Predictive model for dark matter, dark energy, neutrino masses and leptogenesis at the TeV scale

Narendra Sahu and Utpal Sarkar
Theory Division, Physical Research Laboratory, Navarangpura, Ahmedabad, 380 009, India

We propose a new mechanism of TeV scale leptogenesis where the chemical potential of right-handed electron is passed on to the $B-L$ asymmetry of the Universe in the presence of sphalerons. The model has the virtue that the origin of neutrino masses are independent of the scale of leptogenesis. As a result, the model could be extended to explain *dark matter, dark energy, neutrino masses and leptogenesis at the TeV scale*. The most attractive feature of this model is that it predicts a few hundred GeV triplet Higgs scalar that can be tested at LHC or ILC.

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INTRODUCTION

In the canonical seesaw models [1] the physical neutrino masses are largely suppressed by the scale of lepton (L) number violation, which is also the scale of leptogenesis. The observed baryon (B) asymmetry and the low energy neutrino oscillation data then give a lower bound on the scale of leptogenesis to be $\sim 10^9$ GeV [2]. Alternately in the triplet seesaw models [3] it is equally difficult to generate L-asymmetry at the TeV scale because the interaction of $SU(2)_L$ triplets with the gauge bosons keep them in equilibrium up to a very high scale $\sim 10^{10}$ GeV [4]. However, in models of extra dimensions [5] and models of dark energy [6] the masses of the triplet Higgs scalars could be low enough for them to be accessible in LHC or ILC, but in those models leptogenesis is difficult. Even in the left-right symmetric models in which there are both right-handed neutrinos and triplet Higgs scalars contributing to the neutrino masses, it is difficult to have triplet Higgs scalars in the range of LHC or ILC [7]. It may be possible to have resonant leptogenesis [8] with light triplet Higgs scalars [9], but the resonant condition requires very high degree of fine tuning.

In this paper we introduce a new mechanism of leptogenesis at the TeV scale. We ensure that the lepton number violation required for the neutrino masses does not conflict with the lepton number violation required for leptogenesis. This led us to propose a model which is capable of explaining dark matter, dark energy, neutrino masses and leptogenesis at the TeV scale. Moreover, the model predicts a few hundred GeV triplet Higgs whose decay through the same sign dilepton signal could be tested either through the $e^{\pm}e^{\mp}$ collision at linear collider or through the pp collision at LHC.

THE MODEL

In addition to the quarks, leptons and the usual Higgs doublet $\phi \equiv (1,2,1)$, we introduce two triplet Higgs scalars $\xi \equiv (1,3,2)$ and $\Delta \equiv (1,3,2)$, two singlet scalars $\eta^- \equiv (1,1,-2)$ and $T^0 \equiv (1,1,0)$, and a doublet Higgs $\chi \equiv (1,2,1)$. The transformations of the fields are given under the standard model (SM) gauge group $SU(3)_c \times SU(2)_L \times U(1)_Y$. There are also three heavy singlet fermions $S_a \equiv (1,1,0), a = 1,2,3$. A global symmetry $U(1)_X$ allows us to distinguish between the L-number violation for neutrino masses and the L-number violation for leptogenesis. Under $U(1)_X$ the fields $\ell_{iL}^T \equiv (\nu, e)_{iL} \equiv (1,2,-1)$, $e_{iR} \equiv (1,1,-2)$, η^- and T^0 carry a quantum number 1, Δ, S_a, $a = 1,2,3$ and ϕ carry a quantum number zero while ξ and χ carry quantum numbers -2 and 2 respectively. We assume that $M_\xi \ll M_\Delta$ while both ξ and Δ contribute equally to the effective neutrino masses. Moreover, if neutrino mass varies on the cosmological time scale then it behaves as a negative pressure fluid and hence explains the accelerating expansion of the present Universe [10] [1]. With a survival Z_2 symmetry, the neutral component of χ represents the candidate of dark matter [12].

Taking into account of the above defined quantum numbers we now write down the Lagrangian symmetric under $U(1)_X$. The terms in the Lagrangian, relevant to the rest of our discussions, are given by

$$\begin{aligned} -\mathcal{L} \supseteq\ & f_{ij}\xi\ell_{iL}\ell_{jL} + \mu(A)\Delta^\dagger\phi\phi + M_\xi^2\xi^\dagger\xi + M_\Delta^2\Delta^\dagger\Delta \\ & + h_{ia}\overline{e_{iR}}S_a\eta^- + M_{sab}S_aS_b + y_{ij}\phi\overline{\ell_{iL}}e_{jR} + M_T^2T^\dagger T \\ & + \lambda_T|T|^4 + \lambda_\phi|T|^2|\phi|^2 + \lambda_\chi|T|^2|\chi|^2 + f_T\xi\Delta^\dagger TT \\ & + \lambda_{\eta\phi}|\eta^-|^2|\phi|^2 + \lambda_{\eta\chi}|\eta^-|^2|\chi|^2 + V_{\phi\chi} + h.c., \qquad (1) \end{aligned}$$

where $V_{\phi\chi}$ constitutes all possible quadratic and quartic terms symmetric under $U(1)_X$. The typical dimension full coupling $\mu(A) = \lambda A$, A being the acceleron field[2], which is responsible for the accelerating expansion of the Universe. We introduce the $U(1)_X$ symmetry breaking soft terms

$$-\mathcal{L}_{soft} = m_T^2 TT + m_\eta \eta^-\phi\chi + h.c.. \qquad (2)$$

If T carries the L-number by one unit then the first term explicitly breaks L-number in the scalar sector. The second term on the other hand conserves L-number if η^- and χ possess

[1] Connection between neutrino mass and dark energy, which is required for accelerating expansion of the Universe, in large extradimension scenario is discussed in ref. [11]

[2] The origin of this acceleron field is beyond the scope of this paper. See for example ref. [13].

equal and opposite L-number[3]. This leads to the interactions of the fields $S_a, i = 1,2,3$ to be L-number conserving. As we shall discuss later, this can generate the L-asymmetry of the universe, while the neutrino masses come from the L-number conserving interaction term $\Delta^\dagger \xi T T$ after the field T acquires a *vev*.

NEUTRINO MASSES

The Higgs field Δ acquires a very small vacuum expectation value (*vev*)

$$\langle \Delta \rangle = -\mu(A) \frac{v^2}{M_\Delta^2}, \tag{3}$$

where $v = \langle \phi \rangle$, ϕ being the SM Higgs doublet. However, we note that the field ξ does not acquire a *vev* at the tree level.

The scalar field T acquires vev at a few TeV, which then induces a small *vev* to the scalar field ξ. The Goldstone boson corresponding to the L-number violation, the would be Majoron, and the Goldstone boson corresponding to $U(1)_X$ symmetry will have a mass of the order of a few TeV and will not contribute to the Z decay width. The *vev* of the field ξ would give a small Majorana mass to the neutrinos.

The *vev* of the singlet field T gives rise to a mixing between Δ and ξ through the effective mass term

$$-\mathcal{L}_{\Delta\xi} = m_s^2 \Delta^\dagger \xi, \tag{4}$$

where the mass parameter $m_s = \sqrt{f_T \langle T \rangle^2}$ is of the order of TeV, similar to the mass scale of T. The effective couplings of the different triplet Higgs scalars, which give the L-number violating interactions in the left-handed sector, are then given by

$$\begin{aligned} -\mathcal{L}_{\nu-mass} &= f_{ij} \xi \ell_i \ell_j + \mu(A) \frac{m_s^2}{M_\Delta^2} \xi^\dagger \phi\phi + f_{ij} \frac{m_s^2}{M_\xi^2} \Delta \ell_i \ell_j \\ &\quad + \mu(A) \Delta^\dagger \phi\phi + h.c.. \end{aligned} \tag{5}$$

The field ξ then acquires an induced *vev*,

$$\langle \xi \rangle = -\mu(A) \frac{v^2 m_s^2}{M_\xi^2 M_\Delta^2}. \tag{6}$$

The *vev*s of both the fields ξ and Δ will contribute to neutrino mass by equal amount and thus the neutrino mass is given by

$$m_\nu = -f_{ij} \mu(A) \frac{v^2 m_s^2}{M_\xi^2 M_\Delta^2}. \tag{7}$$

[3] If η^- does not possess any L-number then the interaction of S_a explicitly breaks L-number and hence the decay of lightest S_a gives rise to a net L-asymmetry as in the case of right handed neutrino decay [14].

Since the absorptive part of the off-diagonal one loop self energy terms in the decay of triplets Δ and ξ is zero, their decay can't produce any L-asymmetry even though their decay violate L-number. However, the possibility of erasing any pre-existing L-asymmetry through the $\Delta L = 2$ processes mediated by Δ and ξ should not be avoided unless their masses are very large and hence suppressed in comparison to the electroweak breaking scale. In particular, the important erasure processes are:

$$\ell\ell \leftrightarrow \xi \leftrightarrow \phi\phi \quad and \quad \ell\ell \leftrightarrow \Delta \leftrightarrow \phi\phi. \tag{8}$$

If $m_s^2 \ll M_\Delta^2$ then the L-number violating processes mediated through Δ and ξ are suppressed by $(m_s^2/M_\xi^2 M_\Delta^2)$ and hence practically don't contribute to the above erasure processes. Thus a fresh L-asymmetry can be produced at the TeV scale.

LEPTOGENESIS

We introduce the following two cases for generating L-asymmetry which is then transferred to the required B-asymmetry of the Universe.

Case-I:: The explicit L-number violation

First we consider the case where L-number is explicitly broken in the singlet sector. This is possible if η^-, and hence χ, does not possess any L-number. Therefore, the decays of the singlet fermions S_a, $a = 1,2,3$ can generate a net L-asymmetry of the universe through

$$\begin{aligned} S_a &\rightarrow e_{iR}^- + \eta^+ \\ &\rightarrow e_{iR}^+ + \eta^-. \end{aligned}$$

We work in the basis, in which M_{sab} is diagonal and $M_3 > M_2 > M_1$, where $M_a = M_{saa}$. Similar to the usual right-handed neutrino decays generating L-asymmetry [14], there are now one-loop self-energy and vertex-type diagrams that can interfere with the tree-level decays to generate a CP-asymmetry. The decay of the field S_1 can now generate a CP-asymmetry

$$\begin{aligned} \varepsilon &= -\sum_i \left[\frac{\Gamma(S_1 \to e_{iR}^- \eta^+) - \Gamma(S_1 \to e_{iR}^+ \eta^-)}{\Gamma_{tot}(S_1)} \right] \\ &\simeq \frac{1}{8\pi} \frac{M_1}{M_2} \frac{\mathrm{Im}[(hh^\dagger)_{i1}(hh^\dagger)_{i1}]}{\sum_a |h_{a1}|^2}. \end{aligned} \tag{9}$$

Thus an excess of e_{iR} over e_{iR}^c is produced in the thermal plasma. This will be converted to an excess of e_{iL} over e_{iL}^c through the t-channel scattering process $e_{iR} e_{iR}^c \leftrightarrow \phi^0 \leftrightarrow e_{iL} e_{iL}^c$. This can be understood as follows. Let us define the chemical potential associated with e_R field as $\mu_{eR} = \mu_0 + \mu_{BL}$, where μ_{BL} is the chemical potential contributing to $B - L$ asymmetry and μ_0 is independent of $B - L$. At equilibrium thus we have

$$\mu_{e_L} = \mu_{e_R} + \mu_\phi = \mu_{BL} + \mu_0 + \mu_\phi. \tag{10}$$

We see that μ_{eL} is also associated with the same chemical potential μ_{BL}. Hence the $B - L$ asymmetry produced in the right-handed sector will be transferred to the left-handed sector. A net baryon asymmetry of the universe is then produced through the sphaleron transitions which conserve $B-L$ but violate $B+L$. Since the source of L-number violation for the this asymmetry is different from the neutrino masses, there is no bound on the mass scale of S_1 from the low energy neutrino oscillation data. Therefore, the mass scale of S_1 can be as low as a few TeV. Note that the mechanism for L-asymmetry proposed here is different from an earlier proposal of right handed sector leptogenesis [15]. The survival asymmetry in the η fields is then transferred to χ fields through the trilinear soft term introduced in Eq. (2).

Case-II:: Conserved L-number

We now consider the case where L-number is conserved in the singlet sector. This is possible if $\eta^-(\eta^+)$ possesses a L-number exactly opposite to that of $e_R^+(e_R^-)$. Therefore, the decays of the singlet fermions S_a, $a=1,2,3$ can not generate any L-asymmetry. However, it produces an equal and opposite asymmetry between $\eta^-(\eta^+)$ and $e_R^+(e_R^-)$ fields as given by Eq. (9). If these two asymmetries cancel with each other then there is no left behind L-asymmetry. However, as we see from the Lagrangians (1) and (2) that none of the interactions that can transfer the L-asymmetry from η^- to the lepton doublets while e_R is transferring the L-asymmetry from the singlet sector to the usual lepton doublets through $\phi\overline{\ell_L}e_R$ coupling. Note that the coupling, through which the asymmetry between η^- and e_R^+ produced, is already gone out of thermal equilibrium. So, it will no more allow the two asymmetries to cancel with each other. The asymmetry in the η fields is finally transferred to the χ fields through the trilinear soft term introduced in Eq. (2).

DARK MATTER

As the universe expands the temperature of the thermal bath falls. As a result the heavy fields η^- and T^0 are annihilated to the lighter fields ϕ and χ as they are allowed by the Lagrangians (1) and (2). Notice that there is a Z_2 symmetry of the Lagrangians (1) and (2) under which $S_a, a=1,2,3$, η^- and χ are odd while all other fields are even. Since the neutral component of χ is the lightest one it can be stable because of Z_2 symmetry. Therefore, the neutral component of χ behaves as a dark matter.

After T gets a vev the effective potential describing the interactions of ϕ and χ can be given by

$$V(\phi,\chi) = \left(-m_\phi^2+\frac{\lambda_\phi}{f_T}m_s^2\right)|\phi|^2+\left(m_\chi^2+\frac{\lambda_\chi}{f_T}m_s^2\right)|\chi|^2 +\lambda_1|\phi|^4+\lambda_2|\chi|^4+\lambda_3|\phi|^2|\chi|^2+\lambda_4|\phi^\dagger\chi|^2\,, \quad (11)$$

where we have made use of the fact that $m_s=\sqrt{f_T\langle T\rangle^2}$ and λ_ϕ, λ_χ are the quartic couplings of T with ϕ and χ respectively. For $m_\phi^2 > \left(\frac{\lambda_\phi}{f_T}\right)m_s^2 > 0$ and $m_\chi^2, \left(\frac{\lambda_\chi}{f_T}\right)m_S^2 > 0$ the minimum of the potential is given by

$$\langle\phi\rangle = \begin{pmatrix}0\\ v\end{pmatrix} \quad and \quad \langle\chi\rangle = \begin{pmatrix}0\\ 0\end{pmatrix}. \quad (12)$$

The *vev* of ϕ gives masses to the SM fermions and gauge bosons. The physical mass of the SM Higgs is then given by $m_h=\sqrt{4\lambda_1 v^2}$. The physical mass of the real and imaginary parts of the neutral component of χ field are almost same and is given by

$$m^2_{\chi^0_{R,I}} = m_\chi^2+\frac{\lambda_\phi}{f_T}m_s^2+(\lambda_3+\lambda_4)v^2\,. \quad (13)$$

Since χ is odd under the surviving Z_2 symmetry it can't decay to any of the conventional SM fields and hence the neutral component of χ constitute the dark matter component of the Universe. Above their mass scales $\chi^0_{R,I}$ are in thermal equilibrium through the interactions: $\lambda_2{\chi^0_{R,I}}^4$ and $(\lambda_3+\lambda_4){\chi^0_{R,I}}^2h^2$. Assuming that $m_{\chi^0_{R,I}} < m_W, m_h$ the direct annihilation of a pair of $\chi^0_{R,I}$, below their mass scale, to SM Higgs is kinematically forbidden. However, a pair of $\chi^0_{R,I}$ can be annihilated to the SM fields: $f\bar{f}, W^+W^-, ZZ, gg, hh\cdots$ through the exchange of neutral Higgs h. The corresponding scattering cross-section in the limit $m_{\chi^0_{R,I}} < m_W, m_h$ is given by [16]

$$\sigma_h|v| \simeq \frac{\lambda^2 m^2_{\chi^0_{R,I}}}{m_h^4}\,, \quad (14)$$

where $\lambda=(\lambda_3+\lambda_4)$.

We assume that at a temperature T_D, $\Gamma_{ann}/H(T_D)\simeq 1$, where T_D is the temperature of the thermal bath when $\chi^0_{R,I}$ got decoupled and

$$H(T_D) = 1.67 g_*^{1/2}(T_D^2/M_{pl}) \quad (15)$$

is the corresponding Hubble expansion parameter with $g_*\simeq 100$ being the effective number of relativistic degrees of freedom. Using Eq. (14) the rate of annihilation of $\chi^0_{R,I}$ to the SM fields can be given by $\Gamma_{ann}=n_{\chi^0}\langle\sigma_h|v|\rangle$, where n_{χ^0} is the density of $\chi^0_{R,I}$ at the decoupled epoch. Using the fact that $\Gamma_{ann}/H(T_D)\simeq 1$ one can get [17]

$$z_D \equiv \frac{m_{\chi^0_{R,I}}}{T_D} \simeq \ln\left[\frac{N_{ann}\lambda^2 m^3_{\chi^0_{R,I}} M_{pl}}{1.67 g_*^{1/2}(2\pi)^{3/2}m_h^4}\right]\,, \quad (16)$$

where N_{ann} is the number of annihilation channels which we have taken roughly to be 10. Since the $\chi^0_{R,I}$ are stable in the cosmological time scale we have to make sure that it should not over-close the Universe. For this we calculate the energy density of $\chi^0_{R,I}$ at the present epoch. The number density of $\chi^0_{R,I}$ at the present epoch is given by

$$n_{\chi^0_{R,I}}(T_0) = (T_0/T_D)^3\, n_{\chi^0_{R,I}}(T_D)\,, \quad (17)$$

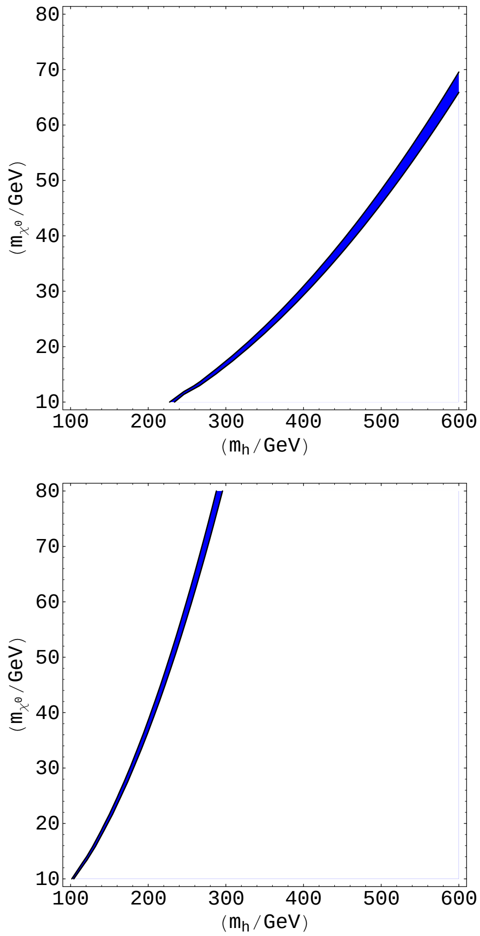

FIG. 1: The allowed region of dark matter at the 1σ C.L. is shown in the plane of m_h versus m_{χ^0} with $\lambda^2 = 0.5$ (upper) and $\lambda^2 = 0.1$ (bottom).

where $T_0 = 2.75^\circ k$, the temperature of present Cosmic Microwave Background Radiation. We then calculate the energy density at present epoch,

$$\rho_{\chi^0_{R,I}} = \left(\frac{0.98\times 10^{-4} eV}{cm^3}\right) \frac{1}{N_{ann}\lambda^2}\frac{(m_h/GeV)^4}{(m_{\chi^0_{R,I}}/GeV)^2}[1+\delta] , \quad (18)$$

where $\delta \ll 1$. The critical energy density of the present Universe is

$$\rho_c = 3H_0^2/8\pi G_N \equiv 10^4 h^2 eV/cm^3 . \quad (19)$$

At present the contribution of dark matter to the critical energy density of the Universe is precisely given by $\Omega_{DM}h^2 = 0.111 \pm 006$ [18]. Assuming that $\chi^0_{R,I}$ is a candidate of dark matter we have shown, in fig. (1), the allowed masses of $\chi^0_{R,I}$ up to 80 GeV for a wide spectrum of SM Higgs masses.

DARK ENERGY AND NEUTRINO

It has been observed that the present Universe is expanding in an accelerating rate. This can be attributed to the dynamical scalar field A [19], which evolves with the cosmological time scale. If the neutrino mass arises from an interaction with the acceleron field, whose effective potential changes as a function of the background neutrino density then the observed neutrino masses can be linked to the observed acceleration of the Universe [10].

Since the neutrino mass depends on A, it varies on the cosmological time scale such that the effective neutrino mass is given by the Lagrangian

$$-\mathcal{L} = \left[f_{ij}\mu(A)\frac{v^2 m_s^2}{M_\xi^2 M_\Delta^2}\nu_i\nu_j + h.c.\right] + V_0 , \quad (20)$$

where V_0 is the acceleron potential. A typical form of the potential is given by [6]

$$V_0 = \Lambda^4 \ln\left(1 + |\overline{\mu}|\mu(A)|\right) , \quad (21)$$

The two terms in the above Lagrangian (20) acts in opposite direction such that the effective potential

$$V(m_\nu) = m_\nu n_\nu + V_0(m_\nu) \quad (22)$$

today settles at a non-zero positive value. From the above effective potential we can calculate the equation of state

$$w = -1 + [\Omega_\nu/(\Omega_\nu + \Omega_A)] , \quad (23)$$

where w is defined by $V \propto R^{-3(1+w)}$. At present the contribution of light neutrinos having masses varying from 5×10^{-4} eV to 1 MeV to the critical energy density of the Universe is $\Omega_\nu \leq 0.0076/h^2$ [18]. Hence one effectively gets $w \simeq -1$. Thus the mass varying neutrinos behave as a negative pressure fluid as the dark energy. For naturalness we chose $\frac{\mu(A)m_s^2}{M_\Delta^2} \sim 1$ eV such that M_ξ can be a few hundred GeV to explain the sub-eV neutrino masses, and $\Lambda \sim 10^{-3}$ eV such that the varying neutrino mass can be linked to the dark energy component of the Universe.

COLLIDER SIGNATURE OF DOUBLY CHARGED PARTICLES

The doubly charged component of the light triplet Higgs ξ can be observed through its decay into same sign dileptons [20]. Since $M_\Delta \gg M_\xi$, the production of Δ particles in comparison to ξ are highly suppressed. Hence it is worth looking for the signature of $\xi^{\pm\pm}$ either at LHC or ILC. From Eq. (5) one can see that the decay $\xi^{\pm\pm} \to \phi^\pm\phi^\pm$ are suppressed since the decay rate involves the factor $\frac{\mu(A)m_s^2}{M_\Delta^2} \sim 1$ eV. While the decay mode $\xi^{\pm\pm} \to h^\pm W^\pm$ is phase space suppressed, the decay mode $\xi^{\pm\pm} \to W^\pm W^\pm$ is suppressed because of the vev of ξ is small which is required for sub-eV neutrino masses as well as to maintain the ρ parameter of SM to be unity. Therefore, once it is produced, ξ mostly decay through the same sign dileptons: $\xi^{\pm\pm} \to \ell^\pm \ell^\pm$. Note that the doubly charged particles can not couple to quarks and therefore the SM background of the process $\xi^{\pm\pm} \to \ell^\pm \ell^\pm$ is quite clean and hence the detection will be unmistakable. From Eq. (5) the decay rate of the process $\xi^{\pm\pm} \to \ell^\pm \ell^\pm$ is given by

$$\Gamma_{ii} = \frac{|f_{ii}|^2}{8\pi} M_{\xi^{++}} \quad and \quad \Gamma_{ij} = \frac{|f_{ij}|^2}{4\pi} M_{\xi^{++}}\,, \qquad (24)$$

where f_{ij} are highly constrained from the lepton flavor violating decays. From the observed neutrino masses we have $f_{ij} x \sim 10^{-12}$ where $x = (\langle\xi\rangle/v)$. If $f_{ij} \gtrsim x$ then from the lepton flavor violating decay $\xi^{\pm\pm} \to \ell_i^\pm \ell_j^\pm$ one can study the pattern of neutrino masses and mixing [21].

CONCLUSIONS

We introduced a new mechanism of leptogenesis in the singlet sector which allowed us to extend the model to explain dark matter, dark energy, neutrino masses and leptogenesis at the TeV scale. This scenario predicts a few hundred GeV triplet scalar which contributes to the neutrino masses. This makes the model predictable and it will be possible to verify the model at ILC or LHC through the same sign dilepton decay of the doubly charged particles. This also opens an window for studying neutrino mass spectrum in the future colliders (LHC or ILC). Since the lepton number violation required for lepton asymmetry and neutrino masses are different, leptogenesis scale can be lowered to as low as a few TeV.

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