Neutrino mixing schemes and neutrinoless double beta decay

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Abstract

We study the possible structure of the neutrino mass matrix taking into consideration the solar and atmospheric neutrinos and the neutrinoless double beta decay. We emphasize on mass matrices with vanishing elements. There are only a very few possibilities remaining at present. We concentrate on three generation scenarios and find that with three parameters there are few possibilities with and without any vanishing elements. For completeness we also present a five parameter four neutrino (with one sterile neutrino) mass matrix which can explain all these experiments and the LSND result.

In recent past there have been many new results in neutrino physics [1]-[6]. All these results are narrowing down the parameter space for the neutrino masses and mixing. The solution to the atmospheric neutrino anomaly [1] is the strongest to constrain the mass squared difference and the mixing angle between the μ and the τ neutrinos. This result is also supported by the K2K result [3] and a combined analysis of both these experiment can now be used to determine the allowed parameter range [7]. The maximal mixing angle restricts the structure of the mass matrix very strongly. Another very strong constrain comes from the CHOOZ result [4] from the non-observation of oscillation of the electron neutrino into any other neutrinos. For the solar neutrinos [2] there are a few possible solutions, which are also narrowing down [8]. The small angle solution of the solar neutrinos is almost ruled out. The situation with the sterile neutrinos is even worse. It is not considered to be a favoured solution to the atmospheric neutrinos and also for the solar neutrinos. However, for the consistency of the LSND result [5] with both the solar and the atmospheric neutrinos we need a sterile neutrino. Thus, although the present popular scheme is to ignore the LSND result and work with a three generation neutrino mass matrix, for completeness we shall also mention a four generation scenario. The three mixing angles of a three generation mass matrix is determined by the mixing angles required for the atmospheric neutrinos, solar neutrinos and the reactor constraint from CHOOZ.

Although the mass squared differences are required to be fairly small compared to what is required for the neutrinos to contribute to the dark matter of the universe [9], the overall masses of the neutrinos could be large enough to constitute the hot dark matter component of the universe, which is required to explain the large scale structure of the universe. The recent indication of the positive evidence for a neutrinoless double beta decay [6] points exactly to this interesting solution of the neutrino masses [10, 11, 12]. This result has now generated some interest in the field [13, 14]. If we include this result, the mass matrix for the three generation scenario will have a very little freedom now.

Among other things, there is now a pattern in the mass matrix [15, 16, 13, 17]. Consider a three generation scenario in the flavour basis, when the charged lepton mass matrix is diagonal $M^{\ell^-} = \mathrm{Diag}[m_e, m_\mu, m_\tau]$. In this case the exact maximal mixing for the atmospheric neutrinos implies that the four elements $M^\nu_{22}, M^\nu_{33}, M^\nu_{23}$ and M^ν_{33} are non-vanishing and constrained. It also implies that $M^\nu_{12} = M^\nu_{13}$. The solar neutrino mixing angle then requires M^ν_{12} and M^ν_{13} to be non-vanishing. Finally M^ν_{11} gives the contribution to the neutrinoless double beta decay. This discussion shows that the neutrino mass matrix is completely determined now and we have very little freedom left. Moreover all the elements are required to be non-vanishing. However, since the atmospheric neutrino mixing could be less than maximal and if all uncertainties in the allowed parameters are considered this simple argument does not work exactly.

Assuming the symmetric neutrino mass matrix to be real there are six parameters. All these six parameters are required to explain the atmospheric neutrino anomaly, solar neutrino problem, the neutrinoless double beta decay and satisfy the CHOOZ constraint. The explanation of the hot dark matter is coupled with the neutrinoless double beta decay result, so this is not considered as an independent constraint. The question we would now like to ask is, can we have a neutrino mass scheme satisfying all the present experimental constraints with less than six parameters? What is the minimum number of parameters we require for a neutrino mass matrix, which can satisfy all these constraints. In particular, can we afford to have any elements of the neutrino mass matrix to be vanishing. Exact zero elements in a mass matrix always makes it convenient for model building, so we try to find out if there exists any neutrino mass matrix with zero elements satisfying all these constraints. As a sequel we present a four generation neutrino mass matrix, which satisfies all these results including the LSND result with only five (or even four) parameters.

The various experimental inputs we consider are the following. The neutrinoless double beta decay experiment constrain the (11) element of the neutrino mass matrix (m_{ee}) in the flavour basis (in which the charged lepton mass matrix is diagonal). We define the mixing matrix $U_{\alpha i}$ to be the one relating the physical states $|\nu_\alpha>$ (in the flavour basis $\alpha = e, \mu, \tau$) to the mass eigenstates $|\nu_i>$ (with masses m_i, $i = 1, 2, 3$)

$$|\nu_\alpha> \;\; = \;\; \sum_i U_{\alpha i} \, |\nu_i> .$$

Then the present evidence of the neutrinoless double beta decay gives

$$m_{ee} = \sum_i |U_{ei}|^2 m_i = (0.05 - 0.86) \text{ eV (at 97\% c.l.)} \tag{1}$$

with a best fit $m_{ee} = 0.39$ eV.

The other constraints on the mass eigenvalues are from the atmospheric and the solar neutrinos and the LSND:

$$\begin{aligned}
\Delta m^2_{atm} &= \{(1.5 - 4.8), 2.7\} \times 10^{-3} \text{ eV}^2 \\
\Delta m^2_{sol-LMA} &= \{(2 - 30), 4.5\} \times 10^{-5} \text{ eV}^2 \\
\Delta m^2_{sol-SMA} &= \{(4 - 6), 4.7\} \times 10^{-6} \text{ eV}^2 \\
\Delta m^2_{LSND} &> 0.2 \text{ eV}^2 .
\end{aligned}$$

The last numbers are the best fit values. For solar neutrinos the large mixing angle MSW solution (LMA) is the preferred solution. However, we mention the small mixing angle MSW solution (SMA) for completeness, which is almost ruled out at the 2σ level. We shall not discuss the SMA solutions in detail. The corresponding mixing angles are:

$$\begin{aligned}
\sin^2 2\theta_{atm} &= \{(0.87 - 1), 1\} \\
\sin^2 2\theta_{sol-LMA} &= \{(0.3 - 0.94), 0.82\} \\
\sin^2 2\theta_{sol-SMA} &= \{(0.001 - 0.004), 0.0015\} \\
\sin^2 2\theta_{LSND} &= (0.001 - 0.04).
\end{aligned}$$

The LSND result will be used only in the four generation scenario with a sterile neutrino. In the three generation case there are three mixing angles. One of them is determined by the atmospheric neutrinos, the second angle is determined by the solar neutrinos. For the third angle the CHOOZ constraint is considered:

$$\Delta m^2_{eX} < 10^{-3}\ \mathrm{eV}^2 \qquad \text{or} \qquad \sin^2 2\theta_{eX} < 0.2$$

We shall not consider any effect of CP violation and hence the mass matrices are assumed to be real.

For a systematic analysis, we start with the possible textures for the mass matrices presented in ref. [15]. We consider all the texture mass matrices, which can explain the maximal mixing for the atmospheric neutrinos and add possible perturbations. Similar to the maximal mixing in the atmospheric neutrino solution, the neutrinoless double beta decay imposes the next strongest constraint on the general structure of the mass matrices. Since the contribution to the neutrinoless double beta decay is larger or equal to the mass squared difference required by the atmospheric neutrino anomaly, the M^ν_{11} must be of the order of any other large elements in the mass matrix. This rules out all the textures with the (11) element 0. In a more general analysis it is possible to consider a scenario with vanishing (11) element for the largest entry in the mass matrix, but for the present purpose of obtaining a simple texture mass matrix we shall not discuss this possibility.

This would mean that the hierarchical neutrino mass schemes are all ruled out [11]. The large angle solutions to the solar neutrinos are also not allowed when the first two mass eigenvalues have opposite signs. As mentioned above, in a more general case it may be possible to have this texture and still satisfy the LMA solution, but it is not possible to have any simple form of the mass matrix. This argument leaves us with three mass textures

$$M_\nu^{A1} = m_0 \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1/2 & 1/2 \\ 0 & 1/2 & 1/2 \end{pmatrix} \qquad M_\nu^{A2} = m_0 \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$M_\nu^{A3} = m_0 \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, \tag{2}$$

which can allow for both LMA as well as the SMA solutions. There are three other textures, which can only allow for SMA solutions, they are

$$M_\nu^{B1} = m_0 \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1/2 & -1/2 \\ 0 & -1/2 & -1/2 \end{pmatrix} \qquad M_\nu^{B2} = m_0 \begin{pmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$M_\nu^{B3} = m_0 \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & -1 & 0 \end{pmatrix}. \tag{3}$$

m_0 is the overall scale in these mass matrices, which is determined by the value of the neutrinoless double beta decay. The B solutions are not very interesting, since they cannot give us LMA solutions.

Let us first study the texture $A1$. The simplest perturbation that may be considered is

$$m_1^{A1} = m_0 \begin{pmatrix} a & b_1 & b_2 \\ b_1 & 0 & 0 \\ b_2 & 0 & 0 \end{pmatrix}. \tag{4}$$

In this case it is possible to have solutions with $b_1 = 0$ and $b_2 = b$ or $b_1 = b$ and $b_2 = 0$, which are exactly equivalent. Different choices for the parameters a and b can now give the SMA or the LMA solutions. This three parameter mass matrix with non zero element depends on the condition, $[m_1^{A1}]_{22} = [m_1^{A1}]_{33} = [m_1^{A1}]_{23} = [m_1^{A1}]_{32}$. A few representative sets of the parameters which allow the LMA and SMA solutions are:

$$
\begin{aligned}
(1) SMA &: \quad m_0 = 0.05; \quad a = 0.001; \quad b = 0.00002; \\
(2) LMA &: \quad m_0 = 0.05; \quad a = 0.003; \quad b = 0.002; \\
(3) LMA &: \quad m_0 = 0.05; \quad a = 0.003; \quad b_1 = b_2 = 0.001;
\end{aligned}
$$

We discuss these solutions briefly. These solutions correspond to the partial degenerate case, where two of the masses are degenerate. In this case it is not possible to have a neutrinoless double beta decay contribution more than the mass required for the atmospheric neutrinos. So, we get an effective mass contributing to the neutrinoless double beta decay to be 0.05. In all the cases the mass squared difference between the states containing the ν_μ and ν_τ is around $\Delta m^2_{atm} \sim 0.0025$ eV, which is required for the atmospheric neutrinos. The mixing angle for the atmospheric neutrinos is maximal or almost maximal ($\sin^2 2\theta_{atm} > 0.95$). In (1), the mass squared difference for solar neutrinos comes out to be $\Delta m^2_{sol} \sim 5 \times 10^{-6}$ eV^2 with a mixing angle $\sin^2 2\theta_{sol} \sim 0.001$. In (2), we get $\Delta m^2_{sol} \sim 2.5 \times 10^{-5}$ eV^2 with a mixing angle $\sin^2 2\theta_{sol} \sim 0.33$. In (3), there are no zero elements, but since $b_1 = b_2$, we get exactly maximal mixing for atmospheric neutrinos $\sin^2 2\theta_{atm} = 1$. For the solar neutrinos the solution is similar to the case (2), $\Delta m^2_{sol} \sim 2.5 \times 10^{-5}$ eV^2 with a mixing angle $\sin^2 2\theta_{sol} \sim 0.33$.

For the texture $A2$, it is not possible to have any elements to be vanishing. The atmospheric neutrino maximal mixing requires that the (23) and the (32) elements are non-vanishing. For maintaining the maximal mixing for atmospheric neutrinos, we also need the (12) and the (13) elements to be equal. Thus one of the possible perturbations to the mass matrix $A2$ is

$$
m_1^{A2} = m_0 \begin{pmatrix} a & b & b \\ b & 0 & b \\ b & b & 0 \end{pmatrix}. \tag{5}
$$

The SMA solution is not possible with any simple form of the perturbations. A representative set for the LMA solution is: $m_0 = 0.4$ eV, $a = 0.0003$

and $b = 0.003$, which gives $\Delta m^2_{sol} \sim 6.3 \times 10^{-5}$ eV^2 with a mixing angle $\sin^2 2\theta_{sol} \sim 0.91$ and $\Delta m^2_{atm} \sim 0.0028$ eV with exactly maximal mixing.

The most interesting case comes out to be the $A3$ case. All the results may be accomodated with one parameter perturbation

$$m_1^{A3} = m_0 \begin{pmatrix} a & 0 & a \\ 0 & 0 & 0 \\ a & 0 & -2a \end{pmatrix}. \tag{6}$$

Now the complete mass matrix $m^\nu = M_\nu^{A3} + m_1^{A3}$ will also have a few zero elements ($m^\nu_{22} = m^\nu_{12} = m^\nu_{21} = 0$). Although the allowed range is not very wide, there exist solutions, like $a = 0.003$ with $m_0 = 0.4$ eV. The predictions are, $\Delta m^2_{sol} \sim 2 \times 10^{-4}$ eV^2 with a mixing angle $\sin^2 2\theta_{sol} \sim 0.33$ and $\Delta m^2_{atm} \sim 0.0023$ eV with almost maximal mixing. Similar results come from two other possible perturbations with this $A3$ texture

$$m_0 \begin{pmatrix} a & a & a \\ a & 0 & 0 \\ a & 0 & -2a \end{pmatrix} \qquad \text{and} \qquad m_0 \begin{pmatrix} a & 0 & a \\ 0 & -a & 0 \\ a & 0 & -a \end{pmatrix} \tag{7}$$

With such simple structures it is not possible to have a wider range of parameters or accomodate the SMA solution.

For the mass matrix with no vanishing elements, it is now possible to give a simple parametrization which guarantees a maximal mixing for the atmospheric neutrinos and gives the neutrinoless double beta decay

$$\mathcal{M}_\nu = m_0 \begin{pmatrix} m_{ee} & a & a \\ a & b+c & b-c \\ a & b-c & b+c \end{pmatrix}. \tag{8}$$

The neutrinoless double beta decay determines the element m_{ee}. The textures $A1$, $A2$ and $A3$ are the limiting cases with $a, c \ll b$; $a \ll b = c$ and $a \ll b = -c$, respectively. The CHOOZ constraint is satisfied and the solar neutrino solutions are obtained with suitable choice of a and c.

In the case of four generations the analysis becomes more involved. Only the mass matrices with minimum number of parameters have been studied in this context [18, 19]. The simplest of these models require four parameters [18], while other four-generation mass matrices require five or more parameters with two identical diagonal elements [19]. We generalise the simplest of these mass matrices [18] to include the neutrinoless double beta decay result and in addition get the correct mixing angle for the solar neutrinos [18]. In the minimal version of this mass matrix only maximal mixing was possible for the LMA solution. We present a mass matrix with only 5 parameters, which can explain all the experiments including the neutrinoless double beta decay and the LSND result.

The mass matrix can be written in the basis $[\nu_e, \nu_\mu, \nu_\tau, \nu_s]$ as

$$\mathcal{M}_{4\nu} = \begin{pmatrix} m & 0 & a & d \\ 0 & c & b & 0 \\ a & b & 0 & 0 \\ d & 0 & 0 & -m \end{pmatrix}. \tag{9}$$

We can further economise by identifying two parameters $m = d$, making it effectively a four-parameter mass matrix.

We discuss the solution briefly. The parameter m determines the amount of neutrinoless double beta decay. The oscillation between the states ν_e and ν_s explains the solar neutrino problem. The mixing angle now becomes, $\sin^2 2\theta_{sol} = d^2/(m^2 + d^2)$. A simple choice of $d = m$ gives $\sin^2 2\theta_{sol} = 0.5$, which is consistent with present data. Restricting ourselves to $c \ll b$ ensures a maximal mixing between ν_μ and ν_τ, as required by the atmospheric neutrinos. The mass squared difference for the atmospheric neutrinos is given by $2bc$ and that for the LSND is $b^2 - d^2 - m^2$. There are no simple expressions for the mass squared difference required for the solar neutrinos and the mixing angle for the LSND result. Numerically these predictions come out as required. For completeness we present a representative set of the values of these parameters

(all parameters are in eV), $a = 0.03$; $b = 0.6$; $c = 0.003$; $d = m = 0.1$; which gives, $m_{ee} = 0.1$ eV,
$\Delta m^2_{sol} = 7 \times 10^{-5}$ eV^2, $\sin^2 2\theta_{sol} = 0.5$,
$\Delta m^2_{atm} = 0.0027$ eV^2, $\sin^2 2\theta_{atm} = 1$,
$\Delta m^2_{LSND} = 0.34$ eV^2, $\sin^2 2\theta_{LSND} = 0.003$.
This appears to be the simplest four-generation mass matrix with texture zeroes, which can explain all experiments in neutrino physics.

We have not mentioned about the LOW, Just-so and Vacuum oscillation solutions of the solar neutrino problem. It has been shown in ref [12] that in a general analysis these solutions are ruled out when the radiative corrections are considered in conjunction with the present result on the neutrinoless double beta decay. However, in a more detailed analysis [13] it has been pointed out that the LOW solution is allowed in some special cases. But from the point of constructing a simple form of the texture mass matrix we could not find any solution, which allows these solutions. In the four generation case the mass matrix we presented can allow for a vacuum solution for some choice of parameters, similar to the earlier analysis of ref [18].

In summary, we try to obtain the simplest forms of the mass matrices consistent with all the present experiments including the neutrinoless double beta decay. We present possible mass matrices with vanishing elements. We also present a simple form of the mass matrix with four parameters, which can reproduce all possible allowed mass matrices. We further present a four generation mass matrix with five parameters, which can explain all the experiments.

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