Test of Special Relativity from K Physics

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Abstract

A breakdown of the Local Lorentz Invariance and hence the special theory of relativity in the Kaon system can, in principle, induce oscillations between the K^0 and $\bar{K}^0$ states. We construct a general formulation in which simultaneous pairwise diagonalization of mass, momentum and weak eigenstates is not assumed and the maximum attainable speeds of the momentum eigenstates are different. This mechanism permits Local Lorentz Invariance violation in a manner that may or may not violate CPT. In the $CPT-$conserving case, we show that violation of special relativity could be clearly tested experimentally via the energy dependence of the $K_L - K_S$ mass difference and we discuss constraints imposed by present experiments. In the CPT-violating case the K^0–$\bar{K}^0$ system also allows the possiblity of testing different Lorentz properties of matter and antimatter.

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The special theory of relativity has been tested to a high degree of precision from various types of experiments [1]. These experiments probe for any dependence of the (non-gravitational) laws of physics on a laboratory's position, orientation or velocity relative to some preferred frame of reference, such as the frame in which the cosmic microwave background is isotropic. Such a dependence would constitute a direct violation of (respectively) Local Position Invariance and Local Lorentz Invariance (LLI), and hence of the Equivalence Principle [2].

A characteristic feature of LLI-violation is that every species of matter has its own maximum attainable speed. This yields several novel effects in various sectors of the standard model [3], including vacuum Cerenkov radiation [4], photon decay [5] and neutrino oscillations [5, 6]. In case of neutrino oscillations constraints on the violation of the equivalence principle [4, 7] can be directly translated into constraints on the violation of LLI [6]. Here we extend these arguments to the particle/antiparticle sector. Specifically, we consider the K^0–$\bar{K}^0$ system and point out that a violation of special relativity here will in general induce an energy dependent $K_L - K_S$ mass difference; an empirical search for such effects can therefore be used to obtain bounds on the violation of LLI in the Kaon sector of the standard model. The approach we will follow to this problem in the Kaon system is phenomenological: we shall assume that the mass or the weak eigenstates are not *a-priori* simultaneously diagonalisable with the momemtum eigenstates and that the maximum attainable velocities of the different momentum eigenstates are different.

The present study of violation of LLI in the Kaon system, in addition to previous studies in other sectors, is motivated by the simple fact that there is no logically necessary reason why special relativity must be valid in all sectors of the standard model of elementary particle physics. Rather its validity must be empirically checked for each sector separately [3]. In particular in the CPT-conserved case we will discuss, where the momentum eigenstates of the Kaon system may have differing maximal speeds, the bound obtained is of the same order as the ones obtained in other sectors of the standard model [1, 5]. On the other hand the K^0–$\bar{K}^0$ system is also a matter-antimatter system in which we can test for possible violations of LLI stemming from different maximum attainable speeds of matter and antimatter (which consequently induce CPT violation). Our formalism contains both the CPT-conserved and $CPT-$violating cases.

More explicitly, for relativistic pointlike Kaons the general form of the effective Hamiltonian associated with the Lagrangian in the (K^0 $\bar{K}^0$) basis will be

$$H = U_W H_{SEW} U_W^{-1} + U_v H_v U_v^{-1} \tag{1}$$

with,

$$H_{SEW} = \frac{(M_{SEW})^2}{2p} = \frac{1}{2p}\begin{pmatrix} m_1 & 0 \\ 0 & m_2 \end{pmatrix}^2 \tag{2}$$

and

$$H_v = \begin{pmatrix} v_1 & 0 \\ 0 & v_2 \end{pmatrix} p \tag{3}$$

to leading order in $\bar{m}^2/p^2$ with p the momentum and $\bar{m}$ the average mass, where for a quantity X, $\delta X \equiv (X_1 - X_2)$, $\bar{X} = (X_1 + X_2)/2$. The constants v_1 and v_2 correspond to the maximum attainable speeds of each eigenstate. If special relativity is valid within the Kaon sector these are both equal to their average $\bar{v}$, which we normalize to unity. If $\bar{v}$ is equal to the speed of electromagnetic radiation then special relativity is valid within the Kaon–photon sector of the standard model. Hence $v_1 - v_2 = \delta v$ is a measure of LLI violation in the Kaon sector. H_{SEW} is the matrix coming from the strong, electromagnetic and weak interactions, whose absorptive (*i.e.* antihermitian) parts we shall neglect for the moment. In the limit $v_1 = v_2$, weak interactions are responsible for $m_1 \neq m_2$, which are interpreted as the K_L and K_S masses.

Since H_{SEW} and H_v are hermitian, U_v and U_W are unitary. From the general form of a 2x2 unitary matrix

$$U = e^{i\chi}\begin{pmatrix} e^{-i\alpha} & 0 \\ 0 & e^{i\alpha} \end{pmatrix}\begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}\begin{pmatrix} e^{-i\beta} & 0 \\ 0 & e^{i\beta} \end{pmatrix}$$

it is straightforward to show that

$$H = pI + \frac{1}{2p}\begin{pmatrix} M_+ & M_{12} \\ M_{12}^* & M_- \end{pmatrix}^2$$

where I is the unit matrix and

$$\begin{aligned} M_\pm &= \bar{m} \pm \frac{cos2\theta_W}{2}\delta m \pm \frac{p^2}{\bar{m}}\frac{\cos 2\theta_v}{2}\delta v \\ M_{12} &= -(e^{-2i\alpha_W}\sin 2\theta_W \delta m + e^{-2i\alpha_v}\frac{p^2}{\bar{m}}\sin 2\theta_v \delta v)/2 \end{aligned} \tag{4}$$

where we have absorbed additional phases into the K^0 and $\bar{K}^0$ wavefunctions. Since in this paper we will be considering effects for which CP-violation is negligible, for simplicity we shall take $\alpha_v = \alpha_W = 0$.[4].

In the basis of the physical states K_L and K_S, the Hamiltonian becomes

$$H = \begin{pmatrix} p + \frac{m_L^2}{2p} & 0 \\ 0 & p + \frac{m_S^2}{2p} \end{pmatrix} = \begin{pmatrix} \tilde{E} & 0 \\ 0 & \tilde{E} \end{pmatrix} + \frac{1}{2} \begin{pmatrix} \Delta E & 0 \\ 0 & -\Delta E \end{pmatrix} \tag{5}$$

where $\tilde{E} = (p + \frac{\bar{m}^2}{2p})$, and

$$\frac{p}{\bar{m}}\Delta E = m_L - m_S = \left[(\delta m)^2 + \left(\delta v \frac{p^2}{\bar{m}} \right)^2 + 2\delta m \delta v \frac{p^2}{\bar{m}} \cos(2(\theta_W - \theta_v)) \right]^{1/2} \tag{6}$$

where m_L and m_S are the experimentally measured masses of K_L and K_S respectively. From the above it is clear that the LLI violation implies that the mass difference $m_L - m_S$ is energy dependent. (The possibility of energy dependence of the various parameters in the Kaon system has been previously considered in different contexts [3, 9]).

The amount of CPT–violation is given by

$$\Delta_{CPT} = M_+ - M_- = \cos(2\theta_W)\delta m + \cos(2\theta_v)\delta v \frac{p^2}{\bar{m}} \tag{7}$$

From this expression we see that it is not possible to conserve CPT for all momenta unless $\theta_W = \theta_v = \frac{\pi}{4}$ (modulo $\frac{\pi}{2}$), thereby separately conserving CPT. In the following we will discuss in detail this $CPT-$conserving case because it is the less constrained case and because our results can be compared with earlier ones where CPT conservation has been assumed. We also consider briefly the interesting maximal $CPT-$violating case where $\theta_W = \pi/4$ and $\theta_v = 0$.

In the $CPT-$conserving case the mass difference is

$$m_L - m_S = \delta m + \delta v \frac{p^2}{\bar{m}} \tag{8}$$

[4]A more general analysis containing $CP-$violating effects will be presented elsewhere [8].

which as noted above is energy dependent. What constraints do present experiments place on δm and δv? In the review of particle properties [10] six experiments were taken into account. Two of them are at high energy [11, 12] with the Kaon momentum p_K between 20 GeV and 160 GeV. The weighted average of these two experiments is [12]: $\Delta m_{LS} = m_L - m_S = (0.5282 \pm 0.0030) \times 10^{10} \hbar s^{-1}$. The four other experiments [13, 14, 15, 16] are at lower energy, with $p_K \approx 5$ GeV, or less. The weighted average of these low energy experiments is $\Delta m_{LS} = (0.5322 \pm 0.0018) \times 10^{10} \hbar s^{-1}$. A fit of equation (8) with the high and low energy value of Δm_{LS} gives : $\delta m = (3.503 \pm 0.012) \times 10^{-12} MeV$ and $\delta v = -(1.6 \pm 1.4) \times 10^{-21} \times \left(\frac{90}{E_{av}}\right)^2$, (where E_{av} is the average energy for the high energy experiment which we take to be 90 GeV). We obtain consequently:

$$|\delta v| \leq 3 \times 10^{-21}.$$

The fitted value above differs from zero by 1.15 standard deviations. While it is certainly premature to regard this as evidence for LLI violation, these values do show that it is possible to test the special theory of relativity in the Kaon sector. A precise fit of mass difference per energy bin in present and future high energy experiments would be extremely useful in constraining the violation of Lorentz invariance parameter δv, particularly since the present experimental situation at low energy is not clear. Indeed one of the low energy experiments [16] published last year found $\Delta m_{LS} = (0.5274 \pm 0.0029 \pm 0.0005) \times 10^{10} \hbar s^{-1}$, a value lower than the weighted average $\Delta m_{LS} = (0.5350 \pm 0.0023) \times 10^{10} \hbar s^{-1}$ of the three (previous) low energy experiments. Without this new experiment, a similar fit of the other five experiments yields $\delta v = -(2.76 \pm 1.54) \times 10^{-21} (90/E_{av})^2$. In this case δv is different from 0 by 1.8 standard deviations. Alternatively taking only the new experiment [16] at low energy we would obtain a value compatible with 0 at less than 1 standard deviation.

We now briefly discuss the case where CPT is conserved in the strong and the electromagnetic sectors ($\theta_W = \pi/4$) but maximally violated by the momentum eigenstates ($\theta_v = 0$). A bound on δv can be obtained from Eq.(7) (with $\theta_W = \pi/4$ and $\theta_v = 0$) and $|M_+ - M_-|/m_K < 9 \times 10^{-19}$ [10] with $p \simeq 100$ GeV:

$$|\delta v| \leq 2.3 \times 10^{-23}.$$

The upper bound we obtain by looking at the energy dependence of $m_L - m_S$, through Eq.(6) as in the $CPT-$conserving case, is relatively less stringent than the one from Eq.(7) by 2 to 3 orders of magnitude.

In the above analysis we have not included the effect of the absorptive part of the Hamiltonian. Inclusion of the absorbtive part entails the replacement of m_i by $m_i - i\Gamma_i/2$. With this change the definitions of $\tilde{E}$ and ΔE are modified to

$$
\begin{aligned}
\tilde{E} &= \left(p + \frac{(\bar{m} - i\bar{\Gamma}/2)^2}{2p}\right) \\
\frac{p}{\bar{m}}\Delta E &= \frac{1}{\sqrt{2}}\left[\sqrt{F^2+G^2}+F\right]^{1/2} + i\frac{1}{\sqrt{2}}\left[\sqrt{F^2+G^2}-F\right]^{1/2} \\
F &= (\delta m)^2 + (\delta v\frac{p^2}{\bar{m}})^2 + 2\delta m\delta v\frac{p^2}{\bar{m}}\cos(2\theta_W - 2\theta_v) - (\frac{\delta\Gamma}{2})^2 \\
G &= -(\delta m\delta\Gamma) - \cos(2\theta_W - 2\theta_v)[\delta\Gamma\delta v\frac{p^2}{\bar{m}}] \qquad (9)
\end{aligned}
$$

We also have,

$$
m_L - m_S = \frac{p}{\bar{m}}\mathrm{Re}(\Delta E) \qquad (10)
$$

$$
\Gamma_S - \Gamma_L = 2\frac{p}{\bar{m}}\mathrm{Im}(\Delta E) \qquad (11)
$$

In deriving these equations we neglected terms in $\delta m\Gamma$, $\delta m\delta\Gamma$ and Γ^2 with respect to the terms in $m\delta m$ or $m\delta\Gamma$. It can be shown that in the $CPT-$conserving case the above mass difference (equation (10)) reduces to equation (8). So in our present analysis the results above are not affected by inclusion of the widths. In this case the difference $\Gamma_S - \Gamma_L = -\delta\Gamma$ is independent of energy. This is consistent with experiment, which indicates that the low and high energy measurements of $\Gamma_S - \Gamma_L$ are fully compatible [10].

To summarize, in constructing our formalism to test the violation of Local Lorentz Invariance in the Kaon sector we have taken a phenomenological approach, making the general hypothesis that momentum eigenstates can be a priori any orthogonal states in the $K^0 - \bar{K}^0$ system, and that these eigenstates have differing momentum eigenvalues. This mechanism can be tested experimentally by searching for an energy dependence in $m_L - m_S$, yielding a stringent bound on LLI violation in this sector. Previous bounds on LLI violation [1, 5, 6] are comparable to the bound obtained from the $K^0 - \bar{K}^0$ system, but occur in different sectors of the standard model. More precise and detailed tests in the Kaon system should provide us with important empirical information on the validity of the special theory of relativity.

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