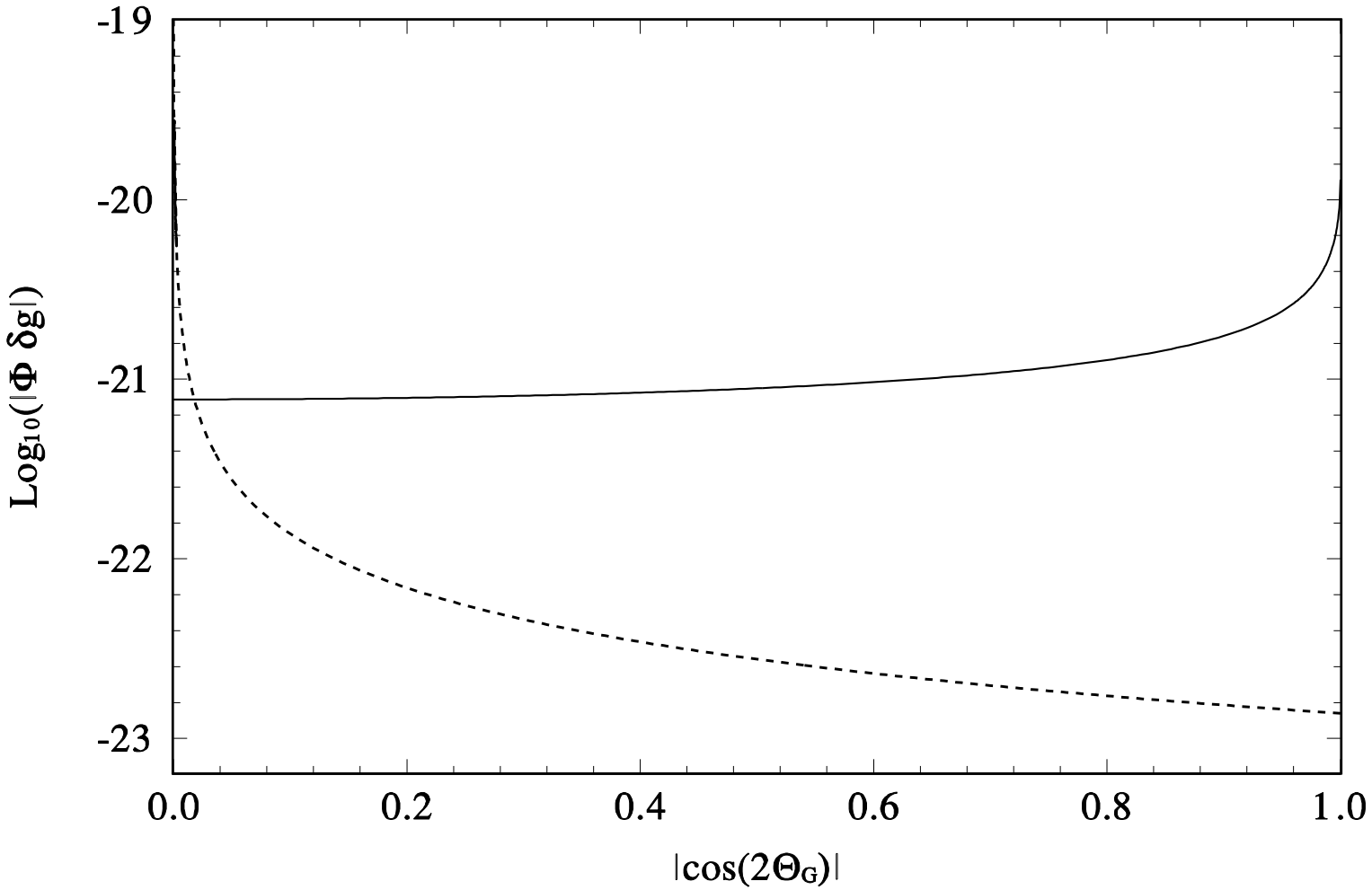
Fig.1: Upper bounds on $|\phi\delta g|$ (for p $\simeq 90GeV$) obtained from Eq.(15) (solid line), Eq.(16)(dashed line) as explained in the text.


-19
-20
-21
-22
-23
0.0
0.2
0.4
0.6
0.8
1.0
Log10(|Φ δg|)
|cos(2ΘG)|