

# Majorana neutrinos with split fermions in extra dimensions

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## Abstract

We propose new solutions to the neutrino mass problem in theories with large extra dimensions in a thick wall scenario. It has recently been argued that our 3-brane could be a thick wall at the boundary of the bulk. The gauge bosons and the Higgs scalars have an almost flat profile on this wall, while fermions could have localized profile with left-handed and right-handed components displaced with respect to each other. We point out that with split fermions it is possible to generate Majorana neutrino masses contributing to the neutrinoless double beta decay. The almost degenerate neutrinos can also come out naturally in this case. Unlike other models of neutrino masses in extra dimensions there are no bulk fields in this scenario.

The recent developments in theories with extra dimensions have changed our conventional idea about physics beyond the standard model [1]. In usual theories the weakness of gravity is attributed to the very small coupling of the gravitational interaction. This leads to the very high Planck scale where gravity could become strong and could then influence the standard model. In theories with large extra dimensions one assumes that there are extra dimensions in which gravity propagates, whereas the ordinary standard model particles live in a 4-dimensional wall at the boundary of the extra dimensions. Then the gravitational interaction strength in the bulk could be strong and the overlap of the graviton wave function in our 3-brane at the boundary makes gravity weak in our world. This would then allow a very low fundamental scale of about TeV replacing the effective Planck scale in the theory.

The main advantage of these theories with extra dimensions and TeV scale gravity is that there is no gauge hierarchy problem, but there is new problem of absence of any large scale. The smallness of neutrino mass is usually attributed to the large lepton number violating scale. But there is no large scale in the theories with extra dimensions, so these mechanisms can not be used. Consider the effective 4-dimensional operator in the standard model, which gives mass to the neutrinos [2]

$$\mathcal{O}_{eff} = \frac{f_{ij}}{M_{\#}} \ell_{iL} \ell_{jL} \phi \phi. \quad (1)$$

Since the highest scale in the theory of extra dimensions is the fundamental scale  $M_*$ , which is of the order of TeV, the effective neutrino mass comes out to be fairly large now, unless we can make the effective coupling constant  $f_{ij}$  to be small.

There are several solutions to this problem, proposed with the same philosophy to make the coupling small [3, 4, 5]. In theories with extra dimensions all standard model particles reside in the four-dimensional wall. Gravity now propagates in the higher-dimensional space and the overlap of the wave function of the gravitons with the four-dimensional wall is very small. So, the gravity coupling to matter in our world is suppressed by the volume of the extra dimensions. Similar to gravity if there are some bulk particles which move in all dimensions (which constitute the bulk of space in the extra dimensions as compared to our wall which is confined only at one end), their overlap in our brane would be small and that can give a small neutrino mass

in our brane [3, 4]. This gives a Dirac mass to the neutrinos. In another scenario lepton number is broken in another distant brane, or in the bulk, which is then conveyed to our brane by a bulk scalar field [3, 5]. The profile of the bulk scalar then can give a small effective coupling constant.

Recently it has been suggested that to solve the fermion mass hierarchy one could consider a thick wall scenario, in which the left and right-handed components are localized at different points with small overlap in the thick wall [6, 7, 8, 9]. Our 3-brane now has a spread (unlike other models where our 3-brane is confined to one point in the extra dimensions) in the extra dimensions. The gauge and the Higgs bosons can propagate anywhere within this thick brane and they have an almost constant profile in our brane. Only outside this thick wall their profile falls off exponentially. However, within this thick wall the fermions are confined at different points with definite profiles. The overlap of the different fields in any interaction then gives the hierarchical Yukawa couplings [6, 7].

We extend this scenario to explain the smallness of the neutrino mass and show that tiny Majorana neutrino masses come out naturally from this scenario. We do not require any bulk fields to make the neutrino mass small. In the present scenario we break lepton number in our brane at the fundamental scale, but because of the small overlap of wave functions of the required fields with each other the effective coupling constant  $f_{ij}$  comes out to be very small naturally.

The thick wall scenario was proposed to solve the problem of hierarchy of fermion masses [6]. The fermions are localized at different points in the higher-dimensional space. This can come from string theory depending on the construction of the p-branes, but there is also a field-theoretical realization of this idea. Consider a five-dimensional example, with  $z = \{x, y\}$  and  $y$  as the coordinate of the fifth dimension, in which a five-dimensional fermion  $\Psi$  and a scalar  $\mathcal{S}$  couple through a Yukawa coupling term  $\sim \int d^5z \mathcal{S} \bar{\Psi} \Psi$ . If the scalar field has a position-dependent vacuum expectation value, which changes sign at a point  $y_0$  in the extra dimension, then the fermion will be localized at  $y_0$  with a Gaussian profile in the extra dimension centered around  $y_0$

$$\Psi(x, y) = A e^{-\mu^2(y-y_0)^2} \psi(x), \quad (2)$$

where  $\psi$  is a normalized four-dimensional massless left-handed fermion,  $A = (2\mu^2/\pi)^{1/4}$  is the normalization and  $\mu = \sqrt{\partial \langle \mathcal{S} \rangle / 2}$  is related to the slope

of the scalar field profile.  $y_0 = 0$  for a massless five-dimensional field  $\Psi$ , but when a mass term is added for a particular fermion field  $\int d^5z M_i \bar{\Psi}_i \Psi_i$ , that field is localized at  $y_0 = y_i = M_i/2\mu^2$ .

A generic five-dimensional Yukawa interaction now gives

$$\mathcal{L}_Y = \int d^5z \sqrt{L} \kappa \Phi \bar{\Psi}_i \Psi_j = \int d^4x \lambda \bar{\psi}_i \psi_j \phi, \quad (3)$$

where  $L$  is the domain wall width and the effective four-dimensional Yukawa coupling constant comes out to be

$$\lambda = \int dy \kappa A e^{-\mu^2(y-y_i)^2} A e^{-\mu^2(y-y_j)^2} = \kappa e^{-\mu^2(y_i-y_j)^2/2}. \quad (4)$$

In general,  $\kappa$  could depend on the indices  $i, j$ , but for purpose of simplification it is assumed that there is only one constant.  $\phi$  is the standard model Higgs doublet contained in the five-dimensional scalar  $\Phi$ . The Gaussian width  $\mu^{-1}$  has to be much larger than the wall thickness  $L$  for this mechanism to work, but for the field theoretic description to work there is a limit  $\mu^2 L < M_*$ , where  $M_*$  is the fundamental scale in the problem. Combining with other constraints, the requirement that the Yukawa coupling to be perturbative at  $M_*$  now gives

$$\mu < M_* < 1000L^{-1} \quad \text{and} \quad L^{-1} < \mu < 30L^{-1}. \quad (5)$$

Constraints from flavor changing neutral currents mediated by the Kaluza-Klein gauge bosons constrain the wall thickness  $L^{-1} \geq 100$  TeV.

Let us now consider the neutrino sector. Although the neutrino masses were discussed in the context of thick wall scenarios [9], our proposed mechanisms differ from them. Here we do not require any bulk particles and the neutrinos get a lepton number violating Majorana mass. Moreover, in this scenario the neutrino masses could be almost degenerate, so that they can contribute to the dark matter of the universe and also explain the neutrinoless double beta decay [10, 11]. In other models of neutrino masses in extra dimensions there is no natural mechanism to explain the almost degenerate neutrinos. Even in ordinary theories it is difficult to accommodate an almost degenerate Majorana neutrino naturally. Starting from a grand unified theory and if one evolves the Yukawa couplings in supersymmetric models, it becomes difficult to maintain the degeneracy [12]. First we propose two different possibilities, each of which has some difficulties. Then we consider

a more general model combining both the mechanisms, which has several interesting features.

First we introduce only right-handed neutrinos for three generations  $N_{\alpha R}$ ,  $\alpha = 1, 2, 3$ , which are singlets under the standard model gauge group. We then allow all possible renormalizable interactions consistent with the standard model gauge symmetry. The Majorana mass term of the right-handed neutrinos will then violate lepton number and set the scale of lepton number violation. The interactions of the right-handed neutrinos are given by

$$\mathcal{L}_N = M_{N\alpha\beta} N_{\alpha R} N_{\beta R} + h_{i\alpha} \bar{l}_{iL} N_{\alpha R} \phi, \quad (6)$$

where  $l_{iL}$  is the left-handed lepton doublet and the Majorana mass of the right-handed neutrinos violate lepton number  $M_N = M_{\cancel{L}}$ . We have written this interaction in terms of four-dimensional fields. The fifth dimension has been integrated out to get these effective coupling constants  $h_{i\alpha}$  and the Majorana mass term  $M_{N\alpha\beta}$ . At this stage, this is exactly similar to the usual see-saw mechanism of neutrino masses [13].

Since any lepton number violating effective interactions of the left-handed scalars can originate only from these two interactions, the lepton number violating mass scale ( $M_{\cancel{L}}$ ) in equation (1) should be given by  $M_N$  and the coefficients  $f_{ij}$  should be determined by  $h_{i\alpha}$ . So, although the Majorana mass term of  $N_R$  would allow the effective lepton number violating operator (1) with very little suppression from the scale of lepton number violation  $M_{\cancel{L}} < M_*$ , the effective coupling now could be very small.

The above mentioned effective four-dimensional interactions come from five-dimensional terms in the Lagrangian

$$\mathcal{L}_{Yuk} = \int d^5z \left[ M_5 \Psi_{N\alpha} \Psi_{N\beta} + \sqrt{L} \kappa \bar{\Psi}_{li} \Psi_{N\alpha} \Phi \right], \quad (7)$$

so that

$$\begin{aligned} M_{N\alpha\beta} &= M_5 \exp \left[ -\mu^2 \frac{(y_{N\alpha} - y_{N\beta})^2}{2} \right], \\ h_{i\alpha} &= \kappa \exp \left[ -\mu^2 \frac{(y_{li} - y_{N\alpha})^2}{2} \right]. \end{aligned} \quad (8)$$

The diagonal elements of  $M_N$  are all the same and equal to  $M_5$ . The right-handed neutrinos could be separated in space so that the mass matrix is

diagonal and given by an identity matrix. But in general they could be neighbours and the mass degeneracy could be broken. We demonstrate this with an example.

Consider the charged lepton mass matrix to be diagonal. Then the required mass hierarchy could be achieved with the configuration of the left-handed lepton doublets  $l_i$  and the right-handed charged leptons  $e_i$  given by (similar to ref. [6])

$$l_i = \mu^{-1} \begin{pmatrix} 12 \\ 0 \\ -1 \end{pmatrix}; \quad \text{and} \quad e_i = \mu^{-1} \begin{pmatrix} 6.87 \\ 3.95 \\ -4.15 \end{pmatrix}. \quad (9)$$

We further assume that the multiplets of the same  $SU(2)_L$  representations to be located at the same place in the fifth dimension. Then both  $e_{iL}^-$  and  $\nu_{iL}$  (contained in  $l_i$ ) will be located at the same place with the same profile, and the configurations of  $\nu_{iL}$  are the same as given for  $l_i$ . If the right-handed neutrinos are now localized at

$$N_\alpha = \mu^{-1} \begin{pmatrix} 7.2 \\ 5 \\ 4.8 \end{pmatrix},$$

the Yukawa couplings and the right-handed Majorana mass matrix will be given by

$$h_{i\alpha} = \begin{pmatrix} 0.093 & 5.7 \times 10^{-7} & 1.3 \times 10^{-7} \\ 5.7 \times 10^{-7} & 0.093 & 0.248 \\ 3.2 \times 10^{-10} & 3.8 \times 10^{-4} & 0.0012 \end{pmatrix}$$

and

$$M_{N\alpha\beta} = M_0 \begin{pmatrix} 1 & 0.135 & 0.0059 \\ 0.135 & 1 & 0.486 \\ 0.0059 & 0.486 & 1 \end{pmatrix},$$

where  $M_0 = M_\mu < M_*$  is the scale of lepton number violation. In this example we considered  $M_0 \sim 10^6$  GeV. The eigenvalues of the left-handed neutrino mass matrix

$$m_{ij}^\nu = \frac{h_{i\alpha} h_{j\beta}^T \langle \phi \rangle^2}{M_{N\alpha\beta}} \quad (10)$$

come out to be  $10^{-9}$  eV, 0.0062 eV and 0.065 eV, which are the masses required to explain the atmospheric neutrino anomaly and the large mixing

angle solution of the solar neutrinos. The correct mixing angles come out only when the charged lepton mass matrix is not diagonal or the neutrinos are spread over more than one extra dimension.

We now consider another possibility of Majorana neutrino mass generation in the thick wall scenario. Instead of a right-handed neutrino we now introduce a triplet Higgs scalar in the theory [14]. The interactions of the five-dimensional triplet Higgs scalar ( $\Xi(z)$ ) now violate lepton number explicitly

$$\mathcal{L}_\Xi = \int d^5z \left[ \sqrt{L} \kappa_l \Xi \Psi_{li} \Psi_{lj} + \mu_\Xi \Xi^\dagger \Phi \Phi + M_\Xi \Xi^\dagger \Xi \right]. \quad (11)$$

From these interactions we can derive the four-dimensional effective interactions

$$\mathcal{L}_\xi = \int d^4x \left[ f_{ij} \xi \psi_{li} \psi_{lj} + \mu_\xi \xi^\dagger \phi \phi + M_\xi \xi^\dagger \xi \right]. \quad (12)$$

This gives a Majorana neutrino mass to the left-handed neutrinos

$$m_{ij}^\nu = -f_{ij} \mu_\xi \frac{\langle \phi \rangle^2}{M_\xi^2}. \quad (13)$$

The effective four-dimensional Yukawa coupling constant  $f_{ij}$  is given by

$$\begin{aligned} f_{ij} &= \int dy \kappa_l e^{-\mu^2(y-y_\xi)^2} e^{-\mu^2(y-y_i)^2} e^{-\mu^2(y-y_j)^2} \\ &= \sqrt{\frac{3}{2}} \kappa_l \exp \left[ -\frac{\mu^2}{2} (y_i - y_j)^2 \right] \cdot \exp \left[ -\frac{2}{3} \mu^2 \left( y_0 - \frac{y_i + y_j}{2} \right)^2 \right]. \end{aligned} \quad (14)$$

$y_i$  and  $y_j$  are the positions at which the leptons are localized and  $y_0$  is the point in the fifth dimension where the triplet Higgs is localized. The first term gives a suppression depending on the separation of the two neutrinos, while the second term gives a suppression depending on the average separation of the neutrinos compared to the triplet scalar. For the diagonal elements, the first term is identity, while for the off-diagonal elements they are almost zero. So, in the basis in which the charged lepton mass matrix is diagonal, all the off-diagonal elements vanish and the neutrino mass matrix always comes out to be diagonal. Since the standard model Higgs doublet has a spread over the entire thick wall, it has complete overlap with the triplet Higgs scalar and there is no suppression for the other coupling  $\mu_\xi$ . The lepton number violating scale is now given by  $M_\psi = M_\xi \sim \mu_\xi < M_*$ .

So in the thick wall scenario only with a triplet Higgs scalar, it is not possible to get a neutrino mass matrix with the required mixing in the flavor basis. However, an interesting case may emerge when we include both the triplet Higgs scalar and the right-handed neutrinos [15]. We shall also assume two extra dimensions, which is anyway required for satisfactory quark mass matrices. Then we consider the possible localized positions of the left-handed and right-handed neutrinos and the triplet Higgs, as shown in figure 1.

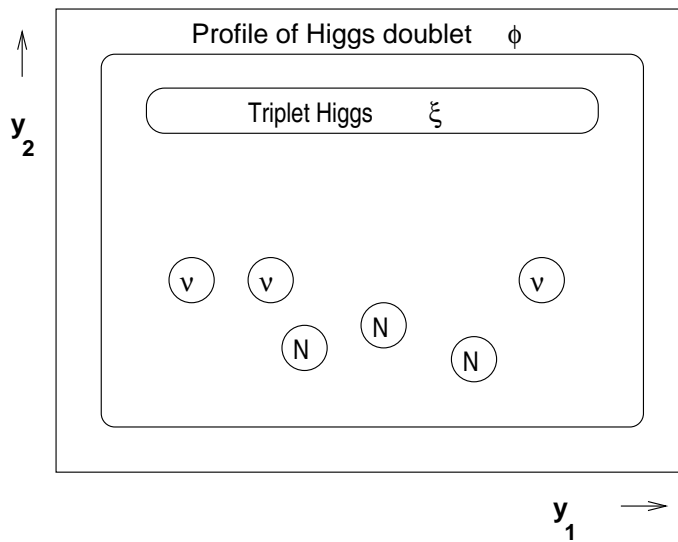


Figure 1: Possible localized positions of neutrinos and the triplet Higgs with two extra dimensions.

The positions of the left-handed neutrinos are determined by the charged lepton mass hierarchy and the possible pattern of the mass matrix. We thus present a scenario of diagonal charged lepton mass matrix. The Higgs doublet  $\phi$  is spread over the entire thick brane and all fields have the same overlap with the Higgs. There will be very little suppression due to the volume of the Higgs profile  $L$  in the extra dimension, but that is the same for all the fields and may be absorbed in the definition of the higher-dimensional coupling constant.

The triplet Higgs now has equal average distance from all the three generations of left-handed neutrinos. As a result, the contributions to the diagonal elements of the Majorana mass matrix of the left-handed neutrinos are equal.



On the other hand, in the basis in which the charged leptons are diagonal, the Majorana mass matrix generated by the triplet Higgs is also diagonal. So, we have an exactly degenerate diagonal mass spectrum for the neutrinos coming from the triplet Higgs. If we now assume that the lepton number violating scale is around  $M_{\mathcal{L}} \sim 10^6$  GeV, then a separation between the mean location of the triplet Higgs to the mean location of the left-handed neutrinos of  $5.35 \mu^{-1}$  would give a neutrino mass matrix

$$m_{\xi ij}^\nu = (0.4 \text{ eV}) \times \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad (15)$$

in a basis in which the charged leptons are diagonal. We assumed that the separation of the left-handed neutrinos is determined by the charged lepton mass hierarchy as discussed earlier.

The right-handed neutrino mass matrix will now be most general. The mixing angle comes out as required for suitable choice of the distances in this two-dimensional plane. There is only one restriction that a hierarchical neutrino mass spectrum is only possible numerically, once the left-handed neutrino locations are determined to get the hierarchy of the charged lepton mass matrix. One possible neutrino mass matrix which could emerge in this scenario is given by

$$m_{Nij}^\nu = (0.025 \text{ eV}) \begin{pmatrix} 0.00004 & 0.005 & 0.01 \\ 0.005 & 1.0 & 0.9 \\ 0.01 & 0.9 & 1.0 \end{pmatrix}. \quad (16)$$

This can explain the solar and atmospheric neutrino anomalies. This mass matrix predicts an almost maximal mixing with mass squared difference of  $0.002 \text{ eV}^2$  to explain the atmospheric neutrinos. A mass squared difference of  $6 \times 10^{-6} \text{ eV}^2$  with a mixing angle of  $\sin^2 2\theta \sim 5 \times 10^{-3}$  solves the solar neutrino problem with the small mixing angle solution.

A more interesting scenario emerges when the see-saw and the triplet Higgs scenarios are combined. In this case an almost degenerate neutrino comes out naturally. It would then be possible to explain the recent observation of the neutrinoless double beta decay [11] by this hybrid model. In this case the neutrino mass matrix becomes,

$$m_{ij}^\nu = m_{\xi ij}^\nu + \tilde{m}_{Nij}^\nu, \quad (17)$$

where  $m_{\xi ij}^\nu$  is given by equation (15). We now require a different see-saw contribution than the one given for the hierarchical neutrino mass scheme. In this case it is not possible to obtain the small mixing angle solution to the solar neutrino problem. With two extra dimensions, it is possible to get a mass matrix of the form

$$\tilde{m}_{Nij}^\nu = (0.0012 \text{ eV}) \begin{pmatrix} 0.1 & 1 & 1 \\ 1 & a & 1 \\ 1 & 1 & a \end{pmatrix}, \quad (18)$$

where  $a < 0.05$ . The complete neutrino mass matrix given by equation (17) then gives almost degenerate mass matrix, with all the masses to be about 0.4 eV. This can explain the neutrinoless double beta decay and can contribute to the hot component of the dark matter of the universe. A mass squared difference between  $\nu_\mu$  and  $\nu_\tau$  of  $2.8 \times 10^{-3} \text{ eV}^2$  with maximal mixing solves the atmospheric neutrino anomaly. The mass difference between  $\nu_e$  and  $\nu_\tau$  comes out (for  $a = 0$ ) to be  $6 \times 10^{-5} \text{ eV}^2$  with mixing angle  $\sin^2 2\theta \sim 0.56$ , which can provide the large mixing angle MSW solution to the solar neutrino problem.

In summary, we proposed models of neutrino masses in a thick wall scenario in which the left-handed and right-handed components have split identity. While the model with a right-handed neutrino can explain the neutrino oscillation experiments, the model with triplet Higgs can give only a diagonal neutrino mass matrix. Combining the two with two extra dimensions, it is possible to obtain an almost degenerate neutrino mass matrix naturally, which can explain the solar neutrinos, atmospheric neutrinos, and also contribute to the dark matter of the universe and explain the neutrinoless double beta decay.

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