# Leptogenesis and low energy CP phases with two heavy neutrinos

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## Abstract

An attractive explanation for non-zero neutrino masses and small matter antimatter asymmetry of the present Universe lies in "leptogenesis". At present the *size* of the lepton asymmetry is precisely known, while the *sign* is not known yet. In this work we determine the sign of this asymmetry in the framework of two right handed neutrino models by relating the leptogenesis phase(s) with the low energy CP violating phases appearing in the leptonic mixing matrix. It is shown that the knowledge of low energy lepton number violating re-phasing invariants can indeed determine the sign of the present matter antimatter asymmetry of the Universe and hence indirectly probing the light physical neutrinos to be Majorana type.

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## I. INTRODUCTION

Within the Standard Model (SM) the neutrinos are massless and hence there is no CP violation in the lepton (L) sector. The current evidence [1, 2, 3] from the neutrino oscillation experiments, on the other hand, suggest that neutrinos are massive, however small, and they mix up. The goal of the present neutrino oscillation experiments is to determine the nine degrees of freedom in the low energy neutrino mass matrix. They are parametrized by three masses, three mixing angles and three CP violating phases out of which two are Majorana and one is Dirac. At present the neutrino oscillation experiments able to measure the two mass square differences, the solar and the atmospheric, and three mixing angles with varying degrees of precision, while there is no information about the phases.

Assuming that the neutrinos are of Majorana type the small masses of the physical left handed neutrinos can be explained by the elegant seesaw mechanism [4] which involves singlet right-handed neutrinos (type-I seesaw) or triplet Higgs (type-II seesaw) or can be both (hybrid seesaw). In the present article we limit ourselves to the case of type-I seesaw models. Although we call them right-handed neutrinos, in the extensions of the SM they are just singlet fermions that transform trivially under the SM gauge group. So, there is no apparent reasons for the number of heavy singlet neutrinos to be same as the number of left-handed neutrinos. So, for the main part of our discussions we restrict ourselves to only two right-handed neutrinos. These results will also be true when there are three right-handed neutrinos, but the third right-handed neutrino do not mix with the other two neutrinos. We start with three right-handed neutrinos and after some general comments work mostly with two right-handed neutrinos.

While there is no information about the absolute mass scales of the physical neutrinos, the currently discovered tiny mass scales; the atmospheric neutrino mass ($\Delta_{atm} = \sqrt{|m_3^2 - m_2^2|}$) in the $\nu_\mu - \nu_\tau$ oscillation and the solar neutrino mass ($\Delta_\odot = \sqrt{m_2^2 - m_1^2}$) in the $\nu_e - \nu_\mu$ oscillation, can be explained by adding at least two right handed neutrinos to the SM Lagrangian. However, with two right-handed neutrinos the seesaw mechanism predicts one of the physical light neutrino mass to be exactly zero which is permissible within the current knowledge of neutrino masses and mixings.

The Majorana mass of the right-handed neutrino violates $L$-number and hence is a natural source of L-asymmetry in the early Universe [5, 6]. A partial L-asymmetry is then converted to baryon (B) asymmetry through the non-perturbative sphaleron processes, unsuppressed above the electroweak phase transition. Currently the $B$-asymmetry has been measured precisely by the Wilkinson Microwave Anisotropy Probe (WMAP)[7] and is given by

$$\left(\frac{(n_B - n_{\bar{B}})}{n_\gamma}\right)_0 \equiv \left(\frac{n_B}{n_\gamma}\right)_0 = \left(6.1^{+0.3}_{-0.2}\right) \times 10^{-10}. \tag{1}$$

It is legitimate to ask if there are any connecting links between leptogenesis and the CP violation in the low energy leptonic sector, in particular neutrino oscillation and neutrinoless double beta decay. In the context of three right-handed neutrino models several attempts have been taken in the literature to connect the CP violation in leptogenesis and neutrino oscillations [8]. It is found that there are almost no links between these two phenomena unless one considers special assumptions [9]. In fact it is shown that leptogenesis can be possible irrespective of the CP violation at low energy [10]. On the other hand, in the two right-handed neutrino models there is a ray of hope connecting leptogenesis with the CP violation in neutrino oscillation [11] and neutrinoless double beta decay processes.

While the magnitude of CP violation is fairly known in the quark sector, it is completely shaded in the leptonic sector of the SM. Therefore, searching for CP violation in the leptonic sector is of great interest in the present days. It has been pointed out that the Dirac phase, being involved in the L-number conserving processes, can be measured in the long baseline neutrino oscillation experiments [12], while the Majorana phase, being involved in the L-number violating processes, can be investigated in the neutrinoless double beta decay [13] processes.

At present the magnitude of B-asymmetry is precisely known, while the sign of this asymmetry is not known yet. However, by knowing the CP violating phases in the leptonic mixing matrix one can determine the sign of the B-asymmetry. This is the study taken up in this work. We consider a minimal extension of the SM by including two singlet right-handed neutrinos which are sufficient to explain the present knowledge of neutrino masses and mixings. We adopt a general parameterization of the neutrino Dirac Yukawa coupling and give the possible links between the CP violation in leptogenesis and neutrino oscillation, CP violation in neutrinoless double beta decay and leptogenesis. It is shown that the knowledge of low energy CP violating re-phasing invariants can indeed determine the sign of the B-asymmetry since the size of this asymmetry is known precisely.

Rest of the manuscript is arranged as follows. In section II we elucidate the canonical seesaw in the framework of three right handed neutrinos. We then display the possible links between leptogenesis and the low energy CP-violating phases appearing in the leptonic mixing matrix in certain special circumstances. It is found that there are almost no links between these two phenomena occurring at two different energy scales. Therefore, in section III we give a parameterization of $m_D$ in the two right-handed neutrino models. In section IV we calculate the neutrino masses and mixings by using the parameterization of $m_D$ given in section III. In section V we estimate the CP violation in leptogenesis. In section VI we consider the re-phasing invariant formalism to study the possible links between the CP violating phases responsible for leptogenesis and the CP violation at low energy phenomena. First we calculate th CP violation in neutrino oscillation and then elucidate its link to leptogenesis. After that we calculate the CP violation in low energy lepton number violating process, i.e., the neutrinoless double beta decay, and then elucidate its link to leptogenesis. We conclude in section VII.

## II. CANONICAL SEESAW AND PARAMETER COUNTING

To account for the small neutrino masses we extend the SM by including right-handed neutrinos. The corresponding leptonic Lagrangian is given by

$$\begin{aligned}\mathcal{L} &= \overline{\ell_{Li}} i\gamma^\mu D_\mu \ell_{Li} + \overline{\ell_{Ri}} i\gamma^\mu \partial_\mu \ell_{Ri} + \overline{N_{R\alpha}} i\gamma^\mu \partial_\mu N_{R\alpha} \\ &- \left( \frac{1}{2} \overline{(N_{R\alpha})^c} (M_R)_{\alpha\beta} N_{R\beta} + \overline{\ell_{Li}} \phi (Y_e)_{ij} \ell_{Rj} + \overline{\ell_{Li}} \tilde{\phi} (Y_\nu)_{i\alpha} N_{R\alpha} + H.C. \right) ,\end{aligned} \tag{2}$$

where $\tilde{\phi} = i\tau^2\phi$ and $i$ runs from 1 to 3, representing the left-handed fields. $\alpha$ represent the right handed neutrino indices. $\ell_{Li}$ represents the $\mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$ doublets, $\ell_{Ri}$ and $N_{R\alpha}$ are right-handed singlets of the theory.

After the electroweak symmetry breaking the canonical seesaw [4] gives the effective neutrino mass matrix

$$m_\nu = -m_D M_R^{-1} m_D^T \,, \tag{3}$$

where $m_D = Y_\nu v$ is the Dirac mass matrix of the neutrinos with $v$ is the vev of SM Higgs and that of $M_R$ is the mass matrix of right handed neutrinos. Without loss of generality we consider $M_R$ to be diagonal and in this basis $m_D$ contains rest of the physical parameters that appears in $m_\nu$.

The diagonalization of $m_\nu$, through the lepton flavor mixing matrix $U_{PMNS}$ [14], gives us three masses of the physical neutrinos. Its eigenvalues are given by

$$D_m \equiv \text{diag.}(m_1, m_2, m_3) = U^\dagger_{PMNS} m_\nu U^*_{PMNS}\,, \tag{4}$$

where the masses $m_i$ are real and positive. The standard PDG parametrization [15] of the PMNS matrix reads:

$$U_{PMNS} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_{13}} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta_{13}} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta_{13}} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta_{13}} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta_{13}} & c_{23}c_{13} \end{pmatrix} . U_{ph} \tag{5}$$

where $U_{ph} = \text{diag.}\,(1, e^{i\eta}, e^{i(\xi+\delta_{13})})$ and $c_{ij}$, $s_{ij}$ stands for $\cos\theta_{ij}$ and $\sin\theta_{ij}$ respectively. The two physical phases $\eta$ and $\xi$ associated with the Majorana character of neutrinos are not relevant for neutrino oscillations. Thus we see that there are three phases in the low energy effective theory responsible for CP violation. However, these phases may not give rise to CP violation at high energy regime, in particular, leptogenesis to our interest. In the following we study this in the framework of three and than two right-handed neutrino models.

In general if $n$ and $n'$ are the number of generations of the left- and right-handed neutrinos that take part in the seesaw then the total number of parameters in the effective theory is estimated to be [16]

$$N_{\text{moduli}} = n + n' + nn'\,, \tag{6}$$

$$N_{\text{phase}} = n(n' - 1)\,. \tag{7}$$

For $n = 3$ and $n' = 3$, $N_{\text{moduli}} = 15$ and $N_{\text{phase}} = 6$, which in the effective theory manifests as three masses of charged leptons, three masses of right-handed neutrinos and remaining 15 parameters including nine moduli and six phases in the Dirac mass matrix $m_D$ in a basis where the charged lepton mass matrix is real and diagonal.

In the bi-unitary parameterization the mass matrix $m_D$ can be given as

$$m_D = U^\dagger_L m_D^{diag} U_R\,, \tag{8}$$

where $U_L$ and $U_R$ are $3\times 3$ unitary matrices. $U_L$ diagonalizes the left-handed sector while $U_R$ is the diagonalizing matrix of $m_D^\dagger m_D$. Any arbitrary $3\times 3$ unitary matrix $U$ can be written as

$$U = e^{i\varphi} P_1 \widetilde{U} P_2 \,, \tag{9}$$

where $\varphi$ is an overall phase and

$$P_1 \;=\; \text{diag.}(1, e^{-i\alpha_1}, e^{-i\alpha_2}) \,, \tag{10}$$

$$P_2 \;=\; \text{diag.}(1, e^{-i\beta_1}, e^{-i\beta_2}) \,, \tag{11}$$

are phase matrices. $\widetilde{U}$ is a CKM like matrix parametrized by three angles and one embedded phase. Now using Eq. (9) in Eq. (8) we get

$$m_D = e^{i(-\varphi_L + \varphi_R)} P_{2L}^\dagger \widetilde{U}_L^\dagger P_{1L}^\dagger m_D^{diag} P_{1R} \widetilde{U}_R P_{2R}. \tag{12}$$

Without loss of generality three of the left phases can be absorbed in the redefinition of charged lepton fields. As a result the effective Dirac mass matrix turns out to be

$$m_D = \widetilde{U}_L^\dagger P_3 m_D^{diag} \widetilde{U}_R P_{2R} \,, \tag{13}$$

where $P_3 = P_{1L}^\dagger P_{1R}$ is an effective phase matrix. Thus in the models with three right-handed neutrinos $m_D$ contains 15 parameters.

In leptogenesis, the CP asymmetry comes in a form $m_D^\dagger m_D$, which contains $P_{2R}$ and $\widetilde{U}_R$, *i.e.*,

$$m_D^\dagger m_D = P_{2R}^\dagger \widetilde{U}_R^\dagger (m_D^{diag})^2 \widetilde{U}_R P_{2R} \,, \tag{14}$$

and hence is independent of $P_3$ and $\widetilde{U}_L$. Although it would be good to know the exact relationship of the phases in $P_{2R}$ and $\widetilde{U}_R$ with the phases appearing in the $U_{PMNS}$ matrix but that is not possible. So, we try with some special cases.

**Case-I:** Let us first consider the case, when $\widetilde{U}_R$ is a diagonal matrix. This is the case when the right-handed neutrino Majorana mass matrix is diagonal to start with. The mass matrix can still contain Majorana phases. In that case, $\widetilde{U}_R$ and $m_D^{diag}$ will commute and hence $m_D^\dagger m_D$ will be real and there will not be any leptogenesis. This already tells us that the phases in leptogenesis crucially depends on the mixing of the right-handed physical neutrinos. Even in this case there will be CP violation at low energy as we shall see below. The light neutrino mass matrix is given by

$$m_\nu = -\widetilde{U}_L^\dagger (P_3)^2 (\widetilde{U}_R)^2 (P_{2R})^2 (m_D^{diag})^2 M_R^{-1} \widetilde{U}_L^*$$

so that the PMNS matrix will become

$$U_{PMNS} = \widetilde{U}_L^\dagger P_3 P_{2R} \,.$$

Thus both the Dirac and Majorana phases at low energy are non-vanishing.

**Case-II:** We shall now consider another special case when there is no leptogenesis. If the diagonal Dirac neutrino mass matrix is proportional to a unit matrix, i.e., $m_D = m \cdot I$ ($I$ is the identity matrix), again there is no leptogenesis,

$$m_D^\dagger m_D = m^2 \cdot I \,.$$

In this case the light neutrino mass matrix becomes

$$m_\nu = -\widetilde{U}_L^\dagger P_3 \widetilde{U}_R P_{2R} m^2 M_R^{-1} P_{2R} \widetilde{U}_R^T P_3 \widetilde{U}_L^* \,,$$

so that the PMNS matrix can be read off to be

$$U_{PMNS} = \widetilde{U}_L^\dagger P_3 \widetilde{U}_R P_{2R} \,.$$

Even in this case the Dirac and Majorana phases are present.

Thus in both these examples, even if CP violation is observed at low energy neutrino experiments, this CP violation may not be related to leptogenesis. Since it is not possible to make any further progress with three heavy neutrinos, we shall now restrict ourselves to models with two heavy neutrinos.

## III. PARAMETERIZATION OF $m_D$ IN 2RH NEUTRINO MODELS

From now on we shall work with only two right-handed (2RH) neutrinos. This result will be applicable when there are only two heavy neutrinos or when there are three heavy neutrinos but one of them do not mix with others and heavier than the other right-handed neutrinos and hence its contribution to the light neutrinos is also negligible. In the present case where we have $n = 3$ and $n' = 2$, from Eq. (6) and (7), we get $N_{\rm moduli} = 11$ and $N_{\rm phase} = 3$. The 14 parameters in the effective theory manifest them as three masses of charged leptons, two masses of right handed neutrinos and remaining nine parameters including six moduli and three phases appear in the Dirac mass matrix $m_D$.

There are various textures and their phenomenological implications of $m_D$ in the 2RH neutrino models that have been considered in the literature [17]. In this article a general parametrization of the $3 \times 2$ mass matrix of the Dirac neutrinos is considered. This is given by

$$m_D = vY_\nu = vUY_{2RH} , \tag{15}$$

where $U$ is an arbitrary Unitary matrix and the Yukawa coupling of the two RH neutrino model is given as

$$Y_{2RH} = \begin{pmatrix} 0 & x \\ z & ye^{-i\theta} \\ 0 & 0 \end{pmatrix} . \tag{16}$$

A derivation of Eq. (16) is given in the appendix A. However, we declare that the texture of $Y_{2RH}$ is not unique. By choosing appropriately the $U$ matrix one can place $x, y, z$ at different positions so as to get the different textures of $Y_{2RH}$ as shown in appendix B. Using (9) in Eq. (15) we get

$$m_D = v\widetilde{U}P_2Y_{2RH} , \tag{17}$$

where $\widetilde{U}$ contains four parameters including three moduli and one phase, $P_2$ contains two phases and $Y_{2RH}$ contains four parameters including three moduli and one phase which all together makes ten parameters in $m_D$. However, by multiplying the phase matrix $P_2$ with $Y_{2RH}$ one can see that one of the phases in the phase matrix $P_2$, i.e., $\beta_2$ becomes redundant and can be dropped without loss of generality. As a result the total number of effective parameters is actually nine and hence consistent with our counting.

Substituting $m_D$, given by Eq. (17), in Eq. (3) we can calculate the effective neutrino mass matrix, $m_\nu$. The diagonalization of $m_\nu$, through the lepton flavor mixing matrix $U_{PMNS}$ [14], then gives us two non-zero masses of the physical neutrinos while setting one of the mass to be exactly zero as shown in the following section.

## IV. NEUTRINO MASSES AND MIXINGS IN 2RH NEUTRINO MODELS

The unitary matrix $\widetilde{U}$, appearing in Eq. (17), can be parameterized as [1]

$$\widetilde{U} = R_{23}(\Theta_{23})R_{13}(\Theta_{13}, \delta'_{13})R_{12}(\Theta_{12}) . \tag{18}$$

[1] This parameterization is usually used for determining the leptonic mixing matrix in the PDG parameterization. Here we have used it for parameterizing $m_D$.

It turns out that this parameterization is useful in determining the leptonic mixing matrix in 2RH neutrino models. Now from Eqs. (3) and (17) we get the effective neutrino mass matrix to be

$$
\begin{aligned}
m_\nu &= -v^2 \widetilde{U} P_2 Y_{2RH} M_R^{-1} Y_{2RH}^T P_2 \widetilde{U}^T \\
&= -v^2 \widetilde{U} P_2 X P_2 \widetilde{U}^T \,,
\end{aligned}
\tag{19}
$$

where

$$
X = Y_{2RH} M_R^{-1} Y_{2RH}^T \,. \tag{20}
$$

For simplicity of the calculation let us take $e^{-i\theta}$ common from 2nd row of $Y_{2RH}$ matrix given by Eq. (16) and absorb it in $P_2$ by redefining $\beta_1$ as $(\beta_1 + \theta) \rightarrow \beta_1$. As a result opposite phase will reappear with $z$. Then the matrix $Y_{2RH}$ turns out to be

$$
Y_{2RH} = \begin{pmatrix} 0 & x \\ z e^{i\theta} & y \\ 0 & 0 \end{pmatrix} . \tag{21}
$$

Using Eq. (21) in the above Eq. (20) we get

$$
X = \begin{pmatrix} \frac{x^2}{M_2} & \frac{xy}{M_2} & 0 \\ \frac{xy}{M_2} & \frac{y^2}{M_2} + \frac{z^2 e^{2i\theta}}{M_1} & 0 \\ 0 & 0 & 0 \end{pmatrix} . \tag{22}
$$

In writing the above equation we have used a diagonal basis of the RH neutrinos where $M_R = \text{diag.}(M_1, M_2)$. For simplicity, we absorb $M_1$ and $M_2$ in $x, y$ and $z$ as $\frac{x}{\sqrt{M_2}} \rightarrow a$, $\frac{y}{\sqrt{M_2}} \rightarrow b$ and $\frac{z}{\sqrt{M_1}} \rightarrow c$. So $X$ can be rewritten as:

$$
X = \begin{pmatrix} a^2 & ab & 0 \\ ab & b^2 + c^2 e^{2i\theta} & 0 \\ 0 & 0 & 0 \end{pmatrix} . \tag{23}
$$

Looking to the effective neutrino mass matrix as given by Eq. (19) we can guess that the diagonalizing matrix would be of the form

$$
U_{PMNS} = \widetilde{U} K \,, \tag{24}
$$

where $K$ is an unitary matrix. Using Eqs. (4) and (24) in Eq. (19) we see that

$$D_m = -K^\dagger P_2 X P_2 K^* \,, \tag{25}$$

which implies that $K$ would diagonalize the matrix $P_2XP_2$. From the structure of $X$ it is clear that one of the light physical neutrinos must be massless. The matrix $K$ can be parameterized as

$$K = P_2 \, R_{12}(\omega, \phi) \, P \,, \tag{26}$$

where $P = \text{diag.}(e^{i\eta_1/2}, e^{i\eta_2/2}, 1)$ and

$$R_{12}(\omega,\phi) = \begin{pmatrix} \cos\omega & e^{i\phi}\sin\omega & 0 \\ -e^{-i\phi}\sin\omega & \cos\omega & 0 \\ 0 & 0 & 1 \end{pmatrix} , \tag{27}$$

with

$$\tan 2\omega \;=\; \left[\frac{2ab\,(a^4 + b^4 + c^4 + 2a^2b^2 + 2b^2c^2\cos 2\theta + 2c^2a^2 cos2\theta)^{1/2}}{(-a^4 + b^4 + c^4 + 2b^2c^2\cos 2\theta)}\right] , \tag{28}$$

and

$$\tan\phi \;=\; \left[\frac{-c^2\sin 2\theta}{a^2 + b^2 + c^2\cos 2\theta}\right] . \tag{29}$$

Since $R_{12}(\omega,\phi)$ diagonalizes the matrix $X$ the resulting diagonal matrix will have complex eigenvalues in general. However, by choosing appropriately the phases of $P$ one can make the eigenvalues of $X$ real. Using Eqs. (28) and (29) we get the eigenvalues $\{\lambda_1, \lambda_2, \lambda_3\}$ of $X$ to be

$$\lambda_1 = a^2 - abe^{i\phi}\tan\omega \,, \quad \lambda_2 = e^{-2i\phi}(a^2 + abe^{i\phi}\cot\omega) \quad \text{and} \quad \lambda_3 = 0 \tag{30}$$

The absolute masses of the physical neutrinos are then given by $\{m_1 = v^2|\lambda_1|, m_2 = v^2|\lambda_2|, m_3 = 0\}$. The MSW effect in the solar neutrino oscillation experiments indicates that $m_2 > m_1$. The corresponding mass scale, giving rise to the $\nu_e - \nu_\mu$ oscillation, is given by

$$\Delta m^2_\odot \equiv m_2^2 - m_1^2 = v^4(|\lambda_2|^2 - |\lambda_1|^2) \,. \tag{31}$$

Using Eq. (30) in the above equation we get the solar neutrino mass scale to be

$$\begin{aligned} \Delta m^2_\odot \;&=\; v^4 \left\{ \left[ (a^2+b^2)^2 + c^4 + 2b^2c^2\cos 2\theta \right]^2 - 4a^4c^4 \right\}^{1/2} \\ &\simeq\; 8\times 10^{-5} eV^2 \,. \end{aligned} \tag{32}$$

The atmospheric mass scale, on the other hand, is given by

$$\Delta m^2_{atm} \equiv |m_2^2 - m_3^2| = v^4(|\lambda_2|^2 - |\lambda_3|^2)\,. \tag{33}$$

Now using Eq. (30) in the above equation we get the atmospheric mass scale to be

$$\begin{aligned}\Delta m^2_{atm} &= \frac{v^4}{2}\Big((a^2+b^2)^2 + c^4 + 2b^2c^2\cos 2\theta \\ &\quad + \Big\{\big((a^2+b^2)^2 + c^4 + 2b^2c^2\cos 2\theta\big)^2 - 4a^4c^4\Big\}^{1/2}\Big)\,, \\ &\simeq 2\times 10^{-3} eV^2\,. \end{aligned} \tag{34}$$

These equations may be inverted to obtain

$$\begin{aligned} v^4\left((a^2+b^2)^2 + c^4 + 2b^2c^2\cos 2\theta\right) &= 2\Delta m^2_{atm} - \Delta m^2_{\odot} \\ a^4c^4v^8 &= \Delta m^2_{atm}(\Delta m^2_{atm} - \Delta m^2_{\odot}). \end{aligned} \tag{35}$$

Now using Eqs. (11) and (27) in Eq. (26) we can rewrite the matrix $K$ as

$$\begin{aligned} K &= R_{12}(\omega, \phi+\beta_1)P' \\ &= \begin{pmatrix} \cos\omega & e^{i(\phi+\beta_1)}\sin\omega & 0 \\ -e^{-i(\phi+\beta_1)}\sin\omega & \cos\omega & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} e^{i\eta_1/2} & 0 & 0 \\ 0 & e^{i(\eta_2/2-\beta_1)} & 0 \\ 0 & 0 & e^{-i\beta_2} \end{pmatrix}. \end{aligned} \tag{36}$$

Thus using Eqs. (36) and (18) in Eq. (24) the PMNS matrix $U_{PMNS}$ is given as

$$U_{PMNS} = R_{23}(\Theta_{23})R_{13}(\Theta_{13}, \delta'_{13})R_{12}(\Theta_{12})R_{12}(w, \phi+\beta_1)P'\,, \tag{37}$$

where

$$R_{12}(\Theta_{12})R_{12}(\omega, \phi+\beta_1) = \begin{pmatrix} \cos\Theta'_{12}e^{i\rho_1} & \sin\Theta'_{12}e^{i\rho_2} & 0 \\ -\sin\Theta'_{12}e^{-i\rho_2} & \cos\Theta'_{12}e^{-i\rho_1} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} e^{i\left(\frac{\rho_1+\rho_2}{2}\right)} & 0 & 0 \\ 0 & e^{-i\left(\frac{\rho_1+\rho_2}{2}\right)} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \cos\Theta'_{12} & \sin\Theta'_{12} & 0 \\ -\sin\Theta'_{12} & \cos\Theta'_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} e^{i\left(\frac{\rho_1-\rho_2}{2}\right)} & 0 & 0 \\ 0 & e^{-i\left(\frac{\rho_1-\rho_2}{2}\right)} & 0 \\ 0 & 0 & 1 \end{pmatrix}. \tag{38}$$

In the above equation we have

$$\cos 2\Theta'_{12} = \cos 2\omega\cos 2\Theta_{12} - \cos(\phi+\beta_1)\sin 2\omega \sin 2\Theta_{12}\,, \tag{39}$$

$$\sin(\rho_2-\rho_1) = \sin(\phi+\beta_1)\tan\omega\left[\cot 2\Theta'_{12} + \frac{\cos 2\Theta_{12}}{\sin 2\Theta'_{12}}\right]\,, \tag{40}$$

$$\sin(\rho_1+\rho_2) = \frac{\sin 2\omega\sin(\phi+\beta_1)}{\sin 2\Theta'_{12}}\,. \tag{41}$$

For further simplification of the PMNS matrix (37) we now compute the matrix product $R_{12}(\Theta_{12})K = R_{12}(\Theta_{12})R_{12}(\omega, \phi+\beta_1)P'$ which is given as

$$R_{12}(\Theta_{12})R_{12}(\omega,\phi+\beta_1)P' = e^{i(\frac{\eta_1}{2}-\rho_2)}$$
$$\begin{pmatrix} e^{i(\rho_1+\rho_2)} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \cos\Theta'_{12} & \sin\Theta'_{12} & 0 \\ -\sin\Theta'_{12} & \cos\Theta'_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i(\rho_2-\rho_1+(\eta_2-\eta_1)/2-\beta_1)} & 0 \\ 0 & 0 & e^{-i(\beta_2-\rho_2+\frac{\eta_1}{2})} \end{pmatrix} \quad (42)$$

Now using Eq. (42) in Eq. (37) the $U_{PMNS}$ matrix can be rewritten as:

$$\begin{aligned} U_{PMNS} &= \tilde{U}K \\ &= R_{23}(\Theta_{23})R_{13}(\Theta_{13},\delta_{13})R_{12}(\Theta'_{12}) \\ &\quad \text{diag.}(1, e^{i(\rho_2-\rho_1+(\eta_2-\eta_1)/2-\beta_1)}, e^{-i(\beta_2-\rho_2+\frac{\eta_1}{2})}) \\ &= V\,.\,V_{ph}\,, \end{aligned} \quad (43)$$

where $V$ is the CKM like matrix and $V_{ph}$ is the Majorana phase matrix. The effective $CP$ violating phase in the $V$ matrix is given by

$$\delta_{13} = \delta'_{13} + (\rho_1+\rho_2)\,. \quad (44)$$

Note that in writing Eq. (43) the overall phase $e^{i(\frac{\eta_1}{2}-\rho_2)}$ has been taken out. Moreover, we absorb the unphysical phase matrix diag.$(1, e^{-(\rho_1+\rho_2)}, e^{-(\rho_1+\rho_2)})$ into the redefinition of charged lepton fields. From Eqs. (5), (18) and (44) we see that, for the chosen parameterization of $Y_{2RH}$, two of the mixing angles $\Theta_{23}$ and $\Theta_{13}$ remains same as of the $(2-3)$ and $(1-3)$ mixing angles in PDG parameterization of the leptonic mixing matrix. Thus we can write $\Theta_{23} \equiv \theta_{23}$ and $\Theta_{13} \equiv \theta_{13}$. While $\Theta_{12}$ gets modified to $\Theta'_{12}$ and is given by Eq. (39), the modified CP violating phase $\delta_{13}$ is given by Eq. (44). At present the best fit value of $\Theta_{23}$ is given to be 45°, while the best fit value with $1\sigma$ error the value of $\Theta'_{12}$ is given to be $33.9° \pm 1.6°$ [18]. The CHOOZ experiment gives a bound on $\Theta_{13}$. Currently the most conservative upper bound on $\Theta_{13}$ at the $3\sigma$ confidence level is given to be [19]

$$sin^2\Theta_{13} < 0.048\,, \quad (45)$$

which gives $\Theta_{13} < 13°$.

## V. LEPTOGENESIS IN 2RH NEUTRINO MODELS

The Majorana mass of the RH neutrino violates L-number and hence is considered to be a natural source of L-asymmetry in the early Universe [5] provided its decay violates CP symmetry, a necessary criteria of Sakharov [20]. In a mass basis where the RH neutrinos are real and diagonal the Majorana neutrinos are defined as $N_i = \frac{1}{\sqrt{2}}(N_{Ri} \pm N_{Ri}^c)$. In this basis the CP asymmetry is given by

$$\epsilon_i = \frac{\Gamma_i - \bar{\Gamma}_i}{\Gamma_i + \bar{\Gamma}_i}\,, \tag{46}$$

where $\Gamma_i$ is the decay rate of $N_i$. If we assume a normal mass hierarchy ($M_1 << M_2$) in the RH neutrino sector then the final L-asymmetry is given by the decay of the lighter RH neutrino, $N_1$. The CP asymmetry parameter, arising from the decay of $N_1$, is then given by

$$\epsilon_1 = \frac{-3}{16\pi v^2}\left(\frac{M_1}{M_2}\right)\frac{Im[(m_D^\dagger m_D)_{12}]^2}{(m_D^\dagger m_D)_{11}}\,. \tag{47}$$

Using Eqs. (17) and (16) in the above Eq. (47) we get

$$\epsilon_1 = \frac{-3}{16\pi}\left(\frac{M_1}{M_2}\right) y^2 \sin 2\theta\,. \tag{48}$$

From the above Eq. (48) it is clear that if $\theta = 0$ then there is no CP violation in leptogenesis. Therefore, $\theta$ can be thought of the phase associated with $M_i$ in a basis where $M_i$'s are complex. Moreover, $\theta$ always hangs around $y$. So $y = 0$ implies no leptogenesis. We will discuss more about it in sec.VI while we compare the CP violation in leptogenesis, neutrino oscillation and neutrinoless double beta decay processes.

We now estimate the magnitude of $L$-asymmetry. A net L-asymmetry arises when $\Gamma_1$ fails to compete with the Hubble expansion parameter, $H = 1.67 g_*^{1/2}(T^2/M_{pl})$, where $g_*$ is the number of relativistic degrees of freedom at the epoch of temperature $T$. In a comoving volume the $L$-asymmetry is defined as

$$Y_L = \epsilon_1 Y_{N_1} d\,, \tag{49}$$

where $d$ is the dilution factor arises due to the competitions between $\Gamma_1$ and $H$ at $T \simeq M_1$. Now using Eq. (48) in the above Eq. (49) we get

$$Y_L = -5.97 \times 10^{-5}\frac{M_1}{M_2}\left(\frac{Y_{N_1} d}{10^{-3}}\right) y^2 \sin 2\theta\,. \tag{50}$$

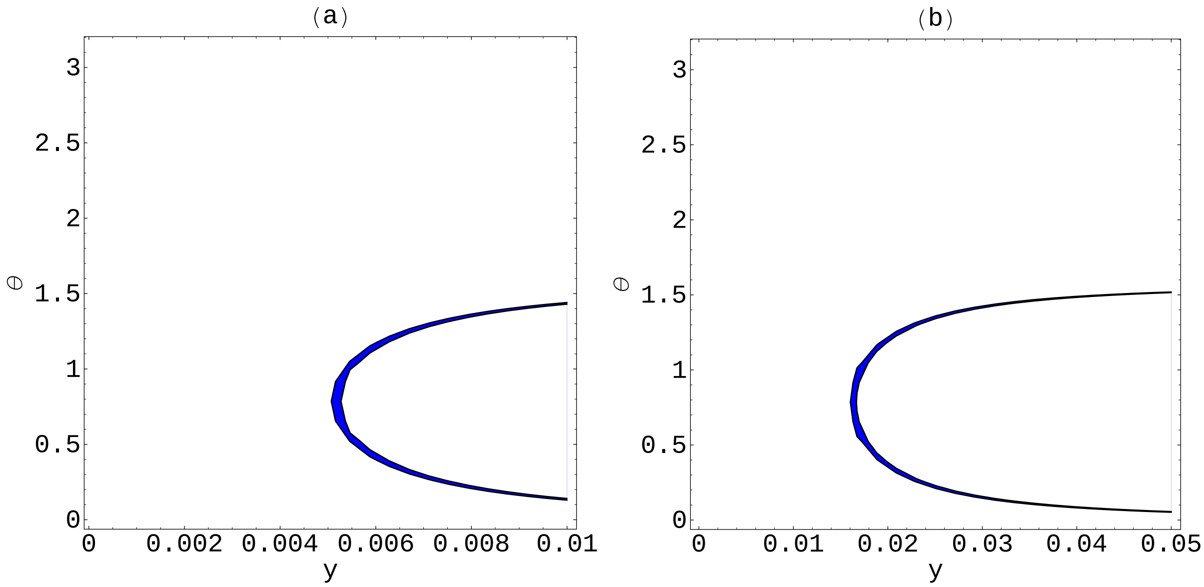


FIG. 1: The allowed values of $y$ are shown against $\theta$ (in rad) for the observed matter anti-matter asymmetry, given by Eq. (1), with (a)$\frac{M_1}{M_2}=0.1$ and (b)$\frac{M_1}{M_2}=0.01$.

A part of the L-asymmetry is then transferred to the B-asymmetry via the sphaleron processes which are unsuppressed above the electroweak phase transition. Taking into account the particle content in the $SM$ the B- and L-asymmetries are related as

$$B=\frac{p}{p-1}L\simeq -0.55L\,, \tag{51}$$

where $p=28/79$ appropriate for the particle content in the $SM$. As a result we get the net B-asymmetry per comoving volume to be

$$Y_B\simeq 3.28\times 10^{-5}\frac{M_1}{M_2}\left(\frac{Y_{N_1}d}{10^{-3}}\right)y^2\sin 2\theta\,. \tag{52}$$

The observed B-asymmetry, on the other hand, is given by

$$\left(\frac{n_B}{n_\gamma}\right)=7Y_B=2.3\times 10^{-4}\frac{M_1}{M_2}\left(\frac{Y_{N_1}d}{10^{-3}}\right)y^2\sin 2\theta\,. \tag{53}$$

Comparing the above Eq. (53) with the observed matter antimatter asymmetry, given by Eq.(1), we get

$$y^2\sin 2\theta=(2.57\quad-\quad 2.78)\times 10^{-6}\frac{M_2}{M_1}\left(\frac{10^{-3}}{Y_{N_1}d}\right)\,. \tag{54}$$

We have shown the allowed values of $y$ in fig. (1), using $(Y_{N1}d)=10^{-3}$, for hierarchical RH neutrinos in the $y-\theta$ plane. It is shown in fig. 1(a) that for $(M_1/M_2)=0.1$ the minimum allowed value of $y$ is $5 \times 10^{-3}$. However, this value is lifted up to $1.7 \times 10^{-2}$ for $(M_1/M_2) = 0.01$ as shown in fig. 1(b).

## VI. CP VIOLATION IN RE-PHASING INVARIANT FORMALISM

It is convenient to study CP violation in a re-phasing invariant formalism. In particular, for the CP violation in the leptonic sector the latter makes it very interesting. The CP violation in any lepton number conserving processes comes out to be of the form [21]

$$J_{abij} = \mathrm{Im}[V_{ai} V_{bj} V_{aj}^* V_{bi}^*], \tag{55}$$

where $V$ is the CKM like matrix in the lepton sector. On the other hand, CP violation in any lepton number violating processes will be of the form [22]

$$t_{aij} = \mathrm{Im}[V_{ai} V_{aj}^* (V_{ph})_{ii}^* (V_{ph})_{jj}]\,. \tag{56}$$

Now one can have as many independent re-phasing invariant measures $t$ as many independent Majorana CP phases. For three generations there are two independent $t$'s (denoted as $J_1$ and $J_2$) and one J (denoted as $J_{CP}$). For example, in the neutrinoless double beta decay the following re-phasing invariant will appear

$$T = \mathrm{Im}[V_{ai} V_{aj} V_{bi}^* V_{bj}^*] \sim t_{aij} t_{bij}^*. \tag{57}$$

It has been shown that the re-phasing invariant CP violating quantity $J_{CP}$ only appears in the neutrino oscillations and that of $J_1$, $J_2$ appears in the neutrinoless double beta decay processes which may be observed in the next generation experiments.

### A. CP-violation in leptogenesis and neutrino oscillation

It has been pointed out that the Dirac phase $\delta_{13}$ can be measured in the long baseline neutrino oscillation experiments [12]. In that case the CP violation arises from the difference of transition probability $\Delta P = P_{\nu_e \to \nu_\mu} - P_{\bar{\nu}_e \to \bar{\nu}_\mu}$. It can be shown that the transition probability $\Delta P$ is proportional to the leptonic Jarlskog invariant

$$J_{CP} = Im[V_{e1} V_{e2}^* V_{\mu 1}^* V_{\mu 2}]\,. \tag{58}$$

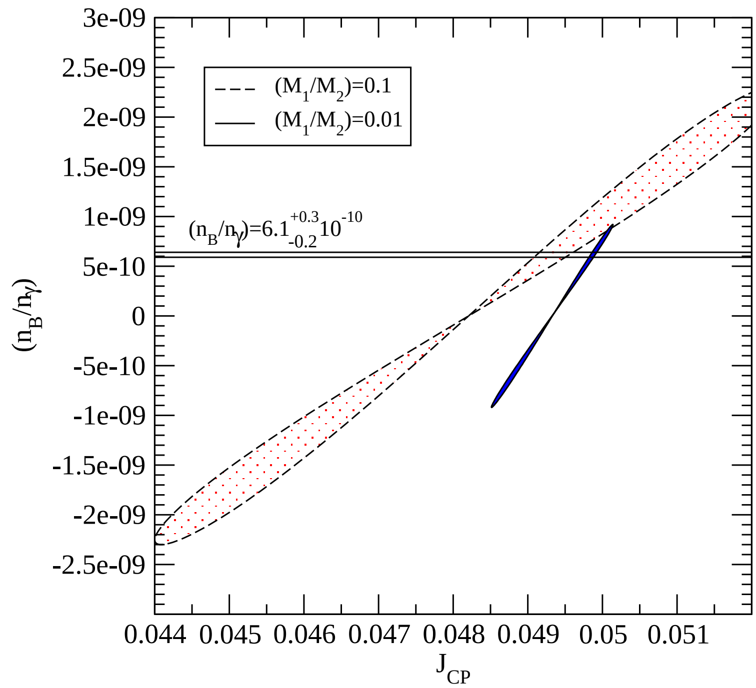


FIG. 2: The overlapping region in the $n_B/n_\gamma - J_{CP}$ plane is shown as $\theta$ (in rad) varies from 0 to $\pi$ with $\Theta_{23} = \pi/4$, $\Theta_{13} = 13^\circ$, $\delta'_{13} = \beta_1 = \pi/2$ and $z = x = 0.01$. The dashed line is obtained for $\Theta_{12} = 33.5^\circ$, $y = 0.01$ and $\frac{M_1}{M_2} = 0.1$, while the solid line is obtained for $\Theta_{12} = 33.8^\circ$, $y = 0.02$ and $\frac{M_1}{M_2} = 0.01$.

Using Eq. (43) the re-phasing invariant $J_{CP}$ can be rewritten as

$$J_{CP} = \frac{1}{8} \sin 2\Theta'_{12} \sin 2\Theta_{23} \sin 2\Theta_{13} \cos\Theta_{13} \sin(\delta'_{13} + \rho_1 + \rho_2)\,. \tag{59}$$

Now using Eqs. (28), (29), (39) and (41) in the above Eq. (59) we get

$$\begin{aligned} J_{CP} &= \frac{1}{8}\frac{\sin 2\Theta_{23} \sin 2\Theta_{13} \cos\Theta_{13}}{\sqrt{\left[(a^2+b^2)^2 + c^4 + 2b^2c^2\cos 2\theta\right]^2 - 4a^4c^4}} \\ &\times \Big[2ab\cos\delta'_{13}\{-c^2 \sin 2\theta \cos\beta_1 + (a^2+b^2+c^2\cos 2\theta)\sin\beta_1\} \\ &+ 2ab\cos 2\Theta_{12} \sin\delta'_{13}\{(a^2+b^2+c^2\cos 2\theta)\cos\beta_1 + c^2 \sin 2\theta \sin\beta_1\} \\ &+ \sin\delta'_{13} \sin 2\Theta_{12}(-a^4 + b^4 + c^4 + 2b^2c^2\cos 2\theta)\Big]\,. \end{aligned} \tag{60}$$

From the above Eq. (60) it is obvious that $J_{CP} = 0$ only if both $\sin\delta'_{13} = 0$ and $b = 0$, while only $b = 0$ (equivalently $y = 0$) implies the condition for "no leptogenesis". This indicates that there is no one-to-one correspondence between the CP violation in neutrino

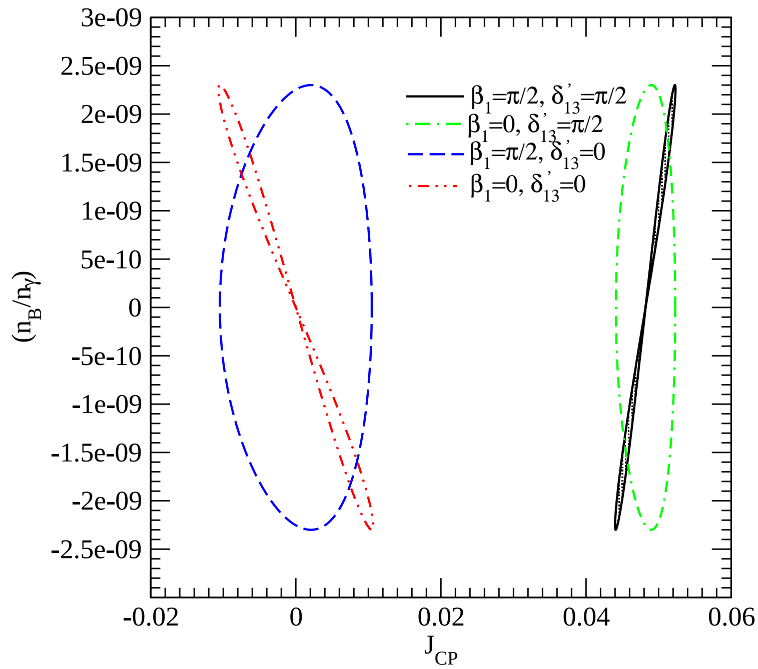


FIG. 3: The variation of $n_B/n_\gamma$ is shown against $J_{CP}$ for different values of $\beta_1$ and $\delta'_{13}$ as $\theta$ (in rad) varies from 0 to $\pi$. We have chosen $\Theta_{23} = \pi/4$, $\Theta_{13} = 13°$, $\Theta_{12} = 33.5°$, $x = y = z = 0.01$ and $\frac{M_1}{M_2} = 0.1$.

oscillation and the CP violation in leptogenesis, even in the 2RH neutrino models. However, it is interesting to see the common regions in the plane of $(n_B/n_\gamma)$ versus $J_{CP}$. This is shown in fig. (2) by taking a typical set of parameters. The main aim is to illustrate the maximal contrast between the positive and negative values of $n_B/n_\gamma$ for a given set of values of $J_{CP}$. This helps us in determining the sign of the asymmetry by knowing the size of the asymmetry. From the fig. (2) it is obvious that for the given set of parameters the positive sign of the asymmetry allows the values of $J_{CP}$ in the range $0.049-0.0495$ for $(M_1/M_2) = 0.1$. However, this range is significantly reduced for $(M_1/M_2) = 0.01$. On the other hand, the negative sign of the asymmetry allows the values of $J_{CP}$ in the range $0.0465 - 0.047$ for $(M_1/M_2) = 0.1$ which is further reduced for $(M_1/M_2) = 0.01$. In this figure the value of $\Theta_{12}$ is used from fig. (4) where we have shown the allowed values of $\Theta_{12}$ as $\theta$ varies from 0 to $\pi$. Note that the above results are true for a non-zero $\Theta_{13}$. Consequently the allowed range of values of $J_{CP}$ may vary depending on the values of $\Theta_{13}$. Thus we anticipate that in the 2RH neutrino models a knowledge of $J_{CP}$ can predict the sign of matter antimatter asymmetry of the Universe. We should note that the predictive power of the model depends on the CP violating phases $\beta_1$ and $\delta'_{13}$. This can be visible from fig. (3) where we have shown the variation of $n_B/n_\gamma$ with $J_{CP}$ for different values of $\beta_1$ and $\delta'_{13}$. In particular, for the choice ($\beta_1 = \pi/2$, $\delta'_{13} = 0$) and ($\beta_1 = 0$, $\delta'_{13} = \pi/2$), the contrast between the positive and negative values of $n_B/n_\gamma$ is almost zero for a given set of values of $J_{CP}$. On the other hand, for the choice ($\beta_1 = \pi/2$, $\delta'_{13} = \pi/2$) and ($\beta_1 = 0$, $\delta'_{13} = 0$), the contrast between the positive and negative values of $n_B/n_\gamma$ is maximal and can be chosen for the present purpose.

### B. CP violation in leptogenesis and neutrinoless double-beta decay

The observation of the neutrinoless double beta decay would provide direct evidence for the violation of total $L$-number in the low energy effective theory and hence probing the left-handed physical neutrinos to be Majorana type. Note that the $L$-number violation at high energy scale is a necessary criteria for leptogenesis. In the canonical seesaw models this is conspired by assuming that the RH neutrinos are Majorana in nature. However, this assumption doesn't ensure that the left-handed physical neutrinos are Majorana type. Assuming that the physical neutrinos are of Majorana type we investigate the connecting links between the two $L$-number violating phenomena occurring at two different energy scales.

In the low energy effective theory with three generations of left-handed fermions, apart from the $J_{CP}$, one can write two more re-phasing invariants $J_1$ and $J_2$ which designates lepton number violation and CP violation [22]. However, in the models with two RH neutrinos one of the eigen values of the physical light neutrino mass matrix is exactly zero. Therefore, the corresponding phase in the Majorana phase matrix can always be chosen so as to set one of the lepton number violating CP violating re-phasing invariant to zero. In the present case $m_3 = 0$ and hence the corresponding phase is arbitrary. This is ensured through $\beta_2$ which is redundant and pointed out in Eq. (17). Therefore, from Eq. (43) we can write the only $L$-number violating CP violating re-phasing invariant as:

$$\begin{aligned} J &= Im\,[V_{e1}V^*_{e2}(V_{ph})^*_{11}(V_{ph})_{22}] \\ &= -\frac{1}{2}\sin 2\Theta'_{12}\cos^2\Theta_{13}\sin(\rho_2 - \rho_1 + \frac{(\eta_2 - \eta_1)}{2} - \beta_1)\,. \end{aligned} \tag{61}$$

Using Eq. (42) the above Eq. (61) can be rewritten as

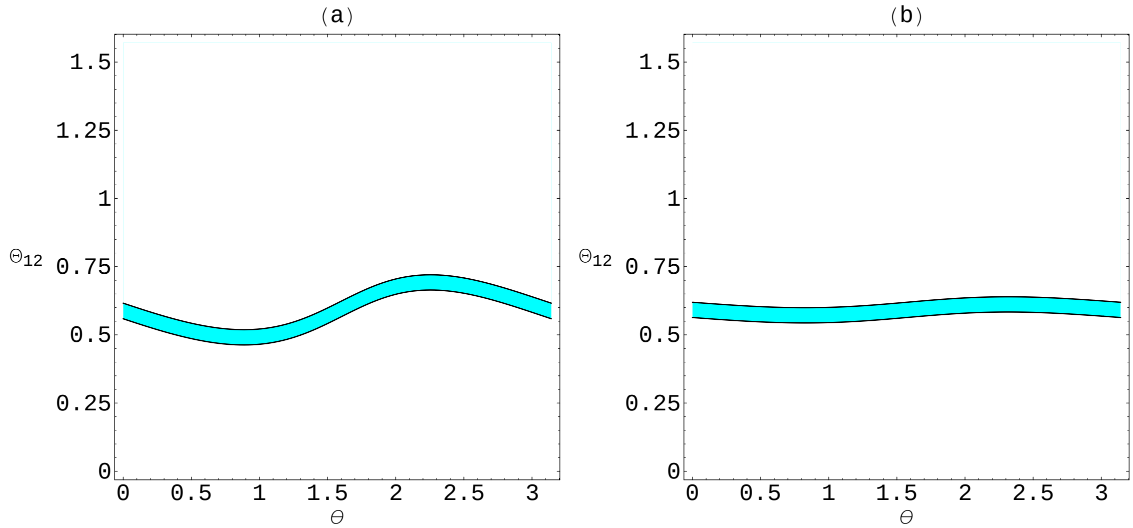


FIG. 4: The allowed range of $\Theta_{12}$ (in rad) in Eq. (63) is shown as $\theta$ (in rad) varies from 0 to $\pi$ for $\Theta'_{12} = (33.9 \pm 1.6)^\circ$, $\beta_1 = \pi/2$, $x = z = 0.01$ (a) $y = 0.01$ and $(M_1/M_2) = 0.1$, and (b) $y = 0.02$ and $(M_1/M_2) = 0.01$.

$$\begin{aligned} J &= -\frac{\cos^2\Theta_{13}}{2}\frac{1}{[(a^2+b^2)^2+c^4+2b^2c^2\cos 2\theta+2c^2a^2\cos 2\theta]} \\ &\times \Big[\sin 2\Theta_{12}\cos\theta\{-c^2\sin 2\theta\cos\beta_1+(a^2+b^2+c^2\cos 2\theta)\sin\beta_1\} \\ &\quad \times\sqrt{(a^2+b^2)^2+c^4+2c^2a^2+2b^2c^2\cos 2\theta} \\ &+ \sin 2\Theta_{12}\sin\theta\{c^2\sin 2\theta\sin\beta_1+(a^2+b^2+c^2\cos 2\theta)\cos\beta_1\} \\ &\quad \times\frac{(-a^4+b^4+c^4+2b^2c^2\cos 2\theta)}{\sqrt{(a^2+b^2)^2+c^4+2c^2a^2+2b^2c^2\cos 2\theta}} \\ &+ \cos 2\Theta_{12}\sin\theta\frac{2ab\{(a^2+b^2)^2+c^4+2b^2c^2\cos 2\theta+2c^2a^2\cos 2\theta\}}{\sqrt{(a^2+b^2)^2+c^4+2c^2a^2+2b^2c^2\cos 2\theta}}\Big]. \end{aligned} \tag{62}$$

In the above Eq. (62) the allowed values of $\Theta_{12}$ is obtained from

$$\begin{aligned} \cos\Theta'_{12} &= \left[\frac{1}{2}\left[1+\frac{(-a^4+b^4+c^4+2b^2c^2\cos 2\theta)\cos 2\Theta_{12}}{\sqrt{((a^2+b^2)^2+c^4+2b^2c^2\cos 2\theta)^2-4a^4c^4}}\right.\right. \\ &\quad \left.\left.-\sin 2\Theta_{12}\frac{2ab\{c^2\sin 2\theta\sin\beta_1+(a^2+b^2+c^2\cos 2\theta)\cos\beta_1\}}{\sqrt{((a^2+b^2)^2+c^4+2b^2c^2\cos 2\theta)^2-4a^4c^4}}\right]\right]^{1/2}, \end{aligned} \tag{63}$$

by fixing $\Theta'_{12} = (33.9 \pm 1.6)^\circ$. This is shown in fig. (4).

From Eq. (62) one can see that $J \neq 0$ as $\theta \to 0$ which is the condition for "no leptogenesis". Thus we see that there is no one-to-one correspondence between the two $L$-number

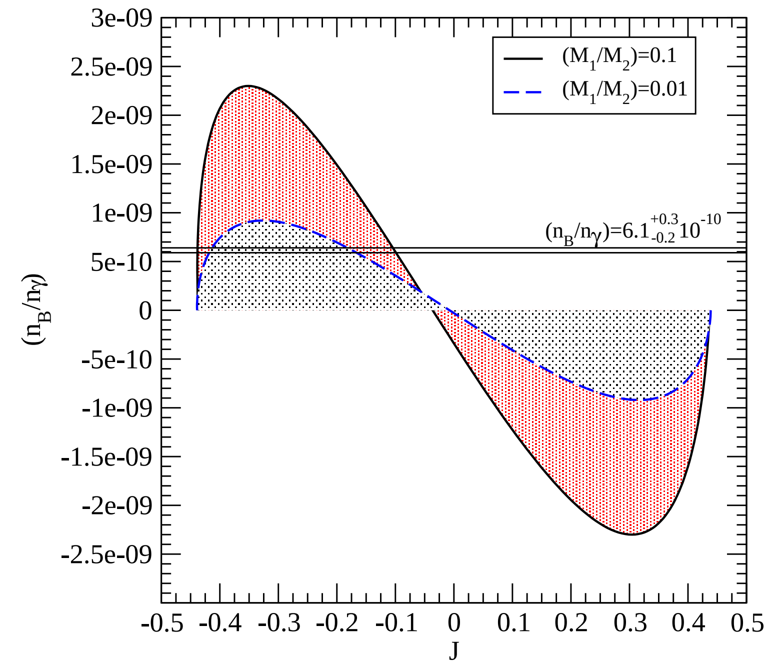


FIG. 5: The overlapping region in the $\frac{n_B}{n_\gamma} - J$ plane is shown as $\theta$ (in rad) varies from 0 to $\pi$ with $\Theta_{13} = 13°$, $\beta_1 = \pi/2$ and $x = z = 0.01$. The solid line is obtained for $y = 0.01$ and $\Theta_{12} = 33.5°$, while the dashed line is obtained with $y = 0.02$ and $\Theta_{12} = 33.8°$.

violating processes occurring at two different energy scales. However, it is always interesting to see the overlapping regions in the plane of $\frac{n_B}{n_\gamma}$ versus $J$ as $\theta$ varies from 0 to $\pi$. This is shown in fig. (5) for a typical set of parameters. From fig. (5) one can see that for positive sign of the $B$-asymmetry the values of $J$ lie in between $-0.45$ to $-0.1$ for $(M_1/M_2) = 0.1$. This range is further reduced to $(-0.4 - -0.15)$ for $(M_1/M_2) = 0.01$. On the other hand, for the negative sign of the $B$-asymmetry the values of $J$ lie in the range $(0.05 - 0.45)$ for $(M_1/M_2) = 0.1$ and in the range $(0.15 - 0.4)$ for $(M_1/M_2) = 0.01$. Thus we see that within the allowed range of parameters the contrast between the positive and negative values of $\frac{n_B}{n_\gamma}$ is maximum for a given set of values of $J$. Therefore, we expect a knowledge of $J$ can precisely determine the sign of $B$-asymmetry since the value of $n_B/n_\gamma$ is known. Finally we note that, unlike $J_{CP}$, $J$ remains non-vanishing even if $\Theta_{13} = 0$ [2]. Now the remaining question to be addressed is how $n_B/n_\gamma$ varies with respect to $J$ for different values of $\beta_1$. This is shown in fig. (6) for a given set of parameters. One can see that for $\beta_1 = 0$ and

[2] In three generations there are two of them. See for example the paper by Y. Liu and U. Sarkar in ref. [22]

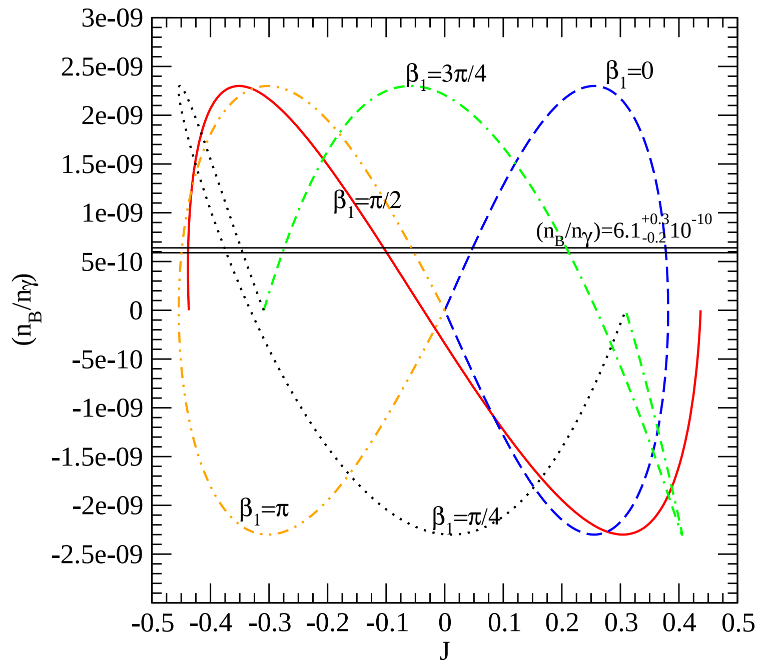


FIG. 6: The variation of $\frac{n_B}{n_\gamma}$ is shown against $J$ for different values of $\beta_1$ as $\theta$ (in rad) varies from 0 to $\pi$. We have chosen $\Theta_{13} = 13°$, $\Theta_{12} = 33.5°$, $x = y = z = 0.01$ and $\frac{M_1}{M_2} = 0.1$.

$\beta_1 = \pi$ both positive and negative values of $n_B/n_\gamma$ correspond to the same set of values of $J$ which is unwelcome for determination of sign of the asymmetry. On the other hand for $\beta_1 \neq 0, \pi$ one can have maximal contrast between the positive and negative values of $n_B/n_\gamma$ for the given set of values of $J$ and hence can be chosen for the present purpose.

## VII. CONCLUSIONS

We have studied the connecting links between the CP violating phase(s) giving rise to leptogenesis, occurring at a high energy scale, and the CP violating phases appearing in the low energy phenomena, i.e., neutrino oscillation and neutrinoless double beta decay processes. This is studied in the framework of two right-handed neutrino models. The low energy leptonic CP violation is studied in a re-phasing invariant formalism. It is shown that there are only two re-phasing invariants; (1) The lepton number conserving CP violating re-phasing invariant $J_{CP}$ which can be determined in the future long-baseline neutrino oscillation experiments, (2) The lepton number violating CP violating re-phasing invariant $J$ which can be determined in the neutrinoless double beta decay experiments. It is found that there is no one-to-one correspondence between these two CP violating phenomena, occurring at two different energy scales, even though the number of parameters involving in the seesaw is exactly same as the number of low energy observable parameters. However, in a suitable parameter space we have shown that the overlapping regions in the plane of $n_B/n_\gamma$ versus $J_{CP}$ and $n_B/n_\gamma$ versus $J$ can indeed determine the *sign* of the matter antimatter asymmetry of the present Universe assuming that the *size* of the asymmetry is precisely known.

## APPENDIX A: PARAMETERIZATION OF $Y_{2RH}$

To parameterize the neutrino Dirac Yukawa coupling in two right-handed neutrino models we follow the same procedure adopted in Ref. [23]. Let $\mathbf{u_1}$, $\mathbf{u_2}$, $\mathbf{u_3}$ be three orthonormal 3 dimensional vectors. Using these basis vectors we can write the most general unitary matrix $U$ as:

$$U = (\mathbf{u_1}\ \mathbf{u_2}\ \mathbf{u_3})\,. \tag{A1}$$

Let us consider an arbitrary $3 \times 2$ matrix $Y$ which in terms of the 3-dimensional vectors $\mathbf{y_1}$ and $\mathbf{y_2}$ can be written as:

$$Y = (\mathbf{y_1}\ \mathbf{y_2})\,. \tag{A2}$$

Without loss of generality we choose $\mathbf{u_2} = \frac{\mathbf{y_1}}{|\mathbf{y_1}|}$. As a result we get

$$U^\dagger Y = \begin{pmatrix} 0 & \alpha_{12} \\ |y_1| & \alpha_{22} \\ 0 & \alpha_{32} \end{pmatrix}, \tag{A3}$$

where $\alpha_{ij} = \mathbf{u_i}^\dagger \cdot \mathbf{y_j}$.

Let $V$ be another unitary matrix which we choose to be of the form:

$$V = \begin{pmatrix} \frac{\alpha_{12}}{\sqrt{|\alpha_{12}|^2+|\alpha_{32}|^2}} & 0 & \beta_{13} \\ 0 & 1 & 0 \\ \frac{\alpha_{32}}{\sqrt{|\alpha_{12}|^2+|\alpha_{32}|^2}} & 0 & \beta_{33} \end{pmatrix}, \tag{A4}$$

where $\beta_{ij}$ must follow $\alpha_{12}\beta_{13}^* + \alpha_{32}\beta_{33}^* = 0$ and $|\beta_{13}|^2 + |\beta_{33}|^2 = 1$. Consequently we have

$$V^\dagger U^\dagger Y = \begin{pmatrix} 0 & \sqrt{|\alpha_{12}|^2 + |\alpha_{32}|^2} \\ |y_1| & \alpha_{22} \\ 0 & 0 \end{pmatrix}, \tag{A5}$$

where we set $V_{32} = 0$ by imposing the unitarity condition of $V$. This implies that we can always write any arbitrary $3 \times 2$ matrix

$$Y = W Y_{2RH} \,, \tag{A6}$$

where $W = VU$ is an unitary matrix and the texture of $Y_{2RH}$, the Yukawa coupling in the two right handed neutrino mass models, is given as

$$Y_{2RH} = \begin{pmatrix} 0 & x \\ z & y e^{-i\theta} \\ 0 & 0 \end{pmatrix} \,, \tag{A7}$$

where $x$, $y$, $z$ and $\theta$ are real numbers. Note that by appropriately choosing the $U$ and $V$ matrices one can construct the $Y_{2RH}$ matrix in twelve possible ways.

## APPENDIX B: POSSIBLE TEXTURES OF $Y_{2RH}$ AND NEUTRINO MIXINGS

In this appendix we specify the various possible textures of $Y_{2RH}$. One of the particular texture of $Y_{2RH}$ has been used in section II for our work. In the table-I we write all the possible textures of $Y_{2RH}$. Each possible $Y_{2RH}$ in table-I will lead to various forms of $X$, apparent from Eq. (20). Accordingly the neutrino masses and mixing angles will be modified through the $m_D$ parameters.


**Acknowledgment**

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TABLE I: Possible textures of $Y_{2RH}$

| | | | | |
|---|---|---|---|---|
| Zeros in the first row | | | | |
| I | $\begin{pmatrix} 0 & 0 \\ z & ye^{-i\theta} \\ 0 & x \end{pmatrix}$ | $\begin{pmatrix} 0 & 0 \\ 0 & x \\ z & ye^{-i\theta} \end{pmatrix}$ | $\begin{pmatrix} 0 & 0 \\ x & 0 \\ ye^{-i\theta} & z \end{pmatrix}$ | $\begin{pmatrix} 0 & 0 \\ ye^{-i\theta} & z \\ x & 0 \end{pmatrix}$ |
| Zeros in the middle row | | | | |
| II | $\begin{pmatrix} z & y^{-i\theta} \\ 0 & 0 \\ 0 & x \end{pmatrix}$ | $\begin{pmatrix} 0 & x \\ 0 & 0 \\ z & ye^{-i\theta} \end{pmatrix}$ | $\begin{pmatrix} y^{-i\theta} & z \\ 0 & 0 \\ x & 0 \end{pmatrix}$ | $\begin{pmatrix} x & 0 \\ 0 & 0 \\ ye^{-i\theta} & z \end{pmatrix}$ |
| Zeros in the bottom row | | | | |
| III | $\begin{pmatrix} z & ye^{-i\theta} \\ 0 & x \\ 0 & 0 \end{pmatrix}$ | $\begin{pmatrix} 0 & x \\ z & ye^{-i\theta} \\ 0 & 0 \end{pmatrix}$ | $\begin{pmatrix} ye^{-i\theta} & z \\ x & 0 \\ 0 & 0 \end{pmatrix}$ | $\begin{pmatrix} x & 0 \\ ye^{-i\theta} & z \\ 0 & 0 \end{pmatrix}$ |


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