Radiative seesaw in left-right symmetric model

Pei-Hong Gu[1*] and Utpal Sarkar[2†]
[1] *The Abdus Salam International Centre for Theoretical Physics, Strada Costiera 11, 34014 Trieste, Italy*
[2] *Physical Research Laboratory, Ahmedabad 380009, India*

There are some radiative origins for the neutrino masses in the conventional left-right symmetric models with the usual bi-doublet and triplet Higgs scalars. These radiative contributions could dominate over the tree-level seesaw and could explain the observed neutrino masses.

PACS numbers: 14.60.Pq, 12.60.Cn, 12.60.Fr

Introduction: Strong evidence from the neutrino oscillation experiments has confirmed the tiny but nonzero neutrino masses. This phenomenon is elegantly explained by the seesaw mechanism [1] in some extensions of the standard model (SM). The seesaw scenario can be naturally embedded into the left-right symmetric models [2] and also the grand unified theories (GUTs).

In this paper, we discuss the neutrino mass generation in a general class of left-right symmetric models with the Higgs sector including one bi-doublet, one left-handed triplet and one right-handed triplet Higgs fields. Our analysis shows that the neutrino masses can originate from some loop diagrams in addition to the tree-level seesaw. We also demonstrate that the radiative neutrino masses could explain the experimental results for some choice of parameters.

The left-right symmetric model: We consider the left-right symmetric extension of the SM with the gauge group $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ and the following Higgs content:

$$\phi(\mathbf{2},\mathbf{2}^*,0)\,,\ \Delta_L(\mathbf{3},\mathbf{1},-2)\,,\ \Delta_R(\mathbf{1},\mathbf{3},-2)\,. \quad (1)$$

A convenient representation of these fields is given by the 2×2 matrices:

$$\phi=\left[\begin{array}{cc}\phi_1^0 & \phi_2^+\\ \phi_1^- & \phi_2^0\end{array}\right]\,,\ \Delta_{L,R}=\left[\begin{array}{cc}\delta^+/\sqrt{2} & \delta^{++}\\ \delta^0 & -\delta^+/\sqrt{2}\end{array}\right]_{L,R}\,. \quad (2)$$

As for the fermion sector, it includes the left- and right-handed quarks:

$$q_L(\mathbf{2},\mathbf{1},\frac{1}{3})=\left[\begin{array}{c}u\\ d\end{array}\right]_L\,,\ q_R(\mathbf{1},\mathbf{2},\frac{1}{3})=\left[\begin{array}{c}u\\ d\end{array}\right]_R \quad (3)$$

and the left- and right-handed leptons:

$$\psi_L(\mathbf{2},\mathbf{1},-1)=\left[\begin{array}{c}\nu\\ \ell\end{array}\right]_L\,,\ \psi_R(\mathbf{1},\mathbf{2},-1)=\left[\begin{array}{c}\nu\\ \ell\end{array}\right]_R\,. \quad (4)$$

Under the left-right (parity) symmetry, we have $\phi\leftrightarrow\phi^\dagger$, $\Delta_L\leftrightarrow\Delta_R$, $q_L\leftrightarrow q_R$ and $\psi_L\leftrightarrow\psi_R$. The parity invariant Yukawa couplings are then given by

$$\begin{aligned}\mathcal{L}\ \supset\ & -\tilde{y}_{q_{ij}}\overline{q_{L_i}}\tilde{\phi}q_{R_j}-y_{q_{ij}}\overline{q_{L_i}}\phi q_{R_j}-\tilde{y}_{\psi_{ij}}\overline{\psi_{L_i}}\tilde{\phi}\psi_{R_j}\\ & -y_{\psi_{ij}}\overline{\psi_{L_i}}\phi\psi_{R_j}-\frac{1}{2}f_{ij}\left(\overline{\psi^c_{L_i}}i\tau_2\Delta_L\psi_{L_j}\right.\\ & \left.+\overline{\psi^c_{R_i}}i\tau_2\Delta_R\psi_{R_j}\right)+\text{h.c.}\,,\end{aligned} \quad (5)$$

where $\tilde{\phi}=\tau_2\phi^*\tau_2$, $y_q=y_q^\dagger$, $\tilde{y}_q=\tilde{y}_q^\dagger$, $y_\psi=y_\psi^\dagger$, $\tilde{y}_\psi=\tilde{y}_\psi^\dagger$ and $f=f^T$. For simplicity, we do not present the most general renormalizable and parity invariant scalar potential which can be found in many early works [3, 4].

The seesaw mechanism: We now review the seesaw mechanism of the neutrino masses in the left-right symmetric model with the choice of Higgs scalars we considered. In this case, the left-right symmetry is broken down to the SM $SU(2)_L\times U(1)_Y$ symmetry after the right-handed triplet Higgs scalar develops its vacuum expectation value (*vev*) $v_R\equiv\langle\Delta_R\rangle$. From Eq. (5), we thus have the following Yukawa couplings and Majorana mass term:

$$\begin{aligned}\mathcal{L}\ \supset\ & -y_{d_{ij}}\overline{q_{L_i}}\tilde{\varphi}d_{R_j}-y_{u_{ij}}\overline{q_{L_i}}\varphi u_{R_j}-y_{\ell_{ij}}\overline{\psi_{L_i}}\tilde{\varphi}\ell_{R_j}\\ & -y_{\nu_{ij}}\overline{\psi_{L_i}}\varphi\nu_{R_j}-\frac{1}{2}f_{ij}v_R\overline{\nu^c_{R_i}}\nu_{R_j}\\ & -\frac{1}{2}f_{ij}\overline{\psi^c_{L_i}}i\tau_2\Delta_L\psi_{L_j}+\text{h.c.}\,.\end{aligned} \quad (6)$$

Here we have defined

$$\phi_1=\left[\begin{array}{c}\phi_1^0\\ \phi_1^-\end{array}\right]\,,\ \phi_2=\left[\begin{array}{c}\phi_2^{0*}\\ -\phi_2^{+*}\end{array}\right]\,, \quad (7)$$

and then

$$\begin{aligned}\varphi &= \phi_1\cos\beta+\phi_2\sin\beta=\left[\begin{array}{c}\varphi^0\\ \varphi^-\end{array}\right]\,, & (8)\\ y_d &= -y_q\sin\beta-\tilde{y}_q\cos\beta\,, & (9)\\ y_u &= y_q\cos\beta+\tilde{y}_q\sin\beta\,, & (10)\\ y_\ell &= -y_\psi\sin\beta-\tilde{y}_\psi\cos\beta\,, & (11)\\ y_\nu &= y_\psi\cos\beta+\tilde{y}_\psi\sin\beta\,, & (12)\end{aligned}$$

where $\beta=\arctan\left(\frac{v_2}{v_1}\right)$ with $v_1\equiv\langle\phi_1\rangle$ and $v_2\equiv\langle\phi_2\rangle$. Obviously, the doublet scalar φ is the SM Higgs. Note

*Electronic address: pgu@ictp.it
†Electronic address: utpal@prl.res.in

that $y_u = -y_d$ and $y_\nu = -y_\ell$ for $\beta = \frac{\pi}{4}$ from $v_1 = v_2$. So we will not consider the case of $v_1 = v_2$ due to the mass differences between the up and down type quarks.

The first line in Eq. (6) will give the masses to the charged fermions after the electroweak symmetry is broken by $v \equiv \langle\varphi\rangle \simeq 174\,\text{GeV}$. As for the second line, the first and the second terms generate the Dirac masses of the neutrinos and the Majorana masses of the right-handed neutrinos, respectively:

$$m_D = y_\nu v\,, \tag{13}$$
$$M_R = f v_R\,. \tag{14}$$

For $M_R \gg m_D$, the left-handed neutrinos can naturally acquire the small Majorana masses,

$$m^I_{\text{tree}} = -m^*_D \frac{1}{M^\dagger_R} m^\dagger_D \sim \mathcal{O}\left(\frac{y^2_\nu}{f}\right)\frac{v^2}{v_R}\,, \tag{15}$$

i.e. the type-I seesaw formula. The third line will also give the left-handed neutrinos a Majorana mass term,

$$m^{II}_{\text{tree}} = f v_L \quad \text{with} \quad v_L \equiv \langle\Delta_L\rangle\,. \tag{16}$$

Here v_L can be determined by minimizing the complete scalar potential [3],

$$v_L \simeq -\frac{\mu\, v^2}{M^2_{\delta^0_L}} \sim -\frac{\mu v^2}{v^2_R}\,, \tag{17}$$

where $\mu \lesssim v_R$ is a product of v_R and some combination of couplings entering in the parity invariant scalar potential. So, we have

$$m^{II}_{\text{tree}} \sim -f\frac{\mu v^2}{v^2_R}\,, \tag{18}$$

which can be comparable to the type-I seesaw contribution. The generation of the small v_L (17) and then the tiny neutrino masses (18) is named as the type-II seesaw.

The radiative generation of neutrino masses: The bi-doublet Higgs scalar contains two iso-doublet scalars: the SM Higgs φ (8) and

$$\eta = -\phi_1 \sin\beta + \phi_2\cos\beta = \left[\begin{array}{c}\eta^0\\ \eta^-\end{array}\right]. \tag{19}$$

η couples to the fermions, but it cannot contribute to any fermion mass at the tree level since it has no *vev*. We shall show that η gives new radiative seesaw contribution to the neutrino masses through some loop diagrams.

For the purpose of demonstration, we deduce the Yukawa couplings of η to the leptons from Eq. (5),

$$\mathcal{L} \supset -h_{\nu_{ij}}\overline{\psi_{L_i}}\eta\nu_{R_j} - h_{\ell_{ij}}\overline{\psi_{L_i}}\tilde{\eta}\ell_{R_j} + \text{h.c.}\,, \tag{20}$$

where

$$\begin{aligned} h_\nu &= -y_\psi \sin\beta + \tilde{y}_\psi\cos\beta \\ &= -\left(y_\ell \sec 2\beta + \frac{1}{2}y_\nu \tan 2\beta\right) \text{ for } \beta \neq \frac{\pi}{4}\,, \end{aligned} \tag{21}$$

$$\begin{aligned} h_\ell &= -y_\psi \cos\beta + \tilde{y}_\psi\sin\beta \\ &= -\left(y_\nu \sec 2\beta + \frac{1}{2}y_\ell \tan 2\beta\right) \text{ for } \beta \neq \frac{\pi}{4}\,. \end{aligned} \tag{22}$$

As shown in Fig. 1, the quartic coupling between φ and η,

$$\mathcal{L} \supset -\lambda(\varphi^\dagger\eta)^2 + \text{h.c.}\,, \tag{23}$$

where $\lambda \lesssim \mathcal{O}(1)$ is some combination of couplings entering in the parity invariant scalar potential, will generate the radiative neutrino masses associated with the first term in Eq. (20) and the Majorana masses of the right-handed neutrinos. We choose the basis in which the Majorana mass matrix (14) of the right-handed neutrinos are real and diagonal and then explicitly calculate the radiative neutrino masses, which have the same forms with those in the two Higgs doublet model [5],

$$\begin{aligned} \left(m^I_{1-\text{loop}}\right)_{ij} &= \sum_{k=1}^{3}\frac{h^*_{\nu_{ik}}h^*_{\nu_{jk}}}{16\pi^2}M_{R_k} \\ &\times\left[\frac{M^2_{\eta^0_R}}{M^2_{\eta^0_R}-M^2_{R_k}}\ln\left(\frac{M^2_{\eta^0_R}}{M^2_{R_k}}\right)\right. \\ &\left.-\frac{M^2_{\eta^0_I}}{M^2_{\eta^0_I}-M^2_{R_k}}\ln\left(\frac{M^2_{\eta^0_I}}{M^2_{R_k}}\right)\right]. \end{aligned} \tag{24}$$

Here η^0_R and η^0_I are defined by $\eta^0 = \frac{1}{\sqrt{2}}\left(\eta^0_R + i\eta^0_I\right)$. Note the quartic coupling (23) guarantees the mass difference between η^0_R and η^0_I,

$$M^2_{\eta^0_R} - M^2_{\eta^0_I} = 4\lambda v^2\,, \tag{25}$$

where λ has been chosen to be real by the proper phase rotation, so that the radiative neutrino masses (24) will not vanish.

The mass splitting between the two doublet scalars ϕ and η are of the order of v_R, since they belong to the same representation of $SU(2)_R$. Therefore, $M_{\eta^0_{R,I}}$ are of the order of v_R for the mass of ϕ is much below v_R. In general, M_{R_k} is smaller than v_R, so we can take $M^2_{R_k} \ll M^2_{\eta^0_{R,I}}$ and then simplify the mass formula (24) as

$$\begin{aligned} \left(m^I_{1\text{-loop}}\right)_{ij} &\simeq \sum_{k=1}^{3}\frac{h^*_{\nu_{ik}}h^*_{\nu_{jk}}}{16\pi^2}M_{R_k}\ln\left(\frac{M^2_{\eta^0_R}}{M^2_{\eta^0_I}}\right) \\ &\simeq \sum_{k=1}^{3}\frac{h^*_{\nu_{ik}}h^*_{\nu_{jk}}}{16\pi^2}M_{R_k}\ln\left[1+\mathcal{O}\left(\lambda\frac{v^2}{v^2_R}\right)\right] \\ &\sim \frac{1}{16\pi^2}\mathcal{O}\left(\lambda h^2_\nu f\right)\frac{v^2}{v_R}\,. \end{aligned} \tag{26}$$

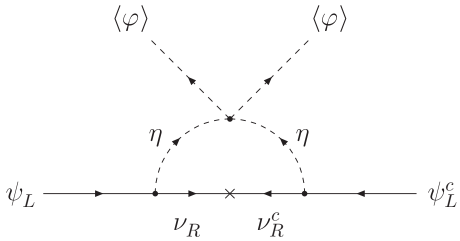

FIG. 1: The one-loop diagram mediated by the right-handed

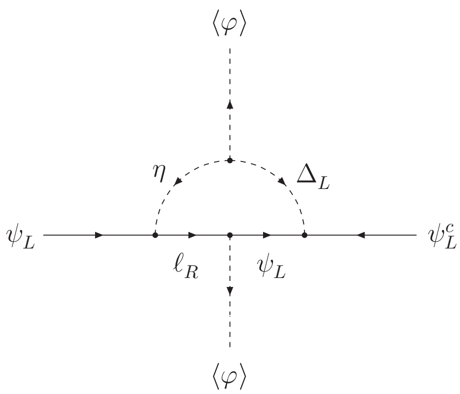

FIG. 2: The one-loop diagram mediated by the left-handed triplet Higgs for generating the radiative neutrino masses.

Now we discuss the contribution from the left-handed triplet Higgs to the radiative neutrino masses. There is a cubic coupling among Δ_L, φ and η,

$$\mathcal{L} \supset -\mu'\eta^T i\tau_2\Delta_L\varphi + \text{h.c.}\,, \tag{27}$$

where $\mu' \lesssim v_R$ is a product of v_R and some combination of couplings entering in the parity invariant scalar potential. Therefore, associated with the third term in Eq. (6) and the second term in Eq. (20), the cubic coupling (27) can generate the radiative neutrino masses as shown in Fig. 2.

For illustration, we write down the mass matrix of δ_L^+ and η^-,

$$\mathcal{L} \supset -\left[\delta_L^{+*}\,,\ \eta^-\right]\left[\begin{array}{cc} M^2_{\delta_L^+} & -\frac{1}{\sqrt{2}}\mu' v \\ -\frac{1}{\sqrt{2}}\mu' v & M^2_{\eta^-}\end{array}\right]\left[\begin{array}{c}\delta_L^+ \\ \eta^{-*}\end{array}\right] \tag{28}$$

Here μ' has been chosen to be real by the proper phase rotation. There are two mass eigenstates S_1 and S_2,

$$\left(\begin{array}{c} S_1 \\ S_2\end{array}\right) = \left(\begin{array}{cc}\cos\vartheta & -\sin\vartheta \\ \sin\vartheta & \cos\vartheta\end{array}\right)\left(\begin{array}{c}\delta_L^+ \\ \eta^{-*}\end{array}\right), \tag{29}$$

with the mixing angle

$$\tan 2\vartheta = \frac{\sqrt{2}\mu' v}{M^2_{\delta_L^+} - M^2_{\eta^-}}\,, \tag{30}$$

and the masses

$$M^2_{S_{1,2}} = \frac{1}{2}\left[M^2_{\delta_L^+} + M^2_{\eta^-} \pm\sqrt{\left(M^2_{\delta_L^+} - M^2_{\eta^-}\right)^2 + 2\mu'^2 v^2}\right]. \tag{31}$$

For $M^2_{\delta_L^+} \sim M^2_{\eta^-} \sim M^2_{\delta_L^+} - M^2_{\eta^-} = \mathcal{O}(v_R^2)$ and $\mu' v \ll v_R^2$, we have

$$\vartheta = \mathcal{O}(\frac{\mu' v}{v_R^2})\,, \tag{32}$$

$$M^2_{S_{1,2}} \sim M^2_{S_2} - M^2_{S_1} = \mathcal{O}(v_R^2)\,. \tag{33}$$

We then calculate the formula of the radiative neutrino masses induced by Fig. 2,

$$\begin{aligned}\left(m^{II}_{1-\text{loop}}\right)_{ij} &= \frac{1}{16\pi^2}\frac{\sin 2\vartheta}{\sqrt{2}}\sum_{k=e,\mu,\tau} f_{ik}h^\dagger_{\ell_{kj}} m_k \\ &\quad\times\left[\frac{M^2_{S_1}}{M^2_{S_1}-m_k^2}\ln\left(\frac{M^2_{S_1}}{m_k^2}\right)\right. \\ &\quad\left.-\frac{M^2_{S_2}}{M^2_{S_2}-m_k^2}\ln\left(\frac{M^2_{S_2}}{m_k^2}\right)\right].\end{aligned} \tag{34}$$

Here we have chosen the basis in which the Yukawa couplings (11) of the charged leptons are real and diagonal and have referred m_k to the masses of the charged leptons. Note the above mass formula is different from that of the Zee model [6] becaue f is symmetric. For $M^2_{S_{1,2}} \gg m_k^2$, we simplify Eq. (34) as

$$\begin{aligned}&\left(m^{II}_{1-\text{loop}}\right)_{ij} \\ &\simeq \frac{1}{16\pi^2}\frac{\sin 2\vartheta}{\sqrt{2}}\sum_{k=e,\mu\,\tau} f_{ik}h^\dagger_{\ell_{kj}} m_k \ln\left(\frac{M^2_{S_2}}{M^2_{S_1}}\right) \\ &\simeq \frac{1}{16\pi^2}\mathcal{O}\left(\frac{\mu' v}{v_R^2}\right)\ln\left[1+\mathcal{O}\left(1\right)\right]\sum_{k=e,\mu,\tau} f_{ik}h^\dagger_{\ell_{kj}} m_k \\ &\sim \frac{1}{16\pi^2}\frac{\mu' v^2}{v_R^2}\mathcal{O}\left(y_\ell h_\ell f\right)\end{aligned} \tag{35}$$

by using Eqs. (32) and (33).

In addition to the two one-loop diagrams, i.e. Figs. 1 and 2, there is a two-loop diagram as shown in Fig. 3 contributing to the radiative neutrino masses due to the following cubic coupling,

$$\mathcal{L} \supset -\mu''\eta^T i\tau_2\Delta_L\eta + \text{h.c.}\,, \tag{36}$$

where $\mu'' \lesssim v_R$ is a product of v_R and some combination of couplings entering in the parity invariant scalar

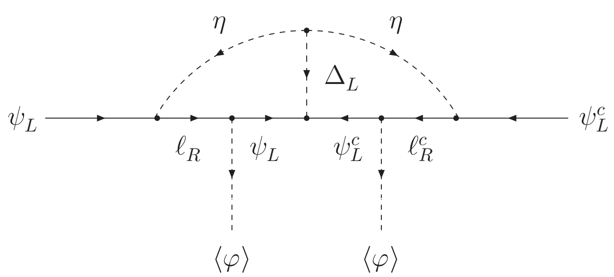

FIG. 3: The two-loop diagram for generating the radiative neutrino masses.

potential. For simplicity, we choose μ'' to be real by the proper phase rotation. Similar to the Zee-Babu model [7], we calculate the two-loop induced neutrino masses as below,

$$
\begin{aligned}
&\left(m_{2-\mathrm{loop}}\right)_{ij}\\
&= \frac{1}{64\pi^4}\sum_{k,n=e,\mu,\tau} h^*_{\ell_{ik}} f_{kn} h^\dagger_{\ell_{nj}} \frac{\mu'' m_k m_n}{M^2_{\delta_L^{++}}}\\
&\quad\times\left\{\sin^4\vartheta\left[\ln\left(1+\frac{M^2_{\delta_L^{++}}}{M^2_{S_1}}\right)\right]^2\right.\\
&\quad+\frac{1}{2}\sin^2 2\vartheta\,\ln\left(1+\frac{M^2_{\delta_L^{++}}}{M^2_{S_1}}\right)\ln\left(1+\frac{M^2_{\delta_L^{++}}}{M^2_{S_2}}\right)\\
&\quad\left.+\cos^4\vartheta\left[\ln\left(1+\frac{M^2_{\delta_L^{++}}}{M^2_{S_2}}\right)\right]^2\right\}\\
&\simeq \frac{1}{64\pi^4}\sum_{k,n=e,\mu,\tau} h^*_{\ell_{ik}} f_{kn} h^\dagger_{\ell_{nj}} \frac{\mu'' m_k m_n}{M^2_{\delta_L^{++}}}\mathcal{O}\left(1\right)\\
&\sim \frac{1}{64\pi^4}\frac{\mu'' v^2}{v_R^2}\mathcal{O}(y_\ell^2 h_\ell^2 f)\,. \qquad (37)
\end{aligned}
$$

Here we have taken $M^2_{\delta_L^{++}} \sim M^2_{S_{1,2}} \sim v_R^2 \gg m^2_{e,\mu,\tau}$ and $\vartheta \ll 1$ into account. Unlike the Zee-Babu model [7], we needn't constrain the Yukawa couplings h_ℓ to be asymmetric.

The radiative neutrino masses versus the tree-level neutrino masses: Now the complete neutrino masses should include five parts,

$$
\begin{aligned}
m_\nu &= m^I_{\mathrm{tree}} + m^{II}_{\mathrm{tree}} + m^I_{1-\mathrm{loop}} + m^{II}_{1-\mathrm{loop}} + m_{2-\mathrm{loop}}\\
&\sim \frac{v^2}{v_R}\left[\mathcal{O}\left(\frac{y_\nu^2}{f}\right) + \frac{\mu}{v_R}\mathcal{O}\left(f\right) + \frac{1}{16\pi^2}\mathcal{O}\left(\lambda h_\nu^2 f\right)\right.\\
&\quad\left.+\frac{1}{16\pi^2}\frac{\mu'}{v_R}\mathcal{O}\left(y_\ell h_\ell f\right) + \frac{1}{64\pi^4}\frac{\mu''}{v_R}\mathcal{O}(y_\ell^2 h_\ell^2 f)\right], \qquad (38)
\end{aligned}
$$

where $v \simeq 174\,\mathrm{GeV}$, $v_R > \mathcal{O}(\mathrm{TeV})$ and $y_\ell \lesssim \mathcal{O}(10^{-2})$ have been known. In the following, we shall show that the five parts can have different weight depending on the choice of the parameters. In particular, for some choice, the radiative contributions could dominate over the tree-level seesaw and could explain the observed neutrino masses.

We now demonstrate that the loop-induced neutrino masses could dominate over the tree-level seesaw for some choice of the parameters. For naturalness, we further assume that there is no cancelation in Eqs. (9) and (10) to generate a quark mass hierarchy so that v_1 and v_2 should not be at the same order since the top quark is much heavier than the bottom quark. For example, we will take $\frac{v_2}{v_1} = \mathcal{O}(10^{-2})$ and hence $h_\nu \sim y_\ell + 10^{-2} y_\nu$ and $h_\ell \sim y_\nu + 10^{-2} y_\ell$ in the quantitative estimation. We then find: (a) $m^I_{1-\mathrm{loop}} \gtrsim m^I_{\mathrm{tree}}$ for $f = \mathcal{O}(0.1)$, $\lambda \lesssim \mathcal{O}(1)$ and $y_\nu \lesssim \mathcal{O}(10^{-4})$; (b) $m^{II}_{1-\mathrm{loop}} \gtrsim m^I_{\mathrm{tree}}$ for $f = \mathcal{O}(0.1)$, $\mu' \lesssim v_R$ and $y_\nu \lesssim \mathcal{O}(10^{-5})$; (c) $m_{2-\mathrm{loop}} \gtrsim m^I_{\mathrm{tree}}$ for $f = \mathcal{O}(0.1)$, $\mu'' \lesssim v_R$ and $y_\nu \lesssim \mathcal{O}(10^{-9})$. Obviously, we can have $m^{I,II}_{1-\mathrm{loop}} \gtrsim m^{II}_{\mathrm{tree}}$ and $m_{2-\mathrm{loop}} \gtrsim m^{II}_{\mathrm{tree}}$ for the proper μ and other parameters.

We then choose some values of the unknown parameters to show that the loop contributions can match the observed neutrino masses: (i) $m^I_{1-\mathrm{loop}} \sim \mathcal{O}(10^{-2}\,\mathrm{eV}) \gg m^I_{\mathrm{tree}} \sim m^{II}_{\mathrm{tree}} \sim m^{II}_{1-\mathrm{loop}} \gg m_{2-\mathrm{loop}}$ for $v_R = \mathcal{O}(10^8\,\mathrm{GeV})$, $\mu = \mathcal{O}(\mathrm{GeV})$, $\mu' \lesssim v_R$, $\mu'' \lesssim v_R$, $\lambda = \mathcal{O}(1)$, $f = \mathcal{O}(0.1)$ and $y_\nu = \mathcal{O}(10^{-5})$; (ii) $m^I_{\mathrm{tree}} \sim m^{II}_{\mathrm{tree}} \sim m^I_{1-\mathrm{loop}} \sim m^{II}_{1-\mathrm{loop}} \sim \mathcal{O}(10^{-2}\,\mathrm{eV}) \gg m_{2-\mathrm{loop}}$ for $v_R = \mathcal{O}(10^4\,\mathrm{GeV})$, $\mu = \mathcal{O}(10^{-6}\,\mathrm{GeV})$, $\mu' = \mathcal{O}(10^2\,\mathrm{GeV})$, $\mu'' \lesssim v_R$, $\lambda = \mathcal{O}(10^{-4})$, $f = \mathcal{O}(0.1)$ and $y_\nu = \mathcal{O}(10^{-6})$; (iii) $m^I_{1-\mathrm{loop}} \sim m^{II}_{1-\mathrm{loop}} \sim m_{2-\mathrm{loop}} \sim \mathcal{O}(10^{-2}\,\mathrm{eV}) \ll m^I_{\mathrm{tree}} \sim m^{II}_{\mathrm{tree}}$ for $v_R = \mathcal{O}(10^4\,\mathrm{GeV})$, $\mu \sim v_R$, $\mu' = \mathcal{O}(0.1\,\mathrm{GeV})$, $\mu'' \sim v_R$, $\lambda = \mathcal{O}(10^{-4})$, $f = \mathcal{O}(0.1)$ and $y_\nu = \mathcal{O}(0.1)$. In the last case, we need the fine-tuned cancelation of m^I_{tree} and m^{II}_{tree} to ensure the complete neutrino masses below the experimental limit.

Summary: We find the new radiative seesaw mechanism for the neutrino masses in the conventional left-right symmetric model with one bi-doublet, one left-handed triplet and one right-handed triplet Higgs scalars. Specifically the neutrino masses can be generated not only by the tree-level seesaw but also by two one-loop diagrams and one two-loop diagram. For some choice of the parameters, the observed neutrino masses can be explained by these loop contributions.

Acknowledgments: We thank Goran Senjanović for helpful discussions.

[1] P. Minkowski, Phys. Lett. B **67**, 421 (1977); T. Yanagida, in *Proc. of the Workshop on Unified Theory and the Baryon Number of the Universe*, ed. O. Sawada and A. Sugamoto (KEK, Tsukuba, 1979), p. 95; M. Gell-Mann, P. Ramond, and R. Slansky, in *Supergravity*, ed. F. van Nieuwenhuizen and D. Freedman (North Holland, Amsterdam, 1979), p. 315; S.L. Glashow, in *Quarks and Leptons*, ed. M. Lévy *et al.* (Plenum, New York, 1980), p. 707; R.N. Mohapatra and G. Senjanović, Phys. Rev. Lett. **44**, 912 (1980); J. Schechter and J.W.F. Valle, Phys. Rev. D **22**, 2227 (1980).

[2] J.C. Pati and A. Salam, Phys. Rev. D **10**, 275 (1974); R.N. Mohapatra and J.C. Pati, Phys. Rev. D **11**, 566 (1975); R.N. Mohapatra and J.C. Pati, Phys. Rev. D **11**, 2558 (1975); R.N. Mohapatra and G. Senjanović, Phys. Rev. D **12**, 1502 (1975).

[3] R.N. Mohapatra and G. Senjanović, Phys. Rev. D **23**, 165 (1981).

[4] N.G. Deshpande, J.F. Gunion, B. Kayser, and F. Olness, Phys. Rev. D **44**, 837 (1991); and references therein.

[5] E. Ma, Phys. Rev. D **73**, 077301 (2006).

[6] A. Zee, Phys. Lett. B **93**, 389 (1980).

[7] A. Zee, Phys. Lett. B **161**, 141 (1985); K.S. Babu, Phys. Lett. B **203**, 132 (1988).