Fermi National Accelerator Laboratory

FERMILAB-Pub-84/118-T
November, 1984

Radiative corrections in supersymmetric gauge theories

Ashoke Sen
Fermi National Accelerator Laboratory
Batavia, P.O.B. 500, IL 60510

ABSTRACT

The effect of radiative corrections in a general supersymmetric gauge theory is studied when the gauge symmetry is partially broken at the tree level. Certain no-renormalization theorems are proved.

Operated by Universities Research Association Inc. under contract with the United States Department of Energy

I. INTRODUCTION

The effect of radiative corrections in supersymmetric theories has been discussed by many authors[1-4]. An extensive discussion of supersymmetric gauge theories where the gauge symmetry and supersymmetry are unbroken at the tree level may be found in Ref.4. The effect of one loop radiative corrections in a supersymmetric gauge theory where supersymmetry is unbroken, but the gauge symmetry is completely broken at the tree level, has been discussed by Ovrut and Wess[3]. In all the cases that have been discussed so far, supersymmetry has been found to be unbroken due to radiative corrections. In this paper we shall discuss the effect of radiative corrections in a general supersymmetric gauge theory, where the original gauge group is spontaneously broken to one of its subgroups H at the tree level. We shall show that although the radiative corrections shift the vacuum expectation values (vev) of various fields, the following no-renormalization theorems hold.

i) Supersymmetry is unbroken even after including the radiative corrections.

ii) The gauge group H is also left unbroken by the radiative corrections.

iii) For every zero eigenvalue of the tree level mass matrix, we have a zero eigenvalue of the renormalized mass matrix. This issue is important in studying the stability of the mass hierarchy.

We use the background field formalism for our analysis. Our result is valid to all orders in perturbation theory, provided we assume that the effective action in the background field formalism reproduces all the physical results correctly to all orders in the perturbation theory. The rest of the paper is organized as follows. In Sec.II we discuss the structure of the scalar and the vector boson mass matrix at the tree level. In Sec.III we analyze the structure of the possible radiatively generated terms in the theory, and discuss their effect on supersymmetry and gauge symmetry breaking, as well as on the scalar mass matrix. We summarize our results in Sec.IV. In appendix A we discuss the Feynmann rules for supersymmetric gauge theories when the gauge symmetry is partially broken at the tree level. Throughout this paper we use the conventions of Ref.4.

II. TREE LEVEL POTENTIAL

Let $\{\phi_i\}$ denote the set of all the chiral superfields in the theory, and z_i be the physical scalar components of ϕ_i. We assume that the theory is described by a superpotential $W(\phi)$ which is invariant under some gauge group G. The generators of G are denoted by T_a. If F_i and D_a denote the auxiliary components of the chiral superfield ϕ_i and the vector superfield V_a respectively, the potential involving the scalar fields is given by,

$$V = -\sum_i \left\{ F_i^* F_i + \left(F_i \frac{\partial W}{\partial z_i} + h.c. \right) \right\} - \sum_a \left\{ \frac{D_a^2}{2} + D_a \sum_{i,j} z_i^\dagger (eT_a)_{ij} z_j \right\} \tag{2.1}$$

[Here we have chosen the Wess-Zumino gauge[5] for the gauge fields]. Eliminating F_i and D_a through their equations of motion, we get the effective potential at the tree level,

$$V = \sum_i \left| \frac{\partial W}{\partial z_i} \right|^2 + \frac{e^2}{2} \sum_a \left(\sum_{i,j} z_i^\dagger (T_a)_{ij} z_j \right)^2 \tag{2.2}$$

We assume that the potential has a supersymmetric minimum at $z_i = z_i^{(0)}$, where,

$$\frac{\partial W}{\partial z_i} = 0 \quad \forall i \quad ; \quad z^\dagger T_a z = 0 \quad \forall a \tag{2.3}$$

The vev of the scalar fields break the gauge group G to one of its subgroup H. Let T_ρ, T_σ,... denote the generators of H, and T_K, T_L,... denote the broken generators of G. Then,

$$\sum_j (T_\rho)_{ij}\, z_j^{(0)} = 0 \qquad \forall\ i \text{ and } \rho \qquad (2.4)$$

Following Ref.6 we shall choose a basis in which the scalar fields z_i are divided into three classes of fields, z_α, z_A and z_K. The fields z_K denote the direction parallel to $T_K z^{(0)}$, while z_α and z_A are orthogonal to this direction. In our convention $\{z_i\}$ will denote the set of all fields $\{z_\alpha,\ z_A,\ z_K\}$. If we choose a basis in which the vector boson mass matrix is diagonal,

$$\mu^2_{KL} \equiv e^2\, z_i^{(0)\dagger}\{T_K\, ,\, T_L\}_{ij} z_j^{(0)} = \mu_K{}^2\, \delta_{KL} \qquad (2.5)$$

then we have,

$$e\,(T_K)_{Li}\; z_i^{(0)} = \frac{1}{\sqrt{2}}\, \mu_K\, \delta_{KL} \qquad (2.6)$$

$$(T_K)_{\alpha i}\, z_i^{(0)} = (T_K)_{Ai}\; z_i^{(0)} = 0 \qquad (2.7)$$

where the sum over i runs over the set α, A, K. The contribution to the scalar mass^2 term from the second term in Eq.(2.2) is given by,

$$\frac{1}{2}\sum_K \mu_K{}^2 \left(\frac{z_K + z_K^*}{\sqrt{2}}\right)^2 \qquad (2.8)$$

On the other hand the contribution to the scalar mass^2 term from the first term in Eq.(2.2) is given by $(M^\dagger M)_{ij}(z_i - z_i^{(0)})^\dagger (z_j - z_j^{(0)})$, where,

$$M_{ij} = \left(\frac{\partial^2 W}{\partial z_i \, \partial z_j}\right)_{z = z^{(0)}} \tag{2.9}$$

Using the invariance of W under a gauge transformation with arbitrary complex parameters[3], we may show that,

$$M_{\alpha K} = M_{AK} = M_{LK} = 0 \qquad \forall\ \alpha, A, L, K \tag{2.10}$$

Hence the fields z_K are eigenvectors of $(M^\dagger M)$ with zero eigenvalue. Fields z_α and z_A are choosen in such a way that z_α's are the additional eigenvectors of $M^\dagger M$ with zero eigenvalue, whereas z_A is an eigenstate of $M^\dagger M$ with eigenvalue M_A. Thus we also have,

$$M_{\alpha A} = M_{\alpha\beta} = 0 \qquad \forall\ \alpha, \beta, A \tag{2.11}$$

whereas M_{AB} is a non-singular matrix. Thus the fields z_α and the imaginary parts of z_K are massless. The imaginary parts of z_K are however eaten up by the gauge fields through higgs mechanism. The real parts of z_K acquire masses from (2.8) equal to those of the gauge bosons, and form scalar parts of complete vector supermultiplet. Hence at the tree level we have a set of massless complex scalar fields z_α, a set of massive complex scalar fields z_A with mass M_A and a set of massive real scalar fields degenerate with the massive vector fields. For later convenience we take all the fields z_α, z_A and z_K to be the unshifted fields. This does not affect Eqs. (2.10) and (2.11) with M_{ij} given by Eq. (2.9).

III. RADIATIVE CORRECTIONS

We shall now study the effect of radiative corrections in this theory using the background gauge formalism. This method has been developed and discussed in detail in Ref.[4] for unbroken gauge theories. Some modifications needed to extend the method to spontaneously broken gauge theories are discussed in appendix A. We split every chiral superfield ϕ_i and the vector superfield V_a into background and quantum superfields as,

$$e^{V_a T_a} = e^{V_a^{(b)} T_a/2} \; e^{V_a^{(q)} T_a} \; e^{V_a^{(b)} T_a/2} \tag{3.1}$$

$$\tilde{\phi} \equiv e^{V_a^{(b)} T_a/2} \phi = \tilde{\phi}^{(b)} + \tilde{\phi}^{(q)} \tag{3.2}$$

$$\bar{\tilde{\phi}} \equiv \bar{\phi} \, e^{V_a^{(b)} T_a/2} = \bar{\tilde{\phi}}^{(b)} + \bar{\tilde{\phi}}^{(q)} \tag{3.3}$$

where the superscripts (b) and (q) denote the background and quantum superfields respectively. Let us define the connections $\Gamma_\alpha^{(b)}$, $\bar{\Gamma}_{\dot\alpha}^{(b)}$ and field strengths $W_\alpha^{(b)}$ and $\bar{W}_{\dot\alpha}^{(b)}$ as,

$$\Gamma_\alpha^{(b)} = i \left(e^{-V_a^{(b)} T_a/2} D_\alpha \, e^{V_a^{(b)} T_a/2} \right) \tag{3.4}$$

$$\bar{\Gamma}_{\dot\alpha}^{(b)} = i \left(e^{V_a^{(b)} T_a/2} \bar{D}_{\dot\alpha} \, e^{-V_a^{(b)} T_a/2} \right) \tag{3.5}$$

$$W_\alpha^{(b)} = e^{V_a^{(b)} T_a/2} \left(\bar{D} \bar{D} \, e^{-V_a^{(b)} T_a} D_\alpha \, e^{V_a^{(b)} T_a} \right) e^{-V_a^{(b)} T_a/2} \tag{3.6}$$

$$\bar{W}_{\dot\alpha}^{(b)} = e^{-V_a^{(b)} T_a/2} \left(D D \, e^{V_a^{(b)} T_a} \bar{D}_{\dot\alpha} \, e^{-V_a^{(b)} T_a} \right) e^{V_a^{(b)} T_a/2} \tag{3.7}$$

where D_α, $\bar{D}_{\dot\alpha}$ are the ordinary covariant derivatives. If we also define,

$$\Phi^{(b)} = e^{-V_a^{(b)} T_a/2}\, \tilde{\Phi}^{(b)} \tag{3.8}$$

and $\bar{\phi}^{(b)}$ to be its complex conjugate field, the action is invariant under the background gauge transformation,

$$\begin{aligned} e^{V_a^{(b)} T_a} &\rightarrow e^{i\bar{\Lambda}_c T_c}\, e^{V_a^{(b)} T_a}\, e^{-i\Lambda_d T_d} \\ \phi^{(b)} &\rightarrow e^{i\Lambda_c T_c}\, \phi^{(b)} \\ \bar{\phi}^{(b)} &\rightarrow \bar{\phi}^{(b)}\, e^{-i\bar{\Lambda}_c T_c} \end{aligned} \tag{3.9}$$

together with some transformations on the quantum fields listed in appendix A. There is another set of transformations on the quantum fields which leaves the action invariant. We shall call this the quantum gauge transformation.

The gauge fixing term given in appendix A is chosen in such a way that it breaks the quantum gauge invariance but not the background gauge invariance. The structure of radiatively generated terms in this theory may be analyzed in the same way as in Ref.4. Some details of this analysis have been given in appendix A. We reach the following conclusions:

i) The full effective action expressed as a function of the background fields $V^{(b)}$, $\tilde{\phi}^{(b)}$ and $\bar{\tilde{\phi}}^{(b)}$, is invariant under the background gauge transformation (3.9).

ii) The radiatively generated terms in the effective action must be of the form,

$$\int d^4\theta \int \left(\prod_r d^4 x_r\right) \; f\left(\{\Phi_i(x_r, \theta)\}\right) \qquad (3.10)$$

where f is some arbitrary function of the superfields ϕ_i and their covariant derivatives at different space-time points x_r but the same fermionic co-ordinate θ.

iii) In two loop order and beyond, there is no explicit dependence of the effective action on the gauge fields $V_\rho^{(b)}$ belonging to the unbroken subgroup H, except through the superfields $\tilde{\phi}^{(b)}$, $\bar{\tilde{\phi}}^{(b)}$, $W_\alpha^{(b)}$, $\bar{W}_{\dot{\alpha}}^{(b)}$, $\Gamma_\alpha{}^{(b)}$ and $\bar{\Gamma}_{\dot{\alpha}}^{(b)}$. At one loop order the radiative corrections may have explicit dependence on $V_\rho^{(b)}$. The contribution comes only from loops of chiral superfields belonging to a complex representation of the gauge group. If the chiral superfields of the theory belong to the real representations of the gauge group H, or occur in complex conjugate pairs so that together they again form a real representation of H, then even the one loop contribution is free from explicit dependence on $V_\rho^{(b)}$.

From now on we shall drop the superscript (b) from various background fields. Any field without a superscript will refer to background field, unless otherwise mentioned. We shall write down all possible radiatively generated terms which may shift the vev of various fields and break supersymmetry. First note that since the effective action is invariant under the background gauge transformation (3.9), we may choose the background gauge field V to be in the Wess-Zumino gauge. The effective potential is then a function of the auxiliary fields F_i, D_a and the scalar fields z_i. In order to saturate the θ integral in Eq.(3.10), all the radiatively generated terms must have at least one power of $F_i^* F_j$ or one power of D_a. Since the minimum of the tree level potential lies at $F_i = D_a = 0$ (and so does the minimum of the full potential, as we shall show), any term quadratic in the auxiliary fields will not shift the vev of various fields or break supersymmetry. Thus the only possible radiatively generated terms that may break supersymmetry is of the form,

$$- e \sum_{a} D_a P_a(z, z^{\dagger}) \tag{3.11}$$

where P_a is some function of the scalar fields. As was shown in Ref.4, P_a is free from quadratic divergences, and is at most logarithmically divergent.

First we shall consider the contribution to P_ρ. One loop contribution to P_ρ from loops of massless chiral superfields vanish in the dimensional regularization scheme. On the other hand, since the generator T_ρ belongs to the unbroken subgroup H, all the massive fields of the theory must be either in the real representation of H, or occur in pairs of complex conjugate representations. This is true for the superfields ϕ_A which acquire their mass from terms in the superpotential $W(\phi)$, as well as the superfields ϕ_K which acquire mass through their mixing with the vector superfields. Combining this result with our previous discussion we see that to all orders in perturbation theory, the radiatively generated terms that are linear in D_ρ depend on Γ_α, W_α, $\tilde{\phi}_i$ and their complex conjugate superfields, but not explicitly on V_ρ. The terms containing D_ρ in Γ_α and W_α are proportional to $\bar{\theta}^2\theta D$ and θD respectively, and cannot saturate the θ integral in (3.10) unless multiplied by some other auxiliary field. Hence the only source of terms linear in D_ρ is the $\exp(V_a T_a/2)$ term in $\tilde{\phi}$ and $\bar{\tilde{\phi}}$. The contribution from such terms to the effective potential may be written as,

$$-e\sum_\rho D_\rho P_\rho = \sum_\rho D_\rho \{ f_i(z, z^\dagger)^* (T_\rho)_{ij} z_j + h.c. \} + O(D^2) \qquad (3.12)$$

where f_i is some function of z, $z^\dagger$. The above equation tells us that P_ρ vanishes at $z^{(0)}$, and more generally, at any point which is invariant under the subgroup H.

Let us now analyze the contribution to P_K. In studying this we shall classify the generators T_K in irreducible representations of the group H. Let $\{T_S\}$ denote the set of generators which are singlets under H, i.e. which commute with every generator of H, and $\{T_N\}$ denote the set of generators which transform non-trivially under H. Then P_N

must vanish at a point z_i which is invariant under the subgroup H. P_S, on the other hand, is of order M^2 where M is the typical mass scale of the theory.

Adding (3.11) to (2.1), and eliminating the auxiliary fields through their equations of motion, we get,

$$F_i = - \frac{\partial W}{\partial z_i} \tag{3.13}$$

$$D_a = - e (z^\dagger T_a z + P_a) \tag{3.14}$$

and the full potential is,

$$V = \sum_i |F_i|^2 + \frac{1}{2} \sum_a D_a^2 \tag{3.15}$$

We seek a solution where F_i and D_a vanish for all values of i and a. We start from the ansatz,

$$\tilde{z}_i^{(0)} = [exp (\sum_s \lambda_s T_s)]_{ij} \; z_j^{(0)} \tag{3.16}$$

λ_S's being the parameters to be determined. Using the invariance of W under gauge transformation with complex parameters, we may show that,

$$\left(\frac{\partial W}{\partial z_i}\right)_{z_i = \tilde{z}_i^{(0)}} = \left(e^{-\lambda_s T_s}\right)_{ji} \left(\frac{\partial W}{\partial z_j}\right)_{z = z^{(0)}} = 0 \tag{3.17}$$

Also,

$$(T_\rho)_{ij} \; \tilde{z}_j^{(0)} = \left(e^{\lambda_s T_s} \; T_\rho \; z^{(0)}\right)_i = 0 \tag{3.18}$$

since T_ρ commutes with T_S. Thus P_ρ, P_N vanish at $\tilde{z}^{(0)}$, and so do D_ρ and D_N.

We shall now show that there always exist solutions for λ_S which make D_S vanish and hence give us a supersymmetric minimum of the full potential. The equation determining λ_S is given by,

$$z^{(0)\dagger} e^{\lambda_{s'} T_{s'}} T_s e^{\lambda_{s''} T_{s''}} z^{(0)} + P_s \left(e^{\lambda_{s'} T_{s'}} z^{(0)}, z^{(0)\dagger} e^{\lambda_{s''} T_{s''}} \right) = 0 \tag{3.19}$$

which may be written as (using Eq. (2.3)),

$$\sum_{s'} z^{(0)\dagger} \{ T_s , T_{s'} \} z^{(0)} \lambda_{s'}$$
$$= - P_s \left(e^{\lambda_{s'} T_{s'}} z^{(0)}, z^{(0)\dagger} e^{\lambda_{s''} T_{s''}} \right)$$
$$- z^{(0)\dagger} \left(e^{\lambda_{s'} T_{s'}} T_s e^{\lambda_{s''} T_{s''}} - T_s - \{ T_s , T_{s'} \} \lambda_{s'} \right) z^{(0)} \tag{3.20}$$

The left hand side of the equation is given by $\mu_S{}^2 \lambda_S / e^2$. We may solve Eq. (3.20) iteratively. In the first stage of iteration we set $\lambda_S = 0$ on the right hand side of the equation, and solve for λ_S. This value of λ_S is then substituted on the right hand side of (3.20), and a new value of λ_S is obtained. Since both λ_S and P_S receive contribution from diagrams involving one or more loops, the correction to the value of λ_S in successive stages of iteration will be of higher and higher powers in the loop expansion. Thus, up to any arbitrary order in perturbation theory, it is always possible to find a solution for λ_S satisfying Eq.(3.19) using the method of iteration.

Next we shall study the effect of radiative corrections on the scalar mass matrix. We shall again ignore terms containing two or more powers of the auxiliary fields, and keep only the terms given in (3.11). It will be shown later that these extra terms do not change any of the results that we shall derive. Let us define new fields z_i'' in terms of the fields z_i as,

$$z_i'' = (e^{-\lambda_s T_s})_{ij} \, z_j \tag{3.21}$$

for i=K, A or α. Thus at the new minimum of the potential z" takes the value

$$z''^{(0)} = e^{-\lambda_s T_s} \tilde{z}^{(0)} = z^{(0)} \tag{3.22}$$

Since W is invariant under gauge transformation with complex parameters, it has the same functional dependence on z" as on z. Thus

$$\left(\frac{\partial^2 W}{\partial z_i'' \, \partial z_j''}\right)_{z = \tilde{z}^{(0)}} = \left(\frac{\partial^2 W}{\partial z_i \, \partial z_j}\right)_{z = z^{(0)}} \tag{3.23}$$

Hence, if we work in the basis $\{z_i''\}$, the contribution to the mass matrix from the F terms of the potential **at** the new minimum involves only the z''_A fields, and not the z''_α or the z''_K fields. The part of the D term of the potential which contributes to the mass matrix may be expressed in terms of the fields z_i'' as,

$$\sum_K D_K{}^2 = \sum_K \{ z''^\dagger e^{\lambda_s T_s} T_K e^{\lambda_{s'} T_{s'}} z'' + P_K (e^{\lambda_s T_s} z'', z''^\dagger e^{\lambda_s T_s}) \}^2 \tag{3.24}$$

Unlike in Sec.II, the contribution from this term to the mass matrix involving z_α'' and z_A'' fields does not vanish, since $\partial D_K/\partial z_\alpha''$ or $\partial D_K/\partial z_A''$ are non-zero in general at $z=\tilde{z}^{(0)}$, with D_K given by Eq.(3.14). The problem may be avoided by defining a new set of fields z_i' such that,

$$z_K'' = z_K' + B_{AK} z_A' + B_{\alpha K} z_\alpha' \tag{3.25}$$

$$z_A'' = z_A' \tag{3.26}$$

$$z_\alpha'' = z_\alpha' \tag{3.27}$$

where $B_{\alpha K}$ and B_{AK} are constants that will be determined shortly. Using Eqs. (2.10) and (3.23) we may show that,

$$\left(\frac{\partial^2 W}{\partial z_i' \, \partial z_j'} \right)_{z=\tilde{z}^{(0)}} = \left(\frac{\partial^2 W}{\partial z_i \, \partial z_j} \right)_{z=z^{(0)}} \tag{3.28}$$

On the other hand,

$$\frac{\partial D_K}{\partial z_i'} = \frac{\partial D_K}{\partial z_i''} + B_{iL} \frac{\partial D_K}{\partial z_L''} \quad \text{for } i = \alpha, A \tag{3.29}$$

where $\partial/\partial z_i'$ denotes differentiation with respect to the fields z_i' keeping all the other z_j' fields fixed. We choose $B_{\alpha K}$ and B_{AK} in such a way that,

$$\left(\frac{\partial D_K}{\partial z_L''}\right)_{z=\tilde{z}^{(0)}} B_{iL} = -\left(\frac{\partial D_K}{\partial z_i''}\right)_{z=\tilde{z}^{(0)}} \quad \text{for } i=\alpha, A \tag{3.30}$$

so that $\partial D_K/\partial z_i'$ vanishes at $z_i = \tilde{z}_i^{(0)}$ and the D term does not contribute to the mass matrix involving the z_A' and the z_α' fields. The solution to (3.30) always exists since $(\partial D_K/\partial z_L'')_{z=\tilde{z}(0)}$ is a non-singular matrix whose lowest order contribution is given by μ_{KL}. $\partial D_K/\partial z_i''$ vanishes at the tree level for $i=\alpha$ or A, but receives contribution at one (or more) loop order. Combining Eqs.(3.24), (3.28), (3.29) and (3.30) we see that the fields z_α' remain massless even after including all the radiative corrections. As in Sec.II, the real part of z_K' acquires a mass from the D terms of the potential, while its imaginary part gets absorbed by the gauge bosons through the higgs mechanism. Although the wave-function renormalization factors for the scalar fields further renormalize the mass matrix, they cannot change the zero eigenvalues of the mass matrix, which represent flat directions in the potential. In other words, for every eigenstate of the tree level mass matrix with zero eigenvalue, we have an eigenstate of the renormalized mass matrix with zero eigenvalue.

Finally we shall discuss the effect of the terms involving two or more powers of the auxiliary fields. As we have already mentioned, these terms do not affect the positions of the new minimum, since they, and their first derivatives with respect to any scalar field, automatically vanish at $\tilde{z}^{(0)}$. The only terms which may contribute to the scalar mass matrix are the ones whose second derivatives with respect to the scalar fields do not vanish at $z=z^{(0)}$. These are the terms quadratic in the auxiliary fields. The most general term of this kind is of the form,

$$h_{ij} F_i^* F_j + (g_{ia} F_i^* + h.c.) + f_{ab} D_a D_b \qquad (3.31)$$

where h, g and f are functions of the scalar fields. Adding this to (2.1) and (3.11) and eliminating the auxiliary fields through their equations of motion, we may write the full potential as,

$$V = \left(\frac{\partial W}{\partial z_i}\right)^* H_{ij} \left(\frac{\partial W}{\partial z_j}\right) + \left\{\left(\frac{\partial W}{\partial z_i}\right)^* G_{ia} (z^\dagger T_a z + P_a) + h.c.\right\}$$

$$+ (z^\dagger T_a z + P_a) F_{ab} (z^\dagger T_b z + P_b) \qquad (3.32)$$

where F_{ab}, G_{ia} and H_{ij} are functions of the scalar fields which may be calculated in terms of the functions f_{ab}, h_{ij}

and g_{ia}. Using Eqs.(3.21) and (3.25) we may express V as,

$$V = \left(\frac{\partial W}{\partial z'_\lambda}\right)^* \mathcal{H}'_{ij} \left(\frac{\partial W}{\partial z'_j}\right) + \left\{ \left(\frac{\partial W}{\partial z'_\lambda}\right)^* G'_{ia} \left(z^\dagger T_a z + P_a\right) + h.c. \right\}$$

$$+ \left(z^\dagger T_a z + P_a\right) F'_{ab} \left(z^\dagger T_b z + P_b\right) \tag{3.33}$$

Using Eqs. (3.28) and (3.30) we may show that,

$$\frac{\partial^2 V}{\partial z'_\alpha \partial z'_i} = \frac{\partial^2 V}{\partial z'^*_\alpha \partial z'^*_i} = \frac{\partial^2 V}{\partial z'_\alpha \partial z'^*_i} = \frac{\partial^2 V}{\partial z'^*_\alpha \partial z'_i} = 0$$

$$\text{at } z = \tilde{z}^{(0)}, \quad \forall\, i, \alpha \tag{3.34}$$

showing that z_α' are massless complex scalar fields.

IV. CONCLUSION

In this paper we have analyzed the effect of radiative corrections in a general supersymmetric gauge theory, where the gauge group G is partially broken to one of its subgroups H at the tree level, but supersymmetry is unbroken. We have done our analysis in the background field gauge and our results are based on the assumption that the effective action in the background field gauge correctly reproduces all the physical results to all orders in perturbation theory. With this assumption we have shown that,

i)Supersymmetry is unbroken to all orders in perturbation theory.

ii)Although the radiative corrections may shift the vev of various scalar fields, the subgroup H is unbroken to all orders in perturbation theory.

iii) For every eigenstate of the tree level scalar mass matrix with zero eigenvalue, we have an eigenstate of the full renormalized mass matrix with zero eigenvalue.

One possible loophole in our analysis may lie in ignoring the possible effects of infrared divergences. When we calculate the effective potential, we keep only those terms which give non-vanishing contribution at zero external momenta. It is, however, possible that the infrared divergences may give rise to inverse powers of external momenta from loop integrals, which cancel some explicit powers of the external momenta in the numerator. Hence graphs, which are naively thought to vanish at zero external momenta may give rise to non-vanishing contribution to the effective potential. Usually, however, the power law infrared divergences are thought to be gauge artifacts, and are not expected to affect any gauge invariant result. A class of non-local gauges, proposed recently[7], may provide a solution to this problem.

Another possible loophole lies in the assumption that the effective action in the background field formalism correctly reproduces all the physical results. Although this is generally believed to be true, there is no rigorous proof to this effect.

If we ignore these two issues, the method used in our analysis, combined with the analysis of Ref.8, may be used for studying the stability of mass hierarchy in supersymmetric gauge theories when supersymmetry is explicitly broken at the tree level through soft terms. Work towards this end is in progress.

APPENDIX A

In this appendix we shall give some details of the Feynman rules for the class of theories discussed in the text. We choose the gauge fixing term to be,

$$- \xi \int d^4x \, d^4\theta \; \bar{\mathcal{F}}_a \mathcal{F}_a \tag{A.1}$$

where,

$$\mathcal{F}_a = \frac{1}{e} \left(\nabla^2 V^{(q)} \right)_a + \frac{e}{\xi} \sum_{i,j} \tilde{\bar{\Phi}}_i^{(b)} \left(T_a \right)_{ij} \left(\nabla^2 \frac{1}{\Box_+} \tilde{\Phi}^{(q)} \right)_j \tag{A.2}$$

$$\nabla_\alpha = e^{-V_a^{(b)} T_a/2} D_\alpha \, e^{V_a^{(b)} T_a/2} = D_\alpha - i \Gamma_\alpha^{(b)} \tag{A.3}$$

$$\bar{\nabla}_{\dot{\alpha}} = e^{V_a^{(b)} T_a/2} \bar{D}_{\dot{\alpha}} \, e^{-V_a^{(b)} T_a/2} = \bar{D}_{\dot{\alpha}} - i \bar{\Gamma}_{\dot{\alpha}}^{(b)} \tag{A.4}$$

$$\Box_+ = \Box - i W^{(b)\alpha} \nabla_\alpha - \frac{1}{2} i \left(\nabla^\alpha W_\alpha^{(b)} \right) \tag{A.5}$$

$$\Box = \frac{1}{2} \{ \nabla_\alpha , \bar{\nabla}_{\dot{\beta}} \} \{ \nabla^\alpha , \bar{\nabla}^{\dot{\beta}} \} \tag{A.6}$$

The first term on the right hand side of (A.2) is the usual gauge fixing term used for unbroken gauge theories. The second term is a generalization of the term used by Ovrut and Wess[3]. The term containing the product of $V^{(q)}$ and $\phi^{(q)}$

in (A.1) cancels a similar term in the original Lagrangian, coming from the $\bar{\tilde{\phi}}\exp(V_a^{(q)}T_a)\tilde{\phi}$ term, except for terms involving covariant derivatives of $\tilde{\phi}^{(b)}$, $\bar{\tilde{\phi}}^{(b)}$ fields. $\mathfrak{F}_a$, as defined in (A.2), is covariant under the background gauge transformation, under which the background fields transform as in Eq.(3.9), while the quantum fields transform as,

$$V_a^{(q)}T_a \rightarrow e^{iK_b(\Lambda)T^b} V_a^{(q)}T_a\, e^{-iK_c(\Lambda)T^c}$$

$$\phi^{(q)} \equiv e^{-V_a^{(b)}T_a/2}\,\tilde{\phi}^{(q)} \rightarrow e^{i\Lambda_a T_a}\,\phi^{(q)}$$

$$\bar{\phi}^{(q)} \equiv \bar{\tilde{\phi}}^{(q)}\, e^{-V_a^{(b)}T_a/2} \rightarrow \bar{\phi}^{(q)}\, e^{-i\bar{\Lambda}_a T_a} \qquad (A.7)$$

where the real superfield $K_a(\Lambda)$ is the solution of the equation,

$$e^{i\bar{\Lambda}_a T_a}\; e^{V_a^{(b)}T_a/2}\; e^{-iK_aT_a}$$

$$= e^{iK_aT_a}\; e^{V_a^{(b)}T_a/2}\; e^{-i\Lambda_a T_a} \qquad (A.8)$$

Thus the gauge fixing term (A.1) is invariant under the background gauge transformation. Then the Faddeev-Popov ghost term in the action, derived from (A.1) must also be invariant under the background gauge transformation. As a result, all the radiatively generated terms in the effective action must be background gauge invariant.

In order to derive the Feynman rules, we must separate the total tree level action into the 'interacting' part and the 'free' part. For this we define the shifted fields,

$$\hat{\phi}_i^{(b)} = \tilde{\phi}_i^{(b)} - z_i^{(0)}$$

$$\bar{\hat{\phi}}_i^{(b)} = \bar{\tilde{\phi}}_i^{(b)} - z_i^{(0)*} \qquad (A.9)$$

and replace $\tilde{\phi}_i^{(b)}$, $\bar{\tilde{\phi}}_i^{(b)}$ by $\hat{\phi}_i^{(b)}+z_i^{(0)}$ and $\bar{\hat{\phi}}_i^{(b)}+z_i^{(0)*}$ everywhere in the tree level action. We define the free part of the effective action to be,

$$S^{(2)}(V^{(q)}) + S^{(2)}(\phi_i^{(q)}, \bar{\phi}_i^{(q)})$$

$$= \frac{1}{2e^2}\int d^4x\, d^4\theta\, V_a^{(q)} \{\nabla^\alpha \bar{\nabla}^2 \nabla_\alpha - \xi(\nabla^2\bar{\nabla}^2 + \bar{\nabla}^2\nabla^2) + \mu^2\}_{ab} V_b^{(q)}$$

$$+ \int d^4x\, d^4\theta\, [\, \bar{\tilde{\phi}}_i^{(q)} \tilde{\phi}_i^{(q)} - \frac{e^2}{\xi}\{ z_i^{(0)\dagger} (T_a)_{ij} (\nabla^2 \frac{1}{\Box_+} \tilde{\phi}^{(q)})_j \}^\dagger$$

$$\{ z_k^{(0)\dagger} (T_a)_{k\ell} (\nabla^2 \frac{1}{\Box_+} \tilde{\phi}^{(q)})_\ell \}] + [\int d^4x\, d^2\theta\, \frac{1}{2} M_{ij}\, \phi_i^{(q)} \phi_j^{(q)}$$

$$+ h.c.] + \text{ghost terms} \qquad (A.10)$$

and call the rest of the action $S_{int.}$. Although S_{int} contains terms quadratic in the quantum fields, these terms are multiplied by explicit factors of $\Gamma_\alpha^{(b)}$, $W_\alpha^{(b)}$, $\hat{\phi}_i^{(b)}$ or their complex conjugate fields, and hence they may be treated as interaction terms.

The Feynman rules may be written down by considering the generating functional,

$$Z(J_a, J_i, \bar{J}_i)$$

$$= \exp\left\{ i S_{int}\left(-i \frac{\delta}{\delta J_a}, -i \frac{\delta}{\delta J_i}, -i \frac{\delta}{\delta \bar{J}_i}\right)\right\}$$

$$\int dV_a^{(q)}\, d\phi_i^{(q)}\, d\bar{\phi}_i^{(q)} \exp\left[i\left\{ S^{(2)}(V^{(q)}) + S^{(2)}(\phi_i^{(q)}, \bar{\phi}_i^{(q)})\right.\right.$$

$$\left.\left. + \int d^4x \left(\int d^4\theta\, J_a V_a^{(q)} + \int d^2\theta\, J_i \tilde{\phi}_i^{(q)} + \int d^2\bar{\theta}\, \bar{J}_i \tilde{\bar{\phi}}_i^{(q)}\right)\right\}\right] \tag{A.11}$$

where J_i, $\bar{J}_i$ are background covariantly chiral and anti-chiral currents satisfying $\bar{\nabla}_{\dot{\alpha}} J = \nabla_\alpha \bar{J} = 0$, and J_a is a real current. For simplicity of notation we have ignored the ghost fields here, they may be treated in the same way as the chiral superfields ϕ_i, $\bar{\phi}_i$. The simplest way to derive the Feynman rules would be to express all the background covariant derivatives in terms of the ordinary covariant derivatives and the fields $\Gamma_\alpha^{(b)}$, $\bar{\Gamma}_{\dot{\alpha}}^{(b)}$, $W_\alpha^{(b)}$ and $\bar{W}_{\dot{\alpha}}^{(b)}$, and express the fields $\tilde{\phi}_i$, $\tilde{\bar{\phi}}_i$ in terms of ϕ_i, $\bar{\phi}_i$ and $V_a^{(b)}$. We may then use the rules of functional differentiation in superspace to derive the Feynman rules, and use the set of manipulations given in Refs.2 and 4 to show that all the radiatively generated terms in the effective action must have the form of Eq.(3.10). However, the Feynman rules derived in this way have explicit dependence on $V_a^{(b)}$ and hence are not useful in proving the proposition (iii) of Sec.III.

We use the doubling trick of Ref.[4] to prove that the effective action does not have any explicit dependence on $V_\rho^{(b)}$ except through the connections, field strengths and the covariant scalar fields $\tilde{\phi}^{(b)}$ and $\bar{\tilde{\phi}}^{(b)}$. We have used this result in the text only in analyzing terms linear in D_ρ, hence we shall prove the result only for such terms. In doing so we may set all components of the background gauge field to zero, except the ones lying in the unbroken subgroup H. The various covariant derivatives ∇_α, $\bar{\nabla}_{\dot{\alpha}}$ are then covariant only with respect to the background gauge transformations with group elements lying in the subgroup H. Since $z_i^{(0)}$ and M_{ij} are invariant under H, we may replace $M_{ij}\phi_i\phi_j$ by $M_{ij}\tilde{\phi}_i\tilde{\phi}_j$ in Eq.(A.10), and $S^{(2)}(\phi_i^{(q)}, \bar{\phi}_i^{(q)})$ given in (A.10) is invariant under local gauge transformations belonging to the subgroup H. Doing some integrations by parts and using the relations,

$$\bar{\nabla}^2 \nabla^2 \tilde{\Phi} = \Box_+ \tilde{\Phi} \tag{A.12}$$

$$\bar{\nabla}_{\dot{\alpha}} \tilde{\Phi} = \nabla_\alpha \bar{\tilde{\Phi}} = 0 \tag{A.13}$$

we get,

$$S^{(2)}(\phi_i^{(q)}, \bar{\phi}_i^{(q)}) = \int d^4x\, d^4\theta \,[\, \bar{\tilde{\phi}}_i^{(q)} \tilde{\phi}_i^{(q)}$$

$$- \frac{e^2}{\xi} \bar{\tilde{\phi}}_i^{(q)} (T_a)_{ij}\, Z_j^{(0)}\, Z_k^{(0)\dagger} (T_a)_{k\ell} \left(\frac{1}{\Box_+} \tilde{\Phi}^{(q)}\right)_\ell$$

$$+ \{ \tfrac{1}{2} M_{ij}\, \tilde{\phi}_i^{(q)} \left(\nabla^2 \frac{1}{\Box_+} \tilde{\Phi}^{(q)}\right)_j + \text{h.c.} \}]$$

$$\equiv \int d^4x\, d^4\theta\, (\tilde{\phi}_i^{(q)} \quad \bar{\tilde{\phi}}_i^{(q)})\, \Delta_{ij}^{(\phi)} \begin{pmatrix} \tilde{\phi}_j^{(q)} \\ \bar{\tilde{\phi}}_j^{(q)} \end{pmatrix} \qquad (A.14)$$

where $\Delta_{ij}^{(\phi)}$ is a 2×2 matrix for fixed i, j. Also we may express $S^{(2)}(V^{(q)})$ as,

$$S^{(2)}(V^{(q)})$$

$$= \int d^4x\, d^4\theta \left(-\frac{1}{2e^2}\right) V_a^{(q)} (\Box - i W^\alpha \nabla_\alpha - i \bar{W}^{\dot\alpha} \bar{\nabla}_{\dot\alpha}$$

$$- (1-\xi)(\nabla^2 \bar{\nabla}^2 + \bar{\nabla}^2 \nabla^2) - \mu^2)_{ab} V_b^{(q)}$$

$$\equiv \frac{1}{2} \int d^4x\, d^4\theta\, V_a^{(q)}\, \Delta_{ab}^{(V)}\, V_b^{(q)} \qquad (A.15)$$

We may now write (A.11) as,

$$Z(J_a, J_i, \bar{J}_i) = \Delta_{\text{Gauge}}\, \Delta_{\text{chiral}}$$

$$\times \left[\exp\{ i S_{int}\left(-i \frac{\delta}{\delta J_a}, -i \frac{\delta}{\delta J_i}, -i \frac{\delta}{\delta \bar{J}_i}\right) \right.$$

$$\exp\Big\{ -i\, J_a \left(\Delta^{(v)}\right)^{-1}_{ab} J_b \;-\; i\left(\left(\nabla^2 \frac{1}{\Box_+} J\right)_i , \left(\overline{\nabla}^2 \frac{1}{\overline{\Box}_+} \overline{J}\right)_i \right) \left(\Delta^{(\phi)}\right)^{-1}_{ij} \begin{pmatrix} \left(\nabla^2 \frac{1}{\Box_+} J\right)_j \\ \left(\overline{\nabla}^2 \frac{1}{\overline{\Box}_+} \overline{J}\right)_j \end{pmatrix} \Big\} \tag{A.16}$$

where,

$$\Delta_{Gauge} = \int dV_a^{(q)} \exp\left(i S^{(2)}\left(V^{(q)}\right)\right) \tag{A.17}$$

$$\Delta_{Chiral} = \int d\phi_i^{(q)}\, d\overline{\phi}_i^{(q)} \exp\left(i S^{(2)}\left(\phi_i^{(q)}, \overline{\phi}_i^{(q)}\right)\right) \tag{A.18}$$

Δ_{gauge} and Δ_{chiral} denote one loop contributions from loops of gauge fields and chiral superfields respectively, with only background gauge fields as external lines.

The Feynman rules for evaluating the J dependent part on the right hand side of (A.16) are obtained by using the rules,

$$\frac{\delta}{\delta J_i(x,\theta)} J_j(x',\theta') = \delta_{ij}\, \delta^{(4)}(x-x')\, \overline{\nabla}^2 \delta^{(4)}(\theta-\theta') \tag{A.19}$$

$$\frac{\delta}{\delta \overline{J}_i(x,\theta)} \overline{J}_j(x',\theta') = \delta_{ij}\, \delta^{(4)}(x-x')\, \nabla^2 \delta^{(4)}(\theta-\theta') \tag{A.20}$$

$$\frac{\delta}{\delta J_a(x,\theta)} J_b(x',\theta') = \delta_{ab}\, \delta^{(4)}(x-x')\, \delta^{(4)}(\theta-\theta') \tag{A.21}$$

$$\frac{\delta J_i}{\delta \bar{J}_j} = \frac{\delta J_i}{\delta J_a} = \frac{\delta \bar{J}_i}{\delta J_j} = \frac{\delta \bar{J}_i}{\delta J_a} = \frac{\delta J_a}{\delta J_i} = \frac{\delta J_a}{\delta \bar{J}_i} = 0 \tag{A.22}$$

Also, in the actual computation we expand $(\Delta^{(V)})^{-1}$ and $(\Delta^{(\phi)})^{-1}$ about the point $V^{(b)}=0$ so that $\Delta^{-1}(V^{(b)})-\Delta^{-1}(V^{(b)}=0)$ term appears as an interaction term with background gauge fields as external lines. A further simplification occurs due to the fact that the fields which receive their mass from the M_{ij} term do not receive any mass from mixing with the gauge fields, and vice versa, as was shown in Sec.II. Since S_{int}, $\Delta_{ij}^{(\phi)}(V^{(b)})$ and $\Delta_{ab}^{(V)}(V^{(b)})$ depend on $V_\rho^{(b)}$ only through the connections, field strengths and the fields $\tilde{\phi}^{(b)}$, $\tilde{\tilde{\phi}}^{(b)}$, the Feynman rules for evaluating the J dependent part of (A.16) also depends on $V^{(b)}$ only through these quantities. The same result is true for the $V^{(b)}$ dependent part of Δ_{gauge}, which may be seen by writing Δ_{gauge} as,

$$\Delta_{Gauge} = \Big[\exp\Big\{ i\int d^4x\, d^4\theta\, \frac{\delta}{\delta J_a} \left(\Delta_{ab}^{(V)}(V^{(b)}) - \Delta_{ab}^{(V)}(V^{(b)}=0)\right) \frac{\delta}{\delta J_b}\Big\}$$
$$\exp\Big\{ -iJ_a \left(\Delta^{(V)}(V^{(b)}=0)\right)^{-1}_{ab} J_b \Big\}\Big]_{J_a=0} \tag{A.23}$$

Since $\Delta_{ab}^{(V)}(V^{(b)})$ does not have any explicit dependence on $V^{(b)}$, the Feynman rules for evaluating $V^{(b)}$ are also free from explicit dependence on $V^{(b)}$.

The same trick cannot be applied for Δ_{chiral}, since $\tilde{\phi}_i$ and $\tilde{\tilde{\phi}}_i$ have explicit dependence on $V^{(b)}$ when expressed in terms of the ordinary chiral and anti-chiral superfields ϕ_i and $\bar{\phi}_i$. We must use the doubling trick of Ref.4 to bring this contribution into a covariant form. Since the covariant derivatives ∇_α and $\bar{\nabla}_{\dot{\alpha}}$ are covariantized only with respect to the subgroup H, the doubling trick may be used for any real (reducible) representation of H. This includes all the chiral superfields which get their mass through their mixing with the gauge fields, since the broken generators of the gauge group form a real representation of H. Let us define,

$$(\mathcal{M}^2)_{i\ell} = e^2 (T_a)_{ij}\, Z_j^{(0)}\, Z_k^{(0)\dagger} (T_a)_{k\ell} \qquad (A.24)$$

so that,

$$S^{(2)}(\Phi_i^{(q)}, \bar{\Phi}_i^{(q)}) = \int d^4x\, d^4\theta\, [\, \tilde{\bar{\Phi}}_i^{(q)}\, \tilde{\Phi}_i^{(q)}$$

$$- \frac{1}{3}\, \tilde{\bar{\Phi}}_i^{(q)} (\mathcal{M}^2)_{ij} \left(\frac{1}{\Box_+}\, \tilde{\Phi}^{(q)}\right)_j + \{\tfrac{1}{2} M_{ij}\, \tilde{\Phi}_i^{(q)} \left(\nabla^2 \frac{1}{\Box_+}\, \tilde{\Phi}^{(q)}\right)_j + h.c.\}]$$

$$(A.25)$$

Using the doubling trick, we may bring Δ_{chiral} in the form,

$$\Delta_{chiral} = \int d\phi_i^{(q)} \exp\left(\frac{i}{2} \int d^4x\, d^2\theta\, \tilde{\Phi}_i^{(q)}\, \tilde{\Delta}_{ij}^{(\phi)}\, \tilde{\Phi}_j^{(q)}\right)$$

$$(A.26)$$

where,

$$\tilde{\Delta}^{(\phi)}_{ij} = \left[\bar{\nabla}^2 \left(I - \frac{\mathcal{M}^2}{\xi} \frac{1}{\Box_+} \right) \nabla^2 \left(I - \frac{\mathcal{M}^2}{\xi} \frac{1}{\Box_+} \right) - M^2 \right]_{ij} \tag{A.27}$$

The Feynman rules for Δ_{chiral} may now be derived by writing (A.26) as

$$\Delta_{chiral} = \Big[\exp\Big\{ \frac{i}{2} \int d^4x\, d^2\theta \left\{ \tilde{\Delta}^{(\phi)}_{ij}(V^{(b)}) - \tilde{\Delta}^{(\phi)}_{ij}(V^{(b)}=0) \right\} \left(-i \frac{\delta}{\delta J_i}\right) \left(-i \frac{\delta}{\delta J_j}\right) \Big\} \exp\Big\{ -\frac{i}{2} \int d^4x\, d^2\theta \left(\tilde{\Delta}^{(\phi)}(V^{(b)}=0) \right)^{-1}_{ij} J_i J_j \Big\} \Big]_{J=0} \tag{A.28}$$

Since $\tilde{\Delta}^{(\phi)}$ depends on $V^{(b)}$ only through its dependence on connections and field strengths, so must be Δ_{chiral}. Thus if all the chiral superfields transform according to real representations of H, terms in the effective action linear in D_ρ do not depend explicitly on $V_\rho^{(b)}$. In the presence of chiral superfields transforming according to a complex representation of H, we could get radiatively generated terms which depend on $V^{(b)}$ explicitly. These terms come from one loop contribution involving chiral superfields, with only background gauge fields as external lines.

However, since H is the unbroken symmetry group of the theory, a chiral superfield belonging to the complex representation of H must be massless. Hence contribution from such one loop graphs linear in the background gauge field involves massless tadpoles, which vanish in the dimensional regularization scheme. As a result, even in the presence of chiral superfields in the complex representation of H, the terms in the effective action linear in the background gauge field do not have explicit dependence on $V_{\rho}^{(b)}$.

REFERENCES

[1] B. Zumino, Nucl. Phys. **B89**, 535 (1975); P. West, ibid. **B106**, 219 (1976); D. M. Capper and M. Ramon Medrano, J. Phys. **G2**, 269 (1976); W. Lang, Nucl. Phys. **B114**, 123 (1976); S. Weinberg, Phys. Lett. **62B**, 111 (1976); E. Witten, Nucl. Phys. **B188**, 513 (1981).

[2] S. Ferrara and O. Piguet, Nucl. Phys. **B93**, 261 (1975); M. Grisaru, M. Rocek and W. Siegel, Nucl. Phys. **B159**, 429 (1979).

[3] B. Ovrut and J. Wess, Phys. Rev. **D25**, 409 (1982).

[4] S. Gates, M. Grisaru, M. Rocek and W. Siegel, 'Superspace or one thousand and one lessons in supersymmetry', Benjamin Pub. 1983.

[5] J. Wess and B. Zumino, Nucl. Phys. **B78**, 1 (1974).

[6] L. Hall, J. Lykken and S. Weinberg, Phys. Rev. **D27**, 2359 (1983).

[7] L. Abbott, M. Grisaru and D. Zanon, Brandeis Univ. report No. BRX-TH-160.

[8] A. Sen, Fermilab report No. FERMILAB-PUB-84/85-T, to appear in Phys. Rev. D.