Neutrino Dark Energy in Grand Unified Theories

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We studied a left-right symmetric model that can accommodate the neutrino dark energy (νDE) proposal. Type III seesaw mechanism is implemented to give masses to the neutrinos. After explaining the model, we study the consistency of the model by minimizing the scalar potential and obtaining the conditions for the required vacuum expectation values of the different scalar fields. This model is then embedded in an SO(10) grand unified theory and the allowed symmetry breaking scales are determined by the condition of the gauge coupling unification. Although $SU(2)_R$ breaking is required to be high, its Abelian subgroup $U(1)_R$ is broken in the TeV range, which can then give the required neutrino masses and predicts new gauge bosons that could be detected at LHC. The neutrino masses are studied in details in this model, which shows that at least 3 singlet fermions are required.

Introduction

During the past couple of decades, astrophysical observations has improved our knowledge of cosmology tremendously. One of the most important discovery resulting from these observations is that of the dark energy[1]. Nature of the dark energy(DE) is one of the most puzzling question of physics. The observations suggest that currently , i.e. around redshift $z \sim 1$, the DE is contributing around 70% of the total energy budget of the universe, while its contribution was sub-dominant in the past($z \gg 1$). Any proposed model of DE is required to satisfy these observational constraints. These models require the mass of the scalar to be very light having scale same as Hubble scale ($\sim 10^{-33} eV$). There exist myriad of such models describing the nature and the dynamics of DE (for recent reviews see Ref. [2]]. One of the very interesting proposal for the DE is based on the fact that typical energy scale of DE $\rho_\Lambda \sim (3 \times 10^{-3}\,\mathrm{eV})^4$ also coincides with the neutrino mass scale $\rho_\Lambda \sim m_\nu^4$. This has led to several attempts to relate the origin of the dark energy with the neutrino masses [3, 4, 5, 6] and this connection can have many interesting consequences [7, 8]. In this scenario a scalar field $\mathcal{A}$ called the acceleron couples with the neutrinos and consequently making the neutrino mass m_ν function of $\mathcal{A}$. Next, it is assumed that the dark energy ρ_{DE} can be written as

$$\rho_{DE} = \rho_\nu + V(\mathcal{A}).$$

Stationary condition on ρ_{DE} then lead to varying the neutrino mass. These type of models are called mass varying neutrino (MaVaN) models [3, 4, 5]. In a typical MaVaN scenario, the standard model is extended by including singlet right-handed neutrinos $N_i, i = 1, 2, 3$, and giving a Majorana mass to the neutrinos which varies with ϕ_a. At present our understanding of MaVaN models is far from being complete, several problems regarding nature origin and nature of the acceleron field, about its stability [4, 9] etc. continue to remain. There has been a significant progress in solving some of these problems in the subsequent works [10, 11], but much more needs to be done before this idea could be considered as a realistic one.

Considering the difficulties involved in constructing a reasonable MaVaNs model, most of the earlier models restricted themselves to start with the standard model and include a singlet right-handed neutrino, or else, include a triplet Higgs scalar. Some time back we constructed a left-right symmetric model with right-handed neutrinos and type-III seesaw neutrino masses, which could explain the dark energy with MaVaNs [12]. In this article we work out some of the details of that model and embed the model in a grand unified theory. The most important feature of this model is that the model justifies the smallness of the very low scale, entering in this model. We have analyzed the consistency of the problem by minimizing the scalar potential and then have found the conditions for the required minima that explains the required mass scales in this MaVaNs model. We also study the gauge coupling unification in the SO(10) GUT, in which this model has been embedded. The neutrino masses have also been studied and some conditions on the number of the singlet fermions have been worked out.

The Model

One of the problems with the original MaVaNs is that the condition from naturalness requires the Majorana masses of the right-handed neutrinos, which varies with the acceleron field, to be in the range of eV. In such models of type

I seesaw, the model becomes void of any seesaw, since the smallness of the neutrino masses can not be attributed to any large lepton number violating scale. Another restrictions of this model is that the model cannot be embedded in any left-right symmetric extension of the standard model, because the equal treatment of the left-handed and the right-handed fields would imply that if neutrino masses vary with the value of the acceleron field, the charged fermion masses would also vary and that would relate the scale of dark energy to the top quark mass scale, which is unacceptable. Although the constraint from the naturalness condition can be softened in the νDEmodels with triplet Higgs scalars [11], this cannot be embedded in a left-right symmetric model. We consider here a left-right symmetric model, where the neutrino masses originate from double seesaw or type III seesaw mechanism and then show how this model can be embedded in a grand unified theory.

In the left-right symmetric models, the standard model gauge group is extended to a left-right symmetric gauge group [13], $G_{LR} \equiv SU(3)_c \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$, so that the electric charge is defined in terms of the generators of the group as:

$$Q = T_{3L} + T_{3R} + \frac{B-L}{2} = T_{3L} + Y\,. \tag{1}$$

The quarks and leptons transform under the left-right symmetric gauge group as:

$$\begin{aligned} Q_L = \begin{pmatrix} u_L \\ d_L \end{pmatrix} \equiv [3,2,1,\frac{1}{3}] \quad , \quad Q_R = \begin{pmatrix} u_R \\ d_R \end{pmatrix} \equiv [3,1,2,\frac{1}{3}]\,, \\ \ell_L = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix} \equiv [1,2,1,-1] \quad , \quad \ell_R = \begin{pmatrix} N_R \\ e_R \end{pmatrix} \equiv [1,1,2,-1]\,, \\ S_R \equiv [1,1,1,0]\,. \end{aligned} \tag{2}$$

The right-handed neutrinos N_R is present in all the left-right symmetric model, which is dictated by the structure of the fermion representations and the gauge group. However, in models with type III seesaw mechanism for neutrino masses one introduces an additional singlet fermion S_R. As the name left-right symmetric model, the model Lagrangian is invariant under the left-right parity transformation given as:

$$\begin{aligned} SU(2)_L &\leftrightarrow SU(2)_R\,, \\ Q_L &\leftrightarrow Q_R\,, \\ \ell_L &\leftrightarrow \ell_R\,. \end{aligned}$$

In this model we have introduced the singlet field S_R, but there is no S_L. But still this model is consistent with left-right parity operation, since the field S_R transform to its CP conjugate state under the left-right parity as: $S_R \leftrightarrow S^c{}_L$. This also ensures that the Majorana mass term is invariant under the parity transformation, because this field S_R transform under the transformation $SU(2)_L \leftrightarrow SU(2)_R$ to itself $S_R \equiv (1,1,1,0) \leftrightarrow (1,1,1,0)$.

The gauge boson (excluding gluons) sector consist of two triplet and one singlet as :

$$W_{\mu L} = \begin{pmatrix} W^+_{L\mu} \\ W^0_{L\mu} \\ W^-_{L\mu} \end{pmatrix} \equiv (1,3,1,0),\; W_{\mu R} = \begin{pmatrix} W^+_{R\mu} \\ W^0_{R\mu} \\ W^-_{R\mu} \end{pmatrix} \equiv (1,1,3,0),\; B_{\mu(B-L)} \equiv (1,1,1,0)$$

There exists several choices of the Higgs scalars, and hence, the choices of symmetry breaking chain. In the present model, the content of the Higgs sector will be chosen according to the following desired symmetry breaking pattern[14]:

$$\begin{aligned} & SU(3)_c \times SU(2)_L \times SU(2)_R \times U(1)_{(B-L)} \qquad [G_{3221D}] \\ & \stackrel{M_R}{\rightarrow} SU(3)_c \times SU(2)_L \times U(1)_R \times U(1)_{(B-L)} \quad [G_{3211}] \\ & \stackrel{m_r}{\rightarrow} SU(3)_c \times SU(2)_L \times U(1)_Y \qquad [G_{321}] \\ & \stackrel{m_W}{\rightarrow} SU(3)_c \times U(1)_Q \qquad [G_{em}]\,. \end{aligned}$$

Breaking of the left-right symmetric group to G_{3211} requires a right triplet Higgs scalars Δ_R transforming as $\Delta_R \equiv (1,1,3,0)$. The triplet does not change the rank of the gauge group and only breaks $SU(2)_R \rightarrow U(1)_R$. Since it does not carry any $U(1)_{B-L}$ quantum number, it cannot give any Majorana masses to the neutral fermions. For the next symmetry breaking stage, $U(1)_R \times U(1)_{B-L} \rightarrow U(1)_Y$, we introduce an $SU(2)_R$ doublet Higgs scalar field

$\chi_R \equiv (1,1,2,1)$ [15, 16]. The *vev* of χ_R could also break $[G_{3221D}] \rightarrow [G_{321}]$, if the field Δ_R were not present. The left-right parity would then require the existence of the fields $\Delta_L \equiv (1,3,1,0)$ and $\chi_L \equiv (1,2,1,1)$. Finally, the standard model symmetry breaking is mediated by a bi-doublet field $\Phi \equiv (1,2,2,0)$, like in any other left-right symmetric model. This field has the Yukawa interaction with the standard model fermions and provide Dirac masses to all of them. We shall introduce one more Higgs bi-doublet scalar $\Psi \equiv (1,2,2,0)$ that is needed for the purpose of our model. We also introduce another singlet scalar field $\eta \equiv (1,1,1,0)$, which acquires a tiny *vev* of the order of the light neutrino masses and generate the mass scale for the dark energy naturally.

Now we write down the explicit forms of all the scalar fields in terms of their components as

$$
\begin{aligned}
\Delta_L &= \begin{pmatrix} \Delta_L^0 & \Delta_L^+ \\ \Delta_L^- & -\Delta_L^0 \end{pmatrix} \ , \ \Delta_R = \begin{pmatrix} \Delta_R^0 & \Delta_R^+ \\ \Delta_R^- & -\Delta_R^0 \end{pmatrix} , \\
\Phi &= \begin{pmatrix} \phi_1^0 & \phi_1^+ \\ \phi_2^- & \phi_2^0 \end{pmatrix} \ , \ \Psi = \begin{pmatrix} \psi_1^0 & \psi_1^+ \\ \psi_2^- & \psi_2^0 \end{pmatrix} , \\
\chi_L &= \begin{pmatrix} \chi_L^+ \\ \chi_L^0 \end{pmatrix} \ , \ \chi_R = \begin{pmatrix} \chi_R^+ \\ \chi_R^0 \end{pmatrix} ,
\end{aligned}
$$

The most general scalar potential has to be constructed in such a way that they respect the left-right parity transformation of the scalar fields listed below:

$$
\begin{aligned}
\chi_L \leftrightarrow \chi_R \ &, \ \Delta_L \leftrightarrow \Delta_R \\
\Phi \leftrightarrow \Phi^\dagger \ &, \ \Psi \leftrightarrow \Psi^\dagger \\
. \ & \ \ \eta \leftrightarrow \eta \, .
\end{aligned}
$$

Under the left-right gauge group transformation, the Higgs fields transform as

$$
\begin{aligned}
\Delta_L \rightarrow U_L \ \Delta_L \ U_L^\dagger \ &, \ \Delta_R \rightarrow U_R \ \Delta_R \ U_R^\dagger \\
\Phi \rightarrow U_L \ \Phi \ U_R^\dagger \ &, \ \Psi \rightarrow U_L \ \Psi \ U_R^\dagger \\
\chi_L \rightarrow U_L \ \chi_L \ &, \ \chi_R \rightarrow U_R \ \chi_R \\
& \ \ \eta \rightarrow \eta \, .
\end{aligned}
$$

In order to write down the scalar potential we also construct the fields $\tau^2 \Phi^* \tau^2$ and $\tau^2 \Psi^* \tau^2$ from Φ and Ψ which transform in the same ways as Φ and Ψ. For convenience, we represent Φ as ϕ_1, $\tau^2 \Phi \tau^2$ as ϕ_2 (and similarly for Ψ) from now on.

Potential Minimization

We first write down the most general renormalizable gauge invariant scalar potential respecting left-right parity and study details of potential minimization. Besides left-right parity, we impose following Z_4 symmetry on only the Higgs potential to avoid few undesired terms

$$
\begin{aligned}
\chi_L \rightarrow i\chi_L \, , &\quad \chi_R \rightarrow -i\chi_R \, , \\
\Delta_L \rightarrow -\Delta_L \, , &\quad \Delta_R \rightarrow -\Delta_R \, , \\
\Phi \rightarrow \Phi \ \ , &\quad \ \ \Psi \rightarrow -\Psi \, , \\
&\quad \ \ \eta \rightarrow \eta \, .
\end{aligned}
\tag{3}
$$

We write the the Higgs potential as a sum of of various parts and write down each part separately as:

$$
\begin{aligned}
V &= V_\phi + V_\psi + V_\Delta + V_\eta + V_\chi + V_{\Delta\phi\psi} + V_{\chi\phi\psi} + V_{\eta\chi\Delta\phi\psi} \\
V_\phi &= -\sum_{i,j} \frac{\mu^2_{\phi ij}}{2}\ tr(\phi_i^\dagger \phi_j) + \sum_{i,j,k,l} \frac{\lambda_{\phi ijkl}}{4}\ tr(\phi_i^\dagger \phi_j)\ tr(\phi_k^\dagger \phi_l) \\
&\quad + \sum_{i,j,k,l} \frac{\Lambda_{\phi ijkl}}{4}\ tr(\phi_i^\dagger \phi_j \phi_k^\dagger \phi_l) \\
V_\psi &= -\sum_{i,j} \frac{\mu^2_{\psi ij}}{2}\ tr(\psi_i^\dagger \psi_j) + \sum_{i,j,k,l} \frac{\lambda_{\psi ijkl}}{4}\ tr(\psi_i^\dagger \psi_j)\ tr(\psi_k^\dagger \psi_l) \\
&\quad + \sum_{i,j,k,l} \frac{\Lambda_{\psi ijkl}}{4}\ tr(\psi_i^\dagger \psi_j \psi_k^\dagger \psi_l) \\
V_\Delta &= -\frac{\mu^2_\Delta}{2}\ [tr(\Delta_L \Delta_L) + tr(\Delta_R \Delta_R)] + \frac{\lambda_\Delta}{4}\ [tr(\Delta_L \Delta_L)^2 + tr(\Delta_R \Delta_R)^2] \\
&\quad + \frac{\Lambda_\Delta}{4}\ [tr(\Delta_L \Delta_L \Delta_L \Delta_L) + tr(\Delta_R \Delta_R \Delta_R \Delta_R)] \\
&\quad + \frac{g_\Delta}{2}\ [tr(\Delta_L \Delta_L)\ tr(\Delta_R \Delta_R)] \\
V_\eta &= \frac{M_\eta^2}{2}\ \eta^2 + \frac{\lambda_\eta}{4}\ \eta^4 \\
V_\chi &= -\frac{\mu^2_\chi}{2}\ [\chi_L^\dagger \chi_L + \chi_R^\dagger \chi_R] + \frac{\lambda_\chi}{4}\ [(\chi_L^\dagger \chi_L)^2 + (\chi_R^\dagger \chi_R)^2] \\
&\quad + \frac{g_\chi}{2}\ [\chi_L^\dagger \chi_L\ \chi_R^\dagger \chi_R] \\
V_{\Delta\phi\psi} &= \sum_{i,j} \alpha_{\phi ij}\ [\Delta_L \Delta_L + \Delta_R \Delta_R]\ tr(\phi_i^\dagger \phi_j) \\
&\quad + \sum_{i,j} \alpha_{\psi ij}\ [\Delta_L \Delta_L + \Delta_R \Delta_R]\ tr(\psi_i^\dagger \psi_j) \\
&\quad + \sum_{i,j} \beta_{\phi ij}\ [\ tr(\Delta_L \Delta_L \phi_i \phi_j^\dagger) + tr(\Delta_R \Delta_R \phi_i^\dagger \phi_j)] \\
&\quad + \sum_{i,j} \beta_{\psi ij}\ [\ tr(\Delta_L \Delta_L \psi_i \psi_j^\dagger + tr(\Delta_R \Delta_R \psi_i^\dagger \psi_j)] \\
&\quad + \sum_{i,j} h_{\Delta\phi ij}\ tr(\Delta_L \phi_i \Delta_R \phi_j^\dagger) + \sum_{i,j} h_{\Delta\psi ij} tr(\Delta_L \psi_i \Delta_R \psi_j^\dagger) \\
V_{\chi\phi\psi} &= \sum_{i,j} h_{\phi\chi ij}\ [\chi_L^\dagger \chi_L + \chi_R^\dagger \chi_R]\ tr(\phi_i^\dagger \phi_j) \\
&\quad + \sum_{i,j} h_{\psi\chi ij}\ [\chi_L^\dagger \chi_L + \chi_R^\dagger \chi_R]\ tr(\psi_i^\dagger \psi_j) \\
V_{\eta\chi\Delta\phi\psi} &= \Big(h_{\eta\chi}\ [\chi_L^\dagger \chi_L + \chi_R^\dagger \chi_R] + h_{\eta\Delta}\ [tr(\Delta_L \Delta_L) + tr(\Delta_R \Delta_R)]\Big)\ \eta^2 \\
&\quad + \left(\sum_{i,j} h_{\eta\phi ij}\ tr(\phi_i^\dagger \phi_j) + \sum_{i,j} h_{\eta\psi ij}\ tr(\psi_i^\dagger \psi_j)\right)\ \eta^2 \\
&\quad + \sum_{i,j} h_{\eta ij}\ \eta\ [tr(\phi_i^\dagger \Delta_L \psi_j) + tr(\phi_i \Delta_R \psi_j^\dagger) + h.c.] \\
&\quad + \sum_{i} h_{\chi i}\ \eta\ [\chi_L^\dagger \phi_i \chi_R + h.c.]\,.
\end{aligned}
$$

We parametrize the true minima of the potential by giving vacuum expectation values to different scalar fields as follows.

$$\phi_1 = \begin{pmatrix} v & 0 \\ 0 & v' \end{pmatrix}, \phi_2 = \begin{pmatrix} v' & 0 \\ 0 & v \end{pmatrix}, \quad \psi_1 = \begin{pmatrix} w & 0 \\ 0 & w' \end{pmatrix}, \quad \psi_2 = \begin{pmatrix} w' & 0 \\ 0 & w \end{pmatrix},$$
$$\chi_L = \begin{pmatrix} 0 \\ v_L \end{pmatrix}, \quad \chi_R = \begin{pmatrix} 0 \\ v_R \end{pmatrix}, \quad \Delta_L = \begin{pmatrix} u_L & 0 \\ 0 & -u_L \end{pmatrix}, \; \Delta_R = \begin{pmatrix} u_R & 0 \\ 0 & -u_R \end{pmatrix}$$
$$\eta = u\,.$$

Since the phenomenological consistency requires $v \gg v'$and $w \gg w'$, we ignore potential terms involving $v'$and w' and write down the general scalar potential in terms of vacuum expectation values of different scalar fields

$$\begin{aligned} V \;=\; & -\frac{\mu_\phi^2}{2}\, v^2 + \frac{\lambda_\phi}{4}\, v^4 - \frac{\mu_\psi^2}{2}\, w^2 + \frac{\lambda_\psi}{4}\, w^4 \\ & -\frac{\mu_\Delta^2}{2}\, (u_L^2 + u_R^2) + \frac{\lambda_\Delta}{4}\, (u_L^4 + u_R^4) \\ & +\frac{M_\eta^2}{2}\, u^2 + \frac{\lambda_\eta}{4}\, u^4 \\ & -\frac{\mu_\chi^2}{2}\, (v_L^2 + v_R^2) + \frac{\lambda_\chi}{4}\, (v_L^4 + v_R^4) + \frac{g_\chi}{2}\, (v_L^2\, v_R^2) \\ & +[(\alpha_\phi + \beta_\phi)v^2 + (\alpha_\psi + \beta_\psi)w^2]\, (u_L^2 + u_R^2) + (h_{\Delta\phi} v^2 + h_{\Delta\psi} w^2)\, u_L u_R \\ & +(h_{\phi\chi} v^2 + h_{\psi\chi} w^2)\, (v_L^2 + v_R^2) \\ & +[h_{\eta\chi}(v_L^2 + v_R^2) + h_{\eta\Delta}(u_L^2 + u_R^2) + h_{\eta\phi} v^2 + h_{\eta\psi} w^2]\, u^2 \\ & +h_\eta\, u(u_L + u_R)vw + h_\chi\, u(v_L v_R)v\,. \end{aligned}$$

For convenience, we have replaced $\lambda_\phi + \Lambda_\phi \rightarrow \lambda_\phi, \quad \lambda_\psi + \Lambda_\psi \rightarrow \lambda_\psi, \quad \lambda_\Delta + \Lambda_\Delta \rightarrow \lambda_\Delta$. The minimization of the potential is studied by taking partial derivatives with respect to *vevs* of all Higgs fields and then separately equating them to zero. Solving all such equations will provide us the desired values. One of the minimization conditions $v_L \left(\frac{\partial V}{\partial v_R}\right) - v_R \left(\frac{\partial V}{\partial v_L}\right) = 0$ leads to the following relation between v_L and v_R:

$$(v_R^2 - v_L^2)\;\; [(\lambda_\chi - g_\chi) v_L v_R - h_\chi u v] = 0\,.$$

Since $(v_R^2 = v_L^2)$ is not desirable phenomenologically, we chose

$$v_L v_R = \frac{h_\chi u v}{(\lambda_\chi - g_\chi)}\,. \tag{4}$$

Using above relation in an another minimization condition $v_L \left(\frac{\partial V}{\partial v_R}\right) + v_R \left(\frac{\partial V}{\partial v_L}\right) = 0$, we get

$$v_L^2 + v_R^2 = -\frac{\mu_\chi^2}{\lambda_\chi}\,. \tag{5}$$

Parametrizing $v_L = A\ \sin\theta$, $v_R = A\ \cos\theta$ and putting them in the two equations 4 and 5 , we find $A = -\mu_\chi^2/\lambda_\chi$ $\sin 2\theta = 2\theta = \frac{2h_\chi uv}{(\lambda_\chi - g_\chi)}$ since μ_χ is a large number compared to the numerator. So we get

$$\begin{aligned} v_R \;&=\; A = \sqrt[2]{-\mu_\chi^2/\lambda_\chi}\,, \\ v_L \;&=\; A\theta = \frac{\lambda_\chi h_\chi}{(g_\chi - \lambda_\chi)}\; \frac{u v v_R}{\mu_\chi^2}\,. \end{aligned}$$

We have chosen the parametrization of v_L and v_R in such a way that v_R gets value equal to breaking scale of G_{3211} and v_L gets a very small value. We could have done other way around but that is not what is phenomenologically allowed. Proceeding with the same kind of analysis for u_L and u_R, i.e., using two minimization conditions $u_L\left(\frac{\partial V}{\partial u_R}\right) - u_R\left(\frac{\partial V}{\partial u_L}\right) = 0$ and $u_L\left(\frac{\partial V}{\partial u_R}\right) + u_R\left(\frac{\partial V}{\partial u_L}\right) = 0$, we get

$$\begin{aligned} u_R &= \sqrt[2]{-\mu_\Delta^2/\lambda_\Delta}\,, \\ u_L &= \frac{\lambda_\Delta h_\Delta}{(g_\Delta - \lambda_\Delta)}\;\frac{(h_{\Delta\phi}v^2 + h_{\Delta\psi}w^2)u_R}{\mu_\Delta^2}\,. \end{aligned}$$

Now using equation 4, the η field can be shown to get vev only by term $h_\eta u(u_L + u_R)$ as only this term is linear in u. The term $h_\chi u(v_L v_R)v$ does not remain linear in u after we substitute the value of $v_L v_R$ from equation 4. Since the mass term for η field is large and positive, we expect very small vev. So we can ignore some of the terms in the potential while solving for u and can easily obtain

$$u = \frac{h_\eta v w(u_L + u_R)}{M_\eta^2 - (h_{\eta\Delta}\mu_\Delta^2/\lambda_\Delta) - (h_{\eta\chi}\mu_\chi^2/\lambda_\chi)}\,.$$

After analyzing the complete scalar potential, we find a consistent solution with ordering

$$u_R \gg v_R > v > w \gg u \gg v_L\,. \tag{6}$$

At this stage we can assume the different mass scales to explain the model. However, when we embed this model in an $SO(10)$ grand unified theory, the gauge coupling unification will impose strong constraints on the different symmetry breaking scales. The left-right parity and the $SU(2)_R$ breaking scale will come out to be above 10^{11} GeV. So, we shall assume $u_R \sim 10^{11}$ GeV. We also assume $m_\eta \sim m_\Delta \sim u_R$. However, it will be possible to keep the G_{3211} symmetry breaking scale to be very low, and hence, we shall assume $m_\chi \sim v_R \sim$ TeV. We find the remaining mass scales to be $v \sim m_w \sim 100$ GeV, $u \sim u_L \sim$ eV and $v_L \sim 10^{-2}$ eV.

Embedding The Model In $SO(10)$GUT

The idea of Grand Unified Theories (GUTs) has emerged as a very attractive idea to go beyond Standard Model (SM) for last three decades. It unifies the three different looking gauge coupling constants of the SM, and in addition, reduces the number of particle irreducible multiplets into lesser number of multiplets. The ad-hoc looking hypercharge assignment in SM gets a predictive framework in GUTs, i.e, the charge quantization remains no more a surprise in GUTs. The smallest GUT $SU(5)$, in its non-supersymmetric version, does not unify the three gauge coupling constants. Out of the higher rank gauge groups containing SM gauge group as a subgroup, the rank four semi-simple group $SO(10)$ has emerged as a very attractive candidate for GUT. It can accommodate the entire SM fermion content in its single 16-dimensional complex irreducible spinor representation including the right handed neutrino with three copies for the three families. Its all irreducible representations are anomaly free providing a natural predictive framework to understand the fermion masses and mixing. Also the seesaw structure gets a natural embedding in $SO(10)$. The left-right symmetry group can also be embedded in $SO(10)$ GUT.

We shall study here the embedding of the present model with all its Higgs content in $SO(10)$ GUT. We consider the following breaking pattern of $SO(10)$ gauge group to first Pati-Salam gauge group $SU(4) \times SU(2)_L \times SU(2)_R$, next to the left-right gauge group and then to the SM gauge group

$$\begin{array}{rlll} SO(10) & \stackrel{M_U}{\rightarrow} & SU(4) \times SU(2)_L \times SU(2)_R & [G_{422D}] \\ & \stackrel{M_C}{\rightarrow} & SU(3)_c \times SU(2)_L \times SU(2)_R \times U(1)_{(B-L)} & [G_{3221D}] \\ & \stackrel{M_R}{\rightarrow} & SU(3)_c \times SU(2)_L \times U(1)_R \times U(1)_{(B-L)} & [G_{3211}] \\ & \stackrel{m_r}{\rightarrow} & SU(3)_c \times SU(2)_L \times U(1)_Y & [G_{321}] \\ & \stackrel{m_W}{\rightarrow} & SU(3)_c \times U(1)_Q & [G_{em}]\,. \end{array}$$

The Higgs multiplets which can provide the masses for all the SM fermions are limited as $16 \times 16 = 10_s + 120_a + \overline{126}_s$. The 10 dimensional Higgs field H_Φ decomposes under left-right gauge group as

$$H_\Phi\,(10) = \Phi(1,2,2;0) \oplus (3,1,1;-\frac{1}{3}) \oplus (\overline{3},1,1;\frac{1}{3})\,.$$

One can easily identify the bi-doublet $\Phi(1,2,2;0)$ appearing in the left-right model contained in $H_\Phi(10)$. To include another bi-doublet $\Psi(1,2,2;0)$ present in the model, a second Higgs field $H_\Psi(10)$.

Although the fermion and gauge sector of the $SO(10)$ GUT model are quite simple, the Higgs sector is quite complicated since it is not only required for generating fermion Masses, but an appropriate Higgs content is also needed for systematic and consistent breaking of the $SO(10)$ gauge group down to the SM gauge group in one or more steps. To break $SO(10)$ gauge group to the Pati-Salam gauge group, one requires Higgs field either $S(54)$ or $\Upsilon(210)$, which decompose under Pati-Salam group as

$$\begin{aligned}
S(54) &= (1,1,1) \oplus (1,3,3) \oplus (20,1,1) \oplus (6,2,2)\,, \\
\Upsilon(210) &= (1,1,1) \oplus (15,1,1) \oplus (6,2,2) \oplus (15,3,1) \\
&\quad \oplus (15,1,3) \oplus (10,2,2) \oplus (\overline{10},2,2)\,.
\end{aligned}$$

Giving *vev* to either of the two fields in the singlet direction will serve the purpose of the desired breaking. The $(15,1,1)$ of Υ also has a singlet under the left-right gauge group which can acquire *vev* to break the Pati-Salam group to the left-right group. The $(15,3,1)$ and $(15,1,3)$ Higgs multiplets of Υ also contain the fields $\Delta_L(1,3,1,0)$ and $\Delta_R(1,1,3,0)$ present the left-right model. Hoever, the Υ singlet under Pati-salam gauge group is odd under D-Parity. If we give *vev* to Υ singlet, the left-right symmetry will be broken at unification scale itself. Since our model is left-right symmetric, we must avoid D-parity breaking until left-right group is broken.

However, the singlet in $S(54)$ field under Pati-Salam gauge group does respect and so can be used to break the GUT group to the Pati-Salam gauge group. But, the breaking with $S(54)$ does not serve the purpose of further breaking to the left-right group. So for the next step breaking, a Higgs Field $A(45)$ is needed along with $S(54)$ which has the decomposition under the left-right group as

$$\begin{aligned}
A(45) &= (1,1,1;0) \oplus \Delta_L(1,3,1;0) \oplus \Delta_R(1,1,3;0) \\
&\quad \oplus (3,1,1;\frac{4}{3}) \oplus (\overline{3},1,1;-\frac{4}{3}) \oplus (8,1,1;0) \\
&\quad \oplus (3,2,2;\frac{2}{3}) \oplus (\overline{3},2,2;-\frac{2}{3})\,.
\end{aligned}$$

The first row of the above decomposition is of our interest as it contains the fields $\Delta_L(1,3,1,0)$ and $\Delta_R(1,1,3,0)$ of our model along with the left-right group singlet. This singlet is even under D-parity and so the left-right symmetry is unbroken until Δ_R acquires *vev* along the singlet direction to the SM gauge group. We will be following this approach in the remaining part of this section.

Now the fields χ_L $(1,2,2,1)$ and $\chi_R(1,1,2,1)$ are still left to be embedded in some tensors of $SO(10)$. The desired quantum numbers indicate that they can be embedded in the spinorial Higgs representation $\left(C(16) \oplus \overline{C(16)}\right)$. Decomposition of the $16 \oplus \overline{16}$ spinor representation under left-right group are given as

$$\begin{aligned}
C(16) &= \chi_L^*(1,2,1,-1) \oplus \chi_R(1,1,2,1) \\
&\quad \oplus (3,2,1,\frac{1}{3}) \oplus (\overline{3},1,2,-\frac{1}{3})\,, \\
\overline{C(16)} &= \chi_L(1,2,1,1) \oplus \chi_R^*(1,1,2,-1) \\
&\quad \oplus (3,1,2,\frac{1}{3}) \oplus (\overline{3},2,1,-\frac{1}{3})\,.
\end{aligned}$$

Having embedded all the Higgs fields of our model into $SO(10)$ tensor fields, we now write vacuum expectation values along the three singlet direction under the SM group of the fields $A(45)$ and $S(54)$ as

$$\begin{aligned}
\langle A \rangle &= M_C \hat{A}_C + M_R \hat{A}_R\,, \\
\langle S \rangle &= M_U \hat{S}\,,
\end{aligned}$$

where $\hat{A}_C$, $\hat{A}_R$ and $\hat{S}$ are the singlet directions under the SM gauge group given as

$$\begin{aligned}
\hat{A}_C &= \left(\hat{A}_{56} + \hat{A}_{78} + \hat{A}_{910}\right) \\
\hat{A}_R &= \left(\hat{A}_{12} + \hat{A}_{34}\right) \\
\hat{S} &= 3 \times \sum_{a=1}^{4} \hat{S}_{aa} - 2 \times \sum_{a=5}^{10} \hat{S}_{aa} \,.
\end{aligned}$$

The indices (1, 2, 3, 4) belong to $SO(4)$ and (5, 6, 7, 8, 9, 10) belong to $SO(6)$ subgroup of the group $SO(10)$. We have not taken care of the normalization factors while writing the directions of the singlets as they are not much relevant for the present discussion. However, we can assume that the normalization factors are absorbed in the corresponding *vev* values and can proceed without worrying about them for an approximate analysis.

Let us denote $H_\Phi = h$, $H_\Psi = H$ for simplicity in notations. Now we write the most general $SO(10)$ invariant Higgs potential:

$$\begin{aligned}
V &= \mu_A^2\, A_{ab}A_{ba} + \mu_S^2\, S_{ab}S_{ba} + \mu_h^2\, h_a h_a + \mu_H^2\, H_a H_a + \mu_C^2\, \left(\bar{C}C\right) + \mu_\eta^2\, \eta^2 + \lambda_\eta\, \eta^4 \\
&+ \lambda_A\, A^2 A^2 + \lambda'_A A^4 + \lambda_S\, S^4 + \lambda_h\, h^4 + \lambda_H\, H^4 + \lambda_c\, \left(\bar{C}C\right)^2 + \lambda'_c\, \left(C^4 + \bar{C}^4\right) \\
&+ g_{AS} A^2 S^2 + g'_{AS} A_{ab}A_{bc}S_{cd}S_{da} + g''_{AS} A_{ab}S_{bc}A_{cd}S_{da} \\
&+ h_a \left(g_{hA}A_{ab}A_{bc} + g_{hS}S_{ab}S_{bc}\right) h_c + \left(g'_{hA}A^2 + g'_{hS}S^2\right) h^2 \\
&+ H_a \left(g_{HA}A_{ab}A_{bc} + g_{HS}S_{ab}S_{bc}\right) H_c + \left(g'_{HA}A^2 + g'_{HS}S^2\right) H^2 \\
&+ \left(g_{hC}h^2 + g_{HC}H^2 + g_{AC}A^2 + g_{SC}S^2\right) \bar{C}C + g_{\eta HC}\eta\, h\, \left(CC + \bar{C}\bar{C}\right)\,.
\end{aligned}$$

The Z_4 symmetry (expression 3) used while writing the Higgs potential invariant under left-right gauge group has also been imposed here on the corresponding $SO(10)$ Higgs multiplets. Moreover, we have prevented some of the terms by applying the discrete symmetry $S \to -S$. The realization of the first three symmetry breaking steps is possible by taking the following structure of the *vev* assignments to the fields $A(45)$ and $S(54)$:

$$\begin{aligned}
\langle A \rangle &= i\tau_2 \otimes diag\left(M_R,\, M_R,\, M_C,\, M_C,\, M_C\right) \\
\langle S \rangle &= I \otimes diag\left(-\frac{3}{2}M_U,\, -\frac{3}{2}M_U,\, M_U,\, M_U,\, M_U\right)\,.
\end{aligned}$$

For the matter of convenience we have just replaced the *vev*s with the corresponding breaking scales. The potential , in terms of the *vev* values of A and S, will be approximately given as

$$\begin{aligned}
V &= \mu_A^2\left(6M_C^2 + 4M_R^2\right) + \mu_S^2\, 15M_U^2 + \left(\mu_C^2 + g_{AC}6M_C^2 + g_{SC}\, 15M_U^2\right)\bar{C}C \\
&+ \left(\mu_h^2 + g_{hS}9M_U^2\right) h_a h_a\, (a = 1-4) + \left(\mu_h^2 + g_{hA}6M_C^2 + g_{hS}6M_U^2\right) h_a h_a\, (a = 5-10) \\
&+ \left(\mu_H^2 + g_{HS}9M_U^2\right) H_a H_a\, (a = 1-4) + \left(\mu_H^2 + g_{HA}6M_C^2 + g_{HS}6M_U^2\right) H_a H_a\, (a = 5-10) \\
&+ \lambda_A\left(6M_C^2 + 4M_R^2\right)^2 + \lambda'_A\left(6M_C^4 + 4M_R^4\right) + \lambda_S M_U^4 + g_{AS}M_U^2\left(6M_C^2 + 9M_R^2\right) \\
&+ \lambda_h h^4 + \lambda_H H^4 + g_{\eta HC}\eta\, h\, \left(CC + \bar{C}\bar{C}\right) + \lambda_c\, \left(\bar{C}C\right)^2 + \lambda'_c\, \left(C^4 + \bar{C}^4\right) + \lambda_\eta\, \eta^4
\end{aligned}$$

We have assumed $M_R \ll M_U \sim M_C$ while writing the final form of the potential. In order to give desired masses (of the order of M_W) to the two left-right bi-doublets , μ_h and μ_H will have to be fine-tuned at the order of scale of M_U. The fine-tuning can produce very large masses to the triplets of $h($ or $H)$ provided the condition $\left(g_{hA}6M_C^2 - h_{hS}3M_U^2\right) \sim \left(+M_U^2\right)$ is satisfied. Another fine-tuning is required in the mass parameter μ_C^2 to provide the desired TeV scale masses to the Higgs fields $C(16) \oplus \overline{C(16)}$. Before ending this section, we would like to notice an important point. If we take the g_{SA} coupling to be very small, we can argue that the appearance of the similar combination $\left(6M_C^2 + 4M_R^2\right)$ everywhere in the potential allows M_C and M_R to take quite different values without disturbing other part of the potential. So the scale of M_C and M_R can be chosen to be different by orders of magnitude to get the desirable breaking.

Gauge Coupling Evolution

In the present section, we will be studying the set of two-loop renormalization group (RG) equations for the evolution of the coupling constants and will be verifying the consistency of the chosen *vev* for different Higgs fields in the context of $SO(10)$ GUT. For simplicity, we assume that the scale M_U and M_C are very close and we ignore the evolution of the coupling constants between the two scales. This is quite preferable as we will see later that the unification scale is very tightly constrained by the current proton decay bound [17] and any substantial difference between the two breaking scales would make it even worse. We start with the following equation for the two-loop evaluation of the coupling constant α_i

$$\frac{d\alpha_i^{-1}(t)}{dt} = -\frac{a_i}{2\pi} - \frac{b_{ij}}{8\pi^2}\left(\frac{1}{\alpha_j^{-1}}\right) \tag{7}$$

where $t = \ln(M_\mu)$ and M_μ is the desired energy scale where the couplings constants, α_i's, are be determined. The a_i's and b_{ij}'s are the one-loop and two-loop beta functions governing the evolution of α_i's and include the contributions from gauge bosons, fermions and scalars in the model.

The fermion contribution to the beta function is taken right from the starting, the electroweak scale (100GeV). The contributions of the gauge bosons to beta functions are straightforward to compute as one can easily determine the expected mass scales of the heavy gauge bosons corresponding to any given gauge group. However, the contribution coming from the Higgs content is not so clear because the heavy Higgs modes can have various possible mass spectrums. We will use the extended survival hypothesis to fix this uncertainty. The extended survival hypothesis is based on the assumption that only minimal number of fine-tunings of the parameters in the Higgs potential are imposed to ensure the hierarchy in various gauge boson masses. According to the extended survival hypothesis, only those scalar multiplets are present at any given intermediate breaking scale M_I of a intermediate gauge group G_I which are either required for breaking the gauge group G_I or needed to further break any other intermediate gauge group below scale M_I.

A list of Higgs multiplets surviving at the breaking scale of a intermediate group G_I, using the extended survival hypothesis, are given in table. A list of both one-loop and two-loop beta coefficients, which include all the contributions, that govern the evolution above the breaking scale of G_I to the next intermediate scale are also listed.

Since our model contains intermediate steps, we require appropriate matching conditions at the corresponding breaking scales. For the tow-loop RG running of the coupling constants, the matching conditions have been derived in [18, 19]. Suppose a gauge group G is spontaneously broken into a sub-group $\prod_i G_i$ with several individual factors G_i, then the following matching condition need to be satisfied for the two-loop analysis

$$\alpha_G^{-1}(M_I) - \frac{C(G)}{12\pi} = \alpha_{G_i}^{-1}(M_I) - \frac{C(G_i)}{12\pi}, \tag{8}$$

where $C(G/G_i)$ is the quadratic Casimir invariant for the group G/G_i. We choose initial starting values of the above three coupling constants (central values) at scale M_W to be $\alpha_{1Y}^{-1}(M_W) = 59.38$, $\alpha_{2L}^{-1}(M_W) = 29.93$, and $\alpha_{3c}^{-1}(M_W) = 8.47$. Now let us write the

The boundary conditions at various breaking scales, using the expression 8, can be written as

1. At scale m_r:

$$\alpha_{1Y}^{-1}(m_r) = \frac{3}{5}\alpha_{1R}^{-1}(m_r) + \frac{2}{5}\alpha_{1(B-L)}^{-1}(m_r).$$

2. At scale M_R:

$$\begin{aligned}\alpha_{1R}^{-1}(M_R) &= \alpha_{2R}^{-1}(M_R) - \frac{2}{12\pi},\\ \alpha_{2R}^{-1}(M_R) &= \alpha_{2L}^{-1}(M_R).\end{aligned}$$

Group G_I	Higgs content	a	b
G_{321}	$\left(1,\,2,\,\frac{1}{2}\right)_{10}\oplus\left(1,\,2,\,-\frac{1}{2}\right)_{10}$ $\left(1,\,2,\,\frac{1}{2}\right)_{10'}\oplus\left(1,\,2,\,-\frac{1}{2}\right)_{10'}$	$\begin{pmatrix}-7\\-3\\\frac{21}{5}\end{pmatrix}$	$\begin{pmatrix}-26 & \frac{9}{2} & \frac{11}{10}\\12 & 8 & \frac{6}{5}\\\frac{44}{5} & \frac{18}{5} & \frac{104}{25}\end{pmatrix}$
G_{3211}	$\left(1,\,2,\,\frac{1}{2}\,0\right)_{10}\oplus\left(1,\,2,\,-\frac{1}{2}\,0\right)_{10}$ $\left(1,\,2,\,,\frac{1}{2}\,0\right)_{10'}\oplus\left(1,\,2,\,-\frac{1}{2},\,0\right)_{10'}$ $\left(1,\,1,\,-\frac{1}{2},\,\frac{1}{2}\right)_{16}+\left(1,\,1,\,\frac{1}{2},\,-\frac{1}{2}\right)_{\overline{16}}$	$\begin{pmatrix}-7\\-3\\\frac{53}{12}\\\frac{33}{8}\end{pmatrix}$	$\begin{pmatrix}-26 & \frac{9}{2} & \frac{3}{2} & \frac{1}{2}\\12 & 8 & 1 & \frac{3}{2}\\12 & 3 & \frac{17}{4} & \frac{15}{8}\\4 & \frac{9}{2} & \frac{15}{8} & \frac{65}{16}\end{pmatrix}$
G_{3221D}	$(1,\,2,\,2,\,0)_{10}$ $(1,\,2,\,2,\,0)_{10'}$ $\left(1,\,2,\,1,\,-\frac{1}{2}\right)_{16}\oplus\left(1,\,2,\,1,\,\frac{1}{2}\right)_{\overline{16}}$ $\left(1,\,1,\,2,\,\frac{1}{2}\right)_{16}\oplus\left(1,\,1,\,2,\,-\frac{1}{2}\right)_{\overline{16}}$ $(1,\,1,\,3,\,0)_{45}$ $(1,\,3,\,1,\,0)_{45}$	$\begin{pmatrix}-7\\-\frac{5}{2}\\-\frac{5}{2}\\\frac{9}{2}\end{pmatrix}$	$\begin{pmatrix}-26 & \frac{9}{2} & \frac{9}{2} & \frac{1}{2}\\12 & \frac{39}{2} & 3 & \frac{9}{4}\\12 & 3 & \frac{39}{2} & \frac{9}{4}\\4 & \frac{27}{4} & \frac{27}{4} & \frac{23}{4}\end{pmatrix}$

Table I: Higgs multiplets at different intermediate breaking scales along with the both one-loop and two-loop beta coefficientss, including all the contributions from fermions, gauge bosons and Higgs bosons, which govern the evolution of coupling constants above breaking scale of G_I to the next breakingscale.

3. At the unification scale M_U

$$\begin{aligned}\alpha_{2L}^{-1}(M_U)-\frac{2}{12\pi} &= \alpha_{2R}^{-1}(M_U)-\frac{2}{12\pi}\\ &= \alpha_U^{-1}(M_U)-\frac{8}{12\pi}\,,\\ \alpha_{3c}^{-1}(M_U)-\frac{3}{12\pi} &= \alpha_U^{-1}(M_U)-\frac{8}{12\pi}\,,\\ \alpha_{B-L}^{-1}(M_U) &= \alpha_U^{-1}(M_U)-\frac{8}{12\pi}\,.\end{aligned}$$

The matching conditions at the unification scale have been written by assuming the Pati-Salam scale to be almost close to the unification scale.

Using the above boundary conditions we have numerically solved the equation 7 for the two-loop RG evolution for all the coupling constants. We have taken the breaking scale of the gauge group G_{3211} to be around 1TeV. The unification scale comes out to be $M_U = 10^{15.4}$GeV and the corresponding coupling constant is estimated as $\alpha_U^{-1}(M_U) = 43.4$. Also the breaking scale of left-right symmetric gauge group, i.e., G_{3221D} turns out to be $M_R = 10^{11.6}$GeV. The running of the various coupling constants with energy scale are shown in figure 1.

However, we find that the scale of the unification along with the $\alpha_U - 1$ are not satisfying the most recent bounds on proton decay, although very close to the limit. The current experimental lower bound of the partial life time for $p \to e^+\pi^0$ is $\tau_p > 8.2\times10^{33}$ years and for $p \to \mu^+\pi^0$ is $\tau_p > 6.6\times10^{33}$ years [17]. The theoretical decay rate of the proton can be estimated as:

$$\Gamma_p \simeq \alpha_{GUT}^2\frac{m_p^5}{M_{X,Y}^4}\,.$$

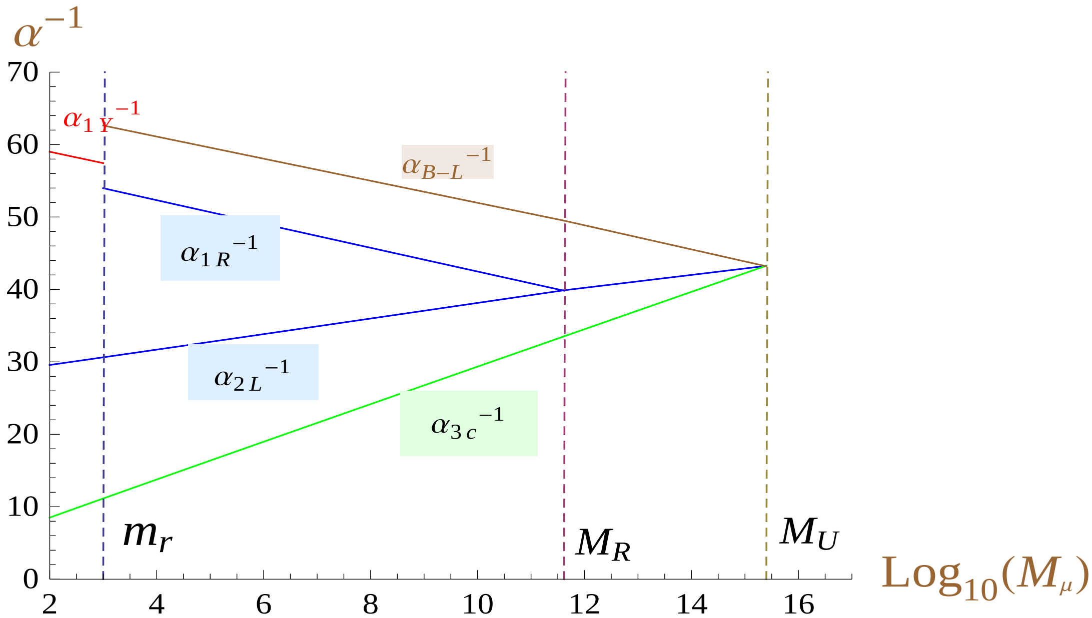

Figure 1: Evolution of coupling constants

SO(10) Higgs Representation	Higgs multiplets contributing to threshold uncertainty (Decomposed under G_{3211})	$\{a_{3c},\, a_{2L},\, a_{1R},\, a_{1(B-L)}\}$
16	$(1,\,1,\,\frac{1}{2},\,\frac{1}{2})_{16} \oplus (1,\,1,\,-\frac{1}{2},\,-\frac{1}{2})_{\overline{16}}$ $(1,\,2,\,0,\,-\frac{1}{2})_{16} \oplus (1,\,2,\,0,\,\frac{1}{2})_{\overline{16}}$	$\{0,\,1,\,\frac{1}{2},\,\frac{9}{4}\}$
45	$(1,\,3,\,1,\,0)_{45}$	$\{0,\,2,\,0,\,0\}$

Table II: Threshold contribution at left-right breaking scale

This can be used to estimate the lower limit of the Heavy gauge boson masses. If the mass scale of super heavy gauge bosons are given as $M_X \simeq 10^n$GeV, the above proton decay bound is equivalent to

$$\kappa = \left(\frac{\alpha_{GUT}}{45}\right) \times 10^{2(n-15)} \gtrsim 11.8\,. \tag{9}$$

What we obtain for the value of κ in our analysis is $\kappa = 6.07$. This is below the lower limit allowed by the proton decay bound as specified in the right-hand side of the expression 9. However, the value of κ is very close to the allowed lower limit and so we will try to explore the viability of our model by allowing threshold uncertainty in the Higgs spectrum at various intermediate breaking scales. It is important to remark at this point that we could get the reported value of κ to be close to the limit only when we optimized certain degrees of freedom in the Higgs sector. For instance, the Higgs-bidoublet Φ has been asumed to arise from a real 10-dimensional $SO(10)$ Higgs H_Φ. So Φ would not be equivalent to two SM Higgs doublets at the electroweak scale but will be equivalent to only one such doublet. Similar asuumption has been also taken for Ψ. However, we would like to emphasize that the results and discussion of the potential minimization will remain almost same.

The threshold uncertainty in the Higgs spectrum arises form the fact that the Higgs bosons becoming heavy at a given breaking scale may not get exactly same masses equal to the energy corresponding to the breaking scale. However, the Higgs mass spectrum is expected to be scattered around the energy of the breaking scale within an small width. For our analysis, we follow a similar approach discussed in [21]. We assume that the masses of the Higgs bosons are scattered around the breaking scale within the factor of $\frac{1}{30}$ to 30. So if the mass of a Higgs multiplet at

SO(10) Higgs Representation	Higgs multiplets contributing to threshold uncertainty (Decomposed under G_{3221D})	$\{a_{3c},\, a_{2L},\, a_{2R},\, a_{1(B-L)}\}$
10	$\left(3,\,1,\,1-\frac{1}{3}\right)_{10} \oplus \left(\overline{3},\,1,\,1,\,\frac{1}{3}\right)_{10}$ $\left(3,\,1,\,1-\frac{1}{3}\right)_{10'} \oplus \left(\overline{3},\,1,\,1,\,\frac{1}{3}\right)_{10'}$	{2, 0, 0, 2}
16	$\left(3,\,2,\,1,\,\frac{1}{6}\right)_{16} \oplus \left(\overline{3},\,2,\,1,\,-\frac{1}{6}\right)_{\overline{16}}$ $\left(\overline{3},\,1,\,2,\,-\frac{1}{6}\right)_{16} \oplus \left(3,\,1,\,2,\,\frac{1}{6}\right)_{\overline{16}}$	{4, 3, 3, 1}
45	$\left(3,\,2,\,2,\,-\frac{1}{3}\right)_{45} \oplus \left(\overline{3},\,2,\,2,\,-\frac{1}{3}\right)_{45}$ $(8,\,1,\,1,\,0)_{45}$	{7, 6, 6, 4}
54	$\left(6,\,1,\,1,\,-\frac{2}{3}\right)_{54} \oplus \left(\overline{6},\,1,\,1,\,\frac{2}{3}\right)_{54}$ $(1,\,3,\,3,\,0)_{54}$ $(8,\,1,\,1,\,0)_{54}$	{8, 6, 6, 8}

Table III: Threshold contribution at the unification scale

the given breaking scale M_I is M_H, then we expect

$$\frac{1}{30} \lesssim \frac{M_H}{M_I} \lesssim 30\,.$$

To include the threshold uncertainty at a given breaking scale, we need to slightly modify our matching conditions at that scale. The matching condition given in expression 8 is modified as

$$\alpha_G^{-1}\left(M_I\right) - \frac{C\left(G\right)}{12\pi} = \alpha_{G_i}^{-1}\left(M_I\right) - \frac{C\left(G_i\right)}{12\pi} - \frac{\lambda_i}{12\pi}\,,$$

where $\lambda_i = a_i \ln\frac{M_H}{M_I}$. So the threshold uncertainty has been included in the matching condition due to presence of the term involving $\ln\left(M_H/M_I\right)$.

To avoid any over estimation of the threshold uncertainty we assume that all the Higgs multiplets, belonging to a single common irreducible Higgs representation of $SO(10)$, becoming heavy at a given breaking scale will have the same mass scale around the breaking scale.

The threshold uncertainty at the breaking scale of gauge group $G_{3211}$is vanishing. The Higgs multiplets, coming from different $SO(10)$ irreducible Higgs, contributing to the threshold uncertainty at remaining two intermediate scales, the left-right breaking scale and the unification scale, are listed in the table II and III, respectively. The corresponding calculated beta-coefficents, (a_i)'s, which include the contribution from all the Higgs multiplets coming from the same $SO(10)$ irreducible representation (as their masses are assumed to be same), are also shown for the two breaking scales.

Now using these calculated a_i's and including uncertainty in M_H/M_I, as discussed before, we have shown a scatterd-plot between coupling constant α_U^{-1} and the corresponding unification scale M_U in figure 2. We have numerically obtained the values for α_U^{-1} and M_U for randomally chosen values for M_H/M_I between the range $\left(\frac{1}{30} - 30\right)$. The random values for all the Higgs multiplets belonging to the same $SO(10)$ ireducible Higgs are taken to be same at one perticular breaking scale but different at the other breaking scale.

Moreover, we have aslo plotted the curve corersponding to the most recent proton decay bound (red solid curve) [17] and relatively older proton decay bound (blue dashed curve) [22] in figure 2 to show the allowed region in α_U^{-1}-M_U plane. Only the right part of the curve is allowed by the bound. It is worth noting that the allowed parameter space is more and more constrained as more updated data on proton decay bound is available. However, we get a resonable allowed region in the figure 2, although small, even after allowing the most conservative threshold uncertainty. So we expect our model to be satisfactory within the tolerable amount of threshold uncertainty as far as proton decay bound is concerned.

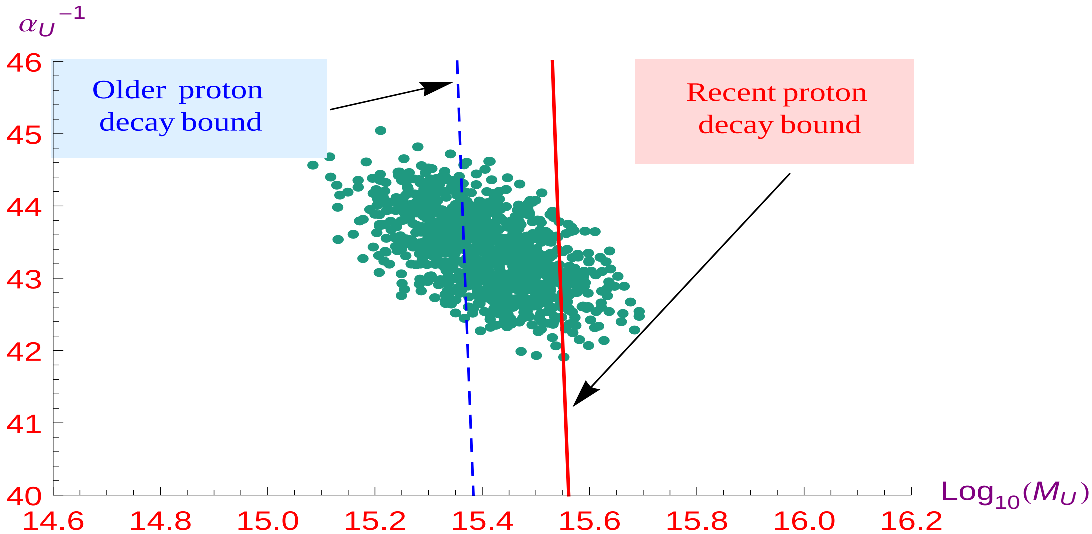

Figure 2: Threshold uncertainty in the unificaton scale.

Yukawa Sector And Neutrino Masses

In the present section, we discuss the origin of neutrino masses in the model. Before proceding further we would like to make it clear that the discussion about neutrino masses in the present section will only move around the left-right symmetric model with few inputs from the $SO(10)$ GUT in motivating about certain patterns for taken Dirac mass matrices for fermions in our analysis. Moreover, the discussion will be mainly focused on the matrix structure of low energy neurino mass matrix allowed with certain assumptions. We will aslo argue, in what follows, that the consistent neutrino mass spectrum is not possible within picture of one or two SO(10) singlet fermions S. We start by writing the Yukawa sector of the model as

$$\mathcal{L}_Y = Y_{ij}\, \overline{\ell_{Li}}\, \ell_{Rj}\Phi + Y'_{ij}\, \overline{\ell_{Li}}\, \ell_{Rj}\Psi + (F_L)_{in}\, \overline{S_{Rn}}\, \ell_{Li}\chi_L + (F_R)_{in}\, \overline{S^c_{Ln}}\ell_{Ri}\, \chi_R \tag{10}$$

$$+ \frac{1}{2}M_{mn}\eta\overline{S^c{}_{Lm}}S_{Rn} \tag{11}$$

The Yukawa couplings Y and Y' are 3×3 matrix, while F_L and F_R are $3\times n$ matrices, if we assume that there are n singlet fermions S. So M is a $n\times n$ matrix. Our study of consistent embedding of the model in $SO(10)$ GUT requires same structure for both F_L and F_R up to the scale of left-right symmetry breaking which, after RG running, can produce small difference at the weak scale. For the present discussion we assume it to be small enough so that it can be safely ignored.

The Dirac masses for all the SM fermions including neutrinos are generated form the the first two terms by giving vev to the bi-doublets as in any other left-right symmetric model. Since Φ and Ψ are coming from two independent and real $SO(10)$ 10-dimensional Higgs, the Dirac mass matrix for neutrinos and charged leptons are independent. However, the Dirac mass matrix for the up-type quarks have the same structure as the Dirac mass matrix for the neutrinos and similarily the Dirac mass matrix for the down-type quarks will have similar structure as the Dirac mass matrix for the charged leptons (simply because all SM fermions are assigned to a multiplet of $SO(10)$ GUT). Although, these similarities in the structures are exact only at the GUT scale, we expect some of its features to be more or less same even at the low scale. So we can well assume that the Dirac mass matrix of the neutrinos would almost appear diagonal in the basis where the charged lepton mass matrix is diagonal. The assumption is based on the observation that the up-type and down-type quarks are simultaneously diagonal in the a basis as the quark mixing matrix is very close to unity. So we borrow the pattern from the quark sector to the lepton sector where the structure of Dirac mass matrix of the neutrinos is not directly known unless neutrinos are Dirac fermions. We expect the following pattern of the Dirac mass matrix of neutrinos in the diagonal basis of the charged leptons

$$M_{\nu D} = vY_{lepton}\left(\frac{m_t}{m_b}\right) = \begin{pmatrix} m_e & 0 & 0 \\ 0 & m_\mu & 0 \\ 0 & 0 & m_\tau \end{pmatrix}\left(\frac{m_t}{m_b}\right) \simeq v\begin{pmatrix} 0.0001 & 0 & 0 \\ 0 & 0.02 & 0 \\ 0 & 0 & 0.3 \end{pmatrix},$$

where m_t and m_b are masses of top and bottom quarks and m_e, m_μ, m_τ are masses of electron, muon and tau leptons.

The part of the Lagrangian relevant for the neutrino mass generation is given as follows,

$$\mathcal{L}_{\nu\ mass} = \begin{pmatrix} \nu, & N^c, & S \end{pmatrix}_L . X . \begin{pmatrix} \nu \\ N^c \\ S \end{pmatrix}_L + H.C. \tag{12}$$

$$= \begin{pmatrix} \nu_i, & N_i^c, & S_m \end{pmatrix}_L \begin{pmatrix} 0 & Y_{ij}v & F_{in}v_L \\ (Y_{ij})^T v & 0 & F_{in}v_R \\ F_{mj}^T v_L & F_{mj}^T v_R & M_{mn}u \end{pmatrix}\begin{pmatrix} \nu_j \\ N_j^c \\ S_n \end{pmatrix}_L + H.C \tag{13}$$

Our first task is to analyze the mass spectrum provided by the matrix X in case of one generation of all fermions. We write the eigenvalue equation as (eigenvalue: λ):

$$\lambda^3 - Mu\,\lambda^2 - F^2 v_R^2 \lambda - 2YF^2 v v_L v_R - MY^2 u v^2 = 0$$

Case 1: $\lambda >> v$**,** we get

$$\lambda\,(\lambda + Fv_R)\ (\lambda - Fv_R) = 0$$

The above eigenvalue equation predicts two TeV scale Majorana fermions. The massless solution contradicts with the condition we started with, and so is unphysical.

Case 2: $\lambda << v$**,** we get

$$\lambda = -\frac{2Yvv_L}{v_R} + \frac{MY^2uv^2}{F^2v_R^2} \tag{14}$$

which is of order of eV. So the two Majorana fermions pick up masses of the order as high as TeV and one remains sufficiently light ($\sim$ eV) to be identified as light neutrino.

To make the discussion some more general, we take three generations for all the SM fermions including the left and right handed neutrinos but only one generation for the singlet S. We look for a possibility whether it can account for the existing picture of three light active neutrinos. To search for any such possibility, we try to find out the mass spectrum, within this scenario, by solving for the eigenvalues of the matrix X. To simplify further, we take all the eigenvalues of the matrix $M_{\nu D}$ to be same with a common value equal to the largest one for initial analysis. This enable us to factor out $\left(\lambda^2 - z^2v^2\right)^2$ from the algebraic expression of $\mathrm{Det}\,(X)$ predicting four Majorana fermions of scale around 10 GeV. The rest of the factors have got the same form as the expression of determinant in case of one generation of all SM fermions, as discussed earlier, leading to the two TeV and one eV scale Majorana fermions. The scenario provides us only one light neutrino and, hence, can not account for the observed neutrino mass spectrum. To explore the effect of some possible hierarchy present in the eigenvalues of the Dirac mass matrix of the neutrino like one present in the charged lepton mass matrix, we take two of the eigenvalues to be same and vary their scale below the third one. We are still able to explicitly get two of the Majorana fermions having mass scale equal to $m_e\left(\frac{m_t}{m_b}\right)$. One may think that the remaining two Majorana fermions might get mass scale as light as eV leading to three light neutrinos. To rule out any such possibility, we have plotted the masses of the two remaining Majorana fermions (which comes out to be same) with the ratio of the two mass scales of the eigenvalues of the Dirac mass mass matrix of the neutrinos in figure 3. We find that the masses do not go below the lightest mass scale of the eigenvalues of $m_{\nu D}$. Even in two generation scenario of S fermions, there is not much progress except we get two eV scale Majorana fermions which is still not sufficient.

We now turn to the case of three generation for S fermions. One obviously expects to get the three light neutrinos. The basic way to get the low energy neutrino mass matrix has been outlined in [15] which is given as

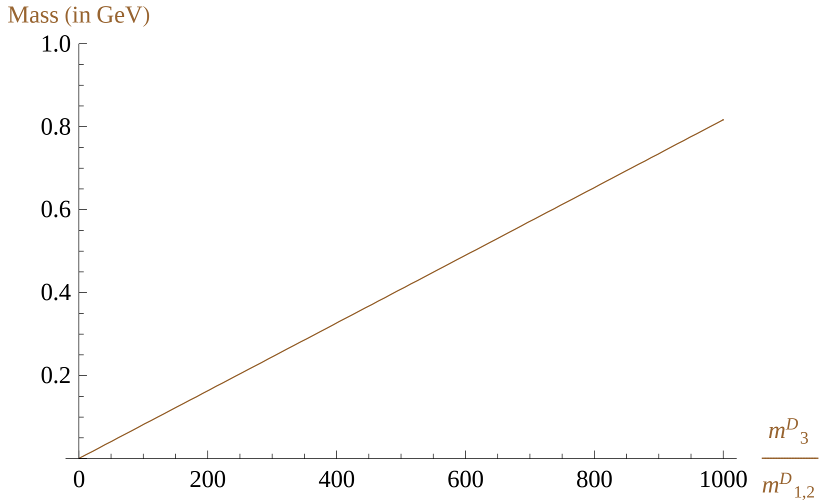

Figure 3: Variation of mass

$$\begin{aligned} m_\nu &= -\left(\frac{vv_L}{v_R}\right)\left(Y+Y^T\right)+\left(\frac{uv^2}{v_R^2}\right)Y\left(FM^{-1}F^T\right)^{-1}Y^T \ , \\ &= -\left(\frac{vv_L}{v_R}\right)\left[\left(Y+Y^T\right)+rY\left(FM^{-1}F^T\right)^{-1}Y^T\right] \ , \end{aligned} \tag{15}$$

as we have $uv^2 = r\ vv_Lv_R$ in our model (expression 4) where $r = (\lambda_\chi - g_\chi)/h_\chi$.

The first term is the type-III seesaw contribution [23] and the second term is the double seesaw contribution. With the choice of the $vevs$, it is obvious that this scenario provides us with three eV neutrinos.

Now we will try to explore the limits of the expression 15 for low energy neutrino mass matrix to check its consitency with current data on neutrino masses and mixing by allowing some very simple form for matrix M. In the basis where charged lepton mass matrix is diagonal, the neutrino mixing matrix (U_{PMNS}) is just the matrix that diagonalizes the m_ν:

$$(U_{PMNS})^T m_\nu U_{PMNS} = m_\nu^{Diag} = Diag\,(m_1, m_2, m_3) \ .$$

The U_{PMNS} mixing matrix is usually parametrized in the literature as

$$U_{PMNS} = R_{23}(\theta_{23})\,R_{13}(\theta_{13},\delta)\,R_{12}(\theta_{12})\,.Dag\left(e^{i\eta_1}, e^{i\eta_2}, 1\right) \ ,$$

where R_{ij} are the rotation matrices in the ij plane with angle θ_{ij}. δ is the CP violating phase associated with 1-3 rotation and η's are the Majorana phases appearing only in the case of Majorana neutrinos. To date, two mass square differences and three angle have been measured while CP violation is completely unknown in the leptonic sector. We take the following observed values for three mixing angle and two mass square differences at 90% confidence level from particle data group [24] as:

$$\begin{aligned} \Delta m_{21}^2 = m_2^2 - m_1^2 &= (8.0 \pm 0.3) \times 10^{-5}\,\mathrm{eV}^2 \\ \Delta m_{21}^2 = m_2^2 - m_1^2 &= 1.9 \text{ to } 3.0 \times 10^{-3}\,\mathrm{eV}^2 \\ sin^2\,(2\theta_{12}) &= 0.86^{+0.03}_{-0.04} \\ sin^2\,(\theta_{23}) &> 0.92 \\ sin^2\,(\theta_{13}) &< 0.19 \end{aligned}$$

We will be mainly using the mean values of the observed parameters in our analysis.

In its most general form, it is straight forward to argue that m_ν can accommodate the existing data on neutrino masses and mixing simply due to the presence of enough number parameters in F and M unless type III term dominates significantly. An interesting thing would be to consider some simpler form of the neutrino mass matrix by reducing appropriate number of parameters with some tolerable assumptions. The basic idea is to explore the possibility of any such simpler structure in light of the current neutrino oscillation data.

We start with the assumption that the three singlet fermions S are blind to their generation within themselves leading to the following democratic structure of matrix M :

$$M = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} u$$

The structure allows us to believe that there is no induced mixing between the left-right neutrinos and the singlets. So, F matrix can be written as product of a unitary matrix and a diagonal matrix. The unitary matrix connects the basis of the democratic structure to the basis where the charged lepton mass matrix becomes diagonal. To get some more simplicity, we are driven to assume that the two basis are identical, i.e., the unitary mass matrix is identity matrix. It leads to the following structure of the low energy neutrino mass matrix:

$$m_\nu = \frac{v v_L}{v_R} \begin{pmatrix} \alpha^2 - 2\frac{m_t}{m_b} m_e & \alpha\beta & \alpha\gamma \\ \alpha\beta & \beta^2 - 2\frac{m_t}{m_b} m_\mu & \beta\gamma \\ \alpha\gamma & \beta\gamma & \gamma^2 - 2\frac{m_t}{m_b} m_\tau \end{pmatrix},$$

where α, β and γ are the final parameters appearing in the neutrino mass matrix after absorbing all the parameters present in F, M and Y. We take the following familiar tri-bimaximal form of [25] of the U_{PMNS} mixing matrix for our discussion and attempt to diagonalize m_ν having above structure:

$$U_{PMNS} = U_{tbm} = \frac{1}{\sqrt{6}} \begin{pmatrix} 2 & \sqrt{2} & 0 \\ -1 & \sqrt{2} & \sqrt{3} \\ 1 & -\sqrt{2} & \sqrt{3} \end{pmatrix},$$

where $\theta_{23} = \pi/4$, $\theta_{13} = 0$, and $\sin^2\theta_{12} = 1/3$.

We attempt to diagonalize m_ν with the tri-biamaximal form of the mixing matrix which requires the following relation of the parameters α, β and γ with masses of the charged leptons as:

$$\begin{aligned} \alpha &= 0 \\ \beta &\simeq \frac{m_\mu}{\sqrt{\frac{m_b}{2m_t}(m_\tau + m_\mu)}} \simeq 0.05 \\ \gamma &\simeq -\frac{m_\tau}{\sqrt{\frac{m_b}{2m_t}(m_\tau + m_\mu)}} \simeq -0.75 \end{aligned}$$

The diagonal neutrino mass matrix comes out of the form:

$$m_\nu^{Daig} \simeq -\frac{2m_t}{m_b} \begin{pmatrix} m_e & 0 & 0 \\ 0 & m_e & 0 \\ 0 & 0 & 2\frac{m_\mu m_\tau}{(m_\mu + m_\tau)} \end{pmatrix} \left(\frac{v v_L}{v_R}\right)$$

So the present form of m_ν and U_{PMNS} produces degenerate masses for the two light neutrinos which is likely to be cured once we slightly deviate from tri-bimaximal form of U_{PMNS}. The deviation can be realized either by taking non-maximal value of θ_{23} or non vanishing value of θ_{13} or both. We take only non-zero value of θ_{13} to be the sole realization of the deviation for our purpose. The deviated form of tri-baimaximal matrix for very small value of θ_{13} can be parametrized as:

$$U_{PMNS} = \frac{1}{\sqrt{6}} \begin{pmatrix} 2 & \sqrt{2} & \theta_{13} \\ -1-\sqrt{2}\theta_{13} & \sqrt{2}-\theta_{13} & \sqrt{3} \\ 1-\sqrt{2}\theta_{13} & -\sqrt{2}-\theta_{13} & \sqrt{3} \end{pmatrix}$$

While trying to diagonalize the m_ν, numerical methods are used to find out the desired values of the free parameters. We find that the degeneracy encountered in the case of tri-bimaximal mixing matrix disappears as soon as finite value of θ_{13} is introduced. This finite value is determined by imposing the condition $\Delta m_{21}^2/\Delta m_{31}^2 \simeq 0.033$ which leads to following value of $\sin\theta_{13}$

$$\sin\theta_{13} = 0.11.$$

The value is well within the allowed value for θ_{13} from oscillation data. The correct scale of the mass square differences is easily achieved by adjusting the over all scale of the neutrino mass matrix. The corresponding values of the other parameters come out to be

$$\begin{aligned} \alpha &= 0.02 \\ \beta &= 0.06 \\ \gamma &= -0.75 \end{aligned}$$

The point we would like to emphasize is that even the simple structure of the mass matrix taken in our analysis is able to account for the existing framework of three active light neutrinos even though the assumptions may not correspond to any real underlying symmetry.

Dark Energy

We shall now show how the model can accommodate the proposal of the mass varying neutrinos (MaVaNs) [3, 4]. The basic idea behind the mass varying neutrinos is that some scalar field, the acceleron, acquires a value of the order of 10^{-3} eV, which gives an effective potential that contributes to the dark energy with the equation of state $\omega = -1$. However, till recently the neutrino masses were contributing to the effective potential much more strongly and the combined fluid of the background neutrinos and the accelerons were behaving as dark matter with the equation of state $\omega = 0$. As the neutrino masses were varying with time, the contribution of the background neutrino density to the effective potential were changing. Only in the recent past, the contribution of the acceleron field to the effective potential became stronger than the background neutrinos, changing the equation of state of the combined fluid, and the universe started accelerating with dark energy domination. This can then explain why the scale associated with the amount of dark energy is comparable to the neutrino masses, why the amount of dark energy is comparable to the ordinary matter, and why the universe is dominated by dark energy only now and for the rest of the time in the past the evolution of the universe was governed by matter.

In spite of these advantages, the MaVaNs scenario are not free of problems. We shall now try to explain how the MaVaNs scenario can be accommodated in a grand unified theory. After describing the generic features of the MaVaNs, following the original proposal [4], we shall explain how our present model answer this question. We shall not restrict ourselves to any particular choice for the acceleron field, and hence, consider the potential for the acceleron field to be same as that considered in the original proposal. At the end we shall mention how the present model can be extended to allow a milli-eV mass pseudo-Nambu-Goldstone Boson (pNGB), which can become the acceleron field.

We shall now mention this possible origin of the acceleron field in an extension of our model. Following the prescription followed in ref. [8], we introduce three η and several Higgs doublets. The *vevs* of the fields η would then give rise to global symmetries, which are allowed by all the Yukawa couplings due to the choice of quantum numbers of the Higgs doublets under these global symmetries. However, when the Higgs doublets acquire *vevs*, the global symmetries will be broken and there will be pNGBs, which couple to the neutrino masses. Although the dynamics of the pNGBs are not specified, the masses and the potentials of the pNGBs are determined by the Coleman-Weinberg potential, as demonstrated in ref. [8]. Since the introduction of several Higgs doublets may not allow the the gauge coupling unification, we shall not discuss this extension any further. Moreover, there could be some other origin of the acceleron field, so from now on we shall only mention the generic features of this model.

In a generic MaVaNs models, the coupling between neutrino mass and $\mathcal{A}$ induces the following effective potential

$$V = (\rho_\nu - 3P_\nu) + V_0(m_\nu) \tag{16}$$

Here the scalar potential $V_0(m_\nu)$ is due to the acceleron field (written as a function of neutrino mass) and P_ν is pressure of the neutrino fluid. In the late time evolution the non-relativistic limit i.e. $m_\nu \gg T$ is of particular interest. In this case $P_\nu \sim 0$ and one can write the effective potential as,

$$V = m_\nu n_\nu + V_0(m_\nu)\,. \tag{17}$$

The acceleron field will be trapped at the minima of the potential, which ensures that as the neutrino mass varies, the value of the acceleron field will track the varying neutrino mass. One can write equation of state in the non-relativistic case for a combined fluid of neutrino + acceleron;

$$w \;=\; P/\rho \;=\; \frac{P_{\mathcal{A}}}{m_\nu n_\nu + \rho_{\mathcal{A}}} \tag{18}$$

One generic feature of this solution is that it gives $\omega \approx -1$ at present. The most important feature of this scenario is that the energy scale for the dark energy gets related to the neutrino mass, which is highly desirable. This also explains why the universe enters an accelerating phase now [26].

We shall now discuss the implementation of the νDE mechanism in our model. For simplicity, we consider only one-generation scenario. The effective scalar field potential of the scalar is of the Coleman-Weinberg type i.e.

$$V_0 = \Lambda^4\ \log(1 + |M_s(\mathcal{A})/\bar{\mu}| \tag{19}$$

where, M_s is the singlet fermion mass. We assume that $M_s(\mathcal{A})/\bar{\mu} \gg 1$.

$$M_s = M\langle\eta\rangle = M\ u$$

depends on the acceleron field $\mathcal{A}$. Thus the neutrino mass becomes a dynamical quantity. When the neutrinos become non-relativistic the dependence of M_s on $\mathcal{A}$ governs the dynamics of the dark energy. Λ is chosen in such a way to yield the dark energy density $\Omega_{\rm DE} \approx 0.7$. This type of potentials are extensively used in the dark energy literature [3, 27]. Now we can write the effective low-energy Lagrangian in our model

$$-\mathcal{L}_{eff} = M_s(\mathcal{A})\frac{Y^2}{F^2}\frac{v^2}{v_R^2}\ \nu_i\nu_j + H.c. + \Lambda^4\log(1 + |M_s(\mathcal{A})/\bar{\mu}|)\,, \tag{20}$$

From the choices we have made about the *vevs*, we have retained only the dominant double seesaw term 14 in the effective Lagrangian. As $u \sim O(eV)$, the mass parameter M_s is of the order of eV. Since the ratio $(v/v_R)^2 \sim 10^{-2} - 10^{-3}$, the Yukawa couplings coupling to be of order unity. Thus the first two terms in equation (14) are comparable to the last term describing the dark energy potential.

The Majorana mass of neutrino varies with the acceleron field through the parameter M_s and the mass scale of this parameter remains near the scale of dark energy *naturally*. The interesting feature of our model is that we do not need any unnaturally small Yukawa couplings or symmetry breaking scale to achieve this naturalness requirement. Also the variation of M_s does not affect charged fermion masses in the model. Moreover, the electroweak symmetry breaking scale v and the $U(1)_R$ breaking scales are comparable and hence the new gauge boson corresponding to the group $U(1)_R$ will have usual mixing with Z and should be accessible at LHC.

Since the local minimum of the potential relates the neutrino mass to a derivative of the acceleron potential, the value of the acceleron field gets related to the neutrino mass. The acceleron field provide an effective attractive force between the neutrinos. When this effective force is stronger than the gravity, perturbations in the neutrino-acceleron fluid become unstable. The source of the free-energy comes from the attractive interaction between the neutrino and the acceleron field. The instability is similar to that of the Jeans instability found in a self-gravitating system. The instability can lead to inhomogeneity and structure formation; the instability would grow till the degeneracy pressure of the neutrinos would arrest the growth. The final state of the instability would produce neutrino lumps or nuggets [27, 28]. The neutrino lumps would then behave as dark matter and will not affect the dynamics of the acceleron field [28]. This instability is a generic feature of MaVaNs scenario, however it can be suppressed if the neutrino become superfluid [29] or if the MaVaNs perturbations become non-adiabatic.

Conclusions

We have constructed a left-right symmetric model of νDE that can be embedded in an $SO(10)$ GUT. After discussing the Higgs content needed for the model, details of potential minimization have been carried out considering all possible allowed terms. In particular, we have tried to explore the possibility of choosing the minima such that only neutral Higgs components get *vev* without constraining the couplings constants. But it turns out that some such constraints are needed in most general form of the potential. The complete analysis allows the desired ordering of the *vev*s. Then we study the embedding of this left-right symmetric model in $SO(10)$ GUT. We show that $SO(10)$ GUT with Higgs multiplets $S(54)$, $A(45)$, two $H(10)$, $C(16) \oplus \overline{C(16)}$, $\eta(1)$ along with an additional fermion singlet is able to accommodate the left-right symmetric model. The embedding allows the Pati-Salam and the left-right symmetry group breaking scales to be different by orders of magnitudes. We have studied the one loop RG running of various couplings constant and have found that the desired assignment for *vev* values for different Higgs fields is consistent with the gauge unification. Then the origin and possible structure of neutrino masses and matrix have been discussed in detail. It has been shown that generation of three light active neutrinos of eV scale is not possible in scenario with one or two $SO(10)$ singlets fermions. In the generic case of three singlets, we have taken a simple structure of neutrino mass matrix with some tolerable assumptions and shown that the structure is consistent with current data on neutrino masses and mixing. Then we described implementation of νDE in the model. The model allows the mass parameter of the singlet, which varies with the acceleron field, to have the same scale as the scale of dark energy satisfying the desired naturalness requirement.

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[1] A. Riess *et al.*, Astron. J. **116**, 1009 (1998); S. Perlmutter *et al.*, Astrophys. J. **517**, 565 (1999); D. Spergel *et al.*, Astrophys. J. Suppl. **148**, 175 (2003).
[2] J. Frieman, M. Turner and D. Huterer, arXiv:0803.0982v1[astro-ph], E.J. Copeland, M. Sami and S. Tsujikawa, Int.J.Mod.Phys.**D15**:1753 (2006).
[3] P. Gu, X. Wang, and X. Zhang, Phys. Rev. D **68**, 087301 (2003).
[4] R. Fardon, A.E. Nelson, and N. Weiner, JCAP **0410**, 005 (2004).
[5] P.Q. Hung, hep-ph/0010126.
[6] C. Wetterich, Nucl. Phys. B **302**, 668 (1988); P.J.E. Peebles and B. Ratra, Astrophys. J. **325**, L17 (1988).
[7] D.B. Kaplan, A.E. Nelson, and N. Weiner, Phys. Rev. Lett. **93**, 091801 (2004); H. Li, Z. Dai, and X. Zhang, Phys. Rev. D **71**, 113003 (2005); V. Barger, P. Huber, and D. Marfatia Phys. Rev. Lett. **95**, 211802 (2005); A.W. Brookfield, C. van de Bruck, D.F. Mota, and D. Tocchini-Valentini, Phys. Rev. Lett. **96**, 061301 (2006); A. Ringwald and L. Schrempp, JCAP **0610**, 012 (2006).
[8] R. Barbieri, L.J. Hall, S.J. Oliver, and A. Strumia, Phys. Lett. B **625**, 189 (2005); C.T. Hill, I. Mocioiu, E.A. Paschos, and U. Sarkar, Phys. Lett. B **651**, 188 (2007); P.H. Gu, H.J. He, and U. Sarkar, Phys. Lett. B **653**, 419 (2007); arXiv:0705.3736 [hep-ph]; P.H. Gu, arXiv:0710.1044 [hep-ph].
[9] N. Afshordi, M. Zaldarriaga, and K. Kohri, Phys.Rev. D **72**, 065024 (2005); J. Vï¿$\frac{1}{2}$liviita, E. Majerotto and R. Maartens, JCAP **0807**, 020 (2008)
[10] R. Takahashi and M. Tanimoto, Phys. Lett. B **633**, 675 (2006); R. Fardon, A.E. Nelson, and N. Weiner, JHEP **0603**, 042 (2006); O.E. Bjaelde, *et al.*, arXiv:0705.2018 [astro-ph]; N. Brouzakis, N. Tetradis, and C. Wetterich, arXiv:0711.2226 [astro-ph].
[11] E. Ma and U. Sarkar, Phys. Lett. B **638**, 356 (2006).
[12] J. R. Bhatt, P. H. Gu, U. Sarkar and S. K. Singh, Phys. Lett. B **663**, 83 (2008)
[13] J.C. Pati and A. Salam, Phys. Rev. Lett. **31**, 661 (1973); Phys. Rev. D **8**, 1240 (1973); *ibid.* **10**, 275 (1974); R.N. Mohapatra and J.C. Pati, Phys. Rev. D **11**, 566, 2558 (1975); R.N. Mohapatra and G. Senjanovic, Phys. Rev. D **12**, 1502 (1975).
[14] M. Malinsky, J.C. Romao, and J.W.F. Valle, Phys. Rev. Lett. 95, 161801 (2005); M. Hirsch, J.W.F. Valle, M. Malinsky, J.C. Romao and U. Sarkar, Phys. Rev. D **75**, 011701 (2007).
[15] S.M. Barr, Phys. Rev. Lett. **92**, 101601 (2004).
[16] C.H. Albright and S.M. Barr, Phys. Rev. D **69**, 073010 (2004); Phys. Rev. D **70**, 033013 (2004).
[17] H. Nishino *et al.* [Super-Kamiokande Collaboration], Phys. Rev. Lett. **102**, 141801 (2009) [arXiv:0903.0676 [hep-ex]].
[18] S. Weinberg, Phys. Lett. B **91**, 51 (1980).
[19] L. J. Hall, Nucl. Phys. B **178**, 75 (1981).
[20] M.K. Parida and P.K. Patra, Phys. Rev. Lett. **66**, 858 (1991); M.K. Parida and P.K. Patra, Phys. Rev. Lett. **68**, 754 (1992); M.K. Parida, Phys. Rev. D **57**, 2736 (1998)

[21] D. G. Lee, R. N. Mohapatra, M. K. Parida and M. Rani, Phys. Rev. D **51**, 229 (1995) [arXiv:hep-ph/9404238]; R. N. Mohapatra and M. K. Parida, Phys. Rev. D **47**, 264 (1993) [arXiv:hep-ph/9204234].
[22] M. Shiozawa *et al.* [Super-Kamiokande Collaboration], Phys. Rev. Lett. **81**, 3319 (1998) [arXiv:hep-ex/9806014].
[23] R. Foot, H. Lew, X.G. He, and G.C. Joshi, Z. Phys. C **44**, 441 (1989); E. Ma, Phys. Rev. Lett. **81**, 1171 (1998).
[24] C. Amsler *et al.* [Particle Data Group], Phys. Lett. B **667**, 1 (2008).
[25] P. F. Harrison, D. H. Perkins and W. G. Scott, Phys. Lett. B **530**, 167 (2002).
[26] C. Wetterich, Phys. Lett. **B 655**, 201 (2007).
[27] O.E. Bjaelde, *et. al.*, arXiv:0705.2018v2[astro-ph] (2008).
[28] D.F. Mota, V. Pettorino, G. Robbers and C. Wetterich, arXiv:0802.1515v1[astro-ph] (2008).
[29] J.R. Bhatt and U. Sarkar, Phys. Rev. D 80,045016 (2009), arXive:0804.2482[hep-ph].
[30] S. Coleman and E. Weinberg, Phys. Rev. D **7**, 1888(1973).