Effects of New Gravitational Interactions on Neutrinoless Double Beta Decay

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Abstract

It has recently been proposed that violations of Lorentz invariance or violations of the equivalence principle can be constrained from the non-observation of neutrinoless double beta decay. We generalize this analysis to all possible new gravitational interactions and discuss briefly the constraints for different cases.

Although there is no evidence for the violation of gravitational laws, lots of work has been done to find out to what accuracy this is true. Many experiments have been performed to test the equivalence principle [1] for ordinary matter and to test local Lorentz invariance [2, 3]. In recent times there has been some effort to test these laws for the gravitational couplings of neutrinos. Assuming that neutrinos of different generations have characteristic couplings to gravity with differing strength and that the gravitational eigenstates differ from the mass eigenstates, one can constrain the amount of violation of equivalence principle (VEP)in the neutrino sector from present neutrino oscillation results [4, 5]. Similar bounds were also obtained for the amount of violation of local Lorentz invariance (VLI), assuming that neutrinos of different generations have characteristic maximum attainable velocities [6, 7]. Recently we have pointed out that in both these cases it is possible to constrain some otherwise unconstrained region in the parameter space from neutrinoless double beta decay [8].

In this article we propose a general framework to study the effect of new gravitational interactions in the neutrinoless double beta decay. This formalism is similar to the one used in the study of $K-$system [9, 10]. We classify all possible interactions as scalar, vector and tensor interactions. Since both the VEP and VLI are tensor interactions, it is expected that in both cases similar constraints should be obtained, as observed. On the other hand, a recent string motivated violation of the equivalence principle a la Damour and Polyakov [11] is a scalar interaction. Thus the constraint in this case is of different nature than in the cases of VEP or VLI considered previously. The possibile fifth force [12] discussed in the literature is a vector interaction and thus also has a different phenomenology. Our analysis can be extended to study the effects of gravitational interactions in neutrino oscillation experiments.

We write down the most general lagrangian for interactions of neutrinos with scalar, vector and tensor fields in the weak basis $[\nu_i]$ following the general framework developed for the $K-$system [9],

$$\mathcal{L} = G_{ij}\nu_i\nu_j + G^{\mu}_{ij}\nu_{i,\mu}\nu_j + G^{\mu\nu}_{ij}\nu_{i,\mu}\nu_{j,\nu} \tag{1}$$

where i, j are generation indices and G_{ij}, G^{μ}_{ij} and $G^{\mu\nu}_{ij}$ are scalar, vector and tensor fields respectively. We shall not work beyond the external field approximation. These fields have some restrictions coming from the symmetry properties and by discarding the total divergence expressions from the lagrangian, which have been discussed in ref. [9] in detail. We further assumed that the gravitational eigenstates could be different from the mass eigenstate as well as the weak eigenstate. For simplicity from now on we shall work in an two generation scenario, $i, j = e, x$ with $x = \mu, \tau, s$.

We now can write down the Feynmann diagrams and hence the self energy matrix in the same way as in ref [9], from which the contribution to the effective hamiltonian can be read off in the weak basis, given by

$$\mathcal{H}_{ij} = G_{ij} + iG^{\mu}_{ij}p_\mu + G^{\mu\nu}_{ij}p_\mu p_\nu. \tag{2}$$

This hamiltonian is related to the effective hamiltonian in the mass and gravitational bases through unitary rotations

$$H = U_m H_m U_m^{-1} + U_G H_G U_G^{-1}. \tag{3}$$

In absence of any new gravitational interactions the neutrino mass matrix in the mass basis $[\nu_1 \;\; \nu_2]$ is given by

$$H_m = \frac{(M_m)^2}{2p} = \frac{1}{2p}\begin{pmatrix} m_1 & 0 \\ 0 & m_2 \end{pmatrix}^2 \tag{4}$$

and the gravitational interaction part of the hamiltonian is

$$H_G = pI + \frac{(M_G)^2}{2p} = pI + \frac{1}{2p}\begin{pmatrix} g_1^a & 0 \\ 0 & g_2^a \end{pmatrix}. \tag{5}$$

Here p denotes the momentum, I represents an unit matrix and $\bar{m}$ the average mass, and for any quantity X we define $\delta X \equiv (X_1 - X_2)$, $\bar{X} = (X_1 + X_2)/2$. $a = S, V, T$ represents scalar, vector and tensor interactions respectively.

The scalar, vector and tensor gravitational interactions can be written in the following forms so as to reproduce the correct dimensions of equation (2),

$$\begin{aligned} g_i^S &= 2{\alpha^S}_i {m_i}^2 \\ g_i^V &= 2{\alpha^V}_i m_i p \\ g_i^T &= 2{\alpha^T}_i p^2 \end{aligned}$$

In the absence of any gravitational interactions ${\alpha^a}_i = 0$, H_G simply becomes the momentum of the neutrinos. Here we are interested in a single virtual neutrino propagating inside the nucleus with a particular momentum. For this reason we assume the momenta of both the neutrinos are p in the absence of any new gravitational interactions. Hence $\alpha_1^a - \alpha_2^a = \delta\alpha^a$ is a measure of the new gravitational interactions in the neutrino sector. To compare our result with the neutrino oscillation experiments we further assume, $\alpha_1^a + \alpha_2^a = 0$, *i.e.*, there is no mean deviation from the gravitational laws and there is only a relative violation given by the measure $\delta\alpha^a$. This approximation will reduce the the number of parameters so that we can compare the bounds from the neutrino oscillation experiments with the ones from neutrinoless double beta decay.

We tried to keep the above discussions as general as possible with the restrictions that we donot go beyond the perturbative regime. We assume that the corrections to gravity comes from interactions with some external scalar, vector or tensor fields only and there is no non-renormalizable higher dimensional operators which modifies gravity with inverse mass scales. Our general parametrization has one drawback that although we are working in the gravitational basis, the masses involved in the expressions for g_i are considered in the mass basis. This can be justified by assuming VEP to be a small effect. In the case of tensor interaction masses donot enter, only in the scalar and vector cases this problem appears. However, as we shall see, the final result for the scalar case comes out to be the same as the one derived from other approaches [19]. Moreover, in the case of some scalar interactions the gravitational basis is equal to the mass basis [1], and then this question will not arise.

We shall not consider any CP violation, and hence H_m and H_G are real symmetric matrices and U_m and U_G are orthogonal matrices $U^{-1} = U^T$. They can be parametrized as $U_i = \begin{pmatrix} \cos\theta_i & \sin\theta_i \\ -\sin\theta_i & \cos\theta_i \end{pmatrix}$, where θ_i represents weak mixing angle θ_m or gravitational mixing angle θ_G. We can now write down the weak Hamiltonian H_w in the weak basis, in which the charged lepton mass matrix is diagonal and the charged current interaction is also diagonal, as

$$H = pI + \frac{1}{2p}\begin{pmatrix} M_+ & M_{12} \\ M_{12} & M_- \end{pmatrix}^2 = pI + \frac{1}{2p}\left(M_1 + \frac{1}{2}M_2^2 M_1^{-1}\right)^2. \tag{6}$$

where,

$$\begin{aligned} M_1 &= U_m M_m U_m^{-1} \\ M_2 &= U_G M_G U_G^{-1} \end{aligned}$$

and we assumed $M_1^2 \gg M_2^2$, so that the gravitational effects are much smaller than the usual neutrino masses. Since no new gravitational effects have been observed so far, we use this formalism to constrain the parameters of the new gravitational interactions, for which this assumption is justified. We then obtain,

$$\begin{aligned} M_\pm &= \bar{m} \pm \frac{cos2\theta_m}{2}\delta m \\ &\pm \left[\mp\bar{g^a}\bar{m} - \delta g^a \bar{m}\frac{\cos 2\theta_G}{2} \pm \frac{\delta m \delta g^a}{4}\cos 2(\theta_G - \theta_m) + \delta m \bar{g^a}\frac{\cos 2\theta_m}{2}\right] / [2(\delta m^2 - \bar{m}^2)] \\ M_{12} &= -\frac{\sin 2\theta_m}{2}\delta m \\ &+ \left[\delta g^a \bar{m}\frac{\sin 2\theta_G}{2} - \delta m \bar{g^a}\frac{\sin 2\theta_m}{2}\right] / 2(\delta m^2 - \bar{m}^2). \end{aligned} \tag{7}$$

[1] this point will be discussed in a forthcoming article, where the dilaton-exchange gravity will be studied by the authors

to a leading order in δg^a.

The decay rate for the neutrinoless double beta decay is given by,

$$[T_{1/2}^{0\nu\beta\beta}]^{-1} = \frac{M_+^2}{m_e^2} G_{01} |ME|^2, \tag{8}$$

where ME denotes the nuclear matrix element, G_{01} corresponds to the phase space factor defined in [13] and m_e is the electron mass. The momentum dependence of M_+ must be absorbed into the nuclear matrix element, so that this quantity contains all the momentum dependence and the remaining part is estimated using zero momentum transfer approximation. Thus, if one ignores the nuclear matrix element, then obviously there cannot be any effect of the vector and tensor type gravitational interactions in neutrinoless double beta decay, which was mistaken in ref. [14]. As it has been discussed earlier [8], the momentum dependence of the tensor type gravitational interactions enters the nuclear matrix element, which then is enhanced by a factor p^2 coming through M_+ in the above expression.

We shall now present a more detail explanation of this analysis.

In ref [14] it is claimed that neither violations of Lorentz invariance nor violations of the equivalence principle may give sizable contributions to neutrinoless double beta decay. The argument discussed is the following: Taking the neutrino propagator

$$\int d^4 q \frac{e^{-iq(x-y)} \langle m \rangle c_a^2}{m^2 c_a^4 - q_0^2 c_a^2 + \vec{q}^2 c_a^2} \tag{9}$$

with the standard $0\nu\beta\beta$ observable $\langle m \rangle$, the neutrino four momentum q and the characteristic maximal velocity c_a. If one would neglect now q_0 and m in the denominator, c_a drops out and the decay rate is independent of c_a.

However, in [8] it has been shown starting from the Hamiltonian level that the propagator (or the $0\nu\beta\beta$ observable) is changed itself violating Lorentz invariance. Since

$$\begin{aligned} H &= \vec{q} c_a + \frac{m^2 c_a^4}{2 \vec{q} c_a} \\ &= \vec{q} I + \frac{m^{(*)2} c_a^4}{2 \vec{q} c_a} \end{aligned} \tag{10}$$

with $c_a = I + \delta v$ and $m^{(*)2} = m^2 + 2\vec{q}^2 c_a \delta v$ an additional contribution is obtained $\propto \vec{q}^2 \delta v$. This mass-like term has a $\vec{q^2}$ enhancement and is not proportional to the small neutrino mass. This consideration answers also the frequently asked question “What is the source of lepton number violation?” in this mechanism. Comparable to a usual mass term, which can be both of Majoran type as well as Dirac type the mass-like term $2\bar{q}^2 c_a \delta v$ can be of Majorana type and act as the source of lepton number violation in this context.

We shall now discuss the three different cases of scalar, vector and tensor interactions and their phenomenology. In the case of tensor interaction the constraint has already been discussed in ref. [8]. The violation of local Lorentz invariance and the violation of the equivalence principle both fall under this category (their equivalence has been pointed out elsewhere [2]). In both these cases the effect of the new gravitational interactions have quadratic momentum dependence.

In case of the tensorial gravitational interaction we have, $\bar{g^T} = 2\bar{\alpha^T} p^2 = 0$ and $\delta g^T = 2\delta\alpha^T p^2$. In particular, for the violation of the equivalence principle we substitute $\delta\alpha^T = 4\delta g\phi$ (following the notation of ref [8]), where ϕ is the Newtonian gravitational potential on the surface of earth. On the other hand, for the violation of the local Lorentz invariance, we substitute instead, $\delta\alpha^T = 2\delta v$. In both these cases bounds were given in ref [8].

To give a bound on tensorial gravitational interactions in the small mixing region (including $\theta_v \sim \theta_m \sim 0$) conservatively $\langle m \rangle \simeq 0$ was assumed. It was also assumed that $\delta m \leq \bar{m}$, and thus $\frac{\delta m}{4\bar{m}}$ may be neglected. Due to the p^2 enhancement the nuclear matrix elements of the mass mechanism have to be replaced by $\frac{m_p}{R} \cdot (M_F^{'} - M_{GT}^{'})$ with the nuclear radius R and the proton mass m_p, which have been calculated in [15]. Inserting the recent half life limit obtained from the Heidelberg–Moscow experiment [16], a bound on the amount of tensorial gravitational interactions as a function of the average neutrino mass $\bar{m}$ was given [8]. It should be stressed also that the GENIUS proposal of the Heidelberg group [17] could improve these bounds by about 1–2 orders of magnitude.

For the vector type gravitational interactions there is a linear momentum dependence. In this case, $\bar{g^V} = 2\bar{\alpha^V} pm = 0$ and $\delta g^V = 2\delta\alpha^V pm$. The fifth force, as discussed by Fishbach et al [12] in the context of $K-$physics is a vector type gravitational interaction. Since no studies of this type of forces exist for neutrino oscillation experiments, with which neutrinoless double beta decay results could be compared, we shall not study this case.

A similar generic structure was considered in a recent analysis of the atmospheric neutrino anomaly [18], where they used the power of momentum dependence as a parameter. From their analysis it becomes apparent that the atmospheric neutrino anomaly may not be explained by either tensorial or vectorial gravitational analysis alone [18].

Recently it has been argued by Damour and Polyakov [11] that string theory may lead to a new scalar type gravitational interaction through interaction of the dilaton field and subsequently its consequence to neutrino oscillation has been studied [19]. Damour and Polyakov have shown that the massless dilaton interaction modifies the gravitational potential energy and there is an additional contribution from an spin-0 exchange, which results in a scalar type gravitational interaction [11]. The resulting theory is of scalar-tensor type with the two particle static gravitational energy

$$V(r) = -G_N m_A m_B (1 + \alpha_A \alpha_B)/r, \tag{11}$$

where G_N is Newton's gravitational constant and α_j denotes the couplings of the dilaton field ϕ to the matter field ψ_j, leading to a gravitational energy of

$$L = m_j \alpha_j \overline{\psi_j} \psi_j \phi. \tag{12}$$

Thus the modified effective mass matrix of the neutrinos are now given by [19]

$$m^{(*)} = m - m\alpha\phi_c \tag{13}$$

where, the classical value of the dilaton field $\phi_c = \phi_N \alpha_{ext}$ is characterised by the α value of the bulk matter producing it and for a static matter distribution proportional to the Newtonian potential ϕ_N.

The effective mass squared difference

$$\Delta m^{(*)2} = -2m^2 \phi_N \alpha_{ext} \delta\alpha \tag{14}$$

(for almost degenerate masses m) gives rise to neutrino oscillation. The corresponding effect for $0\nu\beta\beta$ decay is obtained by replacing $\delta g^S = 2\delta\alpha^S m^2$ (for almost degenerate mass $m_1 \sim m_2 \sim m$). Comparing the arguments in the oscillations propabilities we get

$$M_+ = m + m\alpha_{ext}\Phi_N \delta\alpha \frac{\cos(2\theta_G)}{2}. \tag{15}$$

In this case, it is difficult to obtain any bound from neutrino experiments since for α_{ext} only upper bounds exist. However, to get an idea of the constraints which can come from neutrino experiments in the future if α_{ext} is known, according to ref. [19] we assume $\phi_N = 3 \cdot 10^{-5}$, $\alpha_{ext} = \sqrt{10^{-3}}$ and $m = 2.5$ eV (as an upper bound obtained from tritium beta decay experiments [20]). In this case the quantity $\delta\alpha$ is not constrained from neutrinoless double beta decay.

In summary, we presented a general formalism for the study of effects of new gravitational interactions in neutrinoless double beta decay, which allows to constrain the amount of violation of the gravitational laws. Various scenarios discussed in the literature have been analyzed as special cases of the present formalism.

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