# Majorana Neutrino Superfluidity and Stability of Neutrino Dark Energy

Jitesh R. Bhatt* and Utpal Sarkar
*Physical Research Laboratory, Ahmedabad 380009, India*

We demonstrate that Majorana neutrinos can form Cooper pairs due to long-range attractive forces and show BCS superfluidity in a class of mass varying neutrino dark energy models. We describe the condensates for Majorana neutrinos and estimate the value of the gap, critical temperature and Pippard coherence length for a simple neutrino dark energy model. In the strong coupling regime bosonic degree of freedom can become important and Bose-Einstein condensate may govern the dynamics for the mass varying neutrino models. Formation of the condensates can significantly alter the instability scenario in the mass varying neutrino models.



Some time back neutrino superfluidity was studied for Dirac neutrinos [1], in which the left-handed and the right-handed neutrinos form Cooper pairs due to the attractive force originating from their Yukawa interactions with the Higgs scalar. Unfortunately this interesting concept could not be applied to any realistic situation in astrophysics or cosmology. We extend this formalism for Majorana neutrinos and show that the superfluidity of relic neutrinos could be important when one considers their interactions with very light scalar field, like quintessence. We study the superfluidity of Majorana neutrinos in the context of the mass varying neutrinos (MaVaNs) models [2, 3] and show that it can solve the stability problem [4, 5] of the MaVaNs *naturally*.

The interaction between the neutrinos in a MaVaN scenario is known to be attractive[6, 7, 8] due to the presence of a quintessence field called acceleron. It is well known that these models are unstable when the neutrinos become non-relativistic *i.e.* their pressure $p_\nu \approx 0$ [4, 5]. The instability saturates when the degeneracy pressure balances the attractive force and the final state can evolve as ΛCDM. It should be noted that this instability does not arise in a certain class of models involving super-acceleration [9]

We show that this attractive interaction can lead to neutrino superfluidity in MaVaNs by formation of the Cooper pairs. If the size of the Cooper pairs is smaller than the length scales relevent for the dark energy dynamics, the dynamics of a scalar field describing the Bose-Einstein condensate could be applied for studying the evolution of the system. The inclusion of the condensate dynamics alters the instability scenario significantly. Firstly, there would be no degeneracy pressure in the bosonic system. Moreover the coupling between the neutrinos and the scalar field will be changed. However the attractive force, if any, between the condensates and the acceleron should be balanced by the Heisenberg uncertainty. This kind of stable structures are known in the literature as Boson stars [11]. Next we demonstrate that the stability calculations considered earlier[4, 5] are altered and the new stability criteria can be satisfied by the different models of dark energy potentials. Condensates with Majorana neutrinos has been discussed in the literature [12], but here we develope a statistical formalism following ref. [1].

The first problem one encounters while dealing with the Majorana neutrino supefluidity is the chemical potential. Since the Majorana particles self annihilate, number of particles is not conserved, and hence, their number operator and the chemical potential vanishes in equilibrium. However, in the early universe, these are not of any concern: the Majorana neutrinos have two helicity components which can vary with time according to the helicity-flip rate. One can define the chemical potential to the extent the helicity is conserved. Typically the ratio of the helicity-flip rate to the current Hubble expansion $H$ rates is

$$\frac{G^2\,T^3 g m_\nu^2}{\sqrt{g}T^2/m_p} \sim 10^{-8}, \tag{1}$$

where $g$ is the effective degrees of freedom relevent for the temperature $T$ [13]. Thus the helicity-flipping rate of the Majorana particles ceases below a certain temperature, when the particles move apart from each other due to the expansion of the universe at a faster rate compared to their self annihilation rate [14].

Another important problem is to show how the freely streaming streaming neutrinos can become degenerate and exhibit superfluidity: In any MaVaN model there is an interaction between the scalar field and the neutrinos. Dynamics of the neutrinos can be described by the following kinetic equation[Afshordi et.al. in Ref.[4]]

$$\frac{df}{d\eta} + \mathbf{u}\cdot\nabla f - a\gamma^{-1}\nabla\, m_\nu \cdot \frac{\partial\, f}{\partial\, \mathbf{p}} = 0 \tag{2}$$

where $a$ and $\eta$ are the scale factor and the conformal time, defined in Robertson-Walker metric, respectively. The last term includes the effective neutrino mass variation due to the scalar field. In the absence of the last term on the left hand side, $f$ is given by the usual Fermi-Dirac distribution. However, when small perturbations of the type $\delta f = \Delta(p)exp[i(\mathbf{k}\cdot\mathbf{x} - \omega\eta]$ and $\delta m_\nu = \Sigma exp[i(\mathbf{k}\cdot\mathbf{x} - \omega\eta]$ are considered, the system becomes unstable and eq.(1) gives, in a sub-Hubble regime,

$$\omega\Delta(\mathbf{p}) = \mathbf{k}\cdot\mathbf{u} - \gamma^{-1}\left(\mathbf{k}\cdot\frac{\partial f}{\partial \mathbf{p}}\right)\Sigma \tag{3}$$

Only $\omega/k = c_s$ appear in the equation and there is no preferred scale in this equation. $c_s$ can be found (Afshordi et. al. in Ref.[4]) to be $\pm\sqrt{-1}$ in a nonrelativisitic regime. Thus the instability that grows at smaller length scale. Since there is no preferred length scale in equation (3), the instability can continue to grow on the smaller length scales. However in a realistic situation the instability may saturate when the degeneracy pressure by the neutrinos become important[18]. At this stage the attractive interaction between the degenerate neutrinos may induce the phenomenon of superfluidity.

Consider a Majorana mass term of the left-handed neutrinos in a model that provides an attractive long-range force due to the exchange of a scalar field $\phi$:

$$\mathcal{L}_M = \overline{\nu_M}\ [m_\nu + f_\phi\ \phi]\ \nu_M\ . \tag{4}$$

Although we present the formalism for one gneration, it can be easily extended to the realistic case with three neutrinos.

The Majorana field $\nu_M$, defined in terms of the left-handed neutrinos $\nu_L$ and its CP conjugate field ${\nu^c}_R$:

$$\nu_M = \nu_L + \lambda {\nu^c}_R \tag{5}$$

satisfies the condition:

$$\nu^c_M = [\nu_L + \lambda {\nu^c}_R]^c = \lambda^* \nu_M . \tag{6}$$

where $\lambda$ is the Majorana phase, $|\lambda|^2 = 1$. We work in the Weyl representation:

$$\gamma^\mu \ = \ \begin{pmatrix} 0 & \sigma^\mu \\ \bar{\sigma}^\mu & 0 \end{pmatrix} \quad \text{with} \quad \sigma^\mu = [I_2, \sigma_i];\ \bar{\sigma}^\mu = [I_2, -\sigma_i]$$

where $I_2$ is a $2\times 2$ unit matrix and $\sigma^i$ are the Pauli matrices. In this basis, $\gamma_5 = i\gamma^\circ\gamma^1\gamma^2\gamma^3 = \mathrm{diag}\,(\,-I_2, \quad I_2\,)$ is diagonal, and the left and right-handed fields become:

$$\nu_L = \begin{pmatrix} \psi \\ 0 \end{pmatrix};\nu_R = \begin{pmatrix} 0 \\ \overline{\chi} \end{pmatrix};{\nu^c}_R = \begin{pmatrix} 0 \\ \overline{\psi} \end{pmatrix};{\nu^c}_L = \begin{pmatrix} \chi \\ 0 \end{pmatrix}. \tag{7}$$

The Majorana neutrinos can now be expressed as:

$$\nu_M = \begin{pmatrix} \psi \\ \lambda\bar{\psi} \end{pmatrix};\quad \nu^c_M = \begin{pmatrix} \lambda^*\psi \\ \bar{\psi} \end{pmatrix};\quad \overline{\nu_M}^T = \begin{pmatrix} \lambda^*\overline{\psi}^\dagger \\ \psi^\dagger \end{pmatrix}; \tag{8}$$

so that the Lagrangian density associated with the mass term (equation 4) becomes

$$\mathcal{L}_M \ = \ [m_\nu + f_\phi\phi]\overline{\nu_M}\ \nu_M = [m_\nu + f_\phi\phi]\left[\lambda^*\overline{\psi}^\dagger\psi + \lambda\psi^\dagger\overline{\psi}\right].$$

It ought to be mentioned that the present mass of scalar field differ widely from $10^{-4}eV$ as in Fardon *et. al.* in Ref.[2] to $m_\phi > H$ *e.g* Bjaelde *et. al.* in Ref. [5]. In what follows, we deviate from the initial model of Fardon *et. al.* and consider the case with $m_\phi > H$. For small energy and momentum transfers, interaction term can be written as

$$H_I = -\mathcal{C}\ (\overline{\nu_M}\ \nu_M)\ (\overline{\nu_M}\ \nu_M)\,. \tag{9}$$

In terms of the component fields $\psi$ this becomes

$$\begin{aligned} H_I \ &= \ -\mathcal{C}\left[\lambda^{*2}\ \overline{\psi}^\dagger_a\ \psi_a\ \overline{\psi}^\dagger_b\ \psi_b + \overline{\psi}^\dagger_a\ \psi_a\ \psi^\dagger_b\ \overline{\psi}_b \right. \\ &+ \ \psi^\dagger_a\ \overline{\psi}_a\ \overline{\psi}^\dagger_b\ \psi_b + \lambda^2\ \psi^\dagger_a\ \overline{\psi}_a\ \psi^\dagger_b\ \overline{\psi}_b\ ]\ . \end{aligned} \tag{10}$$

One of the key ingredient in theory of superconductivity is to have an overall attractive interaction between its particles. In the case of a metal the Coulomb interaction between the electrons, in Fourier space is $V_{Coul} = e^2/(k^2 + K_D^2)$, where ${K_D}^{-1}$ is the typical shielding distance. $V_{Coul}$ is always repulsive *i.e.* $V_{coul} > 0$. However the superconductivity arises as the the electrons in the metal also have an attractive interaction $V_{ph}$ arising due to their interaction with the phonons. One can show that for a superconductor $V_{ph} + V_{Coul} < 0$. Under this condition it is energetically more favourable for particles to form pairs. Momenta of the particles in the pair states are directed opposite with each other and have spin in the opposite directions. When the energy minimization is carried out for the wave function containing the pair states either occupied by the two particles or none the gap condition naturally arises [15]. In the case of MaVaN scenario, as implied by Eqs.(9-10) there is an attractive interaction between the neutrons. In fact the famous instability in this scenario arises In fact the instability in MaVaN scenario arises precisely when this interaction dominates over the gravity [4]. Since there is no other interaction that can make up for the overall repulsive interaction, the above condition of superconductivity can be satisfied.

The gauge boson exchange would give repulsive force between two left-handed fields, so the only possible condensate would correspond to a spin-0 pairing of the left-handed neutrinos with the right-handed antineutrinos:

$$\langle\psi_a\ \overline{\psi}^\dagger_b\rangle = \epsilon_{ab}\ D\,. \tag{11}$$

The mean field approximation would then give us the interaction Hamiltonian with the condensate $D$:

$$H^{MF}_1 = -2\ \mathcal{C}\left[\lambda^{*2}\ \overline{\psi}^\dagger_a\ \psi_b\ D + \lambda^2\ \psi^\dagger_a\ \overline{\psi}_b\ D^*\right]\ \epsilon_{ab}\,. \tag{12}$$

We shall now express the Majorana field in terms of the creation and the annihilation operators as

$$\psi_M(x) = \sum_{p,s}\sqrt{\frac{m_\nu}{2\epsilon}}\left(f_{ps}u_{ps}e^{-ipx} + \lambda^* f^\dagger_{ps}v_{ps}e^{ipx}\right)\,. \tag{13}$$

The component fields are then related to the creation and annihilation operators through the relation

$$\psi = \sum_{p,s}\sqrt{\frac{m_\nu}{2\epsilon}}f_{ps}u_{ps}e^{-ipx} \qquad \overline{\psi} = \sum_{p,s}\sqrt{\frac{m_\nu}{2\epsilon}}f^\dagger_{ps}v_{ps}e^{ipx}$$

$$\psi^\dagger = \sum_{p,s}\sqrt{\frac{m_\nu}{2\epsilon}}f^\dagger_{ps}\bar{u}_{ps}e^{ipx} \qquad \overline{\psi}^\dagger = \sum_{p,s}\sqrt{\frac{m_\nu}{2\epsilon}}f_{ps}\bar{v}_{ps}e^{-ipx}\,.$$

The interaction Hamiltonian can then be written in terms of the creation and annihilation operators as:

$$\begin{aligned} H_1^{MF} &= -\mathcal{C}\sum_p \frac{m_\nu}{\epsilon}\left[D\ \lambda^{*2}\ e^{-2i\epsilon t}\left(f_{p\uparrow}f_{-p\downarrow} - f_{p\downarrow}f_{-p\uparrow}\right)\right. \\ &\left. +D^*\ \lambda^2\ e^{2i\epsilon t}\left(f^\dagger_{p\uparrow}f^\dagger_{-p\downarrow} - f^\dagger_{p\downarrow}f^\dagger_{-p\uparrow}\right)\right], \end{aligned} \tag{14}$$

where $\epsilon = \sqrt{p^2 + m_\nu^2}$.

In models of mass varying neutrinos the number density of the Majorana neutrinos becomes proportional to the inverse of neutrino mass. In addition, the effective interaction Hamiltonian also does not conserve particle number. Since the treatment is based on grand canonical ensemble, this requires a self-consistent treatment to determine when the condensates become nonvanishing. The complete Hamiltonian ($H$) is obtained by adding the interaction part $H_1^{MF}$ and the free-particle Hamiltonian

$$H_0 = \sum_p \epsilon\ \left(f^\dagger_{p\uparrow}f_{p\uparrow} + f^\dagger_{p\downarrow}f_{p\downarrow}\right). \tag{15}$$

One can also write the complete Hamiltonian in a so called standard or canonical form in which it resembles with the free particle Hamiltonian in Eq.(15) [16] as

$$\mathcal{H} = \sum_p E\ \left(b^\dagger_{p\uparrow}b_{p\uparrow} + b^\dagger_{p\downarrow}b_{p\downarrow}\right), \tag{16}$$

where $E^2 = (\epsilon - \mu)^2 + \kappa^2$ and $\mu$ is the chemical potential.

For the late universe when the neutrino become nonrelativistic, $T \ll m$, one can write its chemical potential following Ref.[17] as

$$\mu(t) = m + (\mu_D - m)T(t)/T_D$$

where $T_D$ is the decoupling temperature and $T(t)$ can be regarded as the current temperature. In the late universe the second term on the right hand side can be negligible compared to the first term.

One can have a time-dependent transformation that can relate the complete Hamiltonian $H = H_0 + H_1^{MF}$ with the standard form given by Eq.(16) [1, 16]. A relation between the annihilation and the creation operators in both the Hamiltonians is given by

$$\begin{aligned} b_{p\uparrow} &= \cos\theta e^{i(\alpha+\epsilon t)}f_{p\uparrow} - \sin\theta e^{i(\alpha+\epsilon t)}f^\dagger_{-p\downarrow} \\ b_{p\downarrow} &= \cos\theta e^{i(\alpha+\epsilon t)}f_{p\downarrow} + \sin\theta e^{i(\alpha+\epsilon t)}f^\dagger_{-p\uparrow}, \end{aligned} \tag{17}$$

with $D\lambda^{*2} = |D|e^{2i\alpha}$, $\tan 2\theta = \kappa/(\epsilon - \mu)$ and $\kappa = 2\,\mathcal{C}|D|m_\nu/\epsilon$. A consistent solution for the nonvanishing condensate $D \neq 0$ requires $\alpha = \pi/2$, which has contributions from both the condensate $D$ as well as from the Majorana phase $\lambda^*$. The magnitude of the gap is determined by the consistency condition that the value of the condensate is same as that of the value obtained by the canonical transformation. In other words, if we express the condensate in terms of the density matrix ($\rho$) as

$$\langle\psi_a\ \overline{\psi}^\dagger_b\rangle = \rho\psi_a\ \overline{\psi}^\dagger_b, \tag{18}$$

the density matrix ($\rho$) satisfies the consistency condition

$$\rho = \frac{e^{-\beta H - \mu N}}{\sum e^{-\beta H - \mu N}} = \frac{e^{-\beta\mathcal{H}}}{\sum e^{-\beta\mathcal{H}}}. \tag{19}$$

This condition translates into

$$\frac{\mathcal{C}}{2}\int\frac{d^3p}{(2\pi)^3}\frac{m_\nu^2}{\epsilon^2}\frac{1}{\sqrt{(\epsilon-\mu)^2+\kappa^2}} = 1, \tag{20}$$

whose solution gives us the magnitude of the gap. This integral is divergent and it should be cut off with the upper limit $\Lambda$. The condensates form due to the attractive force between the neutrino and the scalar field in the MaVaN scenario. For the early times when the neutrinos were in thermal contact with the other species in the universe, this attractive interaction may not be very important. Thus the values of $\Lambda$ can be estimated from the energy scales below which the attractive interaction can be felt by the neutrinos become important.

Solving this equation we obtain the gap

$$\Delta = 2\sqrt{\frac{2\Lambda}{m_\nu}}\left(3\pi^2 n_\nu\right)^{1/3}e^{-x}, \tag{21}$$

where $x = 2\pi^2/[\mathcal{C}m_\nu^2(3\pi^2 n_\nu)^{1/3}]$. The critical temperature and the Pippard coherent length are given by

$$T_c = \frac{e^\gamma}{\pi}\Delta \approx 0.57\Delta; \qquad \xi = \frac{e^x}{\pi\sqrt{2\Lambda m_\nu}}. \tag{22}$$

This completes the formalism of formation of Cooper pairs with Majorana neutrinos and BCS superconductivity. This may have many applications.

We shall now discuss how the Majorana neutrino superfluidity, can solve the stability problem [4, 5] in neutrino dark energy models [2, 3, 7]. We demonstrate this in a specific two-generation neutrino dark energy model [7]. The standard model is extended with two right-handed neutrinos $N_i, (i = 1, 2)$ and two scalars $\Phi_i, (i = 1, 2)$ with a global $U(1)_1 \times U(1)_2$ symmetry, so that these fields interact as

$$\mathcal{L}_M = \frac{1}{2}\alpha_1\bar{N}_1 N_1^c\Phi_1 + \alpha_2\bar{N}_2 N_2^c\Phi_2. \tag{23}$$

When the fields $\Phi_i$ acquire vacuum expectation values (*vev*) $\langle\Phi_i\rangle = f_i$, we can express them as

$$\Phi_i = \frac{f}{2\sqrt{2}}e^{2i\phi_i/f}, \tag{24}$$

where $\phi_i$ are the massless Nambu-Goldstone bosons and we assumed same decay constant $f_i = f$ for both the fields. Writing $\alpha_i \Phi_i = M_i \exp[2i\phi/f]$, we get the masses of the right-handed neutrinos $N_i$ to be $M_i$.

In this model, the neutrino Dirac mass terms $m_{ij}\bar{\nu}_i N_j$ do not respect the global symmetry, and hence, one combination of the global $U(1)$ symmetries is broken explicitly. As a result, one of the two massless Nambu-Goldstone bosons picks up a small mass, making it a pseudo Nambu-Goldstone boson (pNGB). This pNGB (denoted by $\phi$) can then become the acceleron field that explains the dark energy

After integrating out the heavy right-handed neutrinos, we can write down the mass matrix of the light physical neutrinos $\nu_p^T = (\nu_1 \quad \nu_2)$, as given in equation 4, where $f_\phi = -im^2/2Mf$ is not diagonal, leading to a long-range attractive force, and $m_p = m^2/M$. In these models of MaVaNs, naturalness restricts the mass scales of the model and the right-handed neutrino mass scale is supposed to be as low as eV.

To estimate the parameter $x$ we need to know $\mathcal{C}$ which depends on the acceleron mass $\mathcal{C} \sim \frac{1}{8m_\phi}$. If one takes $m_\phi \sim 6Mpc$ [5], $m_\nu \sim 1eV$ and $n_\nu \sim 56/cm^3$ one finds $x \sim 10^{-56}$. Thus practically the exponential factors in Eqs.(21-23) are unity. So, the relevent scales of this equations are determined by the cut-off $\Lambda$ and neutrino mass $m_\nu$. There remains a great deal of uncertainty over the range of these parameters. For example if one can take $\Lambda$ as the scale when the tracking $\rho_\nu \sim \rho_{DE}$ becomes valid [see Fardon *et.al* in Ref.[2]], one can take $\Lambda$ as the decoupling temperature $1\,MeV$. One can also take $\Lambda$ to be very close to the scale when the neutrinos become non-relativistic i.e. few times $m_\nu$. If we take the neutrino mass around $1\,eV$, $\xi$ has range between $0.36 - 10^4\,cms$. The values of $\xi$ in this entire range still can be smaller than the neutrino lumps [18]. Finally we comment on the so called instability in MaVaN models[4] in this changed scenario with the condensates. The instability is known to arise when the coupling between the scalar field and the neutrinos is stronger than the gravitational force. The coupling can be described by a source term of the type $\beta(\phi)(\rho_\nu - 3p_\nu)$ [5]. In the the relativistic regime $\rho_\nu \sim 3p_\nu$, the coupling is highly suppressed. However, it can become strong in the non-relativistic limit $p_\nu \sim 0$. Size of the Cooper pairs is determined by the interaction strength between the scalar field and the neutrinos. $\phi_s$ dynamics is given by the following Lagrangian density

$$\mathcal{L} = \partial_0\phi_s^\dagger\partial_0\phi_s - \partial_i\phi_s^\dagger\partial_i\phi_s - V(\phi_s), \tag{25}$$

where, $m \simeq 2m_\nu$ represents mass of the condensate. The condensate potential

$$V(\phi_s) = m^2|\phi_s|^2 + g|\phi_s|^4 \tag{26}$$

There can be interaction between the condensate $\phi_s$ and the scalar field $\phi$ given by $V_{int} = g_1\phi\phi_s$. We can write down the perturbation equation for the coupled system $\phi$ and $\phi_s$, following Bjaelde, *et. al.* in Ref.[5], for the minimum of the effective potential tracked by the fields,

$$\delta\ddot{\phi} + 2H\delta\dot{\phi} + \left[k^2 + a^2 V''_\phi\right]\delta\phi \;=\; g_1\delta\phi_s \tag{27}$$

$$\delta\ddot{\phi}_s + 2H\delta\dot{\phi}_s + \left[k^2 + a^2 V''_{\phi_s}\right]\delta\phi_s \;=\; g_1\delta\phi. \tag{28}$$

It should be noted that $g_1$ quantifies the coupling between the condensate field $\phi_s$ and the scalar field $\phi$ and it can be much smaller than the coupling between the neutrinos and the scalar field. The condensates already formed due to the attractive between the Majorana neutrino mediated by the $\phi$.

Eqs.(27-28) contain $V''_\phi$ and $V''_{\phi_s}$ terms which are non-linear functions of $\phi$ and $\phi_s$ respectively. These equations are linearized by making the assumption that they can be written as $\phi \approx \phi_0 + \delta\phi$ and $\phi_s \approx \phi_{s0} + \delta\phi_s$, where, the quantities with suffix 0 represent background quantities and they can vary with time much slowly than the perturbation. From this perturbation in the scalar field

$$\delta\phi = \frac{g_1\delta\phi_s}{-\omega^2 + 2iH + k^2 + a^2 V''_\phi} \tag{29}$$

We can compute $V''_\phi$ in Eq.(24) using two forms of potential $V_\phi$ frequently used in the MaVaN models. First we consider Coleman-Weinberg type of potential [19]

$$V_\phi \;=\; V_0 log(1 \,+\, \kappa\phi) \tag{30}$$

where parameters $V_0$ and $\kappa$ can be selected to yield total dark energy contribution $\Omega_{DE} \simeq 0.7$. Using the above linearization one can find $V''_\phi = \frac{2\kappa^2 V_0}{(1+\kappa\phi_0)^2}$. Secondly we consider an inverse power-law model of the potential given by

$$V_\phi \;=\; \frac{M^{n+4}}{\phi^n} \tag{31}$$

where the parameter $M$ can be fixed by the requirements $\Omega_{DE} \simeq 0.7$ and $m_\phi \gg H$. Thus we write as for $n < 1$,

$$V''_\phi \;=\; \frac{n(n-1)(n-2)M^{n+4}}{\phi_0^{n-1}} \tag{32}$$

Instability is known to occur in a MaVaN scenario for $H < k/a < m_\phi$ [5]. The following dispersion relation can be obtained from Eqs.(27-28):

$$\omega^4 - 4iH\omega^3 - \left[4H^2 + B\right]\omega^2 + 2iHB\omega + B_1B_2 - g_1^2 = 0 \tag{33}$$

where, $B = 2k^2 + a^2\left(V''_\phi + V''_{\phi_s}\right)$, $B_1 = k^2 + a^2 V''_\phi$ and $B_2 = k^2 + a^2 V''_{\phi_s}$. Eq.(30) is quartic in $\omega$ and it can be solved exactly by analytical means. However it is instructive to solve it by approximate methods. The terms that are linear in the expansion rates $H$ will contribute to the damping of the modes. Moreover the instability, in the MaVaN scenario without the condensates, the instability was found to occur in the regime $H < k/a < m_\phi$.

Thus $H$ defines the smallest wave-vector of the perturbations. In what follows we ignore the terms involving $H$ in Eq.(33) to get following biquadratic dispersion relation,

$$\omega^4 - B\omega^2 + B_1 B_2 - g_1^4 \approx 0 \tag{34}$$

Solution of this can be written as

$$\omega^2 \,=\, B \pm \sqrt{B^2 - 4\left(B_1 B_2 - g_1^2\right)} \tag{35}$$

None of the roots of Eq.(32) is imaginary if $B \geq 0$ and $B_1 B_2 - g_1^2 \geq 0$. These are the stability criteria and their validity may depend on the form of the scalar field potential as $B$ and $B_1$ both involve the term $V_\phi''$. If $B \geq 0$ and $B_1 B_2 - g_1^2 < 0$ then the two roots of Eq.(39) are purely imaginary and one of it could give the instability. It is easy to verify that for the Coleman-Weinberg potential given by Eq.(33) $V_\phi''$ is positive for the parmeters values $V_0 \approx 8.6 \times 10^{-13} eV^4$, $\kappa \approx 1 \times 10^{20} M_{pl}^{-1}$ and $\phi_0 \approx 10^{-6} M_{pl}$ taken from Ref. [5]. For the condensate potential with $g > 0$, one can have all the stability conditions satisfied for a sufficiently small coupling strength $g_1$. The stability condition satisfied for the entire regime $H < k/a < m_\phi$. For the power-law kind of potential given by Eq.(34) one can take $M \approx 0.011 eV$, $n = 0.01$ and $\phi_0 \approx 0.001 M_{pl}$ [5],one can have $B_1 > 0$. For this case the stability conditions are again satisfied for a sufficiently small values of the coupling $g_1$ and $g > 0$.

The Majorana neutrino superfluidity we discussed may have some interesting consequences. The effects of the attractive long force, required for the formation of condensates, have been discussed for the neutrino oscillation experiments [20] and also in cosmology [7]. The acceleron potential can also change some of the features of the neutrino oscillations [21], which will be further modified in the presence of the condensates.

We would like to note here that the stability analysis provided in this paper is for the scalar field $\phi_s$ describing neutrino condensates. This is a valid description when the Bose-Einstein condensation (BEC) is formed. However when the coherent length is much smaller than the neutrino interparticle spacing, the phenomenon of superfluidity of Majorana neutrino still occur in BCS as shown in Ref.([1]). Furthermore, the accelerated expansion can make the ratio of the helicity flip rate to the Hubble expansion rate even more smaller than the one we have at present. Thus the condition for defining chemical potential in accelerated universe will remain satisfied in future. However the chemical potential thus defined is a 'dynamical quantity'. But the question of the small variation in the chemical potential with time may not be studied by the formalism presented here.

In summary we have discussed two issues for MaVaN models namely the superfluidity of Majorana neutrinos and the stability of MaVaN dyanmics. We proposed a formalism to have condensates with Majorana neutrinos. Our formalism shows that the neutrino superfluidity naturally arises in MaVaN scenario and is a generic feature of interacting Fermi particles having attractive potential between them at a low temperature [22]. In addition, we have also shown that for the case when the condensate dynamics become important and $m_\phi > H$ the dynamics of the condensate can be stable for a variety of the dark energy potentials.

---

* Electronic address: jeet@prl.res.in


[1] J.I. Kapusta, Phys. Rev. Lett. **93**, 251801 (2004).

[2] P. Gu, X. Wang, and X. Zhang, Phys. Rev. D **68**, 087301 (2003); R. Fardon, A.E. Nelson, and N. Weiner, JCAP **0410**, 005 (2004); P.Q. Hung, hep-ph/0010126; R. D. Peccei, Phys. Rev. **D 71**, 0235727 (2005).

[3] H. Li, Z. Dai, and X. Zhang, Phys. Rev. D **71**, 113003 (2005); V. Barger, P. Huber, and D. Marfatia Phys. Rev. Lett. **95**, 211802 (2005); A.W. Brookfield, C. van de Bruck, D.F. Mota, and D. Tocchini-Valentini, Phys. Rev. Lett. **96**, 061301 (2006); A. Ringwald and L. Schrempp, JCAP **0610**, 012 (2006); R. Barbieri, L.J. Hall, S.J. Oliver, and A. Strumia, Phys. Lett. B **625**, 189 (2005); R. Takahashi and M. Tanimoto, Phys. Lett. B **633**, 675 (2006); R. Fardon, A.E. Nelson, and N. Weiner, JHEP **0603**, 042 (2006); E. Ma and U. Sarkar, Phys. Lett. B **638**, 356 (2006); K. Ichiki and Y. Keum, JHEP, 000(2008); arXiv:0803.3142 (2008).

[4] N. Afshordi, M. Zaldarriaga, and K. Kohri, Phys.Rev. D **72**, 065024 (2005). It is possible that a new kind of instability would arise even in a non-adiabatic regime in dynamical-dark-energy: J. Väliviita, E. Majerotto and R. Maartens, JCAP **0807**, 020 (2008).

[5] O.E. Bjaelde, *et. al.*, arXiv:0705.2018v2[astro-ph]; C. Wetterich, Phys. Lett. **B 655**, 201 (2007); D.F. Mota, V. Pettorino, G. Robbers and C. Wetterich, arXiv:0802.1515v1[astro-ph], see also R. Bean, E.E. Flanagan and M. Trodden, New J.Phys **10**, 03306 (2008).

[6] C.T. Hill, D.N. Schramm, J.N. Fry, Nucl. Part. Phys. **19**, 25 (1989); J.A. Frieman, C.T. Hill, R. Watkins, Phys. Rev. **D 46**, 1226 (1992); A.K. Gupta, C.T. Hill, R. Holman, E.W. Kolb, Phys. Rev. **D 45**, 441 (1992); J. A. Frieman, C. T. Hill, A. Stebbins and I. Waga, Phys. Rev. Lett. **75**, 2077 (1995); E. Masso, F. Rota, G. Zsembinszki, Phys. Rev. **D 70**, 115009 (2004); E. Masso, G. Zsembinszki, JCAP **0602**, 012 (2006); P.Q. Hung, E. Masso, G. Zsembinszki, JCAP **0612**, 004 (2006),

[7] C.T. Hill, I. Mocioiu, E.A. Paschos, and U. Sarkar, Phys. Lett. B **651**, 188 (2007).

[8] P.H. Gu, H.J. He, and U. Sarkar, Phys. Lett. B **653**, 419 (2007); JCAP **0711**, 016 (2007); P.H. Gu, arXiv:0710.1044 [hep-ph].

[9] M. Kaplinghat and A. Rajaraman, Phys. Rev. **D 75**, 103504 (2007).

[10] C. Wetterich, Nucl. Phys. B **302**, 668 (1988); P.J.E. Peebles and B. Ratra, Astrophys. J. **325**, L17 (1988).

[11] D.J. Kaup, Phys. Rev. **C 172**, 172 (1968); F.E. Schunck and E.W. Mielke, Class. Quant. Grav. **20**, R301 (2003).

[12] S. Antusch, J. Kersten, M. Lindner and M. Ratz, Nucl. Phys. **B 658**, 203 (2003); G. Barenboim,

arXiv:0811.2998[hep-ph].

[13] P. Langacker, G. Segré and S. Soni, Phys. Rev. **D 26**, 3425 (1982).

[14] M. Fukugita and T. Yanagida, Phys. Rev. **D 42**, 1285 (1990), M. Li, X. Wang, B. Feng and X. Zhang, Phys. Rev. **D 65** , 1103511 (2002).

[15] R.P. Feynman, *Statistical Mechanics: A set of Lectures*, (Addison-Wesley Publ. Co., Redwood City, 1972).

[16] A.A. Abrisikov, L.P. Gorkov and I.E. Dzyaloshinkii, *Methods of Quantum Field theory in Statistical Physics* (Prentice-Hall, Englewood Cliffs, 1963), L.E. Reichl, *A Modern Course in Statistical Physics* (John Wiley & Sons, New York, 1998).

[17] E. Kolb and M. S. Turner, *The Early Universe*, p.70, (Addison-Wesely Publishing Company, 1990).

[18] N. Brouzakis, N. Tetradis, and C. Wetterich, arXiv:0711.2226,[astro-ph], P.P. Avelino, L.M.G. Beca and C.J.A.P. Martins, Phys. Rev. **D 77**, 101392 (2008).

[19] S.R. Coleman and E. Weinberg, Phys. Rev. **D 7**, 1888 (1973).

[20] J. A. Grifols and E. Masso, Phys. Lett. B **579**, 123 (2004); V. Barger, P. Huber, and D. Marfatia, Phys. Rev. Lett. **95**, 211802 (2005); M. Cirelli, M. C. Gonzalez-Garcia, and C. Pena-Garay, Nucl. Phys. **B719**, 219 (2005); M. C. Gonzalez-Garcia, P. C. de Holanda, E. Masso and R. Zukanovich Funchal, arXiv:hep-ph/0609094.

[21] D.B. Kaplan, A.E. Nelson, and N. Weiner, Phys. Rev. Lett. **93**, 091801 (2004); G. Dvali, Nature **432**, 567 (2004).

[22] R.B. Laughlin, *A Different Universe*, (Basic Books, New York, 2005).