Non-perturbative unification
in the light of LEP results

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Abstract

We consider an alternative to conventional grand unified theories originally proposed by Maiani, Parisi and Petronzio, where owing to the existence of extra fermion generations at some intermediate scale, the gauge couplings become large at high energies. We first comment on how the non-supersymmetric version of this scenario is ruled out; we then consider the two-loop evolution of couplings in the supersymmetric extension of this scenario, and check whether such a scenario is feasible in the light of the precise values of couplings now available from LEP.

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Grand unified theories (GUTs) offer the possibility of unifying the SU(3), SU(2) and U(1) gauge groups of the standard model into one large group at a high energy scale, M_U. This scale is determined as the intersection point of the SU(3), SU(2) and U(1) couplings. The particle content of the theory completely determines the variation of the couplings with energy. Given the particle content of the theory, therefore, one can evolve the couplings determined at low energies to determine whether there is unification.

The determination of the couplings at LEP has important consequences for grand unified theories. The precise determination of the Weinberg angle, θ_W, and the strong coupling, α_s, at the Z-peak has helped in putting rather stringent constraints on unified models [1, 2]. In particular, it has been found [1] that in the standard model with three fermion generations and one Higgs doublet, the couplings do not meet at a single point at high energies. In contrast, in the minimal supersymmetric extension of the standard model (with three generations and two Higgs doublets), a single intersection point obtains at about 10^{16} GeV. The compatibility of this simple supersymmetric GUT with the couplings determined from LEP is remarkable. Nonetheless, it is important to study other models, which are alternatives to grand unification, and see whether they are viable in the light of the available experimental information on couplings.

An interesting alternative to GUTs was proposed by Maiani, Parisi and Petronzio [3] several years ago. In this scheme, the couplings enter a non-perturbative phase at a high energy scale, i.e. the theory is asymptotically divergent. Starting from the renormalisation group equation for a coupling α,

$$\frac{d\alpha}{dt} = \beta(\alpha), \tag{1}$$

where $\beta(\alpha)$ is the beta function and $t = \ln(Q^2/\mu^2)$, μ being some reference scale, we obtain

$$t = \int_{\alpha(\mu)}^{\alpha(Q^2)} \frac{d\alpha}{\beta(\alpha)}. \tag{2}$$

For $\beta(\alpha) > 0$ (asymptotically divergent theory) there is a value of t, given by

$$t = \int_{\alpha(\mu)}^{\infty} \frac{d\alpha}{\beta(\alpha)} < \infty, \tag{3}$$

for which $\alpha \to \infty$. If perturbation theory is to be valid at all energy scales, we require $\alpha(\mu) = 0$, so that $t_c = \infty$, $\alpha(\mu) = 0$ is the infra-red fixed point. But if $\alpha(\mu) \neq 0$ but small, i.e. it is sufficiently close to the infra-red fixed point, then there is a finite cut-off in energy beyond which the theory is non-perturbative.

In Ref. [3], it was assumed that the standard SU(3)×SU(2)×U(1) theory, due to new fermion generations that get switched on around the weak scale $\Lambda_F = 250$ GeV, is asymptotically divergent beyond Λ_F. The couplings $\alpha_{1,2,3}$ are sufficiently close to zero at Λ_F but not quite zero. As a consequence, the theory is cut off at a scale Λ. At this scale, the most interesting situation is that not just one but all three couplings are large, i.e. of O(1). In fact, it has been shown [4] that such a non-perturbative scenario exhibits a "trapping" mechanism, whereby if one of the couplings grows large, the other couplings

will also increase. This effect, by means of which all three couplings are large and of the same order of magnitude at Λ, leads to what is called non-perturbative unification. In Ref. [3] the cut-off scale Λ was assumed to be the Planck scale; however, in subsequent studies [5, 6], Λ was determined to be of the order of 10^{15}–10^{17} GeV. Since the low-energy couplings are close to the infra-red fixed point, they are insensitive to the values of the couplings at the scale Λ.

One natural extension of the above scenario is the inclusion of supersymmetry. This was first considered in Ref. [5], and was later discussed in Refs. [6, 7]. Other than solving the hierarchy problem, the inclusion of supersymmetry is attractive because it provides a framework for the existence of new particles needed to make the theory asymptotically divergent. In the case of the simplest $N = 1$ supersymmetric extension of the scenario, it suffices to consider $n_f = 5$, where n_f is the number of fermion generations.

In this letter, we use the recent LEP values to check whether any strong constraints on the non-perturbative unification scenario can be obtained. The values of $\sin^2\theta_W$ and α_s from LEP are very precise compared to that available from older experiments. One strong constraint is on the number of extra chiral generations. The present limit on the oblique parameters S, T and U allows only three chiral fermion generations, while the vectorial generations are not constrained. Thus in addition to the three chiral fermion generations we are allowed to have only an even number of generations.

We shall first specify the supersymmetric non-perturbative unification scenario in detail. While discussing the results we shall also comment on the results of the non-supersymmetric case. We consider an SU(3)×SU(2)×U(1) supersymmetric gauge theory with the assumption that an $N = 1$ supersymmetry holds above the scale Λ_s. We assume $n_f = 5$ supersymmetric generations and two Higgs supermultiplets. In the discussion of the non-supersymmetric case we shall consider one Higgs scalar and $n_f = 8$ and 9. From the requirement that the Yukawa couplings do not become arbitrarily large, a bound on the fermion masses can be obtained [8, 9]. This bound is that fermion masses are, in general, smaller than 200–250 GeV. We assume that the extra fermion generations, which are required for the theory to be asymptotically divergent, are of the order of 250 GeV in mass.

Having specified the theory we can now address the question of the evolution of the three couplings. The two-loop renormalisation group equations for the couplings are given by the following coupled differential equations:

$$\mu\frac{d\alpha_i(\mu)}{d\mu} = \frac{1}{2\pi}\left[a_i + \frac{b_{ij}}{4\pi}\alpha_j(\mu) + \frac{b_{ik}}{4\pi}\alpha_k(\mu)\right]\alpha_i^2(\mu) + \frac{2b_{ij}}{(4\pi)^2}\alpha_i^3(\mu), \tag{4}$$

where i, j, k = 1, 2, 3 and $i \neq j \neq k$, and a_i and b_{ij} are the one- and two-loop beta function coefficients. In the range of energies between M_Z and the supersymmetric threshold, M_s, we use the non-supersymmetric beta functions to evolve the couplings, whereas from M_s onward the supersymmetric beta functions are effective. We retrieve the result for the non-supersymmetric scenario by taking $M_s = \Lambda_{MPP}$ and large n_f.

In the non-supersymmetric case the one-loop beta function coefficients are [10]

$$b_j = \begin{pmatrix} 0 \\ -\frac{22}{3} \\ -11 \end{pmatrix} + n_f \begin{pmatrix} \frac{20}{9} \\ \frac{4}{3} \\ \frac{4}{3} \end{pmatrix} + n_h \begin{pmatrix} \frac{1}{6} \\ \frac{1}{6} \\ 0 \end{pmatrix} \tag{5}$$

while the two-loop beta functions are

$$a_{ij} = -\begin{pmatrix} 0 & 0 & 0 \\ 0 & \frac{136}{3} & 0 \\ 0 & 0 & 102 \end{pmatrix} + n_f \begin{pmatrix} \frac{95}{27} & 1 & \frac{44}{9} \\ \frac{1}{3} & \frac{49}{3} & 4 \\ \frac{11}{18} & \frac{3}{2} & \frac{76}{3} \end{pmatrix} + n_h \begin{pmatrix} \frac{1}{2} & \frac{13}{6} & 0 \\ \frac{1}{2} & \frac{13}{6} & 0 \\ 0 & 0 & 0 \end{pmatrix} \tag{6}$$

In the supersymmetric case the one-loop beta functions take the form [10]

$$b_j = \begin{pmatrix} 0 \\ -6 \\ -9 \end{pmatrix} + n_f \begin{pmatrix} \frac{10}{3} \\ 2 \\ 2 \end{pmatrix} + n_h \begin{pmatrix} \frac{1}{2} \\ \frac{1}{2} \\ 0 \end{pmatrix} \tag{7}$$

while the two-loop beta functions are

$$a_{ij} = -\begin{pmatrix} 0 & 0 & 0 \\ 0 & 24 & 0 \\ 0 & 0 & 54 \end{pmatrix} + n_f \begin{pmatrix} \frac{190}{27} & 2 & \frac{88}{9} \\ \frac{2}{3} & 14 & 8 \\ \frac{11}{9} & 3 & \frac{68}{3} \end{pmatrix} + n_h \begin{pmatrix} \frac{1}{2} & \frac{3}{2} & 0 \\ \frac{1}{2} & \frac{7}{2} & 0 \\ 0 & 0 & 0 \end{pmatrix} \tag{8}$$

In all these equations, n_f and n_h denote the number of fermion generations and the number of Higgs doublets respectively.

We integrate the coupled differential equations in Eq. (4) numerically, with the initial values of the three couplings $\alpha_{1,2,3}$ taken to be of O(1) at the unification scale Λ. What we do in practice is to evolve downwards using the renormalisation group equations for several values of Λ, and check what the predicted values of the couplings at the scale M_Z are. The extra fermion generations are assumed to contribute to the beta functions for all energies greater than 250 GeV.

We shall first comment on the non-supersymmetric scenario and then present our main result, namely the supersymmetric extension. In this case we find that for $n_f \leq 8$, $\alpha_2(M_z)$ remains too small, and that $\alpha_{1,2}(M_z)$ falls within the experimental bound for $n_f \geq 9$. But for $n_f \geq 9$ the strong coupling constant evolves extremely fast and $\alpha_3(M_z)$ becomes too large. Thus the precision LEP data rule out the non-supersymmetric scenario completely.

The results of the computation for the supersymmetric version are shown in Fig. 1, where $\alpha_{1,2,3}(M_Z)$ are shown as a function of Λ. The solid, dashed and dotted curves are for $M_s = 250$ GeV, 1.2 TeV and 5 TeV, respectively. The horizontal lines in the figures show the upper and lower bounds on the couplings at M_Z as determined by the LEP experiment. These are as follows [11]:

$$\begin{aligned} \alpha_1 &= 0.0101322 \pm 0.000024 \\ \alpha_2 &= 0.03322 \pm 0.00025 \\ \alpha_3 &= 0.120 \pm 0.006. \end{aligned} \tag{9}$$

It is clear from the figure that the non-perturbative unification scheme is certainly viable if we have $M_s = 1.2$ TeV and Λ close to 0.78×10^{17} GeV. We have checked that the range of values allowed is $M_s = 1.2 \pm 0.2$ TeV and $\Lambda = (0.7\text{--}0.8) \times 10^{17}$ GeV. We have also checked that the couplings at M_Z are not sensitive to the choice of the couplings at Λ. We have checked this by varying these from 0.75 to 10.

Let us now summarise our results. We have studied the non-perturbative unification scenario first proposed by Maiani, Parisi and Petronzio. We point out that the non-supersymmetric version of this scenario is ruled out by LEP data. However, the supersymmetric extension of this scenario remains a viable alternative to conventional grand unified theories and is capable of predicting the precision values of couplings determined from LEP. Our numerical results show that the non-perturbative scale, Λ, at which all couplings are large, is around $0.7\text{--}0.8 \times 10^{17}$ GeV, with the supersymmetric threshold M_s around 1.0–1.4 TeV. If the scale M_s gets either larger or smaller it is then not possible to reproduce the values of the couplings at M_Z. We should note that the agreement with the data is obtained only for a constrained range of parameters of this scenario. In principle, the effect of higher-order corrections could be large and this may ruin the agreement. It is also likely that more accurate measurements of the strong coupling α_3 at low energies may be sufficient to either put strong constraints or completely rule out this scenario. It is nevertheless interesting that this scenario, at the two-loop level, is a possible alternative to conventional grand unification.

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Figure caption

Fig. 1 The couplings $\alpha_{1,2,3}(M_Z)$ as a function of the non-perturbative unification scale Λ. The solid, dashed and dotted curves are for M_s=250 GeV, 1.2 TeV and 5 TeV, respectively. The horizontal lines in the figures show the experimentally allowed upper and lower bounds on the couplings at M_Z.