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Critical fatigue behaviour in brittle glasses

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Abstract. The dynamic fatigue fracture behaviour in different glasses under various sub-threshold loading conditions are analysed here employing an anomalous diffusion model. Critical dynamical behaviour in the time-to-fracture and the growth of the micro-crack sizes, similar to that observed in such materials in the case of quasi-static ("instantaneous") failures for above-threshold conditions, are predicted and compared with some of the experimental observations in different glasses.

Keywords. Fracture dynamics; critical behaviour; anomalous diffusion.

1. Introduction

Griffith (1920), equating the released elastic energy from a growing crack inside a solid with the energy of the crack surfaces newly created, came to a quantitative estimate of the fracture strength of a brittle solid containing an already existing fixed geometry micro-crack. Assuming the linear elasticity behaviour up to the breaking point of a brittle solid, and equating the differential increment dE_{el} in the elastic energy $E_{el} \simeq (s^2/Y)l_0^3$ for an elastic solid under stress s, modulus of elasticity Y, containing a micro-crack of length l_0, with the increment dE_s of the crack surface energy $E_s \simeq g l_0^2$, with g denoting the surface energy, as the crack propagates a further length dl, Griffith obtained

$$s_c^0 \simeq \frac{C}{\sqrt{l_0}},\ C \simeq \sqrt{Yg}\ , \qquad (1)$$

for near-equilibrium extension of the crack. For finite rate of growth of the crack, the additional kinetic energy of the crack surface has also to be extracted out of the released elastic energy (Mott 1948). The success of such a simple nucleation theory for the quasi-static growth of the crack within a solid has been checked thoroughly (Lawn 1993). As assumed in the theory, it is found to be valid for brittle solids with a single defect (micro-crack), or in cases where the larger defect is unique and much weaker than the other micro-defects present in the solid. For brittle solids having random defects, where the micro-cracks have disperse geometries and their size distribution becomes continuous, the simple Griffith formula (1) does not hold and one gets non-self-averaging statistical distribution for the fracture strength s_c (Chakrabarti and Benguigui 1997). The critical behaviour of the (phase) transition between the non-nucleating (for small stresses) and nucleating fracture (for stresses just above the threshold) in a random solid is recently being analysed successfully and has also been comprehended satisfactorily employing the percolation theories (see e.g. Chakrabarti and Benguigui 1997; Sahimi 1998).

As mentioned already, the Griffith's theory is applicable for equilibrium or quasi-static fracture growth. The dynamics of fracture in disordered brittle solids like glasses is being investigated intensively these days. Most of these studies are concerned with the growth of the roughness of the fractured surfaces with the increasing velocity of the crack tip, due to the increased stress level ($s > s_c^0$) above the Griffith-stress (s_c^0) for the micro-crack (of length l_0) infected in the solid (Chakrabarti and Benguigui 1997). A particular kind of dynamical fracture in random solids or glasses which also has manifest critical behaviour, viz. the fatigue failures (Lawn 1993), has not yet been analyzed properly by the physicists using their modern statistical physics tools (Sahimi 1998). Extensive literature however exists (Lawn *et al* 1981, 1983; Marshall *et al* 1981; Lawn 1993) for this fatigue behaviour due to the thorough investigations by the materials engineers. In fatigue failures, due to chemically induced stress–corrosion at the crack-tips, the stressed solid eventually fails at a stress level (s) much lower than the Griffith stress (s_c^0) for the static fracture. It is believed (Lawn 1993) that, due to the stressed condition of the solid, the chemically induced creep motions of the micro-crack tips lead to gradual extensions of the existing micro-cracks within the sample, and eventually the external load exceeds the Griffith strength of these extended cracks and the solid fails. Obviously, while for loads just above the static Griffith strength of the solid, the failure occurs almost instantaneously ($t = 0$ for $s > s_c^0$), the time-to-failure t in the fatigue process is nonvanishing and increases significantly as the external load level decreases below s_c^0.

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For a solid containing a typical micro-crack of length l_0, and under a stress s ($< s_c^0$), Lawn and Wilshaw (see Lawn 1993) assumed that due to chemically induced creep, the length $l(t)$ of the micro-crack will increase with time $t(l(t) = l_0$ at $t = 0)$ with a velocity $v = v$ (l_0, s) $\sim \exp(G/T)$, where the stress intensity factor $G \sim s^2 l_0/Y$ and T denotes the ambient temperature. This then suggests that the time-to-fracture $t \sim 1/v_c$ would be given by $t \sim s^{-2} \exp(-s^2 l_0)$. They also investigated how such a formula for t compares with the experimental fatigue data for sodalime glasses in air. Although the formula agrees semi-quantitatively for large time t, significant mismatch occurs near 'instantaneous failures' for s approaching s_c^0 where the observed value of t vanishes while their theory predicts a finite value for t (vanishing only in the limit $s \to \infty$). In a recent book, Lawn gave an extensive discussion on such studies and considered phenomenologically the contributions in the velocity v of the higher order terms which may lead to 'instantaneous' failures for s approaching s_c^0.

Ramanathan and Fisher (1998) has recently argued that the crack propagation velocity v can in fact be taken as an order parameter for the critical dynamics of fracture: similar to the spontaneous magnetization of a ferromagnet changing from its nonzero value below the Curie temperature to the zero value above it, the crack velocity v also changes from its strict zero value for $s \le s_c^0$, to $v \sim (s - s_c^0)^b$ for $s \ge s_c^0$ with an 'universal' exponent b. They gave both mean field and renormalization group estimates for b and related it to the roughness exponent of the corresponding fractured surface (for $s \ge s_c^0$). The apparent success of such a theory establishes clearly the intuitive picture involved: The crack velocity v is zero for stresses less than the threshold value s_c^0 for the sample, and it grows following an universal power law for stress values just above the threshold. This therefore contradicts the model of Lawn and Wilshaw (see Lawn 1993) where they invoked the idea of a finite crack velocity v under sub-threshold conditions ($s \le s_c^0$).

2. Model

In view of the above, and also of several other experimental observations (Marshall *et al* 1981; Lawn *et al* 1981, 1983; Banerjee and Sarkar 1995a, b), in particular of the gradual decay in the 'effective' velocity of the chemically induced crack growth during the fatigue experiments (Lawn *et al* 1983), we develop here an alternative scenario for the fatigue process based on stress-induced diffusion growth of the micro-cracks. We assume that the differential increase dl in the length of the micro-crack occurs due to stress-induced diffusion:

$$dl \sim D(dt)^{\nu}, \tag{2}$$

and not due to any finite velocity of the crack ($\nu \le 1$). Here $D = D(T, s, l_0)$ denotes the diffusion constant and ν denotes the diffusion exponent ($\nu = 1/2$ for normal Brownian diffusion). It may be noted here that similar diffusion induced crack growth (under stress) at the grain boundaries, resulting in fatigue failures, had been considered earlier (Raj and Ashby 1975). There the interest had been in comparing the amount of such diffusion at elevated temperatures and the observed crack growth during indentations. Here we intend to investigate the possible critical behaviour due to such diffusion. This diffusion is expected to be anomalous here for random fractal solids or amorphous glasses, with

$$\nu = d_s/2d_f \le 1/3,$$

where $d_s \simeq 4/3$ denotes the spectral dimension of the fractal with Housdorf dimensionality, d_f (Stauffer and Aharony 1985). If we now assume that the crack length, for which the (applied) stress s would become the Griffith nucleating stress, is l_a:

$$l_a \simeq \frac{C^2}{s^2}, \tag{3}$$

then the fatigue failure will occur when the increase in length Δl due to diffusion (given by (2)) will become equal to $l_a - l_0$, or when

$$\Delta l = C^2\left(\frac{1}{s^2} - \frac{1}{s_c^{0^2}}\right) \simeq \left(\frac{C^2}{s_c^{0^3}}\right)(s_c^0 - s). \tag{4}$$

The fatigue time-to-fracture t will then be given by $\int_0^t dl(t) = \Delta l$, and equating (2) and (4) one gets

$$t \sim C' \, (s_c^0 - s)^{1/\nu}. \tag{5}$$

Such a form for t obviously accommodates the 'instantaneous failure' phenomena: $t \to 0$ for $s \to s_c^0$ and $t = 0$ for $s \ge s_c^0$. It also suggests that t^{ν} ($\nu \le 1/3$) would scale linearly with the stress interval from the Griffith (instantaneous) failure stress s_c^0. Additionally, as mentioned before, the anomalous diffusion assumed in (2) for glasses indicate that the crack tips cannot have a finite velocity consistent with the diffusion picture, and if defined, the effective velocity v would vanish in the long time limit.

3. Experimental and discussion

In order to check some of these features, we analysed some of the previous fatigue study data (Banerjee and Sarkar 1985a, b) and also repeated some of them. Well annealed samples of sodalime and borosilicate glasses of $10 \times 10 \times 4$ mm were taken and polished optically on all the sides. A vickers indenter (Model HMV-2000, Shimadzu) was used for the indentation study. Experiment was carried out at different loads viz. 0·10 N, 0·15 N, 0·25 N, 0·50 N and 1·0 N. Repeated indentation at each load was performed at each contact site without removing the specimen simulating fatigue condition. The contact site could be examined accurately after any number of cycles by interchanging the indenter and the optics. The diagonal lengths ($l(t)$) of the impression were measured after completion of cycles (t) 1, 5, 10 . . . . Also the number of cycles needed to initiate a radial crack was recorded. The procedure was repeated five times to generate the statistics. The experiment was conducted in air at ambient temperature (25 ± 2°C) and humidity (60 ± 5%). The experiment was discontinued when radial crack was initiated. It may be noted here that one school of thought (Lawn *et al* 1981, 1983) argues that due to brittleness of glass, it is essential to incorporate a starter crack to simulate true fatigue. But earlier study by Banerjee and Sarkar (1995a, b) on sodalime, lead-alkali and also on other glasses shows that even without incorporating a crack it is possible to initiate a Griffith crack by repeatedly indenting with subcritical loads. The same experiment was repeated to observe the fatigue behaviour of glasses at subcritical loads ($s < s_c^0$), where we took the number of indentation cycles required for radial fracture as a measure of the time-to-fracture t. At the beginning, the highest load at which cracks were initiated in a single cycle was adopted as a baseline critical load. In this study a baseline load of about 1·0 ± 0·05 N was found to be the critical load (s_c^0) for the unstressed glass samples. Repeated indentation was carried out in the glasses with loads below the critical load. It was observed that during repeated indentation, irrespective of the applied load, the impression size increased with each indentation cycle prior to invitation of surface cracks. The increase in the

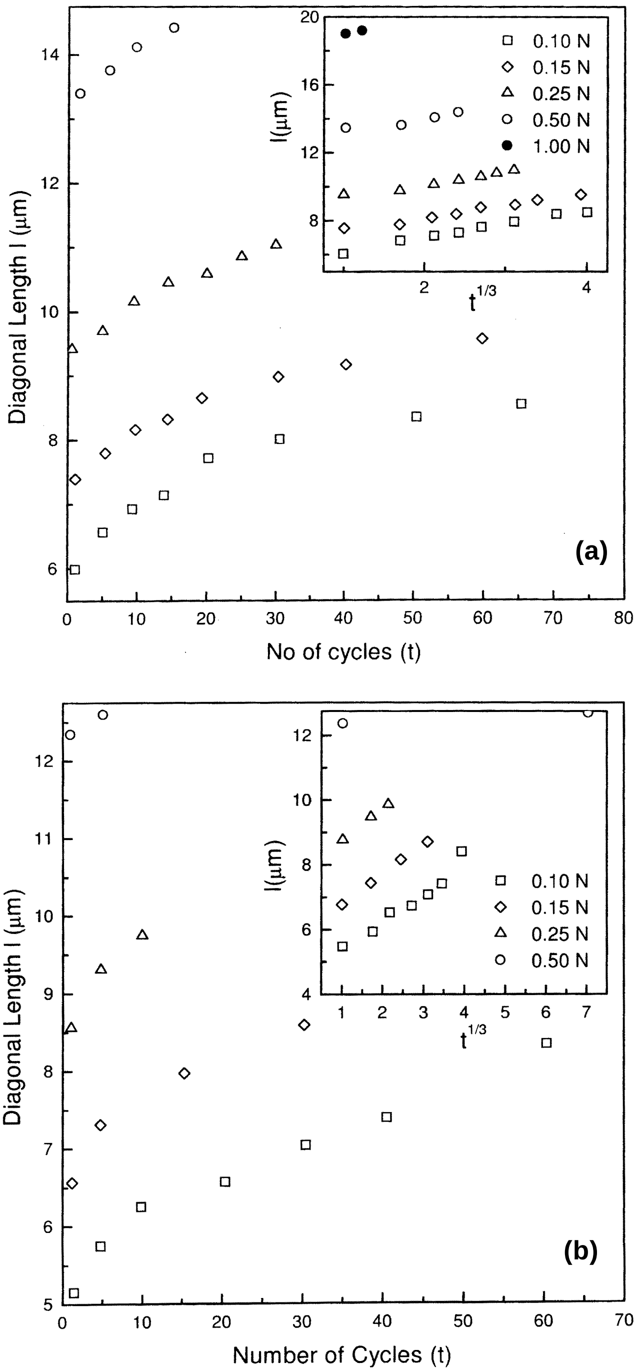

Figure 1. Plots of $l(t)$ vs (t) for different $s < s_c^0$. Various symbols indicate different values of the external load: □ for 0·10 N, ◊ for 0·15 N, Δ for 0·25 N, o for 0·50 N and • for 1·0 N. The last data point in any set indicate the corresponding fracture point. The insets show the corresponding plots of l vs $t^{1/3}$ (**a**) for sodalime and (**b**) for borosilicate glasses.

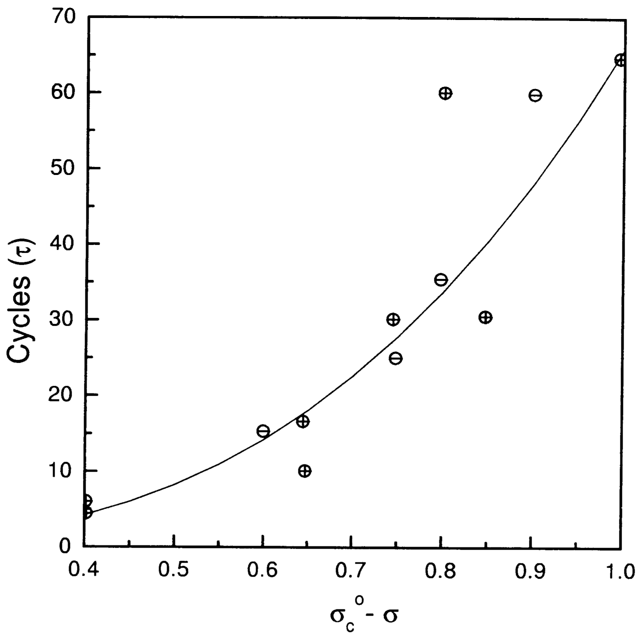

Figure 2. Plots of time-to-fracture t vs load s for different glasses: ⊕ for sodalime, ⊖ for lead-alkali and ⊗ for borosilicate. The solid curve corresponds to (5) with $1/v = 3$ and $s_c^0 \simeq 1{\cdot}0$ N.

impression size ($l(t)$) with number of cycles (t) at each load for different glasses is plotted in figure 1. This gradual decrease in the slope (or the vanishing of the micro-crack tip effective-velocity, v) for larger t in all the cases supports clearly the diffusive picture assumed here (in (2)). The insets, in fact, indicate clearly that $l \sim t^{1/3}$, as suggested by (2). When the time-to-fracture t of these materials, measured in number of cycles, were plotted against the external load s ($< s_c^0$), it was observed that the data fits (see figure 2) more or less to a curve $t \sim (s_c^0 - s)^3$, as given by (5).

4. Conclusions

The observed phenomena can therefore be explained by assuming high stress concentrations developing near the tip of the corners of the impression during indentation. Consequently, a small amount of material gets diffused around the indenter impression, leading to increase in the diagonal length. The process continues with the increasing number of cycles. Assuming the initial crack length, which is less than the Griffith crack corresponding to the applied load, to be given by the microscopic inhomogeneities of the glasses, our model suggests, similar to that proposed by Raj and Ashby (1975), that the subsequent stressed-induced (and thereby significantly enhanced) mass diffusion at the crack-tip extends the crack length, with increasing cycles, to that of the Griffith crack. This gives rise to the fatigue failure and the associated critical behaviour. The model not only validates the experimental results, but can be a formidable tool to explain the phenomena of the fatigue failures in glasses.

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