Test of the Equivalence Principle from Neutrino Oscillation Experiments

R.B. Mann[1]
Department of Applied Mathematics and Theoretical Physics
University of Cambridge, Silver St., Cambridge CB3 9EW
and
U. Sarkar[2]
Theory Group, Physical Research Laboratory
Ahmedabad 380 009
India

February 1, 2008
DAMTP/R-95/28

Abstract

We consider the hypothesis that neutrino oscillation data can be explained if the gravitational couplings of (massless or degenerate mass) neutrinos are flavour non–diagonal, in violation of the equivalence principle. We analyze the various neutrino oscillation laboratory experimental data including the recent LSND observations to constrain the relevant parameter space. We find that there is no allowed region of parameter space which can explain the existing data, implying that the LSND result cannot be explained by oscillations of degenerate-mass neutrinos due to equivalence principle violations.

[1]email: rbm20@amtp.cam.ac.uk   on leave from Physics Department, University of Waterloo, Waterloo, Ontario, Canada N2L 3G1

[2]email: utpal@prl.ernet.in

Empirical evidence supporting neutrino-flavour oscillations continues to mount [1]. At present there are four different solar neutrino experiments [2], each using distinct detection techniques, that consistently find a discrepancy between the measured solar ν_e flux and that predicted by solar models [3]. There are also a number of experiments on atmospheric neutrinos [4] which find that the ratio of the flux of ν_μ to ν_e is significantly smaller than one would expect from standard particle physics models [5]. Most recently, the Liquid Scintillator Neutrino Detector (LSND) group has recently announced an excess $\overline{\nu_e}$ events between the energy range 36 and 60 MeV [6]. If this excess is due to the $\overline{\nu_\mu} \rightarrow \overline{\nu_e}$ oscillation, then it implies an oscillation probability of $(0.34^{+0.20}_{-0.18} \pm 0.07)\%$. The distance traversed by the $\overline{\nu_\mu}$ before being detected as a $\overline{\nu_e}$ is about 30 metres.

Mechanisms underlying neutrino oscillation typically assume that neutrinos have nondegenerate masses, following the original suggestion by Pontecorvo [7]. In this scenario the weak interaction eigenstates of neutrinos are distinct from their mass eigenstates, thereby permitting oscillations between the various flavours.

An alternative neutrino oscillation mechanism was proposed more recently by Gasperini [8] (and independently by Halprin and Leung [9]), in which neutrino weak interaction eigenstates are distinct from their gravitational eigenstates. This mechanism (later referred to as the VEP mechanism [10]) does not require neutrinos to have nonzero masses; instead neutrino oscillations occur in this mechanism due to an assumed flavour non–diagonal coupling of neutrinos to gravity, in violation of the equivalence principle.

From this viewpoint, neutrino oscillation experiments furnish us with a test of the equivalence principle. The VEP mechanism has been explored in a number of papers [11] as a possible explanation of solar neutrino data. A recent analysis [10] has shown that, in the context of a two-flavour model, there are small allowed regions of parameter space at both small and large VEP mixing angles which are compatible with present day solar observations. Extension to a full three-flavour model indicates that the allowed regions of parameter space can widen due to mixing with a third flavour [12].

In the present paper we consider the VEP mechanism in the context of laboratory searches for neutrino oscillations. We first show that LSND data itself can be explained by neutrinos of degenerate or zero mass with flavour non–diagonal gravitational couplings. We then carry our analysis further to include other laboratory experiments [13, 14, 16], which also constrain the allowed VEP parameter space. We find that the combination of these constraints rules out any violation of the equivalence principle in the $e - \mu$ sector, implying that gravity couples to ν_e the same way as ν_μ.

As a consequence, in the absence of other physical mechanisms (such as lepton number violation), previous accelerator data in conjunction with the LSND results can only be explained by assuming neutrinos have differing masses. A repetition of the above analysis in this case would then imply a new bound on the parameter space of the VEP mechanism [8].

In this article we shall not argue in what circumstances the equivalence principle might be violated. Rather, we take a phenomenological approach to this problem and try to constrain the parameter space only from an analysis of the existing data on neutrino oscillations. For the sake of simplicity we work in the two generation scenario. At the end we shall comment on the role other oscillation data plays in constraining VEP.

We turn now to the question of whether or not the LSND results can be understood solely in the context of the VEP mechanism. In this mechanism, the gravitational eigenstates $|\nu_G\rangle = (\nu_{1G}, \nu_{2G})$ are related to the weak eigenstates $|\nu_W\rangle = (\nu_e, \nu_\mu)$ by an $SO(2)$ rotation $R(\theta_G)$

$$|\nu_W\rangle = R(\theta_G)|\nu_G\rangle \tag{1}$$

where θ_G is the mixing angle. The gravitational eigenstates are solutions to the Dirac equation in a Schwarzchild background. For a spherically symmetric metric, choosing the trajectory of the neutrino in the radial direction, we can write down the diagonal components of the Hamiltonian,

$$H_i^G = -2|\phi(r)|E_i(1 + f_i) \tag{2}$$

which governs the evolution of the neutrinos. Here the f_i are the flavour dependent gravitational parameters, which determine the magnitude of the violation of the Weak Equivalence Principle. The evolution of the weak eigenstates will be governed by the equation

$$i\frac{d}{dr}\begin{pmatrix}\nu_e\\ \nu_\mu\end{pmatrix} = 2E|\phi(r)|\Delta f\begin{pmatrix}0 & \frac{1}{2}\sin 2\theta_G\\ \frac{1}{2}\sin 2\theta_G & \cos 2\theta_G\end{pmatrix}\begin{pmatrix}\nu_e\\ \nu_\mu\end{pmatrix} \tag{3}$$

where $|\phi(r)|$ is the Newtonian gravitational potential and $\Delta f \equiv f_2 - f_1$. If the equivalence principle is not violated then $f_1 = f_2$.

In this paper we shall be discussing small scale terrestrial laboratory experiments, for which $\phi(r)$ may taken to be constant. Although a natural choice for ϕ would be the earth's gravitational potential ($\sim O(10^{-9})$), another choice is to consider the potential due to all forms of distant matter. The dominant contribution is from the local supercluster which has been estimated to be 3×10^{-5} [17]. For our purposes the choice of ϕ is irrelevant, since to find the allowed parameter space we shall consider $|\phi|\Delta f$ as the relevant parameter. Particular limits on Δf that arise from given experiments may be found by substituting the above values for ϕ.

Consider a beam of muon neutrinos that traverses a distance of L meters. The probability that a muon neutrino will get converted to an electron neutrino is given by

$$P(\nu_\mu \to \nu_e) = \sin^2 2\theta_G \sin^2 \frac{\pi L}{\lambda_G}, \tag{4}$$

where $\lambda_G = \frac{\pi}{E|\phi(r)|\Delta f}$ is the oscillation length. Although this oscillation mechanism has a number of similarities with that for neutrinos of non-degenerate mass, the oscillation length has a markedly different energy dependence, varying inversely with energy in (4), whereas it is proportional to energy in the massive case [1]. As we shall see, although there exists a fairly wide parameter space for neutrinos of non-degenerate mass which can explain all the laboratory experiments, the VEP mechanism for degenerate mass neutrinos cannot explain all these experiments.

To analyse any neutrino oscillation data, it is useful to divide the parameter space into three regions: $L << \lambda$, $L >> \lambda$ and $L \sim \lambda$. where λ is the oscillation length. In the large λ regime, $\sin^2 2\theta$ can be as large as unity, and the data constrain λ as a function of the mixing angle θ. As θ decreases, the maximum excursion for this part of the curve occurs at approximately $\sin^2 2\theta = \langle P \rangle$, for which $\lambda = 2L$. In the case of the LSND experiment, if one interprets the excess $\overline{\nu_e}$ events as neutrino oscillations, then the above oscillation probability would mean both lower and upper bounds on λ_G, and hence on $|\phi|\Delta f$. In the small λ region large numbers of oscillations take place before the beam reaches the detector, and so the value of $\sin^2 \frac{\pi L}{\lambda}$ may be assumed to be its average value $\frac{1}{2}$. In this regime, observation of a small (or null) neutrino oscillation probability puts both lower and upper (or just upper) bounds on $\sin^2 2\theta$ for large $|\phi|\Delta f$. In the intermediate region the oscillation length is comparable to the travel length L and so $\sin^2 \frac{\pi L}{\lambda}$ varies slowly between 0 and 1, putting limits on $\sin^2 2\theta$ that are quite sensitive to the actual value of $|\phi|\Delta f$.

For the LSND experiment, the oscillation probability (4) in the VEP mechanism can be simplified to

$$P = \sin^2 2\theta_G \sin^2(7.62 \times 10^{15} |\phi(r)|\Delta f) \tag{5}$$

where we have taken $\langle L \rangle = 30$ meters and the average neutrino energy is taken to be 50 MeV. The change on the bounds on $|\phi(r)|\Delta f$ for large $\sin^2 2\theta_G$ due to the uncertainty in the distance or neutrino energy are negligible. For $\sin^2 2\theta_G = 1$, the allowed region (95% C.L. from LSND [6]) for the violation of the equivalence principle is

$$9.76 \times 10^{-18} > |\phi(r)|\Delta f > 5.03 \times 10^{-18}. \tag{6}$$

The small λ region occurs when

$$|\phi|\Delta f > 1.35 \times 10^{-16} \tag{7}$$

for which the LSND data yields the bound

$$.0029 < \sin^2 2\theta_G < .011. \tag{8}$$

Thus with the above bounds on the VEP parameter $|\phi|\Delta f$, one can explain the LSND result. This would apparently mean that LSND result does not imply a non-trivial neutrino mass matrix. We shall now demonstrate that these allowed regions have already been ruled out by other accelerator experiments.

The E776 experiment at the Brookhaven National Laboratory did not see any statistically significant excess number of ν_e ($\overline{\nu_e}$) events over the background at a distance of 1 km from the source in a wide-band ν_μ ($\overline{\nu_\mu}$) beam. Most of the events are above 1 GeV and peaked around 1.4 GeV. Only 19 events with an expected background of $25 \pm 5 \pm 3 \pm 3$ were observed and from this an upper limit on the probability of neutrino oscillations was determined at 90% C.L. [13].

For large $\sin^2 \theta_G$ this gives a lower limit on the mass-squared difference consistent with the LSND result for a neutrino oscillation scenario of neutrinos of non-degenerate mass [6]. However, because of the difference in energy between the LSND and the E776 experiments, the 1 GeV null result of the E776 gets translated to

$$|\phi|\Delta f < 3.0 \times 10^{-21} \tag{9}$$

in the VEP mechanism. This bound is not consistent with the LSND result (6). In table 1, we present the upper and lower bounds on $|\phi|\Delta f$ that are permitted within the limits of error from the E776 and LSND experiments respectively for several values of $\sin^2 2\theta_G$. Over the entire range we find that the upper bound on the $|\phi|\Delta f$ from the E776 experiment is much less than the lower bound of $|\phi|\Delta f$ as allowed by the LSND result. As a result there does not exist any region of the parameter space of the VEP mechanism which can explain both the LSND result as well as the E776 experiment for $\sin^2 2\theta_G > .003$.

We now concentrate on the other regions of the parameter space. The small λ_G region (for which $|\phi|\Delta f$ is so large that many oscillations occur within the experimental beam length) sets in for the E776 experiment at

$$|\phi|\Delta f > 8.03 \times 10^{-20} \tag{10}$$

which is consistent with (7), and implies a bound of

$$\sin^2 2\theta_G < .003 \tag{11}$$

on the mixing angle. Thus combining the allowed regions of the E776 experiment with the LSND result implies that both these experiments can only be explained by degenerate mass neutrinos through the VEP mechanism for very large $|\phi|\Delta f > 1.35 \times 10^{-16}$ and for $0.0029 < \sin^2 2\theta_G < 0.003$.

Even this marginally consistent result may be ruled out as follows. If we further argue that gravity couples with matter and antimatter in the same way (so that gravitational interactions conserve total lepton number), then we can compare the allowed parameter space of the VEP mechanism with the $\nu_\mu \to \nu_e$ oscillation limit as obtained by the SKAT experiment at Serpukhov [14], which provides the most stringent upper limit on the mixing angle in the small λ region at 90% C.L. [15]. SKAT measures the ratio of ν_e to ν_μ induced charged current reactions as observed in a bubble chamber exposed to a wide band neutrino beam with energies between 3 and 30 GeV and a neutrino beam length of 270 meters. The small λ_G region sets in for the SKAT average beam energy at $|\phi|\Delta f > 2.3 \times 10^{-19}$ and the bound on the

Table 1: Bounds on $|\phi(r)|\Delta f$ from E776 (upper) and LSND (lower)

$\sin^2 2\theta_G$	$\lvert\phi(r)\rvert\Delta f$	
	upper bound from E776 experiment	lower bound from LSND experiment
0.002	8.03E-20	1.35E-16
0.003	6.06E-20	1.02E-16
0.004	5.09E-20	8.55E-17
0.005	4.48E-20	7.52E-17
0.006	4.05E-20	6.8E-17
0.007	3.72E-20	6.25E-17
0.008	3.46E-20	5.81E-17
0.009	3.25E-20	5.46E-17
0.01	3.08E-20	5.16E-17
0.02	2.15E-20	3.6E-17
0.03	1.75E-20	2.93E-17
0.04	1.51E-20	2.53E-17
0.05	1.35E-20	2.26E-17
0.06	1.23E-20	2.06E-17
0.07	1.14E-20	1.91E-17
0.08	1.06E-20	1.78E-17
0.09	1.00E-20	1.68E-17
0.1	9.51E-21	1.6E-17
0.2	6.71E-21	1.13E-17
0.3	5.48E-21	9.19E-18
0.4	4.75E-21	7.96E-18
0.5	4.24E-21	7.12E-18
0.6	3.87E-21	6.5E-18
0.7	3.59E-21	6.02E-18
0.8	3.35E-21	5.63E-18
0.9	3.16E-21	5.31E-18
1	3.00E-21	5.03E-18

mixing angle is $\sin^2 2\theta_G < .0025$. This fully rules out the the VEP mechanism for degenerate mass neutrinos.

We can thus conclude that the LSND result cannot be explained by neutrinos of degenerate mass if other laboratory bounds on neutrino oscillations are taken into consideration. This result holds regardless of the gravitational potential at the earth's surface. In particular, for massless neutrinos there is no allowed region of the parameter space of the VEP mechanism which can explain all the experiments.

If we admit the possibility of a non-trivial neutrino mass matrix, then all of the above experiments only put bounds on the VEP parameters. As discussed in ref. [8], in the expression for the neutrino oscillation probability in a two-flavour scenario the oscillation length λ is now a function of $|\phi|\Delta f$, Δm^2 and two mixing angles, and the LSND experiment will limit a combination of these parameters in the large oscillation region via an appropriate generalization of eqs. (5) and (6). The other experiments will provide further constraints on the parameter space, and there will be some minimal Δm^2 which is consistent with all empirical constraints. We intend to relate details of this analysis in a forthcoming paper.

In summary, we have shown that the LSND result in conjunction with other laboratory experiments rules out the possibility of the VEP mechanism for neutrinos of degenerate mass (this includes massless neutrinos as well). This situation arises because of the particular energy dependence of neutrino oscillations in the VEP mechanism in combination with the differing neutrino energies employed in each experiment. In the absence of other physical mechanisms for introconversion of neutrino species, these results imply that neutrinos must have different nonvanishing masses. A naturalness argument would then imply that if the gravitational couplings of ν_e and ν_μ are the same, then the gravitational coupling of ν_τ should be equal to these, making the VEP mechanism an unlikely candidate for a neutrino oscillation mechanism. Once we admit the possibility that neutrinos are massive, there is little motivation to consider the VEP mechanism as the mechanism chiefly responsible for neutrino oscillations. Present experiments can at best put bounds on the relevant parameter space.

Acknowledgements

This work was supported in part by the Natural Sciences and Engineering

Research Council of Canada. R.B.M. would like to thank the members of D.A.M.T.P. and of P.R.L. for their hospitality.

References

[1] A. Joshipura, *preprint* PRL/94-41 (1994).

[2] R. Davis, D.S. Harmer and K.C. Hoffman, Phys. Rev. Lett **20** (1986) 1205; K. Hirata *et. al.* Phys. Rev. Lett. **65** (1990) 1297; *ibid¿* **65** (1990) 1301; J.N. Abdurashidov *et. al.*, Phys. Lett. **B328** (1994) 234; P. Anselmann *et. al.* Phys. Lett. **B327** (1994) 377.

[3] S. Turck-Chieze *et. al.* Phys. Rep. **230** (1993) 57; J. Bahcall and R.K. Ulrich, Rev. Mod. Phys. **60** (1989) 297.

[4] M.C. Goodman, Nucl. Phys. **B38** (Proc. Supp.) (1995) 337; K.S. Hirata *et. al.*, Phys. Lett. **B280** (1992) 146; D. Casper *et.al.* Phys. Rev. Lett. **66** (1991) 2561.

[5] B.C. Barish, Nucl. Phys. **B38** (Proc. Supp.) (1995) 343.

[6] C. Athanassopoulos *et. al.*, LA-UR-95-1238, nucl-ex-9504002 (1995); B.T. Cleveland *et. al.* Nucl. Phys. **B38** (Proc. Supp.) (1995) 47.

[7] B.M. Ponetcorvo, Sov. Phys. JETP **34** (1958) 247.

[8] M. Gasperini, Phys. Rev. **D38** (1988) 2635; *ibid.* **D39** (1989) 3606.

[9] A. Halprin and C.N. Leung, Phys. Rev. Lett. **67** (1991) 1833; Nucl. Phys. **B28A** (Proc. Supp.) (1992) 139.

[10] J. N. Bahcall, P. I. Krastev, and C. N. Leung, IASSNS-AST 94/54, UDHEP-10-94 (October 1994).

[11] M. N. Butler *et al.*, Phys. Rev. **D47**, 2615 (1993); A. Halprin and C. N. Leung, Phys. Rev. Lett. **67**, 1833 (1991); J. Pantaleone, A. Halprin, and C. N. Leung, Phys. Rev. **D47**, R4199 (1993); K. Iida, H. Minakata and O. Yasuda, Mod. Phys. Lett. **A8** (1993) 1037.

[12] J. Mureika and R.B. Mann, hep-ph-9501340 (1995).

[13] L. Borodovsky *et. al.*, Phys. Rev. Lett. **68** (1992) 274.

[14] V.V. Ammosov *et. al.*, Z. Phys. **C40** (1988) 487.

[15] Review of Particle Properties, Phys Rev **D50**, (1994) 1428.

[16] L.A. Ahrens *et. al.*, Phys. Rev. **D31** (1985) 2732; L.S. Durkin *et. al.*, Phys. Rev. Lett. **61** (1988) 1811; C. Angelini *et. al.*, Phys. Lett. **B179** (1986) 307; G. Zacek *et. al.*, Phys. Rev. **D34** (1986) 2621.

[17] R.J. Hughes, Phys. Rev. **D46** (1992) R2283; I.R. Kenyon, Phys. Lett. **B237** (1990) 274.