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Gravitational uncertainties from dimension six operators on supersymmetric GUT predictions

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Abstract

We consider the gravity induced dimension six terms in addition to the dimension five terms in the SUSY GUT Lagrangian and find that the prediction for α_s may be washed out completely in supersymmetric grand unified theories unless the triplet higgs mass is smaller than 7×10^{16} GeV.

Recently, Hall and Sarid,[1] and Langacker and Polonsky[2] have shown that the prediction of the strong coupling constant α_s in the minimal supersymmetric $SU(5)$ grand unified theory is smeared out when dimension five non-renormalizable operators arising from gravity is included (Recently Planck scale effects have also been considered by A.Vayonakis[2]). In this brief report we point out that for high GUT scale higher dimensional operators can be as significant as dimension five operators. In particular we show that these operators can wash out the prediction for α_s completely.

In the case of non-supersymmetric GUTs it was shown[3] that by considering dimension five operators alone it is not possible to make minimal $SU(5)$ GUT consistent with the LEP data and proton decay limit. Whereas by considering both dimension five and dimension six operators one can make the minimal $SU(5)$ GUT consistent with LEP data and satisfy the proton decay limit.[4]

We use the notation of Hall and Sarid and include the GUT threshold corrections to compare our result with that of Ref.1. We include both dimension 5 and dimension 6 operators, which might originate from non-renormalizable quantum gravity effect, and write

$$\delta\mathcal{L} = \frac{c}{2\hat{M}_P}\mathrm{tr}(GG\Sigma)+\frac{1}{2\hat{M}_P^2}[d_{11}\frac{1}{2}\mathrm{tr}(G\Sigma^2G) + d_{12}\frac{1}{2}\mathrm{tr}(G\Sigma G\Sigma)+d_2\mathrm{tr}(G^2)\mathrm{tr}(\Sigma^2)+d_3\mathrm{tr}(G\Sigma)\mathrm{tr}(G\Sigma)] \tag{1}$$

where $\hat{M}_P = (8\pi G_N)^{-1/2} \simeq 2.4 \times 10^{18}$ GeV is the reduced Planck mass.

Then these terms will modify the kinetic energy terms of the standard model gauge bosons to

$$\begin{aligned}
\mathcal{L}_{gauge} \quad = \quad & -\frac{1}{4}(FF)_{U(1)}\left[1+\frac{c}{2}\frac{v}{\hat{M}_P}\left(-\frac{1}{2\sqrt{15}}\right)+\frac{v^2}{2\hat{M}_P^2}\left(\frac{1}{15}\right)\left(d_1\frac{7}{4}+d_2\frac{15}{2}+d_3\frac{15}{2}\right)\right] \\
& -\frac{1}{4}(FF)_{SU(2)}\left[1+\frac{c}{2}\frac{v}{\hat{M}_P}\left(-\frac{3}{2\sqrt{15}}\right)+\frac{v^2}{2\hat{M}_P^2}\left(\frac{1}{15}\right)\left(d_1\frac{9}{4}+d_2\frac{15}{2}\right)\right] \\
& -\frac{1}{4}(FF)_{SU(3)}\left[1+\frac{c}{2}\frac{v}{\hat{M}_P}\left(\frac{1}{\sqrt{15}}\right)+\frac{v^2}{2\hat{M}_P^2}\left(\frac{1}{15}\right)\left(d_1+d_2\frac{15}{2}\right)\right]
\end{aligned} \tag{2}$$

where we have defined $d_1 = (d_{11} + d_{12})/2$ as the the first two operators in eqn.(1) always contribute equally. Note that in principle one can also include operators of dimensions higher than six in our analysis but their contributions to $\vec{\epsilon}$, where $\vec{\epsilon}{\alpha_G}^{-1}$ is the amount by which $\vec{\alpha_G}^{-1}$ gets modified in the evolution equations for the coupling constants, can be included by absorbing them in the co-efficients d_1, d_2 and d_3. Since we are interested only in gauge coupling evolutions it is thus sufficient to confine our analysis to just dimension five and dimension six operators for minimal supersymmetric $SU(5)$ GUT and see how they can affect the predictions of α_s. At the one loop level the gauge coupling, evaluated at the Z mass $\vec{\alpha}^{-1} \equiv \vec{\alpha}^{-1}(m_Z) \equiv ({\alpha_1}^{-1}, {\alpha_2}^{-1}, {\alpha_3}^{-1})$ will be related to the GUT scale (M_G) gauge coupling constant

$$\vec{\alpha}^{-1} = \alpha_G^{-1}\left(\vec{1}+\vec{\epsilon}_5+\vec{\epsilon}_6\right) - \sum_a \vec{\beta}_a \ln\left(\frac{M_a}{M_G}\right)$$

where, $\vec{\epsilon}_5 \equiv \frac{c}{2}\frac{v}{\hat{M}_P}\left(\frac{1}{\sqrt{15}}\right)\left(-\frac{1}{2}, -\frac{3}{2}, 1\right)$ and $\vec{\epsilon}_6 \equiv \frac{d}{30}\frac{v^2}{\hat{M}_P^2}\left(d_1\frac{7}{4}+d_2\frac{15}{2}+d_3\frac{15}{2}, d_1\frac{9}{4}+d_2\frac{15}{2}, d_1+d_2\frac{15}{2}\right)$.

Then, following Hall and Sarid[1] the modified unification equations are given by

$$\begin{aligned}
\frac{2}{\alpha_s}+\frac{6}{5\pi}\ln\frac{M_{tr}}{m_Z}-\sqrt{\frac{12}{5}}\frac{c}{2}\frac{v}{\hat{M}_P}\frac{1}{\alpha_G}+[\frac{1}{5}d_1-\frac{1}{2}d_3]\frac{v^2}{2\hat{M}_P^2}\frac{1}{\alpha_G} &= f_1(s^2,m_0,m_{\frac{1}{2}},\mu,m_H) \\
\frac{2}{\alpha_s}+\frac{9}{\pi}\ln\frac{5}{12}+\frac{12}{\pi}\ln g_5+\frac{6}{\pi}\ln\lambda_{24}+\frac{18}{\pi}\ln\frac{v}{m_Z}+\frac{5}{2}\frac{d_3}{2}\frac{v^2}{\hat{M}_P^2}\frac{1}{\alpha_G} &= f_2(s^2,m_0,m_{\frac{1}{2}}) \qquad (3)
\end{aligned}$$

where M_{tr} is the mass of the color triplet higgs.

Subtracting one of the equations in (3) from the other we obtain an equation for M_{tr} which can be written as

$$\frac{-84}{5\pi}\ln t = w_1t^2+w_2t+b \qquad (4)$$

where

$$t=\frac{M_{tr}}{\hat{M}_P}\ ,\ w_1=\frac{1}{\alpha_G{\lambda_5}^2}[\frac{18}{5}d_3-\frac{6}{25}d_1]\ ,\ w_2=\frac{6}{5\alpha_G\lambda_5}c$$
$$b=f_1-f_2+\frac{6}{\pi}\ln\frac{\lambda_{24}}{{\lambda_5}^3}+\frac{6}{\pi}\ln 4\pi\alpha_G+\frac{84}{5\pi}\ln\frac{\hat{M}_P}{m_Z}$$

Defining, $x=M_{tr}/\lambda_5\hat{M}_P$ we can rewrite the first eqn in (3) as

$$\frac{2}{\alpha_s} = f_1(s^2,m_0,m_{\frac{1}{2}},\mu,m_H)-\frac{6}{5\pi}\ln\frac{M_{tr}}{m_Z}+\frac{6}{5}\frac{c}{\alpha_G}x+[\frac{3}{5}d_3-\frac{6}{25}d_1]\frac{x^2}{\alpha_G} \qquad (5)$$

We now numerically solve eqn.(4) for t and then use eqn.(5) to calculate α_s. We use the same mass spectrum and ranges of parameters (s^2, m_0, $m_{\frac{1}{2}}$, μ, m_{H_2},λ_5,λ_{24},c) as in Ref.1. In other words we vary the light superpartner masses and the second higgs doublet mass between 100 GeV and 1 TeV, s^2 between 0.2314 and 0.2324 [5], λ_5 and λ_{24} between .1 and 3 while we constrain $|c|<1$. The co-effecients d_1, d_2 and d_3 are unknown, but we see from eqn.(4) and eqn.(5) that only d_1 and d_3 contribute to the equations for M_{tr} and α_s. We also observe from eqn.(4) that d_3 has a much larger coefficient . We can now consider two scenarios, one with $|d_1|<1; d_3=0$ and $|d_1|=0; |d_3|<1$. There may be multiple solutions to eqn.(3) and we have chosen the lowest solution in our analysis. To select the lowest solution we define two critical solutions t_1 and t_2 which are given by

$$t_1 = \frac{t_{ex}}{2}[1+\sqrt{(1-2y)}] \qquad (6)$$
$$t_2 = \frac{t_{ex}}{2}[1-\sqrt{(1-2y)}] \qquad (7)$$

where $t_{ex} = -w_2/2w_1$, $a = 84/5\pi$ and $y = a/w_1 t_{ex}^2$. For $w_2 = 0$ we have one critical solution t_{cr} given by

$$t_{cr} = \sqrt{\frac{-a}{2w_1}} \tag{8}$$

The critical solutions correspond to points where the tangent to the logarathmic function on the left hand side of eqn.(4) equals the tangent to the parabola on the right hand side of eqn.(4). When t_1 and t_2 are both real and positive and distinct from one another we can have at most three solutions, one below t_1, one between t_1 and t_2 and one above t_2. If instead the critical solutions are real and positive, but equal then we can have at most two solutions. For $w_1 < 0$ there is always one real positive critical solution and so there can be up to two solutions one on either side of the critical solution. When there is no real, positive critical solution there can be up to one solution to eqn.(4). For $w_1 = 0$, as observed in Ref.1, there can be only one solution for w_2 greater than 0 while for w_2 less than 0 there can be upto two solutions lying on either side of the critical solution $t_{critical} = -a/w_2$.

Results For the case where $|d_1| < 1; d_3 = 0$, the effect of dimension 6 operators are found to be negligible. However for the case where $|d_1| = 0; |d_3| < 1$, the effect of dimension six operator can be significant. In fig.1 (a) we show a plot of the solutions in the $\alpha_s - M_{tr}$ plane. Although we cut off the figure at $\alpha_s = 1$, we mention that there are solutions for larger values of α_s [1]. In table.1 we show the ranges of α_s for different M_{tr}. Fig.1 (b) is a blow up of fig.1 (a) for $\alpha_s \leq 0.12$. Here, we have used a much smaller grid size for λ_5 in our numerical computation; as a result, some solutions that do not show up in fig.1 (a) now appear in fig.1 (b). We observe that for $M_{tr} \geq 7 \times 10^{16}$ GeV the range of the solutions for α_s is greatly increased. We also note that with dimension 6 operators it is now possible to get values of α_s below 0.11 which was not possible with pure dimension 5 term. This could be of interest if in the future the central value of $\alpha_s = 0.120 \pm 0.007 \pm 0.002$ [5] shifts down by $\sim 1.5\sigma$. (It is interesting to note that such a low value of α_s (0.108 ± 0.004) is indeed obtained in an analysis of LEP data by Maxwell et al. [6] where it is claimed that the standard perturbative QCD analyses used to extract α_s from LEP data do not correctly take into account higher order NNLO corrections which can be sizeable for some of the LEP observables used in the determination of α_s.) We found that solutions with large values of α_s and small values of α_s (less than 0.11) correspond to small values of λ_5 in the range 0.1 to 0.3 indicating a high value for M_X (or x) and consequently large gravitational corrections.

[1]Of course, the equations themselves cease to be valid if α_s is too large.

When the unification scale is close to the Planck scale the magnitude of the terms induced by the higher dimensional operators in eqn.(5) can become comparable to the combination of the first two terms, resulting in a much wider range for α_s. In our calculations we have constrained the heavy masses to be less than $\hat{M}_P$. To compare to the results with only the dimension five operator included, we note that in that case, the parameter x always is of the order of 10^{-2}. However the inclusion of the dimension six operators allows x to be an order of magnitude higher indicating a higher unification scale close to $\hat{M}_P$ (Note $\frac{M_X}{\hat{M}_P} = \sqrt{8\pi\alpha_G}x \sim x$ for $\alpha_G = \frac{1}{25}$; where M_X is the vector boson mass) and therefore it is not surprising that the effects of the higher dimensional operators are significant.

In summary, we have shown that the inclusion of dimension 6 operators may totally wash out the predictions for the strong coupling constant and further, that the correlation between α_s and M_{tr} is also destroyed unless we constrain the triplet higgs mass $M_{tr} < 7 \times 10^{16}$ GeV because as we see from Table.1 the range of α_s increases significantly from the point $M_{tr} = 7 \times 10^{16}$ GeV onwards. Turning this around, if we require that SUSY-GUT make calculable predictions at the electroweak scale in the presence of gravity induced non-renormalizable operators we may infer more restrictive bounds on the triplet higgs mass than are available in the literature [7].

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References

[1] L.J. Hall and U. Sarid, Phys. Rev. Lett. **70**, 2673 (1993).

[2] P. Langacker and N. Polonsky, Phys. Rev. **D 47** (1993) 4028 ; A.Vayonakis, Phys. Lett. **B 307**,318 (1993).

[3] C.T. Hill, Phys. Lett. **B 135**, 47 (1984); C. Wetterich, Phys. Lett. **B 110**, 384 (1982); J. Ellis and M.K. Gaillard, Phys. Lett. **B 88**, 315 (1979); Q. Shafi and C. Wetterich, Phys. Rev. Lett. **52**, 875 (1984).

[4] M.K. Parida, P.K. Patra and A.K. Mohanty, Phys. Rev. **D 39**, 316 (1989); B. Brahmachari, P.K. Patra, U. Sarkar and K. Sridhar, Mod. Phys. Lett. **A 8**, 1487 (1993).

Table 1: Allowed ranges of α_s for various M_{tr} for $|d_1| = 0$; $|d_3| < 1$

$M_{tr} \times 10^{16}$ GeV	α_s(max)	α_s(min)
1	0.144	0.115
2	0.146	0.118
3	0.146	0.119
4	0.147	0.120
5	0.147	0.119
6	0.147	0.120
7	0.404	0.124
8	0.705	0.121
9	1.408	0.121
10	2.95	0.121
14	2.82	0.101
18	1.742	0.0660
22	3.89	0.0590
26	3.82	0.0570
30	3.36	0.0560

[5] P. Langacker and J. Erler, Paricle Data Group, Phys.Rev. **D 50**, 1304 (1994).

[6] C.J. Maxwell, proceedings of the XXVI International Conference on High Energy Physics, Dallas (1992), edited by James.R. Sanford, p.905 ; D.T. Barclay, C.J. Maxwell and M.T. Reader, UICHEP-TH/93-14, DTP/93/68, hep-ph/9310203.

[7] J. Hisano, H. Murayama and T. Yanagida, Nucl. Phys. **B 402** 46, (1993).

0.1 Figure Captions

- **fig.1 (a)**: The predictions for α_s in minmal SU(5) SUSY GUT as a function of the color-triplet higgs mass M_{tr} in GeV.
- **fig.1 (b)**: Predictions for α_s below 0.12. The numerical calculations for this figure is done with a smaller grid size for λ_5 than was used for fig.1 (a).