A Model for the Three Lepton Decay Mode of the Proton

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Abstract

An extension of the left–right symmetric model has been constructed which gives in a natural way the three lepton decay modes of the proton which have been suggested as an explanation for the atmospheric neutrino anomaly. We write down the potential which after minimization gives the proper choice of the Higgs spectrum. With

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this Higgs spectrum we then study the evolution of the gauge coupling constants and point out that for consistency one has to include effects of gravity.

1 Introduction

One expects to see produced in the atmosphere twice as many muon neutrinos as electron neutrinos since detectors cannot distinguish between neutrinos and antineutrinos. The two water–Cerenkov detectors give a result which is a factor of two smaller for the ratio $R = N(\nu_\mu)/N(\nu_e)$, a ratio in which many systematic uncertainties are expected to cancel. The results are

$$R_{obs}/R_{MC} = 0.60 \pm 0.07 \pm 0.05$$

from the Kamiokande experiment [1] (based on 6.1 Kton year) and

$$R_{obs}/R_{MC} = 0.54 \pm 0.05 \pm 0.12$$

from IMB [2](based on 7.7 Kton Year). The experiments look for "contained" single prong events which are caused by neutrinos with energies below 2 GeV. The ratio R is estimated from the relative rates of sharply defined single rings (muon–like) and diffused single rings (electron–like). (Other reported values [3] for this ratio are: Frejus [4] 0.87 ± 0.21 (1.56 Kton Year), NUSEX [5] 0.99 ± 0.40 (0.4 Kton year), SOUDAN II [6] 0.69 ± 0.19 (1 Kton year)).

Although this atmospheric neutrino anomaly has a popular explanation within the neutrino oscillation framework [7], there is an alternative explanation based on three lepton decays of the proton [8]. The single ring events have been analysed within the proton decay interpretation where it is argued that, if the proton decays into a positron and two neutrinos with a lifetime of $\tau(P \to e^+\nu\nu) \sim 4 \times 10^{31}$ years, then the excess observed electron events could be due to proton decay events [8]. The lifetime for this particular decay mode of the proton [9] is consistent with the present limit [10] for the expected dominant decay mode of the proton $\tau(P \to e^+\pi^\circ) > 5 \times 10^{32}$ yr. The possibility that this particular decay mode might dominate over other usual decay modes was considered earlier on general grounds [11, 12, 13] where it

was pointed out that it is difficult to have light neutrinos in the final decay product.

In most models the left–handed neutrinos are light and the right handed–neutrinos are heavy. Thus the decay modes are restricted to

$$P \rightarrow e^+ \nu_L \nu_L \quad \text{or} \quad P \rightarrow e^+ \nu_L{}^c \nu_L \quad \text{or} \quad P \rightarrow e^+ \nu_L{}^c \nu_L{}^c. \tag{1}$$

Recently [13] it has been pointed out that these decay modes are allowed in the framework of certain left–right symmetric models. In this paper we construct an explicit model in which three lepton decay mode of the proton is the dominant one. We write down the general form of the potential and show that the minima of the potential are consistent with the choice of Higgs structure. We then study the evolution of the gauge coupling constants including non-renormalizable interactions which may arise from the Planck scale physics [14, 15].

2 A Proton Decay Mechanism

We work in the framework of the left–right symmetric model [11, 12, 16] and start with the symmetry breaking chain

$$SU(4) \times SU(2)_L \times SU(2)_R \, [\equiv G_{PS}]$$

$$\xrightarrow{M_{PS}} \quad SU(3)_c \times SU(2)_L \times SU(2)_R \times U(1)_{(B-L)} \, [\equiv G_{LR}]$$

$$\xrightarrow{M_R} \quad SU(3)_c \times SU(2)_L \times U(1)_Y \, [\equiv G_{std}]$$

$$\xrightarrow{M_W} \quad SU(3)_c \times U(1)_{em}.$$

In the minimal left–right symmetric model [11, 12, 16] the Higgs scalars consist of the following fields. The group G_{PS} is broken by the vacuum expectation value (*vev*) of the field H which transforms as (**15**,**1**,**1**) under the group G_{PS}. The right handed group is broken by the *vev* of a right handed triplet Higgs field $\Delta_R \equiv$ (**1**,**1**,**3**,-2) $\subset$ (**10**,**1**,**3**). By left–right parity this will imply the existence of the left handed triplet field $\Delta_L \equiv$ (**1**,**3**,**1**, -2) $\subset$ (**10**,**3**,**1**), which gives Majorana mass to the left–handed neutrinos and whose *vev* should be ≤ 1 GeV. (Where there are four numbers, the first three correspond to the representations of $SU(3)_c \times SU(2)_L \times SU(2)_R$ while the last shows the $U(1)$ quantum numbers).

Finally the electroweak symmetry breaking takes place through the *vev* of a doublet scalar field $\phi \equiv$ (**1**,**2**,**2**,0) $\subset$ (**1**,**2**,**2**). This field ϕ also gives masses to the fermions. However this does not reproduce the right quark–lepton mass ratios. For the right magnitude of the quark–lepton mass ratios we require yet another field $\xi \equiv$ (**1**,**2**,**2**,0) $\subset$ (**15**,**2**,**2**) [11]. The $SU(3)_c$ singlet component of this field ξ, which acquires *vev*, has different Clebsch–Gordon coefficients for the $SU(3)_c$ and the $U(1)$ part of $SU(4)$. Hence they

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contribute to the quark and lepton masses with different coefficients. As a result, suitable combinations of ϕ and ξ can reproduce the right quark–lepton mass ratios.

It was pointed out in ref [13] that with this minimal scalar content it is possible to get the decay mode required to explain the atmospheric neutrino problem. For this we need the $SU(3)_c$ color triplet components of the fields Δ_L and ξ, which we represent by Δ_L^3 and ξ^3 respectively. (Δ_L is $(\mathbf{10},\mathbf{3},\mathbf{1})$ under G_{PS} and the $(\mathbf{1},\mathbf{3},\mathbf{1},-2)$ component acquires a *vev*; the $\mathbf{10}$ representation of $SU(4)$ decomposes under $SU(3)_c$ as $\mathbf{6} + \mathbf{3} + \mathbf{1}$. Similarly, the $\mathbf{15}$ representation of $SU(4)$ decomposes under $SU(3)_c$ as $\mathbf{8} + \mathbf{3} + \bar{\mathbf{3}} + \mathbf{1}$). Then the Yukawa couplings,

$$\mathcal{L}_{Yuk} = f_{ql}(\overline{q_L^c}l_L)\Delta_L^{3*} + f_{dl}(\overline{d_R^c}l_L{}^c)\xi^{3*} \tag{2}$$

and the quartic scalar coupling,

$$\mathcal{L}_s = \lambda^{pr}\Delta_L^3\Delta_L^3\xi^3\xi^1 \tag{3}$$

give the $(B-L)$ conserving proton decay $P \rightarrow e_L{}^+\nu_L\nu_L{}^c$ through the diagram of figure 1. This diagram will also give, with equal probability, the decay mode, $P \rightarrow \mu_L{}^+\nu_L\nu_L{}^c$.

Such a proton decay mechanism will give equal number of sharp single rings (muon–like events) and diffuse single rings (electron–like events). Since the proton decay events contribute to both electron– and muon–like events this seems to imply that the reduction of the ratio R cannot be explained by proton decay events. However, the weighted average of the two processes with ratios $R(proton\ decay) = 1$ and $R(atmospheric\ neutrino) = 2$ (the theoretical expected ratio for the muon–to–electron events if they have their origin only from the atmospheric neutrinos) can in fact explain the atmospheric neutrino anomaly.

To see this we note the observed numbers of electron–like [muon–like] events $n_e(obs)$ [$n_\mu(obs)$] are the sum of the numbers of electrons [muons] pro-

duced by the atmospheric electron [muon] neutrinos ν_e $[\nu_\mu]$ through scattering inside the detector $n_e(atm)$ $[n_\mu(atm)]$ and from the decays of the protons into $e^+\nu_L\nu_L{}^c$ $[\mu^+\nu_L\nu_L{}^c]$ inside the detectors $n_e(prot)$ $[n_\mu(prot)]$. That is,

$$R_{obs} = \frac{n_\mu(obs)}{n_e(obs)} = \frac{n_\mu(atm) + n_\mu(prot)}{n_e(atm) + n_e(prot)} \sim 0.6 R_{MC} \sim 1.2.$$

In ref [8] it was assumed that the proton decays into $e^+\nu_L\nu_L{}^c$ (and not muons), *i.e.*, $n_\mu(prot) = 0$. They found that, by doing a Monte Carlo simulation to obtain the proton lifetime, the atmospheric neutrino anomaly could be achieved with a proton lifetime of $\tau_p \sim 4 \times 10^{31}$ years. In the above relation this corresponds to $n_e(prot) \sim (2/3)n_e(atm) \sim (1/3)n_\mu(atm)$.

In the present scenario the proton decays into both electrons and muons so that $n_e(prot) \sim n_\mu(prot)$. Thus for the explanation of the atmospheric neutrino anomaly we require $n_e(prot) \sim 4n_e(atm) \sim 2n_\mu(atm)$. Since the number of proton decays is increased to give the same R_{obs} there will be a reduction in the proton lifetime by a factor of 6. Thus in this scenario we can explain the atmospheric neutrino anomaly with a proton lifetime $\tau_p \sim (2/3) \times 10^{31}$ years, which is still consistent with present experiments on proton decay.

The amplitude for the process is given by

$$\mathcal{A} = \frac{\lambda^{pr} f_{ql}^2 f_{dl} \langle \xi^1 \rangle}{m_{\xi^3}^2 m_{\Delta^3}^4}. \tag{4}$$

where, $\langle \xi^1 \rangle = \langle \phi \rangle = 250$ GeV, λ^{pr} is the strength of the quartic coupling defined in Eq. (3) and the f_{ql}, f_{dl} are the Yukawa coupling constants.

Then, taking reasonable values for the quartic and the quadratic Yukawa coupling parameters, $\lambda^{pr} \sim 10^{-2}$ and $f \sim 10^{-3}$, say, requires m_{ξ^3} and m_{Δ^3} to be relatively light. For the proton decay mode $P \rightarrow e_L{}^+\nu_L\nu_L{}^c$ to be 10^{31} years to explain the atmospheric neutrino anomaly, it has been argued [11, 12] that the mass m_{ξ^3} can be as light as about a TeV, which requires $m_{\Delta^3} \sim$ few TeV. This can also be achieved naturally [13].

3   A General Left–Right Symmetric Potential

We now concentrate on the masses m_{ξ^3} and m_{Δ^3}. In earlier references [11, 12, 13] two new mechanisms were proposed which could give rise to appropriate masses for these fields. Here we check the consistency of these two different mechanisms when the complete potential with all the scalar fields is written and minimized. First we describe the two mechanisms which keep these two fields m_{ξ^3} and m_{Δ^3} light.

For light m_{ξ^3} it was argued [11, 12] that if there exists a field $\xi' \equiv (\mathbf{15}, \mathbf{2}, \mathbf{2})$ which can mix with the field ξ, then fine tuning can give a large mass to one combination of the fields ξ and ξ' and keep the other combination with a light mass. However, this will also keep the masses of the color octet and the color singlet light, which is undesirable from the point of view of evolution of the gauge coupling constants. This problem is avoided [12] if instead of ξ' we introduce a field $\chi \equiv (\mathbf{6}, \mathbf{2}, \mathbf{2})$ under G_{PS}. If the symmetry group G_{PS} is embedded in the unified group $SO(10)$ then this field is contained in a $\mathbf{54}$–plet of $SO(10)$, which is required to break the symmetry of the large group. The mixing of the field χ with ξ can then give a mass matrix which may be fine tuned to give only a light color triplet field. We shall discuss the details of this mechanism at a later stage. In the rest of the article we shall use the field χ and not ξ'.

For the field Δ^3 to remain light we have to alter the way in which the left–right symmetric model gets broken, although, as we shall see later, this particular method will make the mechanism in the last paragraph consistent with the minimization of the general potential. For this purpose we introduce a singlet field $\eta \equiv (\mathbf{1,1,1},0) \subset (\mathbf{1,1,1})$, to break the left–right parity (usually this is referred to as D–parity) at a different scale from the left–right symmetry breaking scale M_R [17]. This field transforms under D as $\eta \rightarrow -\eta$. The scalar and the fermionic fields transform under D–parity as $\Delta_{L,R} \rightarrow \Delta_{R,L}$ and $\psi_{L,R} \rightarrow \psi_{R,L}$, while ϕ and ξ stay the same. Then with the field η we can

add new terms to the lagrangian,

$$\mathcal{L}_{\eta\Delta} = -M_\eta \eta (\Delta_R^\dagger \Delta_R - \Delta_L^\dagger \Delta_L) - \lambda_\eta \eta^2 (\Delta_L^\dagger \Delta_L + \Delta_R^\dagger \Delta_R) \qquad (5)$$

In theories where the triplet Higgs breaks D–parity along with $SU(2)_R$ we have $m_{\Delta_L} = m_{\Delta_R}$. The masses of the fields Δ_L and Δ_R are not be the same when D–parity is broken by the *vev* of the field η. When η gets a non–zero vacuum expectation value, the masses are given by,

$$m_{\Delta_L}^2 = m_\Delta^2 - M_\eta \langle\eta\rangle + \lambda_\eta \langle\eta\rangle^2 \ \text{ and } \ m_{\Delta_R}^2 = m_\Delta^2 + M_\eta \langle\eta\rangle + \lambda_\eta \langle\eta\rangle^2$$

where m_Δ is the mass at which the left–right symmetry breaking of $SU(2)_R$ is broken spontaneously. So, in the absence of the η field, both these Δ fields will have mass $\sim m_\Delta$. With $< \eta >$ present the parameters in the three terms can be tuned to make m_{Δ_L} vanish. The field Δ_L will then acquire mass of the order of a TeV from radiative corrections. The same sets of parameter will also make m_{Δ_R} heavy and lead to a solution

$$\langle\eta\rangle \sim \langle\Delta_R\rangle \gg \langle\Delta_L\rangle \text{ and } M_\eta \approx m_{\Delta_R} \approx \langle\Delta_R\rangle \approx m_\Delta \text{ and } m_{\Delta_L} \ll \langle\Delta_R\rangle \quad (6)$$

Thus we can have $m_{\Delta^3} \sim m_{\Delta_L} \sim$ few TeV even when $m_{\Delta_R} \sim \langle\Delta_R\rangle$ is as large as 10^{10} GeV.

We now write the most general potential with all the fields present in the minimal left–right symmetric model with the additional fields ξ, χ and the D–parity odd–singlet field η. We then show the two mechanisms required to keep the color triplet fields light are consistent with the minima of the potential. To simplify the expression we define,

$$\phi_1 \ \equiv \ \phi \ ; \ \ \phi_2 \equiv \tau_2 \phi_1^* \tau_2 \ ; \ \ \xi_1 \equiv \xi \ ; \ \ \xi_2 \equiv \tau_2 \xi_1^* \tau_2$$

The most general potential with all the fields is,

$$\begin{aligned} V(\phi_1, \phi_2, \Delta_L, \Delta_R, \xi_1, \xi_2, \eta, \chi) \ = \ & V_\phi + V_\Delta + V_\eta + V_\xi + V_\chi \\ & + V_{\eta\phi} + V_{\eta\Delta} + V_{\Delta\phi} + V_{\phi\xi} + V_{\Delta\xi} + V_{\eta\xi} + V_{\chi\xi} \end{aligned}$$

$$(7)$$

where the different terms in this expression are given by,

$$V_\phi = -\sum_{i,j} \mu_{ij}^2 \ tr(\phi_i^\dagger \phi_j) + \sum_{i,j,k,l} \lambda_{ijkl} \ tr(\phi_i^\dagger \phi_j) \ tr(\phi_k^\dagger \phi_l)$$
$$+ \sum_{i,j,k,l} \lambda'_{ijkl} \ tr(\phi_i^\dagger \phi_j \phi_k^\dagger \phi_l)$$

$$V_\Delta = -\mu^2 \ [tr(\Delta_L^\dagger \Delta_L) + tr(\Delta_R^\dagger \Delta_R)] + \rho_1 \ [tr(\Delta_L^\dagger \Delta_L)^2 + tr(\Delta_R^\dagger \Delta_R)^2]$$

$$+\rho_2 \ [tr(\Delta_L^\dagger \Delta_L \Delta_L^\dagger \Delta_L) + tr(\Delta_R^\dagger \Delta_R \Delta_R^\dagger \Delta_R)] + \rho_3 \ tr(\Delta_L^\dagger \Delta_L \Delta_R^\dagger \Delta_R)$$

$$V_\eta = -\mu_\eta^2 \ \eta^2 + \beta_\eta \ \eta^4$$

$$V_\xi = \sum_{i,j} m_{ij}^2 \ tr(\xi_i^\dagger \xi_j) + \sum_{i,j,k,l} n_{ijkl} \ tr(\xi_i^\dagger \xi_j \xi_k^\dagger \xi_l) + \sum_{i,j,k,l} p_{ijkl} \ tr(\xi_i^\dagger \xi_j) \ tr(\xi_k^\dagger \xi_l)$$

$$V_\chi = M_\chi^2 tr(\chi^\dagger \ \chi) + \lambda_1^\chi [tr(\chi^\dagger \ \chi)]^2 + \lambda_1^\chi tr(\chi^\dagger \ \chi \chi^\dagger \ \chi)$$

$$V_{\Delta\phi} = +\sum_{i,j} \alpha_{ij} \ [tr(\Delta_L^\dagger \Delta_L) + tr(\Delta_R^\dagger \Delta_R)] \ tr(\phi_i^\dagger \phi_j) + \sum_{i,j} \beta_{ij} \ [\ tr(\Delta_L^\dagger \Delta_L \phi_i \phi_j^\dagger)$$
$$+tr(\Delta_R^\dagger \Delta_R \phi_i^\dagger \phi_j)] + \sum_{i,j} \gamma_{ij} \ tr(\Delta_L^\dagger \phi_i \Delta_R \phi_j^\dagger)$$

$$V_{\eta\Delta} = -M_\eta \ \eta \ [tr(\Delta_L^\dagger \Delta_L) - tr(\Delta_R^\dagger \Delta_R)] + \lambda_\eta \ \eta^2 \ [tr(\Delta_L^\dagger \Delta_L) + tr(\Delta_R^\dagger \Delta_R)]$$

$$V_{\eta\phi} = \sum_{i,j} \delta_{ij} \ \eta^2 \ tr(\phi_i^\dagger \phi_j)$$

$$V_{\phi\xi} = \sum_{i,j,k,l} u_{ijkl} \ tr(\phi_i^\dagger \phi_j \xi_k^\dagger \xi_l) + \sum_{i,j,k,l} v_{ijkl} \ tr(\phi_i^\dagger \phi_j) \ tr(\xi_k^\dagger \xi_l)$$

$$V_{\Delta\xi} \;=\; +\sum_{i,j} a_{ij} \; [\; tr(\Delta_L^\dagger \Delta_L) + tr(\Delta_R^\dagger \Delta_R)] \; tr(\xi_i^\dagger \xi_j)$$

$$+\sum_{i,j} b_{ij} \; [\; tr(\Delta_L^\dagger \Delta_L \xi_i \xi_j^\dagger) + tr(\Delta_R^\dagger \Delta_R \xi_i^\dagger \xi_j)]$$

$$+\sum_{i,j} c_{ij} \; tr(\Delta_L^\dagger \xi_i \Delta_R \xi_j^\dagger)$$

$$+\sum_{i,j} \lambda_{ij}^{pr} [tr(\Delta_L \Delta_L \xi_i \xi_j) + tr(\Delta_R \Delta_R \xi_i \xi_j) + tr(\Delta_L \Delta_R \xi_i \xi_j)]$$

$$V_{\eta\xi} \;=\; \sum_{i,j} d_{ij} \; \eta^2 \; tr(\xi_i^\dagger \xi_j)$$

$$V_{\chi\xi} \;=\; P \; \eta[tr(\xi\chi\Delta_R) - tr(\xi\chi\Delta_L)] + M \; [tr(\xi\chi\Delta_R) + tr(\xi\chi\Delta_L)]$$

We have not written the $SU(4)$ indices explicitly for simplicity. For example, if we include the $SU(4)$ index, the term $\rho_2 tr(\Delta_L^\dagger \Delta_L \Delta_L^\dagger \Delta_L)$ in our notation will actually mean two terms, $\rho_2^a tr(\Delta_L^{\dagger\alpha}\Delta_{L\alpha}\Delta_L^{\dagger\beta}\Delta_{L\beta})$ and $\rho_2^b tr(\Delta_L^{\dagger\alpha}\Delta_{L\beta}\Delta_L^{\dagger\beta}\Delta_{L\alpha})$. However, as far as the minimization and the consistency of the model is concerned, we only have to replace ρ_2 by $(\rho_2^a + \rho_2^b)$. Otherwise the rest of the analysis will be unaltered. A more detailed analysis with explicit $SU(4)$ indices will not constrain or relax any of the constraints in this model.

The vacuum expectation values (vev) of the fields have the following form:

$$< \phi > \;=\; \begin{pmatrix} k & 0 \\ 0 & k' \end{pmatrix} \;;\quad < \Delta_L >= \begin{pmatrix} 0 & 0 \\ v_L & 0 \end{pmatrix} \;;$$

$$< \tilde{\phi} > \;=\; \begin{pmatrix} k' & 0 \\ 0 & k \end{pmatrix} \;;\quad < \Delta_R >= \begin{pmatrix} 0 & 0 \\ v_R & 0 \end{pmatrix} \;;$$

$$< \xi > \;=\; \begin{pmatrix} \tilde{k} & 0 \\ 0 & \tilde{k}' \end{pmatrix} \;;\quad < \tilde{\xi} >= \begin{pmatrix} \tilde{k}' & 0 \\ 0 & \tilde{k} \end{pmatrix} \;;$$

$$< \eta > \;=\; \eta_0 \;;\quad\; < \chi >= 0$$

The notation needs some clarification. For the fields ϕ and ξ we have used the representation in which rows correspond to the $SU(2)_L$ quantum numbers $(+\frac{1}{2}, -\frac{1}{2})$ and columns correspond to the $SU(2)_R$ quantum numbers

$(-\frac{1}{2}, \; +\frac{1}{2})$. The field ϕ is a singlet under the group $SU(4)$ and hence it has a $SU(4)$ matrix representation $diag(1, 1, 1, 1)$. On the other hand the field ξ transforms under $SU(4)$ as a **15** representation. Under the $SU(3)$ subgroup of $SU(4)$ the **15** decomposes as $\mathbf{8} + \mathbf{3} + \bar{\mathbf{3}} + \mathbf{1}$. The $SU(4)$ matrix representation of the singlet is a traceless diagonal matrix which is a unit matrix in the $SU(3)$ space. Hence the $SU(4)$ matrix representation of the component of ξ, which is a singlet under both the $SU(3)_C$ and the $U(1)$ subgroups of $SU(4)$ and which acquires a *vev*, is $diag(1, 1, 1, -3)$. The $SU(2)$ representations are as above. Thus these fields ϕ and ξ contribute differently to the quarks and leptons masses, and hence a proper combination of the two fields give the correct mass relations between the quarks and leptons [11]. For the fields Δ_L and Δ_R we used the 2×2 triplet representations of $SU(2)$. Thus the components $\tau^1 - i\,\tau^2$, which has the isospin $+1$ and hence charge neutral, acquire *vev*. (The electric charge is $T_{3L} + T_{3R} + (B - L)/2$. For these representations $B - L = -2$. So the charge neutral component should have T_{3L} or $T_{3R} = +1$, meaning they should contract with $\tau^1 - i\,\tau^2$).

4 Minimization of the Potential

It is almost impossible to minimize the potential with respect to all the fields and then find the absolute minima. For this purpose one needs to simplify the problem considerably. As a first approximation one can extremize the potential with respect to all the fields and then substituting the *vev*s of the different fields to check if there is any inconsistency. In the minimal left–right symmetric potential, *i.e.,* without the field η, there are no linear terms in any field so the usual practice is to replace the various fields by their *vev*s then extremize it with respect to these *vev*s and finally check for consistency. We shall also follow the same procedure, but we need to take care of the extra linear terms present in the potential. For these linear terms, we shall afterwards minimize the potential with respect to those fields which do not acquire any *vev*. The vanishing of these derivatives after substituting for the *vev*s will then impose new constraints which also have to be satisfied.

After the spontaneous symmetry breaking, when the fields acquire a *vev*, the potential contains terms with k, k', $\tilde{k}$, $\tilde{k}'$, v_L and v_R. We need only terms involving v_L and v_R. These are given by,

$$
\begin{aligned}
V &= -\mu^2 \ (v_L^2 + v_R^2) + \frac{\rho}{4} \ (v_L^4 + v_R^4) + \frac{\rho'}{2} \ (v_L^2 v_R^2) + 2 v_L v_R [(\gamma_{11} \\
&\quad + \gamma_{22}) k k' + \gamma_{12}(k^2 + k'^2)] + (v_L^2 + v_R^2) \ [(\alpha_{11} + \alpha_{22} + \beta_{11}) \ k^2 \\
&\quad + (\alpha_{11} + \alpha_{22} + \beta_{22}) \ k'^2 + (4\alpha_{12} + 2\beta_{12}) \ k k'] \\
&\quad - M_\eta \ \eta_0 \ (v_L^2 - v_R^2) + \lambda_\eta \ \eta_0^2 \ (v_L^2 + v_R^2) \\
&\quad + (v_L^2 + v_R^2) \ [(a_{11} + a_{22} + b_{11} + \lambda_{11}^{pr}) \ \tilde{k}^2 + (a_{11} + a_{22} + b_{22} + \lambda_{22}^{pr}) \ \tilde{k}'^2
\end{aligned}
$$

$$+(4a_{12} + b_{12} + \lambda_{12}^{pr}) \; \tilde{k}\tilde{k}'] + 2v_L v_R[(c_{11} + c_{22} + \lambda_{11}^{pr} + \lambda_{22}^{pr})\tilde{k}\tilde{k}'$$

$$+(c_{12} + \lambda_{12}^{pr})(\tilde{k}^2 + \tilde{k}'^2)] \tag{8}$$

where we have defined the new parameters $\rho = 4(\rho_1 + \rho_2)$ and $\rho' = 2\rho_3$.

The minimization of this potential gives a constraint on v_L and v_R. Instead of minimizing this potential with respect to the fields v_L and v_R separately, we consider a combination, $\frac{\partial V}{\partial v_L}v_R - \frac{\partial V}{\partial v_R}v_L = 0$, which gives a relation among the fields v_L and v_R

$$v_L v_R = \frac{\beta_1 k^2 + \beta_2 \tilde{k}^2}{[\rho - \rho' - \frac{4M_\eta \eta_0}{(v_L^2 - v_R^2)}]}. \tag{9}$$

where, $\beta_1 = 2\gamma_{12}$; and $\beta_2 = 2(c_{12} + \lambda_{12}^{pr})$ and we assumed $k' << k$ and $\tilde{k}' << \tilde{k}$. This allows us to have a very tiny *vev* for the left–handed triplet field Δ_L while keeping the *vev* of the right–handed triplet field Δ_R very large. This is in agreement with what we required in Eq. (6).

Now consider the linear terms involving χ given by $V_{\chi\xi}$. These terms allow for the correct mixing between the color triplet components of the fields ξ and χ. The mass matrix for ξ^3 and χ^3 is now given by

$$\mathcal{M} = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \tag{10}$$

where, $a = m^2$, $d = M_\chi^2$ and $b = c = (P\eta_0 + M)v_R$, and where we assumed all m_{ij}s are equal to m. If we now fine tune parameters to make $\det\mathcal{M} = 0$, *i.e.*,

$$(mM_\chi)^2 = (P\eta_0 + M)^2 v_R^2$$

then one of the mass eigenvalues is zero. This fine tuning requires that M must be negative and $(P\eta_0 + M)$ be very small and negative. In fact, $|P\eta_0 + M|$ has to be of the order of $\sim \frac{M_W^2}{M_R}$. This massless field will get a mass of the order of $\sim$ TeV after radiative corrections during the electroweak symmetry breaking are included.

The terms linear in χ required for this mechanism have another effect which was not transparent when we did the minimization of the potential with respect to the *vev*s of the various fields. If we first minimize with respect to the field χ and then substitute for the *vev*s of the various fields (which is not usually done since that complicates the calculation), then there is an additional constraint,

$$v_L = \frac{P\eta_0 + M}{P\eta_0 - M} v_R \tag{11}$$

This means that to satisfy Eq. (9), we require $|P\eta_0 + M| << |P\eta_0 - M|$. In other words, for $v_L \sim \frac{M_W^2}{M_R}$, we need $\left|\frac{P\eta_0 + M}{P\eta_0 - M}\right| \sim \frac{M_W^2}{M_R^2}$. This is consistent with the fine tuning used to keep the color triplet component of ξ light, for which $|P\eta_0 + M| \sim \frac{M_W^2}{M_R}$ and $|P\eta_0 - M| \sim M_R$. This would not have been possible if the field η were not present. For example, in the original paper [12] where the field χ was introduced and the field η was not required, this method would have led to an inconsistency. In the absence of the field η minimization of the potential with respect to the field χ would have given a constraint $v_L = -v_R$, which is inconsistent with the LEP data. Here we need the field η to keep the left–handed color triplet light and the fine tuning makes it consistent.

We now turn to the question of light Δ^3. As mentioned earlier, to have $M_\Delta \sim$ few TeV, we require the coupling of η and Δ as in Eq. (5). Also, in the most general potential the only term which contributes at the level of $\eta_0 \sim M_{\Delta_R}$ is $V_{\eta\Delta}$, which is exactly the same as in (Eq. 5). Thus the field $\Delta^3 \sim \Delta_L$ can be massless at that level. Then during the electroweak phase transition this will again acquire mass through radiative corrections of the order of a few TeV. This is more natural in supersymmetric theories where the radiative corrections induce mass of the order of supersymmetry breaking scale, which are usually of the order of a few TeV.

The mechanism just mentioned to make Δ^3 light has one drawback. It makes the other components of Δ_L also very light. For example, the Δ^6 can

now mediate $n - \bar{n}$ oscillation, which has to be suppressed. This problem is similar to the doublet-triplet splitting of any other grand unified models. In the present scenario we assume that although this field is light the coupling of Δ^6 is very small, which can make the model safe. However, this is not the best choice and if one can find some good solution to the doublet-triplet splitting in other GUTs, then one has to incorporate the same mechanism here in future. With our present assumption that the Yukawa coupling of Δ^6 is very small, we now have to check the consequences of these fields in the evolution of the gauge coupling constants.

Finally, we point out that the quartic coupling (given by $V_{\Delta\xi}$) required for generating the required diagram is also present in the general potential.

5 Evolution of the Coupling Constants

There have to be many light scalars for the present scenario to work. These scalars may destabilize the unification of the gauge coupling constant at the unification scale. In the evolution of the gauge coupling constant with these Higgs scalars included and with the mass scales as above it is impossible to have unification of the gauge coupling constants using the LEP constraints [18] on $\sin^\theta_w$ and α_s. However higher dimensional operators, which might originate from Planck scale physics such as quantum gravity or compactification of Kaluza-Klein theories or Superstring theories, can save the situation[14, 15]. Thus if the three lepton decay mode of the proton survives all of the experimental tests, then we may have an indication that Planck scale physics is actually modifying the boundary conditions of the gauge coupling constants near the unification scale.

In our analysis we include the effect of the non-renormalizable terms arising from Planck scale physics from the beginning using the notation and method of reference [15]. We write down the generalized renormalization group equations in which the Planck scale effects are parametrized in terms of four extra parameters. We recover the usual relations between the coupling constants in the absence of gravity by setting the extra parameters to zero. It is when we do this we obtain a contradiction and the equations fail to provide unification.

The evolution of gauge coupling constants with the energy scale is governed by Renormalization Group Equations (RGE). Here we consider the RGE in one loop approximation i.e. the gauge fields fermionic fields and the scalar fields contribute to the evolution of the gauge couplings via one loop graphs only [19]. In this approximation the renormalization group equation takes the form,

$$\mu \frac{\mathrm{d}\alpha_i(\mu)}{\mathrm{d}\mu} = 2b_i \alpha_i^2(\mu)$$

where, $\alpha_i = \frac{g_i^2}{4\pi}$ and the beta function is given in the following generic form.

$$b = \frac{1}{4\pi}\left[-\frac{11}{3}N + \frac{4}{3}n_f + \frac{1}{6}T_s\right] \qquad (12)$$

Here $N = 1, 2, 3$ or 4, the number of neutrinos is always 3 and the scalars take on the values discussed below. Since there are a large number of scalar fields present in our model their contribution will be substantial despite the suppression by a factor of 6 in the beta function of the scalar term (T_s). We list the scalar fields that contribute to the RGE at different energy scales in Table. 1. For simplification of notation we write M_c for M_{PS} in this section.

We embed the Pati-Salam group G_{PS} into a larger GUT group SO(10). The $SO(10)$ symmetry is broken by a **54**-plet of Higgs field Σ at the scale M_U. The Σ is a traceless symmetric field of the $SO(10)$ and the *vev*s of Σ which mediates this symmetry breaking are given by,

$$\langle\Sigma\rangle = \frac{1}{\sqrt{30}}\ \Sigma_0\ \mathrm{diag}(1, 1, 1, 1, 1, 1, -\frac{3}{2}, -\frac{3}{2}, -\frac{3}{2}, -\frac{3}{2}). \qquad (13)$$

where, $\Sigma_0 = \sqrt{\frac{6}{5\pi\alpha_G}}M_U$ and $\alpha_G = g_0^2/4\pi$ is the GUT coupling. The *vev* of a 45-plet field H breaks the symmetry group G_{PS} to G_{LR},

$$\langle H\rangle = \frac{1}{\sqrt{12}\,i}\ H_0 \begin{pmatrix} 0_{33} & 1_{33} & 0_{34} \\ -1_{33} & 0_{33} & 0_{34} \\ 0_{43} & 0_{43} & 0_{44} \end{pmatrix} \qquad (14)$$

where, 0_{mn} is a $m \times n$ null matrix and 1_{mm} is a $m \times m$ unit matrix.

The **(1,1,1)** component of the **54**-plet field Σ breaks the $SO(10)$ group at the scale M_U and hence does not affect the RGE. The **(15,1,1)** component of the **45**-plet field H breaks G_{PS} and so contributes to the RGE between the scale M_c and M_U. The color singlet part of the **(10,1,3)** component of the **126**-plet field Δ_R breaks the G_{LR} symmetry. Then by extended survival hypothesis the color singlet component contribute to the RGE between the scale M_R to M_c and all the components **(10,1,3)** contribute between the scales M_c and M_U [19]. On the other hand our proposed mechanism, allows all of

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the components of the field $\Delta_L \equiv (\mathbf{10,3,1})$ to remain light and contribute to the RGE at all energies between M_W and M_U. The bidoublet $(\mathbf{1,2,2})$ field breaks the electroweak symmetry group and contributes at all energies to the RGE . For the correct quark-lepton mass relation we also require the bidoublet color singlet component of $(\mathbf{15,2,2})$ to acquire a *vev* at the electroweak scale. For the potential we have, the $(\mathbf{6,2,2})$ field will mix with the color triplet and anti-triplet components of the $(\mathbf{15,2,2})$ field and one combination of these color triplet and anti-triplet fields will remain light. The color singlet and one combination of the triplet and anti-triplet component will then contribute to the RGE at all energies, while the other combination of the color triplet and anti-triplet will become heavy and contribute only between the energies M_c and M_U.

$M_U \to M_c$	$M_c \to M_R$	$M_R \to M_W$
(1,1,1)		
(15,1,1)		
(10,1,3)	$(1,1,3,\sqrt{\frac{3}{2}})$	
(10,3,1)	$(6,3,1,-\sqrt{\frac{3}{2}})+(3,3,1,\sqrt{\frac{1}{6}})+(1,3,1,\sqrt{\frac{3}{2}})$	$(6,3,-\sqrt{\frac{3}{5}})+(3,3,\frac{1}{3}\sqrt{\frac{3}{5}})+(1,3,\sqrt{\frac{3}{5}})$
(1,2,2)	$(1,2,2,0)$	$(1,2,\frac{1}{2}\sqrt{\frac{3}{5}})+(1,2,\frac{-1}{2}\sqrt{\frac{3}{5}})$
(15,2,2)	$(3,2,2,\sqrt{\frac{2}{3}})+(\bar{3},2,2,-\sqrt{\frac{2}{3}})+(1,2,2,0)$	$(3,2,\frac{1}{6}\sqrt{\frac{3}{5}})+(3,2,\frac{7}{6}\sqrt{\frac{3}{5}})+(1,2,\frac{1}{2}\sqrt{\frac{3}{5}})$ $(\bar{3},2,\frac{-1}{6}\sqrt{\frac{3}{5}})+(\bar{3},2,\frac{-7}{6}\sqrt{\frac{3}{5}})+(1,2,\frac{-1}{2}\sqrt{\frac{3}{5}})$
(6,2,2)		

Table 1: Higgs scalars at various symmetry breaking scales. The $U(1)$ quantum numbers are normalized from their embedding in $SO(10)$.

The normalization of the U(1) quantum numbers at the right handed breaking scale is fixed by the relation,

$$Y = \sqrt{\frac{3}{5}}T_R^3 + \sqrt{\frac{2}{5}}Y_{B-L}.$$

Combining results of Eq. 12 and Table. 1 it is easy to write down the following explicit form of the beta functions that regulate the evolutionary behaviour of the gauge couplings at various energy scales. We assume that the number of fermion generations is three.

$M_U \to M_c$	$M_c \to M_R$	$M_R \to M_z$
b_{4uc}= -11/3	b_{3cr}= -29/6	b_{3crw} = -29/6
b_{2Luc}= 11/3	b_{2Lcr}= 4/3	b_{2Lrw} = 4/3
b_{2Ruc}= 11/3	b_{2Rcr}= 3/2	b_{1Yrw}= 49/6
	b_{1B-Lcr}= 13	

Table 2: The modified beta functions ($\widetilde{b}_N = 4\pi b_N$) for the various groups at different energy scales. In the table we use the notation $b_N xy$, where N represents the group (N for $SU(N)$ and $1S$ for $U(1)_S$) and xy means the beta functions within the scales M_x and M_y.

We consider both symmetry breaking scales, M_U and M_c, to be very large so that Planck scale effects are not negligible. We start with the renormalizable $SO(10)$ invariant lagrangian,

$$L = -\frac{1}{2}\text{Tr}(F_{\mu\nu}F^{\mu\nu}) \tag{15}$$

and then include the non-renormalizable higher dimensional terms which have their origin in Planck scale physics. We consider only terms of dimension 5 and 6, given by

$$L^{(5)} = -\frac{1}{2}\frac{\eta^{(1)}}{M_{Pl}}\text{Tr}(F_{\mu\nu}\phi F^{\mu\nu}) \tag{16}$$

$$L^{(6)} = -\frac{1}{2}\frac{1}{M_{Pl}^2}\left[\eta_a^{(2)}\{\text{Tr}(F_{\mu\nu}\phi^2 F^{\mu\nu}) + \text{Tr}(F_{\mu\nu}\phi F^{\mu\nu}\phi)\} +\right.$$

$$\left.\eta_b^{(2)}\text{Tr}(\phi^2)\text{Tr}(F_{\mu\nu}F^{\mu\nu}) + \eta_c^{(2)}\text{Tr}(F^{\mu\nu}\phi)\text{Tr}(F_{\mu\nu}\phi)\right] \tag{17}$$

where $\eta^{(n)}$ are dimensional couplings of the higher dimensional operators. When any Higgs scalar ϕ acquires *vev* ϕ_0, these operators induce effective dimension 4 terms modifying the boundary conditions at the scale ϕ_0.

The symmetry breaking at M_U shifts the boundary condition of the $SU(4)$ coupling constant with respect to the $SU(2)$ couplings whereas the *vev*s of the **45**-plet field H contribute to the relative couplings of the $SU(3)$ and the $U(1)$ constants. The G_{PS} invariant effective lagrangian, modified by these higher dimensional operators, is given by,

$$-\frac{1}{2}(1 + \epsilon_4)\,\text{Tr}(F_{\mu\nu}^{(4)}\,F^{(4)\mu\nu}) - \frac{1}{2}(1 + \epsilon_2)\,\text{Tr}(F_{\mu\nu}^{(2L)}\,F^{(2L)\mu\nu})$$

$$-\frac{1}{2}(1 + \epsilon_2)\,\text{Tr}(F_{\mu\nu}^{(2R)}\,F^{(2R)\mu\nu}) \tag{18}$$

where,

$$\epsilon_4 = \epsilon^{(1)} + \epsilon_a^{(2)} + \frac{1}{2}\epsilon_b^{(2)}$$

$$\epsilon_2 = -\frac{3}{2}\epsilon^{(1)} + \frac{9}{4}\epsilon_a^{(2)} + \frac{1}{2}\epsilon_b^{(2)}.$$

and

$$\epsilon_i^{(n)} = \left[\left\{\frac{1}{25\pi\alpha_G}\right\}^{\frac{1}{2}}\frac{M_U}{M_{Pl}}\right]^n\eta_i^{(n)}.$$

Then the usual G_{PS} lagrangian can be recovered with the modified coupling constants,

$$\begin{aligned}
g_4^2(M_U) &= \bar{g}_4^2(M_U)(1 + \epsilon_4)^{-1}\\
g_{2L}^2(M_U) &= \bar{g}_{2L}^2(M_U)(1 + \epsilon_2)^{-1}\\
g_{2R}^2(M_U) &= \bar{g}_{2R}^2(M_U)(1 + \epsilon_2)^{-1}
\end{aligned} \tag{19}$$

where the $\bar{g}_i$ are the coupling constants in the absence of the nonrenormalizable terms and g_i are the physical coupling constants that evolve below M_U. The modified boundary condition then reads,

$$g_4^2(M_U)(1 + \epsilon_4) = g_{2L}^2(M_U)(1 + \epsilon_2) = g_{2R}^2(M_U)(1 + \epsilon_2) = g_0^2. \qquad (20)$$

At M_c the symmetry group $SU(4)_c$ breaks down to $SU(3)_c \times U(1)_{B-L}$ when the $(15,1,1)$ component of the 45-plet of Higgs field H acquires a *vev*. The $SU(3)_c \times U(1)_{B-L}$ invariant lagrangian is given by,

$$-\frac{1}{2}(1 + \epsilon_3') \operatorname{Tr}(F_{\mu\nu}^{(3)} F^{(3)\mu\nu}) - \frac{1}{2}(1 + \epsilon_1') \operatorname{Tr}(F_{\mu\nu}^{(1)} F^{(1)\mu\nu})$$

where,

$$\epsilon_3' = \epsilon_a'^{(2)} + 12\epsilon_b'^{(2)}$$

$$\epsilon_1' = 7\epsilon_a'^{(2)} + 12\epsilon_b'^{(2)} + 12\epsilon_c'^{(2)}.$$

and

$$\epsilon_i'^{(2)} = \frac{\eta_i'^{(2)} \phi_0^2}{24M_{Pl}^2} = \left[\frac{1}{20\pi\alpha_4}\left[\frac{M_I}{M_{Pl}}\right]^2\right]\eta_i'^{(2)}$$

where, $i = a, b, c$. Then the boundary condition at M_c becomes,

$$g_{1(B-L)}^2(M_c)(1 + \epsilon_1') = g_{3c}^2(M_c)(1 + \epsilon_3') = g_4^2(M_c).$$

The matching conditions at the scale M_R are not modified by the Planck scale effects and are given by,

$$\begin{aligned} g_{1Y}^{-2}(M_R) &= \frac{3}{5}g_{2R}^{-2}(M_R) + \frac{2}{5}g_{1(B-L)}^{-2}(M_R) \\ g_{2L}^{-2}(M_R) &= g_{2R}^{-2}(M_R) \end{aligned} \qquad (21)$$

Using the above boundary conditions and the one loop renormalization group equation the unification coupling α_U can be related to the three couplings at the W mass scale (M_W) through the following relations [15] (we have defined $m_{ij} = \ln\frac{M_i}{M_j}$),

$$\alpha_y^{-1}(M_W) = \alpha_G^{-1}\left(1 + 3/5\epsilon_2 + 2/5\epsilon_4 + 2/5(\epsilon'(1+\epsilon_4))\right) + \left(6/5\, b_{2ruc} + 4/5(1+\epsilon_1')\, b_{4uc}\right) m_{uc}$$
$$+\, (6/5\, b_{2rcr} + 4/5\, b_{1blcr})\, m_{cr} + 2b_{1yrw}\, m_{rw}$$
$$\alpha_2^{-1}(M_W) = \alpha_G^{-1}\left(1 + \epsilon_2\right) + 2\, b_{2luc}\, m_{uc} + 2\, b_{2lcr}\, m_{cr} + 2\, b_{2lrw}\, m_{rw}$$
$$\alpha_3^{-1}(M_W) = \alpha_U^{-1}\left(1 + \epsilon_4\right) + 2\, b_{4uc}\, m_{uc} + 2\, b_{3cr}\, m_{cr} + 2\, b_{3crw}\, m_{rw}$$

We define,
$$A = \alpha_Y^{-1}(M_W) - \alpha^{-1}(M_Z)$$

and
$$B = \alpha_{2L}^{-1}(M_W) + \frac{5}{3}\alpha_Y^{-1}(M_Z) - \frac{8}{3}\alpha_{3c}^{-1}(M_W)$$

and relate them to the experimental numbers through the following equations,

$$\sin^2_{\theta_W} = \frac{3}{8} - \frac{5}{8}\alpha A$$
$$1 - \frac{3}{8}\frac{\alpha}{\alpha_s} = \alpha B. \tag{22}$$

If we now take all the ϵ s to be zero, then we have

$$m_{rw} = -36.7 + 5.7 m_{uc}$$
$$m_{cr} = 147.3 - 11.2 m_{uc}\,.$$

There is no solution with positive m_{rw} and m_{cr} for any value of M_{uc} with the constraints, $\sin^2\theta_W = 0.2334$, $\alpha_s = 0.12$, and with the unification scale below the Planck scale . In other words this means that if we do not consider the effect of gravity, then in the presence of so many light Higgs scalars it is not possible to have unification of the gauge coupling constants. Thus if the three lepton decay mode of the proton is the explanation of the atmospheric neutrino problem, gravity effects modify the low energy predictions of the grand unified theories.

We now consider the effects of the Planck scale. For several choices of the ϵ s it is possible to have a mass scale solution which may explain the atmospheric neutrino problem. To demonstrate this we present a few representative solutions in table 2. The light scalars contribute from the scale $M_z \sim 100$ GeV in the RG equation. The unification scale M_U and the G_{PS} breaking scale M_c are very close to each other (*i.e.*, $m_{uc} = \ln\frac{M_U}{M_c}$ is very small) and is considered here to be around $M_U \sim M_c \sim 10^{18}$ GeV (*i.e.*, $m_{rw} + m_{cr} + m_{uc} = 39$); the right handed breaking scale is around $M_R \sim 10^{13}$ GeV (*i.e.*, $m_{rw} \sim 28$). With these values of the mass scales the three lepton decay mode of the proton would be the most dominant decay mode (with $\tau(P \to e^+\nu\nu) \sim 10^{31}$ yrs). Since the unification scale is quite high now, conventional proton decay modes are very much suppressed (with $\tau(P \to e^+\pi^\circ) \sim 10^{39}$ yrs.). Thus even though the three lepton decay mode of the proton explains the atmospheric neutrino anomaly there is no conflict with the non-observation of proton decay in other experiments.

ϵ'_1	ϵ_2	ϵ'_3	ϵ_4	α_G^{-1}	m_{rw}	m_{cr}	m_{uc}
-1	$-.75$	1	-1	55	28.32	9.73	$.93$
-1	$-.75$	1	-1	57	28.04	10.34	$.61$
-1	$-.75$	1	-1	59	27.75	10.95	$.28$
$-.99$	$-.75$	1	-1	59	27.76	10.95	$.28$
$-.90$	$-.75$	1	-1	59	27.76	10.94	$.28$
$-.80$	$-.75$	1	-1	59	27.77	10.93	$.28$

Table 3: Allowed ranges of parameters for unification

6 Conclusions

We have presented an extension of the left–right symmetric model where the most dominant proton decay mode is through three leptons. The lifetime for this decay mode is large enough to explain the atmospheric neutrino anomaly. We have minimized the complete potential to check the consistency of the model. In the end we have carried out a renormalization group analysis to estimate the mass scales of the model. We have shown that when the gravity induced effects coming from the Plank scale physics are included in the renormalization group analysis, the couplings unify; an estimation of the several mass scales of the model then becomes possible.

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Figure 1: Diagram giving $P \rightarrow e_L{}^+\nu_L\nu_L{}^c$

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