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CP violation in the mass matrix of heavy neutrinos

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Abstract

We discuss the question of $CP-$violation in the effective Hamiltonian approach in models of leptogenesis through heavy right handed neutrino decays. We first formulate the problem in four component notation and then point out that before the heavy neutrinos have decayed away, the universe becomes $CP-$asymmetric in the heavy neutrinos. However, the lepton asymmetry generated after they completely decay are independent of this asymmetry.

The baryon asymmetry of the universe could be created from a lepton asymmetry of the universe before the electroweak phase transition [1, 2, 3]. In these scenarios the lepton number violation at some very high energy generates a lepton asymmetry of the universe, which then get converted to a baryon asymmetry when the sphaleron mediated processes are in equilibrium. A popular scenario is the one in which the right handed neutrinos ($N_{Ri}, i = e, \mu, \tau$) decay into light left-handed leptons and anti-leptons [1].

$CP-$asymmetries are calculated in the heavy neutrinos decay from the interference of tree level and one-loop vertex and self energy corrections. The self-energy [4] was initially considered as one additional contribution to CP asymmetry, but later it was pointed out that this may be interpreted as an oscillation type of indirect CP violation [5]. Since then several works contributed towards the understanding of this effect [6, 7, 8, 9] and a resonant phenomenon was observed [7, 9] for nearly degenerate right handed neutrinos.

The self-energy contribution to the $CP-$violation has been interpreted as $CP-$violation in the mass matrix. This gives a difference between the amplitudes of $N_{Ri} \rightarrow N^c_{Rj}$ and $N^c_{Ri} \rightarrow N_{Rj}$. In analogy to the $K-$system, we call this $CP-$violation as the indirect $CP-$violation and the vertex contribution to the $CP-$violation in the decays of the right handed neutrinos as the direct $CP-$violation. In earlier treatments of this effect, the neutrinos and antineutrinos were treated as different two component objects [5, 7], although finally the physical states were identified with four component physical Majorana states. We formulate this problem in the conventional method of treating a Majorana particle as a four component object, which will clarify several features of the problem. We then discuss the question of $CP-$violation for Majorana particles and point out that a $CP-$asymmetric universe in the heavy neutrinos is created first, which is different from the $CP-$asymmetry created after these heavy neutrinos decay. We point out the essential role played by the non-hermiticity of the Hamiltonian in this problem, which was not appreciated before.

To simplify the problem we work in a two generation framework and include two right handed neutrinos ($N_i,\ i = 1, 2$) and write down their Majorana mass term and their interactions with the light fields,

$$\begin{aligned}\mathcal{L}_{int} &= \sum_i M_i[\overline{(N_{Ri})^c} N_{Ri} + \overline{N_{Ri}}(N_{Ri})^c] + \sum_{\alpha,i} h^*_{\alpha i}\,\overline{N_{Ri}}\,\phi^\dagger\,\ell_{L\alpha} + \sum_{\alpha,i} h_{\alpha i}\,\overline{\ell_{L\alpha}}\phi\,N_{Ri} \\ &+ \sum_{\alpha,i} h^*_{\alpha i}\,\overline{(\ell_{L\alpha})}^c\,\phi^\dagger\,(N_{Ri})^c + \sum_{\alpha,i} h_{\alpha i}\overline{(N_{Ri})^c}\,\phi\,(\ell_{L\alpha})^c \qquad (1)\end{aligned}$$

where $\phi \equiv (1, 2, 1/2)$ is the usual higgs doublet of the standard model and $\ell_{\alpha L}, \alpha = e, \mu, \tau$ are the light left-handed leptons; $h_{\alpha i}$ are the Yukawa couplings and in general complex. We work in a basis in which the Majorana mass matrix is real and diagonal with eigenvalues M_i.

Once the phases of N_i are fixed by the choice of the basis, we cannot absorb any phases contained in the couplings $h_{\alpha i}$. A phase transformation of the light leptons can apparently absorb some of these phasses, but they will then appear in the charged current interactions. So, if there is $CP-$violation in the leptonic sector, then phase transformations of the light leptons cannot absorb any of the remaining phases. In addition, measurable quantities should not depend on the choice of phase of the light leptons. So to understand the question of $CP-$violation, it is convenient to consider rephasing invariant quantities [10, 11], which are the combinations of the couplings which remains invariant under any change of phases of the light leptons.

In the quark sector there is only one $CP-$phase in the CKM-matrix V_{aj} and one can construct only one rephasing invariant quantity, which is the Jarlskog invariant $J = V_{ai} V_{bj} V^*_{aj} V^*_{bi}$. In the leptonic sector [10] the Majorana nature of the neutrinos allow new types of $CP-$invariant quantities, which may not have any analogy in the quark sector. Consider the phase transformation of the light leptons,

$$l_\alpha \rightarrow e^{i\delta_\alpha} l_\alpha$$

which will imply a phase tranformation to the Yukawa couplings,

$$h_{\alpha i} \rightarrow e^{i\delta_\alpha} h_{\alpha i}.$$

Under this phase transformation the combinations of the Yukawa couplings which remains invariant and could be complex are,

$$t_{\alpha ij} = \mathrm{Im}\ (h_{\alpha i} h^*_{\alpha j}) \tag{2}$$

and

$$T_{\alpha\beta ij} = \mathrm{Im}\ (h_{\alpha i} h_{\beta j} h^*_{\alpha j} h^*_{\beta i}). \tag{3}$$

Thus if any of the above combination of the Yukawa couplings enter in some process, there can be $CP-$violation.

The quantity $T_{\alpha\beta ij}$ is similar to the Jarlskog invariant in the quark sector. However, there is no analog of the other rephasing invariant quantity $t_{\alpha ij}$ in the quark sector since there are no Majorana particles. In the decays of the right handed neutrinos, the direct and the indirect

$CP-$asymmetry depends on the same rephasing invariant quantity Im $[t_{\alpha ij}t_{\beta ij}]$ [11]. In our present formalism we shall demonstrate that first we have a $CP-$asymmetric universe in N_{Ri} and the amount of N_{Ri} asymmetry depends on the rephasing invariant quantity Im $[t_{\alpha ij}]$. As a result it is possible to imagine a situation with Re $[t_{\alpha ij}] = 0$, but with Im $[t_{\alpha ij}] \neq 0$, when the universe could be $CP-$asymmetric in N_{Ri} at higher temperature, but after the heavy neutrinos decay the universe becomes $CP-$symmetric.

We shall now discuss another conceptually important question of how the N_{Ri} asymmetry is generated before they decayed away. For Majorana particles we donot have to consider the N_{Ri} and N^c_{Ri} as independent, since they are related by Majorana condition. The mass matrices are also same for both. However, when there is $CP-$violation, the mass matrices are no longer the same, but still they remain related by simple phase rotation.

The situation changes when these particles have a decay width. This point has been discussed in details in the field theoretic language extensively [8], but here we shall discuss this point in an effective Hamiltonian language. Consider the tree level mass matrix in the $[N_1 \quad N_2]$ basis,

$$\begin{pmatrix} M_1 & 0 \\ 0 & M_2 \end{pmatrix} \tag{4}$$

Since CP is conserved at this level, the mass matrix is same as above in the $[N_1^c \quad N_2^c]$ basis, so that hermitian conjugate states may be obtained by operating with CP on the physical states.

We can then write down the total effective Hamiltonian including the one loop self energy corrections, in the bases $[N_1 N_2]$ and $[N_1^c N_2^c]$ respectively as,

$$\mathcal{H}^+ = \begin{pmatrix} M_1 & h^+ \\ h^+ & M_2 \end{pmatrix} \qquad \text{and} \qquad \mathcal{H}^- = \begin{pmatrix} M_1 & h^- \\ h^- & M_2 \end{pmatrix} \tag{5}$$

where we have absorbed the dispersive part in the wave function and mass renormalisation; redefined the diagonal elements as $M_a \rightarrow \; M_a + h_a$; with,

$$h^+ = -\frac{i}{32\pi}\left[M_i \sum_\alpha h^*_{\alpha i} h_{\alpha j} + M_j \sum_\alpha h_{\alpha i} h^*_{\alpha j}\right] \tag{6}$$

$$h^- = -\frac{i}{32\pi}\left[M_i \sum_\alpha h_{\alpha i} h^*_{\alpha j} + M_j \sum_\alpha h^*_{\alpha i} h_{\alpha j}\right] \tag{7}$$

and

$$h_a = -\frac{i}{32\pi}\left[2M_i \sum_\alpha h_{\alpha i} h^*_{\alpha i}\right] \tag{8}$$

neglecting terms of order $O\left(m_\alpha^2/p^2\right)$, $O\left(m_\phi^2/p^2\right)$ with $p^2 \geq M_i^2$.

Because of the absorbtive part the mass matrices of the N_{Ri} and N_{Ri}^c are no longer related by just phase rotation. This needs some explanation. Because of the decay width of the heavy neutrinos N_{Ri}, their mass matrix takes the form $M - \frac{i}{2}\Gamma$, where the decay width Γ comes from the the absorbtive part and hence it is anti-hermitian. As a result, the mass matrices of the charge conjugate states N_{Ri}^c now becomes $M^* - \frac{i}{2}\Gamma^*$ which is not related by simple phase rotation to the mass matrix of the states N_{Ri}.

It was not emphasized earlier that this anti-hermitian decay width Γ implicitly gives CPT violation at the *effective theory level*, and hence the conventional CPT conserving results do not hold. Consider the theorem which tells us that in thermal equilibrium the number density of particles with non-zero charge Q would be same as the number density of the antiparticles since the expectation value of the conserved charge, given by,

$$< Q >= \frac{\mathrm{Tr}\left[Qe^{-\beta H}\right]}{\mathrm{Tr}\left[e^{-\beta H}\right]}$$

vanishes since any conserved charge Q is odd while H is even under CPT. Consider now a non-hermitian Hamiltonian,

$$H = M\overline{\Psi^c}\Psi$$

where Ψ carries a charge $Q = 1$ and there is no hermitian conjugate part of the Hamiltonian. This Hamiltonian now satisfies, $[Q, H] = 2H$. It is now obvious that this charge Q cannot be odd or H cannot be even under CPT, because then this commutation relation is not invariant under CPT. On the other hand, if Q is not odd or H is not even, then the above theorem does not hold and one can have a non-zero expectation value for the charge Q. Because of this reason, when the decay width is included in the mass matrix along with its CP non-conservation, an asymmetry was obtained for N_{Ri}.

It is similar to the fact, that the total decay width of any particle is same as that of the total decay width of the antiparticle. But the partial widths for them could be different. In fact, the partial decay widths are just equal and opposite to each other for the different decay modes of particles and antiparticles. In the same way, if we consider the total system, CPT will be conserved and we cannot get any asymmetry. As a result, the lepton asymmetry can be generated in this system only when the N_{Ri} decays out-of-equilibrium.

As an illustration of the discussions of the previous paragraph consider a scalar X, which has two decay modes,

$$X \to a + b \qquad\qquad X \to c + d.$$

CPT invariance would then imply that the total decay widths satisfy, $\Gamma_{total}(X) = \Gamma_{total}(\bar{X})$. Whereas, CP violation would imply that the partial decay widths of X and $\bar{X}$ are different, $\Gamma(X \to a + b) \neq \Gamma(\bar{X} \to \bar{a} + \bar{b})$, although $\Gamma(X \to a + b) = \Gamma(\bar{X} \to \bar{c} + \bar{d})$. In analogy, when CPT and unitarity is respected, we have

$$\sum_f \Gamma(S_i \to S_f) = \sum_f \Gamma(S_f \to S_i), \tag{9}$$

where S_i and S_f are the initial and final states. However, when the system departs from equilibrium, this is no longer true locally. Consider the case, when the two real intermediate states X_S and X_L have very short and long lifetimes, which are combinations of the states X and $\bar{X}$. In a short interval of time, only X_S will have time to decay, while X_L will decay after the universe has expanded further. So at any given time equation (9) is not valid, which has been discussed in details in ref [8].

In words, if N_{Ri} decays and inverse decays are slow enough to satisfy the out-of-equilibrium condition, the difference in the decay widths of the physical states can generate an asymmetry in N_{Ri}. This is because although the state with fast decay rate can recombine again, the slow decaying particle cannot recombine. These rates for the particles and antiparticles are different because of the non-hermiticity of the decay matrix Γ. However, if the physical states decay very fast, then the decays and inverse decays of these states will be in equilibrium and we have to treat the system as a whole, which conserve CPT and hence there will not be any N_{Ri} asymmetry.

We shall now calculate the amount of $CP-$asymmetry in N_{Ri} in this approach. In the present effective Hamiltonian language, the mass matrices of N_{Ri} and N_{Ri}^c being different implies that due to the different Majorana masses the transition $N_{Ri} \to N_{Rj}^c$ (which is same as $N_{Rj} \to N_{Ri}^c$ by the CPT theorem) is not the same as that of $N_{Rj}^c \to N_{Ri}$ (which is same as that of $N_{Ri}^c \to N_{Rj}$ by CPT). This produces an $N-$asymmetric universe even before their decay generates an asymmetry in the left-handed leptons.

The eigenvalues for these Hamiltonians are

$$\lambda_1^{\pm} = \frac{1}{2}(M_1 + M_2 + \sqrt{S^{\pm}}) \quad and \quad \lambda_2^{\pm} = \frac{1}{2}(M_1 + M_2 - \sqrt{S^{\pm}}) \tag{10}$$

where

$$S_{\pm} = (M_1 - M_2)^2 + 4h^{\pm 2}. \tag{11}$$

The corresponding physical states are,

$$\Psi^+_{\frac{1}{2}} = a^+_{\frac{1}{2}} N_1 + b^+_{\frac{1}{2}} N_2 \tag{12}$$

$$\Psi^-_{\frac{1}{2}} = a^-_{\frac{1}{2}} N_1^c + b^-_{\frac{1}{2}} N_2^c \tag{13}$$

with,

$$\begin{pmatrix} a_1^\pm & b_1^\pm \\ a_2^\pm & b_2^\pm \end{pmatrix} = \begin{pmatrix} \frac{\mathcal{C}^\pm}{\mathcal{N}^\pm} & \frac{1}{\mathcal{N}_\pm} \\ \frac{1}{\mathcal{N}_\pm} & -\frac{\mathcal{C}^\pm}{\mathcal{N}^\pm} \end{pmatrix}. \tag{14}$$

and

$$\mathcal{C}^\pm = \frac{2h^\pm}{M_2 - M_1 + \sqrt{\mathcal{S}^\pm}}; \qquad and \qquad \mathcal{N}^\pm = \sqrt{1 + (\mathcal{C}^\pm)^2}. \tag{15}$$

The physical states $\Psi^\pm_{1,2}$ will now evolve with time. So even if we start with an N symmetric universe, we will end up in a N asymmetric universe. The physical states will decay and recombine continuously. However, since the decay widths for the two physical states are different, this process will create an asymmetry in N_{Ri}, which will be compensated by an asymmetry in the decay products. If one calculates the amount of asymmetry in N_{Ri} and the decay products, the CPT theorem will tell us that there cannot be any net asymmetry. However, when we calculate the asymmetry of only N_{Ri} when the decay rates are slow enough to satisfy the out-of-equilibrium condition, there will be an asymmetry in N_{Ri}. The amount of N_{Ri} asymmetry is given by,

$$\Delta_N = \sum_{i=1,2} \frac{\Gamma_{\psi_i^+ \to N} - \Gamma_{\psi_i^- \to N^c}}{\Gamma_{\psi_i^+ \to N} + \Gamma_{\psi_i^- \to N^c}} = \frac{A - B}{A + B}, \tag{16}$$

where, $A = |a_i^+ + b_i^+|^2 = 2|\mathcal{N}^+|^{-2}[|\mathcal{C}^+|^2 + 1]$ and $B = |a_i^- + b_i^-|^2 = 2|\mathcal{N}^-|^{-2}[|\mathcal{C}^-|^2 + 1]$.

In the approximation, $h^\pm \ll |M_2 - M_1|$ we can find an expression for the N asymmetry to be,

$$\Delta_N = \frac{1}{8\pi} \frac{M_1 - M_2}{M_1 + M_2} \mathrm{Im} \sum_\alpha h^*_{\alpha i} h_{\alpha j} \tag{17}$$

where the rephasing invariant CP violating quantity is of the type $\mathrm{Im}[t_{\alpha ij}]$.

In the present approach we can calculate the final lepton asymmetry in the same way. The physical states ψ_i^+ will decay into only leptons, whereas the states ψ_i^- will decay into only antileptons. Since the mass matrices for the states N_{Ri} and N^c_{Ri} are not related by just phase rotation, there will be an asymmetry. In the field theoretic language this has been clarified and demonstrated recently [8].

The amount of lepton number asymmetry is given by,

$$\delta = \sum_{i=1,2} \frac{\Gamma_{\psi_i^+ \to l} - \Gamma_{\psi_i^- \to l^c}}{\Gamma_{\psi_i^+ \to l} + \Gamma_{\psi_i^- \to l^c}}, \tag{18}$$

where,

$$\Gamma_{\psi_i^+ \to l} = \sum_\alpha |a_i^+ h_{\alpha 1} + b_i^+ h_{\alpha 2}|^2 \tag{19}$$

and

$$\Gamma_{\psi_i^- \to l^c} = \sum_\alpha |a_i^- h_{\alpha 1}^* + b_i^- h_{\alpha 2}^*|^2 \tag{20}$$

and hence,

$$\Delta = \sum_i \frac{\mathcal{X}_-^i}{\mathcal{X}_+^i} \tag{21}$$

where

$$\begin{aligned} \mathcal{X}_\pm^i &= (|a_i^+|^2 \pm |a_i^-|^2) \sum_\alpha |h_{\alpha 1}|^2 + (|b_i^+|^2 \pm |b_i^-|^2) \sum_\alpha |h_{\alpha 2}|^2 \\ &\quad +2\ Re[\sum_\alpha h_{\alpha 1}^* h_{\alpha 2} (a_i^{+*} b_i^+ \pm a_i^- b_i^{-*})] \end{aligned} \tag{22}$$

In the approximation $|M_2 - M_1| \gg |h^\pm|$, it is given by the expression as obtained by other methods,

$$\delta = -\frac{1}{8\pi} \frac{M_1 M_2}{M_2^2 - M_1^2} \frac{\text{Im}\ \left[\sum_\alpha (h_{\alpha 1}^* h_{\alpha 2}) \sum_\beta (h_{\beta 1}^* h_{\beta 2})\right]}{\sum_\alpha |h_{\alpha 1}|^2} \tag{23}$$

where the rephasing invariant CP violating phase is of the form Im $[t_{\alpha 12} t_{\beta 12}]$. As a result, it is possible to create a N_{Ri} asymmetry, without creating a lepton asymmetry of the universe. However, before generating lepton asymmetry of the universe through heavy Majorana neutrino decay, we cannot avoid having a $CP-$asymmetric universe in the right handed neutrinos. When a $CP-$asymmetric universe is created in N_{Ri}, an equal and opposite amount of light left-handed lepton asymmetry is also generated to compensate. So, with the same assignment of the lepton numbers for N_{Ri} and the light leptons, the total lepton asymmetry of the universe before the decay of the heavy neutrinos would vanish because of the conservation of CPT. On the other hand, after the heavy neutrinos decay the generated asymmetry will depend on the amount of $CP-$violation and more crucially depend on the departure of the system from thermal equilibrium.

The Boltzmann equation has been solved for the same system in details in the literature [8], and it has been pointed out that when the heavy right handed neutrinos decay away from equilibrium, the CP asymmetry produces a lepton asymmetry of the universe. Because of a difference in the decay width of the two physical states, when proper weight is given to the different real intermediate states, the amount of CP asymmetry we calculated will produce a lepton asymmetry of the universe. During the electroweak phase transition, this lepton asymmetry will get converted to a baryon asymmetry of the universe.

To summarize, we have studied the question of lepton asymmetry of the universe in heavy neutrinos decay. We have developeded the effective hamiltonian formalism with four component neutrinos for this problem. We point out that before generating a lepton asymmetry, the universe becomes $CP-$asymmetric in the heavy neutrinos even before the inverse decay stops. We then calculate the amount of lepton asymmetry finally created after the heavy neutrinos decayed away completely, which is different from the CP asymmetry of the heavy neutrinos.

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