Scale of $SU(2)_{\rm R}$ symmetry breaking and leptogenesis

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Models of leptogenesis often invoke the out-of-equilibrium decays of heavy right-handed neutrinos in order to create a baryon asymmetry of the universe through the electroweak phase transition. Their presumed existence argues strongly for the presence of an $SU(2)_{\rm R}$ gauge symmetry. We study the equilibrating effects of the resulting additional right-handed interactions and find that successful leptogenesis requires $m_N \gtrsim 10^{16}$ GeV if $m_N > m_{W_{\rm R}}$, and $m_{W_{\rm R}} \gtrsim 2\times10^5$ GeV $(m_N/10^2\ {\rm GeV})^{3/4}$ if $m_N < m_{W_{\rm R}}$, where m_N is the mass of the lightest right-handed neutrino. A better bound $m_{W_{\rm R}} \gtrsim 3\times10^6$ GeV $(m_N/10^2\ {\rm GeV})^{2/3}$ is obtained if leptogenesis occurs at $T > m_{W_{\rm R}}$. We show also that the $m_N > m_{W_{\rm R}}$ option is excluded in a supersymmetric theory with gravitinos.

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It is now accepted that neutrinos do have small masses, thus accounting for the atmospheric neutrino anomaly [1] and the solar neutrino puzzle [2], and perhaps also providing a fraction of the dark matter of the universe [3]. A natural solution to the smallness of neutrino masses is to consider them as Majorana particles. The nonconservation of lepton number at some large scale M would induce an effective dimension-5 operator [4] $h\ell_{\rm L}\ell_{\rm L}\phi\phi/M$, where ϕ is the usual Higgs doublet. As ϕ acquires a nonzero vacuum expectation value (*vev*), $\langle\phi\rangle = v$, to break the electroweak gauge symmetry, neutrinos obtain small masses as well: $m_\nu = hv^2/M$.

An additional appeal of this solution is that it offers an elegant mechanism for generating the baryon asymmetry of the universe. As is well known, interactions which violate $B+L$ while conserving $B-L$ are unsuppressed by sphaleron processes at high temperatures [5]. Thus any primordial L asymmetry would be (partly) converted into a B asymmetry — a process termed leptogenesis. The required lepton asymmetry can be generated in two possible ways through the lepton number violation responsible for neutrino masses [6,7]. One can introduce right-handed singlet neutrinos which acquire large Majorana masses, resulting in small Majorana masses for the left-handed neutrinos, through the so-called seesaw mechanism [8]. Decays of the right-handed neutrinos do not conserve lepton number, thereby generating a primordial lepton asymmetry [6]. Alternatively, one can extend the standard model to include very heavy Higgs triplet scalars whose couplings break lepton number explicitly. They would naturally acquire tiny seesaw *vev*s, thereby inducing small masses for the left-handed neutrinos and their decays would also generate the primordial lepton asymmetry [7].

In both routes, the scale of lepton number violation M is arbitrary. It is thus natural to consider a left-right symmetric model where both possibilities are realized and M is related to the left-right symmetry breaking scale [9]. This is a natural framework for explaining parity nonconservation at low energies; it may also be embedded in interesting grand unified theories. Within this broader context, an important change in the conditions of leptogenesis occurs because the right-handed neutrinos must now interact with the $SU(2)_{\rm R}$ gauge bosons.

In this Letter, we examine the effect of the interactions of the right-handed gauge bosons $W_{\rm R}$ on the generation of the primordial lepton asymmetry of the universe. We conclude that for this to be phenomenologically successful, $W_{\rm R}$ should be heavier than the lightest right-handed neutrino: $m_{W_{\rm R}} \gtrsim 2\times10^5$ GeV $(m_N/10^2\ {\rm GeV})^{3/4}$, unless $m_N \gtrsim 10^{16}$ GeV in which case $m_{W_{\rm R}}/m_N \gtrsim 0.1$. A better bound $m_{W_{\rm R}} \gtrsim 3\times10^6$ GeV $(m_N/10^2\ {\rm GeV})^{2/3}$ is obtained if leptogenesis occurs at $T > m_{W_{\rm R}}$.

In left-right symmetric models the quarks and leptons transform under the group $SU(3)_{\rm c}\otimes SU(2)_{\rm L}\otimes SU(2)_{\rm R}\otimes U(1)_{B-L}$ as

$$\begin{aligned} q_{i\rm L} &\equiv \begin{pmatrix} u_{i\rm L} \\ d_{i\rm L} \end{pmatrix} \sim (3,2,1,1/3), \\ q_{\alpha\rm R} &\equiv \begin{pmatrix} u_{\alpha\rm R} \\ d_{\alpha\rm R} \end{pmatrix} \sim (3,1,2,1/3), \\ \ell_{i\rm L} &\equiv \begin{pmatrix} \nu_{i\rm L} \\ e_{i\rm L} \end{pmatrix} \sim (1,2,1,-1), \\ \ell_{\alpha\rm R} &\equiv \begin{pmatrix} N_{\alpha\rm R} \\ e_{\alpha\rm R} \end{pmatrix} \sim (1,1,2,-1). \end{aligned} \tag{1}$$

The Higgs bidoublet $\phi \equiv (1,2,2,0)$ breaks electroweak symmetry, whereas the left-right symmetry is broken by the $SU(2)_{\rm R}$ Higgs triplet $\Delta_{\rm R} \equiv (1,1,3,-2)$. Left-right parity also requires the existence of an $SU(2)_{\rm L}$ Higgs triplet $\Delta_{\rm L} \equiv (1,3,1,-2)$.

To begin with, we shall ignore the effects of the triplets for the generation of the lepton asymmetry and consider the interactions of the leptons alone:

$$\mathcal{L} = f_{i\alpha}\overline{\ell_{i\rm L}}\ell_{\alpha\rm R}\phi + f_{{\rm L}ij}\overline{\ell_{i\rm L}{}^c}\ell_{j\rm L}\Delta_{\rm L} + f_{{\rm R}\alpha\beta}\overline{\ell_{\alpha\rm R}{}^c}\ell_{\beta\rm R}\Delta_{\rm R}$$

$$+\frac{1}{2}g_{\rm L}\overline{\ell_{i{\rm L}}}\gamma_\mu\tau^a_{ij}\ell_{j{\rm L}}W^{a\mu}_{\rm L}+\frac{1}{2}g_{\rm R}\overline{\ell_{\alpha{\rm R}}}\gamma_\mu\tau^a_{\alpha\beta}\ell_{\beta{\rm R}}W^{a\mu}_{\rm R}. \quad (2)$$

Note that $B-L$ is now a *local* symmetry, hence it cannot be directly violated at high energies. Its violation occurs only when the left-right symmetry is broken by the *vev* $v_{\rm R}$ of the right-handed Higgs triplet $\Delta_{\rm R}$. This gives Majorana masses to the right-handed neutrinos, and lepton number is violated in their decays:

$$N_{\alpha{\rm R}}\to\ell_{i{\rm L}}+\phi^\dagger \ \text{ and } \ {\ell_{i{\rm L}}}^c+\phi. \quad (3)$$

If the couplings $f_{i\alpha}$ are complex and if these decays satisfy the out-of-equilibrium condition, then they can generate a primordial $B-L$ asymmetry. There are two contributions to the magnitude of this asymmetry, the first coming from the interference of the tree-level diagrams with the vertex diagrams [6,10,11] and the second from that of the former with the self-energy diagrams [12]. Although one starts with real and diagonal masses for the heavy neutrinos, the loop diagrams introduce complex phases and hence an amount of CP violation, denoted as η in the following.

We assume the neutrino masses to be hierarchical ($M_{N_{3\rm R}}\gg M_{N_{2\rm R}}\gg M_{N_{1\rm R}}=m_N$). It is the decay of the *lightest* right-handed neutrino which will determine the final lepton asymmetry, hence we shall only be concerned with the scale m_N. The lepton asymmetry $n_L\equiv n_l-n_{l^c}$ will evolve according to the transport equation [13]:

$$\frac{{\rm d}n_L}{{\rm d}t}+3Hn_L=\eta\Gamma_N[n_N-n_N^{\rm eq}]-\frac{1}{2}\left(\frac{n_L}{n_\gamma}\right)n_N^{\rm eq}\Gamma_N \quad (4)$$
$$-2n_\gamma n_L\langle\sigma|v|\rangle,$$

where Γ_N is the thermally-averaged decay rate of the right-handed neutrinos $N_{\alpha\rm R}$, and n_N their number density, with the equilibrium value

$$n_N^{\rm eq}=\begin{cases}(45/2\pi^4)sg_*^{-1}, & m_N\ll T,\\ (45/4\pi^4)sg_*^{-1}\sqrt{\pi/2}(m_N/T)^{3/2}{\rm e}^{-m_N/T}, & m_N\gg T.\end{cases} \quad (5)$$

In the above, s is the entropy density, n_γ is the photon density, and g_* the effective number of interacting relativistic degrees of freedom. The second term on the left-hand side (lhs) of Eq. (5) accounts for the expansion of the universe (where $H\simeq1.7g_*^{1/2}T^2/M_{\rm P}$ is the Hubble rate) and the term $\langle\sigma|v|\rangle$ is the thermally-averaged lepton-number violating scattering cross section.

Similarly, the density of the heavy particles satisfies the equation

$$\frac{{\rm d}n_N}{{\rm d}t}+3Hn_N=-\Gamma_N(n_N-n_N^{\rm eq})-({n_N}^2-{n_N^{\rm eq}}^2)\langle\sigma_N|v|\rangle. \quad (6)$$

The first term on the right-hand side (rhs) accounts for the decays (and inverse decays) of the heavy right-handed neutrinos. The second term on the rhs, although usually neglected in discussions of out-of-equilibrium decays, is in fact crucial in the present context. This is the lepton-number *conserving* thermally-averaged scattering cross section of the right-handed neutrinos $N_{\alpha\rm R}$. It is instrumental in initially equilibrating the number density of the right-handed neutrinos but, as we shall see, it severely depletes the amount of lepton asymmetry generated by their decays.

For convenience we define the parameters $K\equiv\Gamma_N/H$ and $K_N\equiv2n_\gamma\langle\sigma_N|v|\rangle/H$ at $T=m_N$. For $K\ll1$ at $T\sim m_N$, the system is far from equilibrium, hence the last two terms in Eq. (5) (the ones responsible for the depletion of n_L) are negligible. In this limit, if $K_N\ll1$, the asymptotic solution is $n_L^{\rm asym}/s\simeq\eta/g_*$. However, for $K_N\gg1$ there is a strong suppression of the abundance. For $K_N\sim1$ there is already a suppression $n_L\sim0.04n_L^{\rm asym}$, while in the range $1<K_N<10^3$ the suppression may be approximated as $n_L\sim0.04n_L^{\rm asym}/K_N$. Beyond this range the suppression is somewhat faster than linear in K_N.

We shall now assume conservatively that for an adequate lepton asymmetry to be generated, in addition to the usual out-of-equilibrium condition [14], viz. $K\lesssim1$, we also require $K_N\lesssim1$. The difference is that if the former condition is not fulfilled, any primordial asymmetry will also be erased. This does not happen if the latter condition is not satisfied, since K_N measures the approach to *kinetic* rather than chemical equilibrium. Nevertheless, a large value of K_N suppresses the generation of a lepton asymmetry and in the following we show that $W_{\rm R}$ mediated scattering processes are important in this context.

We first consider the case $m_N>m_{W_{\rm R}}$. At the time when $N_{1\rm R}$ decays, it is still interacting with $W_{\rm R}$. If these interactions are sufficiently fast, equilibrium of $N_{1\rm R}$ with the decay products will be maintained, thus *preventing* the generation of any lepton asymmetry. The requirement for the $SU(2)_{\rm R}$ interactions

$$e^-_{\rm R}+W^+_{\rm R}\to N_{\rm R}\to e^+_{\rm R}+W^-_{\rm R} \quad (7)$$

to fall out of equilibrium is

$$\frac{g_{\rm R}^2}{8\pi}T\lesssim1.7g_*^{1/2}\frac{T^2}{M_{\rm P}} \qquad \text{at } \ T=m_N, \quad (8)$$

so that for generating a lepton asymmetry we require

$$m_N\gtrsim10^{16}\ {\rm GeV}, \quad (9)$$

where we take $g_{\rm R}^2=g_{\rm L}^2=0.4$, $g_*\sim10^2$, and $M_P\sim10^{19}$ GeV. Note that this stringent bound comes from the fact that the gauge coupling $g_{\rm R}$ is of order unity. In the usual leptogenesis scenario without $SU(2)_{\rm R}$ interactions, the corresponding coupling is a Yukawa coupling which may be very much suppressed. Since $m_{W_{\rm R}}/m_N\sim g_{\rm R}/f$, where $f \lesssim \sqrt{4\pi}$ is a reasonable assumption, we conclude that $m_{W_{\rm R}}/m_N \gtrsim 0.1$ in this case.

The above lower bound is in conflict, however, with an independent upper bound on m_N if the theory is supersymmetric and includes a gravitino, as we discuss below. It is of course necessary that leptogenesis occurs *after* inflation. In supersymmetric theories the temperature at the begining of the radiation-dominated era following inflation is restricted from considerations of the thermal production of massive gravitinos [15]. Since they interact only gravitationally and thus decay after nucleosynthesis, the abundances of the synthesized elements can be drastically altered, in conflict with observations [16,17]. This imposes a bound on the 'reheating' temperature at the begining of the radiation-dominated era which is usually quoted to be of $\mathcal{O}(10^9)$ GeV. However, a recent reevaluation of the gravitino production rate [18], in conjunction with the nucleosynthesis constraints [17] strengthens this to $T_{\rm reheat} \lesssim 10^7$ GeV for weak-scale gravitinos. Even taking into account that particles of mass as high as $\sim 10^3 T_{\rm reheat}$ may be produced with sufficient abundance for successful leptogenesis [19], this argument severely restricts the maximum possible mass of the right-handed neutrino:

$$m_N \lesssim 10^{10}\ \text{GeV}\ . \qquad (10)$$

Combined with the lower bound of Eq. (9) on m_N, this rules out the possibility of leptogenesis if $m_N > m_{W_{\rm R}}$. It would seem that this argument can be evaded if the gravitino is in fact the lightest supersymmetric particle, and is thus stable. Even so there would be a constraint from the requirement that they do not 'overclose' the universe which relaxes the upper bound on the reheat temperature to $T_{\rm reheat} \lesssim 10^{11}$ GeV [16,18]. This requires $m_N \lesssim 10^{14}$ GeV for leptogenesis, so there is still a conflict with the lower bound of Eq. (9).

We now discuss the effect of $W_{\rm R}$ interactions when $m_N < m_{W_{\rm R}}$. In this case, we must consider both $T = m_N$ and $T = m_{W_{\rm R}}$. The scattering processes

$$e^{\pm}_{\rm R} + N_{\rm R} \to W^{\pm}_{\rm R} \to e^{\pm}_{\rm R} + N_R \qquad (11)$$

are important for bringing the right-handed neutrinos into equilibrium. The condition that this reaction departs from equilibrium when the right-handed neutrinos decay is

$$\frac{g_{\rm R}^4}{16\pi}\frac{T^5}{m_{W_{\rm R}}^4} \lesssim 1.7 g_*^{1/2}\frac{T^2}{M_P} \qquad \text{at}\ \ T = m_N, \qquad (12)$$

which translates into

$$m_{W_{\rm R}} \gtrsim 2\times 10^5\ \text{GeV}\left(\frac{m_N}{10^2\ \text{GeV}}\right)^{\frac{3}{4}}. \qquad (13)$$

Another important process is the scattering of $W_{\rm R}$'s into $e_{\rm R}$'s through $N_{\rm R}$ exchange:

$$W^{\pm}_{\rm R} + W^{\pm}_{\rm R} \to e^{\pm}_{\rm R} + e^{\pm}_{\rm R}. \qquad (14)$$

This is the analog of the standard-model process $W^{\pm}_{\rm L} + W^{\pm}_{\rm L} \to e^{\pm}_{\rm L} + e^{\pm}_{\rm L}$ through $\nu_{\rm L}$ exchange [20]. The condition that this reaction departs from equilibrium is

$$\frac{3g_{\rm R}^4}{32\pi}\frac{m_N^2 T^3}{m_{W_{\rm R}}^4} \lesssim 1.7 g_*^{1/2}\frac{T^2}{M_{\rm P}} \qquad \text{at}\ T = m_{W_{\rm R}}, \qquad (15)$$

which translates into

$$m_{W_{\rm R}} \gtrsim 3\times 10^6\ \text{GeV}\left(\frac{m_N}{10^2\ \text{GeV}}\right)^{\frac{2}{3}}. \qquad (16)$$

We note that Eq. (15) applies [21] if leptogenesis occurs at $T \simeq m_N$ and Eq. (18) applies if it occurs at $T > m_{W_{\rm R}}$. Recognizing that Eq. (18) is a better bound than Eq. (15) and noting that m_N should exceed the electroweak breaking scale of 10^2 GeV, we have an absolute lower bound of 3×10^6 GeV. This may be further improved if m_N is of $\mathcal{O}(10^3)$ GeV [22], in which case $m_{W_{\rm R}} \gtrsim 10^7$ GeV. A similar bound was mentioned previously [11] in the context of having $m_N \sim 10^7$ GeV. Note also that the analog of Eq. (16) for the neutral $SU(2)_{\rm R}$ gauge boson, *i.e.* $Z_{\rm R} + Z_{\rm R} \to N + N$, is less important because m_N is heavy.

Consequently, if a right-handed gauge boson is observed with a mass below $\sim 10^7$ GeV, it will necessarily imply that right-handed neutrinos *cannot* have generated the lepton asymmetry of the universe. Moreover, since the interactions of $W_{\rm R}$ would have erased all primordial $(B-L)$ asymmetry, the observed baryon asymmetry must have been generated at a scale lower than the $SU(2)_{\rm R}$ symmetry breaking scale $M_{\rm R}$.

Finally we discuss the contributions of the Higgs triplet scalars to leptogenesis. As mentioned earlier, left-right parity breaks along with the $SU(2)_{\rm R}$ symmetry when the field $\Delta_{\rm R}$ acquires a *vev*. The masses M_Δ of $\Delta_{\rm L}$ and $\Delta_{\rm R}$ are initially equal, but their *vev*s are rather different, being related by $v_{\rm L} \sim v^2/v_{\rm R}$. Beyond the terms given in Eq. (2), we also have $\Delta_{\rm L}\Delta_{\rm R}\phi\phi$. Since $v_{\rm L}$ is very small, $\Delta_{\rm R}$ will dominantly decay into 2 leptons but rarely into 2 scalars, so it cannot create a lepton asymmetry. However, since $v_{\rm R}$ is of $\mathcal{O}(M_{\rm R})$, the decays of $\Delta_{\rm L}$ can contribute significantly to the lepton asymmetry of the universe [7,23]. Moreover, since $\Delta_{\rm L}$ does not interact with $W_{\rm R}$, the existence of right-handed gauge bosons does not change this conclusion. The lepton-number conserving interaction, $\Delta^{\dagger}_{\rm L} + \Delta_{\rm L} \to W^{\dagger}_{\rm L} + W_{\rm L}$, is the most efficient one at bringing the number density of $\Delta_{\rm L}$ into equilibrium. So for the lepton asymmetry to survive, this interaction should be out of equilibrium, implying a lower bound on M_Δ of $\sim 10^{13}$ GeV [7].

As an example, in a realistic supersymmetric SO(10) grand unified theory, where two 10-plet and one 126-plet contribute to the fermion masses [24] and the Majorana Yukawa couplings of the right-handed neutrinos can be calculated, the left-right symmetry breaking scale is very high and the Yukawa couplings are also quite large. In this case, $m_N > m_{W_R}$ and the bound of Eq. (9) applies. As discussed earlier, this means that it will not be possible to create enough right-handed neutrinos after reheating without also creating an unacceptable abundance of gravitinos. However it may well be possible to have a scalar potential which allows $M_\Delta \sim 10^{13}$ GeV, so that decays of the Higgs triplets can generate a lepton asymmetry leading to successful baryogenesis through the electroweak phase transition.

To conclude, we have shown that W_R interactions will in general bring the number density of the right-handed neutrinos into equilibrium. Consequently, it is not possible to generate the baryon asymmetry of the universe through leptogenesis in supersymmetric models where $m_N > m_{W_R}$, or when $m_{W_R} \lesssim 10^7$ GeV.

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