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# Baryogenesis from a Lepton Asymmetric Universe

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## Abstract

We demonstrate that  $CP$ -violation in the Majorana mass matrices of the heavy neutrinos can generate a  $CP$ -asymmetric universe. The subsequent decay of the Majorana particles generates a lepton number asymmetry. During the electroweak phase transition the lepton asymmetry is converted into a baryon asymmetry, which survives down to this time.

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The cosmological baryon excess can be generated from the initial condition  $B = 0$  through the baryon number violating decays of very heavy particles [1]. This requires violation of both  $C$  and  $CP$  and, in addition, these processes must happen during an epoch which brings the universe out of equilibrium. This mechanism has two major drawbacks. In most grand unified theories, baryon changing decays conserve  $(B - L)$  and anomalous electroweak interaction induced by sphalerons can wash out the asymmetry [2]. Second, the asymmetry generated in most models is much smaller than the ratio

$$\frac{n_B}{n_\gamma} = (4 - 7) \times 10^{-10}$$

required by observations and big-bang cosmology. A second approach relies on the violation of baryon number through the global  $(B - L)$ -anomaly which together with  $C$ - and  $CP$ -violation generates a baryon excess at the electroweak phase transition, provided it is first order [3]. The search for a consistent extension of the standard model is under active investigation.

At present, a very attractive scenario consists of generating a lepton number asymmetry through lepton number violating decays of heavy neutrinos. The baryon asymmetry is then generated from the lepton asymmetry during the electroweak phase transition, mentioned above. Some time ago, Fukugita and Yanagida proposed this mechanism [4-7] where the lepton number violation was introduced explicitly through a Majorana mass term of the right-handed neutrinos. In all these models the  $CP$ -violation is introduced through the interference of tree-level and one-loop diagrams in the decays of the heavy neutrinos.

Attempts were made [8, 9, 10] to generate the baryon asymmetry in models, where the  $CP$ -violation comes through the mixing of scalars. The pos-

sibility of mixing scalar particles in order to obtain large  $CP$ -violation was discussed some time back [8]. A similar idea was recently exploited in the context of GUTs [9, 10], where the oscillation of scalar particles was proposed. In these scenarios, the GUTs were extended to include new scalars which allow such interactions and introduce  $\epsilon$ -type  $CP$ -violating effects in the mass matrix of the scalars. In the above models  $CP$ - and baryon-violation occur in the same diagrams. In a recent article [11],  $\epsilon$ -type effects have been considered in a different way. Through the scattering of heavy bosons it has been shown that it is possible to create first a  $CP$ -asymmetric universe (to be precise  $CP$ -asymmetric density of scattering states) before the heavy particles decay. When they begin to decay they generate the baryon asymmetry. The main difference of this mechanism from the previous ones lies in the origin of  $CP$ -violation. In the latter mechanism [11] the  $CP$ -violation arises from the phases in the couplings of the matter-anti-matter oscillations in which a quantum number changes by two units, in contrast to references [8, 9, 10] where the  $CP$ -violation occurs in decays of the heavy particles and the quantum numbers change by one unit. If this oscillation violates baryon number by two units, and at the same time there is  $CP$ -violation, then the oscillation will generate a  $CP$ -asymmetric and also baryon asymmetric universe.

In the case of heavy Majorana neutrinos, there exists one loop self-energy diagrams introducing a correction to the Majorana mass of the particles. These diagrams will not conserve  $CP$  and will produce neutrino-anti-neutrino oscillations violating lepton number by two units, which can contribute to the lepton number asymmetry of the universe. In the case of three generation models, the  $CP$ -violating phases which enter in the self energy diagrams are usually not the ones appearing in the interference terms of the

decays of the heavy neutrinos [12]. Thus this contribution of  $C-$ ,  $CP-$  and  $(B-L)$ -violation to leptogenesis is distinct from the ones considered in the literature. In addition, the contributions are larger than the earlier ones for several choices of parameters.

We consider a theory with the same particle content as in ref [4], that is, the particle content of the standard model plus three Majorana neutrinos  $N_i$  with  $i = 1, 2, 3$ , one for each generation. The new Majorana neutrinos are singlets under the standard model, i.e.  $SU(3)_c \times SU(2)_L \times U(1)_Y$ . The new interactions of these neutrinos with leptons and the Majorana mass term are given by,

$$\mathcal{L} = M_{ij}\bar{N}_i^c N_j + h_{\alpha i}\bar{N}_i l_\alpha \phi^\dagger + \text{h.c.} \quad (1)$$

where  $\phi$  is the standard model higgs doublet, which breaks the electroweak symmetry and gives mass to fermions.

The lepton number and  $(B-L)$  are violated through the Majorana mass of the right-handed neutrinos  $N_i$ 's. In all other interactions lepton number is conserved. We shall work in the density matrix formalism [13] to estimate the contribution of neutrino-anti-neutrino oscillations to the lepton number generation of the universe. The neutrino state  $|N\rangle$  and the anti-neutrino state  $|N^c\rangle$  can be distinguished by their decay properties. The state  $|N\rangle$  decays into a light lepton  $l_i$  and a higgs  $\phi^\dagger$ , whereas the state  $|N^c\rangle$  decays into a light anti-lepton  $l_i^c$  and a higgs  $\phi$ . The only interaction which relates the two states and violates lepton number is the Majorana mass term, which also causes mixing of the two states. We shall discuss the question of  $CP$ -violation in such a mixing later on. We shall assume  $CPT$ -invariance, but violation of  $CP$ . For the sake of simplicity, we consider two generations of Majorana neutrinos where the indices  $i$  and  $j$  take the values 1 and 2. We

define a basis of states as  $(N_1^c \ N_2^c \ N_1 \ N_2)$  and the Hamiltonian takes the form,

$$\begin{pmatrix} 0 & 0 & H_{11} & H_{12} \\ 0 & 0 & H_{12} & H_{22} \\ H_{11} & H_{21} & 0 & 0 \\ H_{21} & H_{22} & 0 & 0 \end{pmatrix}.$$

Without loss of generality, we work in the basis where the Majorana mass matrix is diagonal and real. Then the terms  $H_{11}$  and  $H_{22}$  are the mass terms in equation (1). In this basis, the element  $H_{12}$  is generated through the diagrams of figure 1. Similar diagrams produce  $H_{21} = M_{21} - \frac{i}{2}\Gamma_{21}$  with  $M_{21} = M_{12}^*$ ,  $\Gamma_{21} = \Gamma_{12}^*$ .

We define the  $4 \times 4$  density matrix in the basis of states defined above. It satisfies the equation,

$$\frac{d\rho}{dt} = -i(H\rho - \rho H^\dagger).$$

The solutions are obtained by diagonalizing  $H$ . The expectation value of any observable,

$$\mathcal{O}_f = \langle f | \begin{pmatrix} |N_1^c \rangle \langle N_1^c| & |N_1^c \rangle \langle N_2^c| & \dots & |N_1^c \rangle \langle N_2| \\ |N_2^c \rangle \langle N_1^c| & \dots & \dots & \dots \\ \dots & \dots & \dots & \dots \\ |N_2 \rangle \langle N_1^c| & \dots & \dots & |N_2 \rangle \langle N_2| \end{pmatrix} |f \rangle,$$

is given by,

$$\langle \mathcal{O}_f \rangle = \frac{\text{Tr}(\mathcal{O}_f \rho)}{\text{Tr} \rho}, \quad (2)$$

where  $|f \rangle$  is the final states of leptons or anti-leptons produced through the decays of these density of state  $|N \rangle \rightarrow l\phi^\dagger$  and  $|N^c \rangle \rightarrow l^c\phi$ . The observables corresponding to leptons and antileptons in the final states are, respectively,

$$\mathcal{O}_{l_\alpha} \propto \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & h_{\alpha 1} h_{\alpha 1}^* & h_{\alpha 1} h_{\alpha 2}^* \\ 0 & 0 & h_{\alpha 2} h_{\alpha 1}^* & h_{\alpha 2} h_{\alpha 2}^* \end{pmatrix} \quad \text{and} \quad \mathcal{O}_{\bar{l}_\alpha} \propto \begin{pmatrix} h_{\alpha 1} h_{\alpha 1}^* & h_{\alpha 1} h_{\alpha 2}^* & 0 & 0 \\ h_{\alpha 2} h_{\alpha 1}^* & h_{\alpha 2} h_{\alpha 2}^* & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}.$$

Hence a lepton asymmetry is generated through the  $CP$ -violating phase and is given by,

$$\delta = \frac{\Gamma_l - \Gamma_{\bar{l}}}{\Gamma_l + \Gamma_{\bar{l}}}. \quad (3)$$

The above asymmetry  $\delta$  calculated later on is different than the asymmetry generated from the decay of heavy neutrinos [10], because the elements of the density matrix already contain  $CP$ -violation. In the terminology of  $K$ -meson decays it originates in the mass-matrix, that is the density matrix described above (similar to that of ref.[11])<sup>2</sup>.

In the present case, the physical states are pure states and can also be described in the wave function formalism. For the mass matrix under consideration, we give the mass eigenstate for one of the eigenvalues as,

$$\psi_1 \approx |N_1 + N_1^c \rangle + b|N_2 \rangle + d|N_2^c \rangle$$

The decay widths for the decays of  $\psi_1$  into leptons and antileptons is given by,

$$\begin{aligned} \Gamma_l &\propto |h_{\alpha 1} + b h_{\alpha 2}|^2 \\ \Gamma_{\bar{l}} &\propto |h_{\alpha 1}^* + d h_{\alpha 2}^*|^2, \end{aligned}$$

with,

$$\begin{aligned} b &\approx \frac{H_{12}H_{11} + H_{21}H_{22}}{H_{11}^2 - H_{22}^2} \\ \text{and } d &\approx \frac{H_{21}H_{11} + H_{12}H_{22}}{H_{11}^2 - H_{22}^2} \end{aligned}$$

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<sup>2</sup>We thank Dr. H. So for pointing out that the specific mechanism in ref.[11] is suppressed by the general property of GUTs, according to which  $(B - L)$ -changing operators are suppressed relative to the  $(B + L)$ -changing operator. In ref. [11] the suppression is implied by the conservation of weak isospin.

derived from the Hamiltonian perturbatively. In a similar way we can now write down the decay rates for the states  $\psi_2, \psi_3$  and  $\psi_4$ . The quantities  $b$  and  $d$  are determined from the loop diagram in figure 1. The final asymmetry obtained either in the density matrix or the wave function formalism. The oscillation of a  $j = 2$  neutrino to an  $i = 1$  neutrino and its decay to the  $\alpha$ th light lepton or antilepton produces the asymmetry,

$$\delta = 4\sqrt{2}\pi \left( \frac{M_i}{M_j} \right) \frac{\text{Re}(h_{\alpha j} h_{\alpha i}^*) \text{Im}(h_{\alpha j} h_{\alpha i}^*)}{|h_{\alpha i}|^2 + |h_{\alpha j}|^2} \quad (4)$$

We shall include this lepton asymmetry in the evolution of the lepton number determined by the Boltzmann equation.

The number density of the left-handed leptons  $n_l$  will evolve in time following the equation,

$$\frac{dn_l}{dt} + 3Hn_l = \left[ \Delta_l n_j \Gamma_i + \frac{1}{2}(1 + \epsilon') n_i \Gamma_i - \mathcal{D}_i \right] \quad (5)$$

The second term on the left side of the equation comes from the expansion of the universe, where  $H$  is Hubble's constant. We now discuss the origin of the various terms on the right of this expression.

The first term contains the number density  $n_j$  of initial states  $|N_j \rangle$ , which is converted to the final state  $|N_i^c \rangle$ , which subsequently decays into light leptons with a decay width  $\Gamma_i = \frac{h_{ii}^2}{16\pi^2} M_i$ . Here  $M_i$  is the Majorana mass eigenstate of the  $i$ -th neutrino. The factor  $\Delta_l$  contains the  $CP$ -conserving and  $CP$ -violating terms of the  $|N \rangle - |N^c \rangle$  oscillations. The second term in the expression (5) arises from the  $CP$ -violation in the decays of the heavy leptons. The interference of the tree level diagram and the one loop penguin-type diagrams in the decays of the neutrinos  $N_i$  contains the  $CP$ -violating phase and the absorptive part of the integral. This contribution is the usual one which was studied in the literature in detail [4, 5, 6]. The measure of the

$CP$ -violation is now given by  $\epsilon'$ . In the absence of the new contribution, *i.e.*, of the term proportional to  $\Delta_l$ , the asymptotic value of the lepton number asymmetry is proportional to  $\epsilon'/g_*$ .

The evolution of the light anti-leptons will be given by similar equations, with  $\Delta_l \rightarrow \Delta_{\bar{l}}$ ,  $\epsilon' \rightarrow -\epsilon'$  and  $l \rightarrow \bar{l}$ . This way we arrive at a Boltzmann equation for the lepton difference  $n_L s = n_l - n_{\bar{l}}$ , where  $s = g_* n_\gamma$  is the entropy density,  $g_*$  the number of active degrees of freedom and  $n_\gamma$  is the number of photons. In the difference occurs a term proportional to  $\delta = \Delta_l - \Delta_{\bar{l}}$  computed in equation (5).

There are also various processes which deplete the number densities of the left-handed leptons. They come from the reactions  $l + \phi^+ \rightarrow l^c + \phi$ . We can denote by  $D_l$  the depletion of lepton and  $D_{\bar{l}}$  the depletion of the antileptons. The difference which occurs in the asymmetry equation is,

$$\mathcal{D} = D_l - D_{\bar{l}} = (2\delta + \epsilon')n^{eq}(\Gamma_i + \tilde{\sigma}_v) + n^{eq}(\Gamma_i + \tilde{\sigma}_v)\frac{n_L}{n_\gamma} \quad (6)$$

The term with  $\Gamma_i$  comes from the  $|N\rangle$  or  $|N^c\rangle$  neutrinos in the intermediate state and  $\tilde{\sigma}_v$  is produced from the remaining intermediate states. Since this contribution is, in general, smaller than  $\Gamma$  it will be neglected. Direct and indirect  $CP$ -violation in the intermediate states produce the first term. The difference  $|\mathcal{A}(l\phi^c \rightarrow l^c\phi)|^2 - |\mathcal{A}(l^c\phi \rightarrow l\phi^c)|^2$  will also contribute to the depletion through the term proportional to  $\frac{\mu_l - \mu_{\bar{l}}}{T} = \frac{n_l - n_{\bar{l}}}{n_\gamma}$ . When  $\tilde{\sigma}_v$  is neglected, the last term in this expression is due to the recombination of the left-handed leptons and the higgs to form the heavy neutrinos. All these processes are rapid enough to occur in thermal equilibrium and hence the number density entering here is the equilibrium number density  $n^{eq}$  of  $N$ .

We now introduce  $Y_i = n_i/s$ ,  $Y^{eq} = n^{eq}/s$ , the dimensionless variable,  $x = M_1/T$  and make use of the relations,  $t = 1/2H = x^2/2H(x=1)$  with



$H(x = 1) = 1.73\sqrt{g_*}\frac{M_1^2}{M_{Pl}}$  (from now on we assume that  $M_1 < M_2 < M_3$  and the lepton number asymmetry is generated just before  $T \sim M_1$ ). We also define the parameter,

$$K = \frac{\Gamma_i(x = 1)}{H(x = 1)},$$

which is the measure of the out-of-equilibrium condition. For the usual extensions of the standard model, one can take  $g_* = 400$ . With these redefinitions we obtain a simplified equation for the evolution of the lepton number asymmetry of the universe,

$$\frac{dn_L}{dx} = Kx^2 [(2\delta + \epsilon')(Y_i - Y^{eq}) - Y^{eq}g_*n_L]. \quad (7)$$

The  $Y_i$  satisfy the Boltzmann equations,

$$\frac{dY_i}{dx} = -Kx^2(Y_i - Y^{eq}). \quad (8)$$

To solve these equation we assume that although the universe is out-of-equilibrium, it stays close to it, so that  $\frac{d(Y_i - Y^{eq})}{dx} = 0$ . Then it is easier to solve the equations and obtain an estimate of the lepton number generated. For very large time, the solution of a differential equation has an asymptotic value, which is almost independent of the constant  $K$  for  $1 \leq K \leq 50$ , and is given approximately by,

$$n_L = \frac{1}{g_*}(2\delta + \epsilon'). \quad (9)$$

If  $\delta = 0$ , we recover the expression for the scenarios [4, 5], where the heavy neutrinos can only decay. We compare next the two  $CP$ -violating contributions in these scenarios.

The  $CP$ -violation in the decays of heavy neutrinos has been discussed in the literature and for the model under consideration it is given by [4, 5, 6],

$$\epsilon' = \frac{1}{2\pi} \frac{\text{Im}(h_{\alpha 1} h_{\alpha j} h_{\beta 1}^* h_{\beta j}^*)}{h_{\alpha 1} h_{\alpha 1}^*} \frac{M_1}{M_j}. \quad (10)$$

From this expression it is clear that the present contribution  $\delta$  is different from the contribution considered earlier, given by  $\epsilon'$ . There exist various choices of parameters for which  $\delta \gg \epsilon'$ . In all these cases, the matter–anti-matter oscillations will first create a  $CP$ –asymmetric universe with unequal densities of Majorana neutrinos and antineutrinos, whose decays generate a lepton asymmetric universe.

In the presence of sphalerons, this lepton number asymmetry will be converted to the baryon number asymmetry. After the electroweak phase transition, the baryon number of the universe will be given by,

$$n_B = - \left( \frac{8N_g + 4N_H}{22N_g + 13N_H} \right) n_{(B-L)} \sim \frac{1}{3} n_L \sim \frac{2\delta}{3g_*},$$

where  $N_H$  is the number of Higgs doublets and  $N_g$  the number of generations. For  $h_{13} \sim h_{11}$  and  $g_* \sim 400$ , the required amount of baryon asymmetry can be generated with a  $CP$ –violating phase  $\delta \sim 10^{-5} - 10^{-7}$ , which is a very natural choice.

We have shown that  $CP$ –violation in the mass matrix of heavy Majorana neutrinos produces a  $CP$  and lepton asymmetric universe. The same mass terms break the  $(B-L)$  symmetry and produce a net excess of this quantum number which survives all the way down to our epoch. The asymmetries created on the mass matrix are relatively large and are transformed into an excess of Baryons, when the electroweak phase transition takes place.

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Figure 1: One loop diagrams contributing to the  $CP$  violating  $|N_1 \rangle \rightarrow |N_2^c \rangle$  transition.