Neutrino Dark Energy and Baryon Asymmetry from Higgs Sector

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We propose a new model to explain the neutrino masses, the dark energy and the baryon asymmetry altogether. In this model, neutrinos naturally acquire small Majorana masses via type-II seesaw mechanism, while the pseudo-Nambu-Goldstone bosons associated with the neutrino mass-generation mechanism provide attractive candidates for dark energy. The baryon asymmetry of the universe is produced from the Higgs triplets decay with CP-violation.

I. INTRODUCTION

The atmospheric, solar and laboratory neutrino oscillation experiments [1] have confirmed that neutrinos have tiny but nonzero masses, of the order of $10^{-2}\,$eV. This phenomenon is elegantly explained by the seesaw mechanism [2], in which neutrinos acquire small Majorana [2] or Dirac [3] masses naturally. Furthermore, the observed matter-antimatter asymmetry in the universe can be generated through leptogenesis [4] in the neutrino seesaw scenario.

On the other hand, various cosmological observations [1] provide strong evidence that the expansion of our universe is accelerating due to the existence of dark energy. One possible explanation for the dark energy has its origin in a dynamical scalar field, such as the quintessence [5] with an extremely flat potential. It was shown [6] that the pseudo-Nambu-Goldstone boson (pNGB) provides an attractive realization of the quintessence field.

A striking coincidence between the scales of neutrino masses and dark energy ($\sim (10^{-3}\,\text{eV})^4$), inspires us to consider them in a unified scenario, i.e., the neutrino dark energy model which generically predicts neutrino-mass variation [7]. There have been lots of recent activities studying the neutrino dark energy models. A possible connection between the neutrinos and the pNGB dark energy was explored in the type-I seesaw scenario [8, 9].

In this paper, we propose a new neutrino dark energy model to simultaneously explain the generation of neutrino masses and the origin of dark energy from the Higgs sector. In particular, the pNGBs associated with neutrino mass-generation provide the consistent candidates of dark energy while the small neutrino masses depending on the dark energy field are realized through the type-II seesaw. Furthermore, the CP-violation and out-of-equilibrium decays of the Higgs triplets produce the baryon asymmetry in the universe.

II. THE MODEL

We extend the $SU(2)_L \times U(1)_Y$ standard model (SM) with triplet and singlet Higgs scalars,

$$
\begin{aligned}
\psi_{Li} &= \begin{pmatrix} \nu_{Li} \\ l_{Li} \end{pmatrix} \ (\mathbf{2}, -\tfrac{1}{2}), \quad H = \begin{pmatrix} H^0 \\ H^- \end{pmatrix} \ (\mathbf{2}, -\tfrac{1}{2}), \\
\xi_{ij} &\equiv \xi_{ji} = \begin{pmatrix} \frac{1}{\sqrt{2}}\delta_{ij}^+ & \delta_{ij}^{++} \\ \delta_{ij}^0 & -\frac{1}{\sqrt{2}}\delta_{ij}^+ \end{pmatrix} \ (\mathbf{3}, 1), \\
\chi_{ij} &\equiv \chi_{ji} \ (\mathbf{1}, 0), \quad (\text{for } i, j = 1, 2, 3),
\end{aligned}
\tag{1}
$$

are left-handed lepton doublets, Higgs doublet, Higgs triplets and Higgs singlets, respectively. The full Lagrangian is supposed to be invariant under a global $U(1)^6$ symmetry, generated by the independent phase transformations of each Higgs triplet among the six ξ_{ij} fields. Transformations of the Higgs singlets χ_{ij} under this $U(1)^6$ are determined by requiring the invariance of the following scalar interactions,

$$
\chi_{ij}H^T i\tau_2 \xi_{ij} H + \text{h.c.},
\tag{2}
$$

where we have defined the SM Higgs doublet H as a singlet under $U(1)^6$. The Higgs triplets have the Yukawa couplings to the left-handed lepton doublets,

$$
\overline{\psi_{Li}^c} i\tau_2 \xi_{ij} \psi_{Lj} + \text{h.c.},
\tag{3}
$$

which explicitly break the $U(1)^6$ down to its subgroup $U(1)^3$. So we have only three massless Nambu-Goldstone bosons (NGBs) after the six χ_{ij}'s acquire their vacuum expectation values (VEVs).

We write down the relevant Lagrangian for the Higgs and lepton-Yukawa interactions,

$$
\begin{aligned}
\mathcal{L} \supset &- \sum_{ij} \left(\overline{\mu}_{ij}^2 + \sum_{kl} \lambda_{ij,kl} \chi_{kl}^\dagger \chi_{kl} \right) \text{Tr}\left(\xi_{ij}^\dagger \xi_{ij} \right) \\
&- \sum_{ij \neq kl} \lambda_{ij,kl}^{'} \chi_{ij}^\dagger \chi_{kl} \text{Tr}\left(\xi_{ij}^\dagger \xi_{kl} \right) \\
&- \sum_{ij} \left(\frac{1}{2} y_{ij} \overline{\psi_{Li}^c} i\tau_2 \xi_{ij} \psi_{Lj} - h_{ij} \chi_{ij} H^T i\tau_2 \xi_{ij} H \right. \\
&\left. \qquad\qquad + \text{h.c.} \right),
\end{aligned}
\tag{4}
$$

where $\lambda_{ij,kl}^{(')} \equiv \lambda_{ji,kl}^{(')} \equiv \lambda_{ij,lk}^{(')}$, $y_{ij} \equiv y_{ji}$ and $h_{ij} \equiv h_{ji}$ are dimensionless while μ_{ij} has mass-dimension equal one. After the Higgs singlets get their VEVs, $\langle \chi_{ij} \rangle \equiv \frac{1}{\sqrt{2}} f_{ij}$, we can write

$$\chi_{ij} = \frac{1}{\sqrt{2}} \left(f_{ij} + \sigma_{ij} \right) \exp \left(i\varphi_{ij}/f_{ij} \right) \qquad (5)$$

with $\sigma_{ij} \equiv \sigma_{ji}$, $\varphi_{ij} \equiv \varphi_{ji}$ $(i, j = 1, 2, 3)$ being the neutral bosons and the NGBs, respectively. Among these six NGBs, three of them will acquire nonzero masses via the Coleman-Weinberg potential (due to the small explicit breaking of global symmetries, $U(1)^6 \to U(1)^3$) and thus become pNGBs. The other three remain massless as the result of spontaneous breaking of the subgroup $U(1)^3$.

For convenience, we redefine the Higgs triplets as

$$\exp \left(i\varphi_{ij}/f_{ij} \right) \xi_{ij} \rightarrow \xi_{ij}. \qquad (6)$$

The mass matrix M for the physical triplet scalar fields is now given by

$$M_{ij,kl} \equiv \left[\left(\overline{\mu}_{ij}^2 + \frac{1}{2} \sum_{kl} \lambda_{ij,kl} f_{kl}^2 \right) \delta_{ij,kl} + \frac{1}{2} \lambda_{ij,kl}^{'} f_{ij} f_{kl} \right]^{\frac{1}{2}}. \qquad (7)$$

The VEVs of χ_{ij}'s generate the effective trilinear interactions between the Higgs triplets and doublet,

$$\frac{1}{\sqrt{2}} h_{ij} f_{ij} H^T i\tau_2 \xi_{ij} H + \text{h.c.} \equiv \mu_{ij} H^T i\tau_2 \xi_{ij} H + \text{h.c.}, \qquad (8)$$

where the cubic couplings μ_{ij} will be set as real after proper phase rotations. From Eqs. (6), (7) and (8), we derive the Lagrangian (4) as below,

$$\begin{aligned}
\mathcal{L} \supset &- \sum_{ij,kl} M_{ij,kl}^2 \,\text{Tr} \left(\xi_{ij}^\dagger \xi_{kl} \right) \\
&- \sum_{ij} \left[\frac{1}{2} y_{ij} \exp \left(-i\varphi_{ij}/f_{ij} \right) \overline{\psi_{Li}^c} i\tau_2 \xi_{ij} \psi_{Lj} \right. \\
&\left. \qquad - \mu_{ij} H^T i\tau_2 \xi_{ij} H + \text{h.c.} \right].
\end{aligned} \qquad (9)$$

We still have the freedom to redefine the phases of the three lepton doublets, which can remove three of the fields φ_{ij} from the lepton-Higgs Yukawa interactions. Without loss of generality, we choose the rephasing,

$$\exp \left[-i\varphi_{ii}/(2f_{ii}) \right] \psi_{Li} \rightarrow \psi_{Li} \qquad (10)$$

which transforms the Lagrangian (9) into a new form,

$$\begin{aligned}
\mathcal{L} \supset &- \sum_{ij,kl} M_{ij,kl}^2 \text{Tr} \left(\xi_{ij}^\dagger \xi_{kl} \right) \\
&- \left[\frac{1}{2} \sum_i y_{ii} \overline{\psi_{Li}^c} i\tau_2 \xi_{ii} \psi_{Li} \right. \\
&\quad + \frac{1}{2} \sum_{i \neq j} y_{ij} \exp(i\phi_{ij}/f) \overline{\psi_{Li}^c} i\tau_2 \xi_{ij} \psi_{Lj} \\
&\left. \quad - \sum_{ij} \mu_{ij} H^T i\tau_2 \xi_{ij} H + \text{h.c.} \right]
\end{aligned} \qquad (11)$$

where

$$\frac{\phi_{ij}}{f} = -\frac{\varphi_{ij}}{f_{ij}} + \frac{1}{2} \frac{\varphi_{ii}}{f_{ii}} + \frac{1}{2} \frac{\varphi_{jj}}{f_{jj}}, \qquad (12)$$

and f is of the order of f_{ij}. It is not possible to remove ϕ_{ij} $(i \neq j)$ from Eq. (11) by any further transformations and hence these ϕ_{ij}'s will become the pNGBs with tiny masses and can naturally serve as the candidates of dark energy. Note that from (6), (10) and the subsequent phase rotations on the right-handed charged leptons, the three massless NGBs, φ_{ii}, will only have the derivative interactions to other fields, but they are highly suppressed by $1/f_{ii}$ and thus escape from experimental constraints at low energy scales.

III. NEUTRINO MASS AND MIXING

The electroweak symmetry breaking takes place with the VEV of the Higgs doublet, which induces small VEVs to the triplets,

$$\langle H \rangle \equiv \frac{1}{\sqrt{2}} \begin{pmatrix} v \\ 0 \end{pmatrix}, \quad \langle \xi_{ij} \rangle \equiv \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 0 \\ u_{ij} & 0 \end{pmatrix}, \qquad (13)$$

with $i, j = 1, 2, 3$, where the VEVs of the Higgs triplets ξ_{ij} are deduced as

$$u_{ij} \simeq \frac{v^2}{\sqrt{2}} \sum_{kl} \mu_{kl} \left(M^{-2} \right)_{kl,ij}. \qquad (14)$$

Inspecting Eqs. (7) and (8), it is natural to take the masses $M_{ij,kl}$ around the same order as the scalar cubic couplings μ_{ij} since they are both controlled by the singlet VEVs (f_{ij}). Hence, the triplet VEVs in (14) are seesaw-suppressed by the ratio of the electroweak scale v over the heavy mass M, i.e., $u_{ij} = \mathcal{O}\left(v^2/M\right) \ll v$.

These small VEVs will then generate the Majorana masses for the neutrinos,

$$\mathcal{L}_m = -\frac{1}{2} \sum_{ij} (m_\nu)_{ij} \overline{\nu_{Li}^c} \nu_{Lj} + \text{h.c.}, \qquad (15)$$

where
$$(m_\nu)_{ij} = \begin{cases} m_{ij}, & \text{(for } i = j\text{)}, \\ m_{ij} \exp \left(i\phi_{ij}/f \right), & \text{(for } i \neq j\text{)}, \end{cases} \qquad (16)$$

with $m_{ij} \equiv \frac{1}{\sqrt{2}} y_{ij} u_{ij}$. With the 3×3 symmetric mass-matrix (16), we can readily realize the neutrino mass-spectrum and mixings, consistent with the neutrino oscillation experiments. Moreover, the interactions of the neutrinos with the pNGBs will induce a small long-range force, which can have direct consequences in cosmology [10, 11] and neutrino oscillation experiments [12].

IV. ORIGIN OF DARK ENERGY

Three of the NGBs, ϕ_{ij} $(i \neq j)$ as defined in (12), will acquire small masses due to the Yukawa couplings
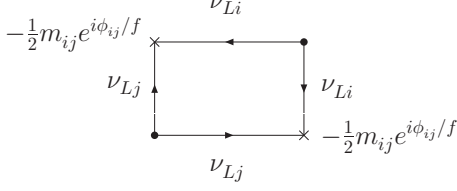
FIG. 1: The one-loop diagram contributing to the Coleman-Weinberg potential of the pNGBs.

between the left-handed lepton doublets and the Higgs triplets, and thus become the pNGBs. As shown in Fig. 1, the leading loop diagram will contribute a Coleman-Weinberg effective potential for ϕ_{ij}. Similar to [8], we explicitly calculate the potential,

$$V(\phi_{12}, \phi_{23}, \phi_{31}) = -\frac{1}{32\pi^2} \sum_{k=1}^{3} m_k^4 \ln \frac{m_k^2}{\Lambda^2}, \qquad (17)$$

where m_k as a function of ϕ_{ij} is the kth eigenvalue of the neutrino mass matrix m_ν, and Λ is the ultraviolet cutoff. A typical term in V that contributes to the potential of a pNGB field Q has the form,

$$V(Q) \simeq V_0 \cos(Q/f) \qquad (18)$$

with $V_0 = \mathcal{O}(m_\nu^4)$. It is well known that with f of the order of Planck mass M_{Pl}, the pNGB Q will acquire a mass of order $\mathcal{O}(m_\nu^2/M_{\mathrm{Pl}})$ and thus provides a consistent candidate for the quintessence dark energy.

Finally, we also note that after the electroweak symmetry breaking, the explicit breaking of the $U(1)^3$ symmetry is only generated by the dimension-3 soft mass term in Eq. (15). So, in this model there is no dimension-4 term which explicitly breaks $U(1)^3$, and thus there are no higher order correction that could contribute a divergent term to spoil the renormalizability and the original symmetry of the theory. This point can be checked by explicit calculations as well. For instance, at the two-loop order, we have

$$\begin{aligned} V_2 &\approx \left(\frac{1}{16\pi^2}\right)^2 \Lambda^2 \,\mathrm{Tr}\left(m_\nu m_\nu^\dagger\right) \left[\mathrm{Tr}\left(yy^\dagger\right) \right.\\ &\left. + \left(\frac{1}{2} + \frac{1}{4\cos^2\theta_W}\right) g^2\right]. \end{aligned}$$

Since $\mathrm{Tr}\left(m_\nu m_\nu^\dagger\right)$ is independent on the pNGBs as well as $\mathrm{Tr}\left(yy^\dagger\right)$, we see that the two-loops have no contribution to the effective potential for the pNGBs. Similarly, there is no contribution from other higher loops.

V. BARYON ASYMMETRY

The decays of Higgs triplets,

$$\xi_{ij} \rightarrow \begin{cases} \psi_{Li}^c \psi_{Lj}^c, & (L = -2), \\ H^* H^*, & (L = 0), \end{cases} \qquad (19)$$

can break the lepton number. As shown in Fig. 2, the mass-mixings in (7) among different Higgs triplets contribute the tree-level and one-loop diagrams that interfere to generate CP asymmetry in these decays. The decays of the triplet Higgs will then produce enough lepton asymmetry before the electroweak phase transition, which can successfully explain the observed matter-antimatter asymmetry in the universe through the sphaleron processes [13] which convert the lepton asymmetry into the existing baryon asymmetry.

To calculate the CP-asymmetry, we define the Higgs triplets in their mass-eigenbasis,

$$\widehat{\xi}_a \equiv \sum_{ij} U_{a,ij} \xi_{ij} \qquad (20)$$

with the diagonalized mass-eigenvalues,

$$\widehat{M}_a \equiv \sum_{ij,kl} U_{a,ij} M_{ij,kl} U_{a,kl}. \qquad (21)$$

Thus we can rewrite the Lagrangian (11) as

$$\begin{aligned} \mathcal{L} &\supset -\sum_a \widehat{M}_a^2 \mathrm{Tr}\left(\widehat{\xi}_a^\dagger \widehat{\xi}_a\right) - \left(\frac{1}{2}\sum_{a,ij} \widehat{y}_{ij}^a \overline{\psi_{Li}^c} i\tau_2 \widehat{\xi}_a \psi_{Lj}\right.\\ &\left. -\sum_a \widehat{\mu}_a H^T i\tau_2 \widehat{\xi}_a H + \mathrm{h.c.}\right) \end{aligned} \qquad (22)$$

with the definition

$$\widehat{\mu}_a \equiv \sum_{ij} U_{a,ij}\mu_{ij} \qquad (23)$$

and

$$\widehat{y}_{ij}^a \equiv \begin{cases} U_{a,ij}y_{ij}, & (\text{for } i = j), \\ U_{a,ij}y_{ij}\exp\left(i\phi_{ij}/f\right), & (\text{for } i \neq j). \end{cases} \qquad (24)$$

Here U is an orthogonal rotation matrix. The proper CP-asymmetry parameter is then described by [14]

$$\begin{aligned} \varepsilon_a &\equiv 2 \times \frac{\sum_{ij}\left[\Gamma\left(\widehat{\xi}_a^* \to \psi_{Li}\psi_{Lj}\right) - \Gamma\left(\widehat{\xi}_a \to \psi_{Li}^c\psi_{Lj}^c\right)\right]}{\Gamma_a} \\ &= \frac{1}{\pi}\sum_{b \neq a} \frac{\mathrm{Im}\left\{\mathrm{Tr}\left[\left(\widehat{y}^b\right)^\dagger \widehat{y}^a\right]\right\}\widehat{\mu}_b\widehat{\mu}_a}{\mathrm{Tr}\left[\left(\widehat{y}^a\right)^\dagger \widehat{y}^a\right]\widehat{M}_a^2 + 4\widehat{\mu}_a^2} \frac{\widehat{M}_a^2}{\widehat{M}_a^2 - \widehat{M}_b^2} \end{aligned} \qquad (25)$$

with

$$\Gamma_a = \frac{1}{8\pi}\left\{\frac{1}{4}\mathrm{Tr}\left[\left(\widehat{y}^a\right)^\dagger \widehat{y}^a\right] + \frac{\widehat{\mu}_a^2}{\widehat{M}_a^2}\right\}\widehat{M}_a \qquad (26)$$

being the total decay width of $\widehat{\xi}_a$ or $\widehat{\xi}_a^*$.
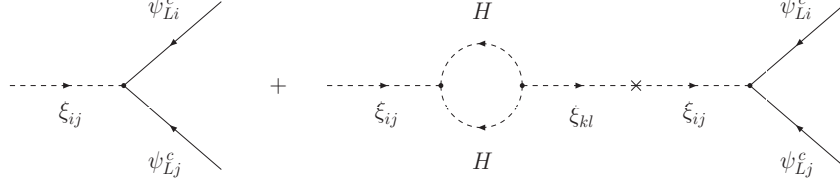
FIG. 2: The Higgs triplets decay to the leptons at one-loop order.

For illustration, we use $\widehat{\xi}_a$ to denote the lightest Higgs triplet and hence its contribution is expected to dominate the final baryon asymmetry, which is given by the approximate relation [15],

$$Y_B \equiv \frac{n_B}{s} \simeq -\frac{28}{79} \times \begin{cases} \dfrac{\varepsilon_a}{g_*}, & \text{(for } K \ll 1), \\[2mm] \dfrac{0.3\,\varepsilon_a}{g_* K \,(\ln K)^{0.6}}, & \text{(for } K \gg 1), \end{cases} \tag{27}$$

with the factor $28/79$ being the value of $B/(B-L)$ and the parameter K defined by

$$K \equiv \left. \frac{\Gamma_a}{2H(T)} \right|_{T=\widehat{M}_a} \tag{28}$$

as a measurement of the departure from equilibrium. Here $H(T) = (8\pi^3 g_*/90)^{\frac{1}{2}} T^2/M_{\rm Pl}$ is the Hubble constant with the Planck mass $M_{\rm Pl} \simeq 1.2 \times 10^{19}$ GeV and the relativistic degrees of freedom $g_* \simeq 106.75$. For simplicity, we further consider

$$r_{ba} \equiv \frac{\widehat{M}_b}{\widehat{M}_a} = \frac{\widehat{\mu}_b}{\widehat{\mu}_a} \gg 1 \quad \text{and} \quad \widehat{y}^b \equiv \widehat{y}^a e^{-i\delta_{ba}}, \tag{29}$$

where δ_{ba} is the relative phase between $\widehat{y}^b$ and $\widehat{y}^a$. We thus neglect the contribution from the heavier triplets to the neutrino masses and conveniently express K as

$$K = \left[\frac{(4\pi)^5 g_*}{45}\right]^{-\frac{1}{2}} \left(B_\psi B_H\right)^{-\frac{1}{2}} \frac{M_{\rm Pl}\overline{m}}{v^2}. \tag{30}$$

Here the quadratic mean of the neutrino masses ($\overline{m}$) is defined by

$$\overline{m}^2 \equiv \sum_{k=1}^{3} m_k^2 \equiv \mathrm{Tr}\left(m_\nu^\dagger m_\nu\right) \simeq \frac{1}{4}\mathrm{Tr}\left[(\widehat{y}^a)^\dagger \widehat{y}^a\right] \frac{\widehat{\mu}_a^2 v^4}{\widehat{M}_a^4}$$

$$= (8\pi)^2 B_\psi B_H \Gamma_a^2 \frac{v^4}{\widehat{M}_a^4} \tag{31}$$

and $(B_\psi,\, B_H)$ are the branching ratios of the tree-level decays of $\widehat{\xi}_a$ into the lepton and Higgs doublets, which always hold the relationship,

$$B_\psi + B_H \equiv 1, \quad \Longrightarrow \quad B_\psi B_H \leqslant \frac{1}{4}. \tag{32}$$

Then, we compute the CP-asymmetry (25) as

$$\varepsilon_a = -\frac{1}{\pi} \frac{\mathrm{Tr}\left[(\widehat{y}^a)^\dagger \widehat{y}^a\right] \widehat{\mu}_a^2}{\mathrm{Tr}\left[(\widehat{y}^a)^\dagger \widehat{y}^a\right] \widehat{M}_a^2 + 4\widehat{\mu}_a^2} \sum_{b\neq a} \frac{r_{ba}\sin\delta_{ba}}{r_{ba}^2-1}$$

$$\simeq -\frac{1}{\pi} \left(B_\psi B_H\right)^{1/2} \frac{\widehat{M}_a\overline{m}}{v^2} \frac{\sin\delta_{ba}}{r_{ba}}. \tag{33}$$

Inputting $v \simeq 246$ GeV, $B_\psi B_H = 1/4$, $\widehat{M}_a = 4 \times 10^{12}$ GeV, $\overline{m} = 0.1$ eV, $\sin\delta_{ba} = 0.1$ and $r_{ba} = 10$, we derive the sample predictions: $\varepsilon_a \simeq -1.1 \times 10^{-5}$ and $K \simeq 46$. Finally, we deduce, $n_B/s \simeq 10^{-10}$, consistent with the cosmological observations.

VI. CONCLUSION AND DISCUSSION

In this paper, we propose a new model to unify the neutrino dark energy and baryon asymmetry by extending only the SM Higgs sector with triplet and singlet Higgs scalars. The Higgs triplets naturally acquire tiny VEVs and give small Majorana masses for the neutrinos. The model contains three pNGBs associated with the neutrino mass-generation, which provide consistent candidates for the quintessence field. The matter-antimatter asymmetry in the universe is produced by the out-of-equilibrium decays of the Higgs triplets with CP-violating couplings. We can readily accommodate the dark matter as well in our construction by adding a darkon field [16] or an inert Higgs doublet [17].

In our model, the neutrino masses are functions of the dark energy field. Being a dynamical component, the dark energy will evolve with time and/or in space. Accordingly, the neutrino masses will vary instead of being constants. The prediction of the neutrino-mass variation could be verified in the present and future experiments, such as the observations on the cosmic microwave background and the large scale structures [10], the measurement of the extremely high-energy cosmic neutrinos [11], and the analysis of the neutrino oscillation data [12].

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