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A Non-perturbative Description of the Gimon-Polchinski Orientifold

Ashoke Sen[1 2]

Mehta Research Institute of Mathematics
and Mathematical Physics
10 Kasturba Gandhi Marg, Allahabad 211002, INDIA

Abstract

A T-dual version of the Gimon-Polchinski orientifold can be described by a configuration of intersecting Dirichlet seven branes and orientifold seven planes in the classical limit. We study modification of this background due to quantum corrections. It is shown that non-perturbative effects split each orientifold plane into a pair of nearly parallel seven branes. Furthermore, a pair of intersecting orientifold planes, instead of giving rise to two pairs of intersecting seven branes, gives just one pair of seven branes, each representing a pair of nearly orthogonal seven branes smoothly joined to each other near the would be intersection point. Interpretation of these results from the point of view of the dynamics on a three brane probe is also discussed.

[1]On leave of absence from Tata Institute of Fundamental Research, Homi Bhabha Road, Bombay 400005, INDIA

[2]E-mail: sen@mri.ernet.in, sen@theory.tifr.res.in

1 Introduction and Summary

Orientifolds[1, 2] are generalization of orbifolds where the group by which we mod out the theory includes a world-sheet parity transformation, possibly in conjunction with some other internal and / or space-time symmetry transformation. These models have been of interest lately since they give rise to many new classes of string compactifications, some of which are related to more conventional string compactifications by strong-weak coupling duality transformation. In particular, type I theory, which can be viewed as the quotient of type IIB theory by the world-sheet parity transformation, has been conjectured to be dual to SO(32) heterotic string theory[3].

Much can be learned about orientifolds by compactifying type I theory on a torus and then making $R \to (1/R)$ duality transformation on some / all directions of the torus. Under this T-duality transformation, the world-sheet parity transformation gets converted to a new transformation which is a combination of the world-sheet parity transformation, change of sign of some of the space coordinates, and possibly some internal symmetry transformation. The hyperplanes in space time which are left invariant by the space-time part of the transformation are known as orientifold planes. These orientifold planes turn out to carry charge under a gauge field originating in the Ramond-Ramond (RR) sector of the theory. In order to cancel this charge, we need to place appropriate number of Dirichlet branes[2] (D-branes) − which are also known to carry RR charges[4] − parallel to the orientifold planes.

A special class of orientifolds, obtained by compactifying type I theory on a two dimensional torus, and then performing $R \to (1/R)$ duality transformation in both circles, was analysed in detail in ref.[5]. This T-dual version can be obtained by modding out type IIB theory on T^2 by a combination of the world-sheet parity transformation, an internal symmetry transformation that changes the sign of all Ramond sector states in the left moving sector of the string world-sheet, and a space-time transformation that changes the sign of both coordinates of the torus. The resulting theory contains four orientifold seven planes transverse to the torus directions, each carrying -4 units of RR charge, and sixteen Dirichlet seven branes parallel to them cancelling their RR charges. It was shown in ref.[5] that while this picture is valid in the classical limit, and also to all orders in the open string perturbation theory, non-perturbative corrections change this picture. In particular, each orientifold plane splits into two seven branes, which are related to the ordinary Dirichlet seven branes by SL(2,Z) transformation of the type IIB theory.

In [6] Gimon and Polchinski constructed a more complicated class of orientifold models describing N=1 supersymmetric theories in six dimensions. (See also ref.[7] for an earlier construction of these models in a different formalism.) These theories are obtained by compactifying type IIB theory on a four dimensional torus, and modding out the theory by a $Z_2 \times Z_2$ symmetry, with the first Z_2 generated by the world-sheet parity transformation, and the second Z_2 generated by a geometric transformation that changes the sign of all the coordinates of T^4 (which we shall denote by $x^6, \ldots x^9$). The result is a configuration of orientifold five planes and nine planes (filling up the whole space-time), and Dirichlet five branes and nine branes. We shall analyse a T-dual version of this model, obtained by making an $R \to (1/R)$ duality in two of the coordinates of the torus (which we shall take to be x^6 and x^7 for definiteness). The effect of this T-duality transformation is to make the orientifold nine plane and the Dirichlet nine branes into orientifold seven planes and Dirichlet seven branes transverse to x^6 and x^7. On the other hand the orientifold five planes and Dirichlet five branes are also transformed into orientifold seven planes and Dirichlet seven branes respectively, but these are transverse to x^8 and x^9. Thus the result is a set of intersecting orientifold seven planes and Dirichlet seven branes.

The question that we ask is again, how does the non-perturbative effects in the orientifold theory modify this picture? For a class of Gimon-Polchinski models we are able to answer this question by analysing various consistency requirements. The basic idea, as in ref.[5],is to use these consistency conditions to determine the type IIB string coupling (the axion-dilaton modulus) as a function of the space-time coordinates. This in turn gives us the locations of the seven branes in space-time. We find that these corrections do not modify the configuration of intersecting Dirichlet seven branes, but it does modify the configuration of orientifold planes. First of all, as in ref.[5] each of the orientifold planes splits into two seven branes. Thus naively we would expect that a pair of intersecting orientifold planes will be described by two pairs of intersecting seven branes. What we find instead is that a pair of such seven branes, which are asymptotically orthogonal to each other, now join smoothly near the expected point of intersection[3] to give one smooth seven brane. Thus a pair of intersecting orientifold planes gets transformed to pair of seven branes. In the general case these two seven branes further join smoothly at their

[3]Throughout the paper, when we refer to the 'point of intersection' of two seven branes, we really mean a five dimensional submanifold.

would be intersection points to give a single seven brane; but in special cases they can remain as two distinct seven branes.

Our results can also be interpreted as giving information about the non-perturbative dynamics on a three brane probe in this orientifold background[9]. In particular it gives the U(1) gauge coupling on this three brane probe as a function of various moduli. At the classical level the N=1 supersymmetric world volume field theory on this three brane probe has some novel features. For example, at a generic point in the moduli space, there is a single U(1) gauge group, which gets enhanced to SU(2) in different regions of the moduli space. If we bring this three brane near the point of intersection of the two orientifold planes, then in the classical limit the three brane world volume theory seems to contain two different SU(2) gauge groups sharing this single U(1) group. The classical Lagrangian for this system, based on an $SU(2) \times SU(2)$ gauge group, can be constructed along the lines of ref.[8]. Once quantum corrections are taken into account, however, the $SU(2) \times SU(2)$ symmetry is not restored near the region of intersection of the two orientifold planes. Instead, in the infrared limit, the low energy theory is an N=2 superconformal field theory with U(1) vector multiplet, and a massless charged hypermultiplet.

The rest of the paper is organised as follows. In section 2 we describe the T-dual version of the Gimon-Polchinski model that will be the focus of our attention. This gives a description of the background in the classical limit. In section 3 we show how non-perturbative quantum corrections modify this picture.

2 A T-Dual Version of Gimon-Polchinski Orientifold

The Gimon-Polchinski orientifold[6] is obtained by modding out type IIB string theory on T^4 by a $Z_2 \times Z_2$ symmetry. If we denote by Ω the world-sheet parity transformation, and by $\mathcal{I}_{6789}$ the transformation that changes the sign of all four coordinates x^6, x^7, x^8, x^9 labelling the torus, then the first Z_2 is generated by Ω and the second Z_2 is generated by $\mathcal{I}_{6789} \cdot \Omega$. If we now perform an $R \to (1/R)$ duality transformation in x^6 and x^7 directions then the generators of the two Z_2 transformations get transformed to

$$g \equiv \mathcal{I}_{67} \cdot (-1)^{F_L} \cdot \Omega, \qquad \text{and} \qquad h \equiv \mathcal{I}_{89} \cdot (-1)^{F_L} \cdot \Omega, \tag{2.1}$$

respectively, where $\mathcal{I}_{mn}$ denotes the transformation $x^m \to -x^m$, $x^n \to -x^n$, and $(-1)^{F_L}$ is the transformation that changes the sign of all the Ramond sector states of the left-moving sector of the string world sheet without affecting the right-moving and/or Neveu-Schwarz sector states. This is the model on which we shall focus our attention. Note that this description only specifies the action of the Z_2 transformation on the untwisted sector closed string states. Different models have been constructed by exploiting the ambiguity in the action of the Z_2 transformations on the twisted sector and open string states[10, 11, 12] (see also [13, 14]), but for us the action of the Z_2 transformation on all the states is fixed completely by demanding that this is the T-dual of the model discussed in ref.[6]. We shall concentrate on the sector of the theory that is connected to the $U(16)\times U(16)$ point, *i.e.* the sector with no half five-branes in the language of refs.[6, 15].

To start with, let us set up some notations. We define complex coordinates on T^4

$$w = x^6 + ix^7, \qquad\qquad z = x^8 + ix^9. \tag{2.2}$$

These coordinates change sign under g and h respectively. We also introduce coordinates

$$u = w^2, \qquad\qquad v = z^2, \tag{2.3}$$

which are single valued on the orientifold.

The theory described above has $N = 1$ supersymmetry in six dimensions. The spectrum of massless twisted sector / open string states in this theory, as determined in ref.[6], is as follows:

1. For each of the sixteen points on the torus fixed under gh we get a hypermultiplet of the $N = 1$ supersymmetry algebra from closed string states twisted under gh.

2. Due to the modding out by g, each of the four orientifold seven planes fixed under $\mathcal{I}_{67}$ carries -4 units of charge under an RR gauge field. This charge is cancelled by putting 16 ($= 4 \times 4$) Dirichlet seven branes transverse to the 67 plane. This guarantees that the RR charge is neutralised globally. It is neutralised locally when the 16 D- seven branes are grouped into four groups of four each, and each group is localised at an orientifold plane. In order to describe the spectrum of massless open string states for such a configuration, it is best to focus on one of the orientifold planes, since each of them gives identical spectrum. If we ignore the projection by the group element h, then the open string states starting and ending on these D-branes and their images under g give rise to $SO(8)$ gauge fields, and a complex scalar (representing the motion of the seven brane in two transverse directions) in

the adjoint representation of $SO(8)$. However, projection by h acts on this gauge group as a gauge transformation

$$\mathcal{M} = \begin{pmatrix} M_4 & 0 \\ 0 & -M_4 \end{pmatrix} , \tag{2.4}$$

where

$$M_{2n} = \begin{pmatrix} 0 & I_n \\ -I_n & 0 \end{pmatrix} , \tag{2.5}$$

I_n being the $n \times n$ identity matrix. Note that our notation differs from that of [6, 15] by a simple rearrangement of basis. This breaks $SO(8)$ to a group that commutes with (2.4), *i.e.* to $U(4)$. We shall denote this group by $U(4)_v$ since it lives on the seven branes transverse to the u-plane, *i.e.* parallel to the v-plane. One loop anomaly effect breaks this further to $SU(4)_v$[15], but we shall continue to refer to the corresponding gauge groups by $U(4)_v$ rather than $SU(4)_v$ keeping in mind that we are refering to the unbroken gauge symmetry *in the classical limit.*

The action of h on the adjoint complex scalar fields (which we shall denote by ϕ_v) living on these seven branes takes the form

$$\phi_v(z) \to -\mathcal{M}\phi_v(-z)\mathcal{M}^{-1} . \tag{2.6}$$

For $z(\equiv \sqrt{v})$ independent ϕ_v, this gives two complex massless scalars in the **6** representation of $SU(4)_v$, carrying $U(1)_v$ charge ± 1. These complex scalars, together with the components of the $U(4)_v$ gauge fields along the v plane which survive the h projection, and the fermionic open string states, give rise to two hypermultiplets of the N=1 supersymmetry algebra in six dimensions.

We shall consider breaking the gauge group further by giving vacuum expectation value (vev) to ϕ_v of the form:

$$\langle \phi_v \rangle = \begin{pmatrix} 0 & \mathbf{m_d} \\ -\mathbf{m_d} & 0 \end{pmatrix} \tag{2.7}$$

where

$$\mathbf{m_d} = \begin{pmatrix} m_1 & & & \\ & m_2 & & \\ & & m_3 & \\ & & & m_4 \end{pmatrix} . \tag{2.8}$$

Physically this corresponds to moving the four seven branes away from the orientifold plane, with $\pm m_i$ denoting the locations of the seven branes and their images under g in the w plane. The projection (2.6) now requires

$$m_3 = m_1, \qquad m_4 = m_2. \tag{2.9}$$

In other words, the four seven branes, instead of being allowed to move freely, can move only in pairs.[4] This vev breaks the $U(4)_v$ gauge group to $SU(2)_v \times SU(2)'_v$. A special case of this is $m_1 = m_2$; in this case the unbroken gauge group is $Sp(4)_v$.

As has already been pointed out, due to the presence of four orientifold planes transverse to u plane, the above story is repeated four times. Thus, for example, in the case where four seven branes (and their images) sit at each orientifold plane, the net gauge group arising from the D-branes transverse to the u-plane is $(U(4)_v)^4$.

3. The same story is repeated for the orientifold plane and the seven branes transverse to the v plane. Thus, for example, each of the four orientifold planes fixed under $\mathcal{I}_{89}$ carries -4 units of RR charge which is cancelled by placing 16 D- seven branes parallel to these orientifold planes. The charge is neutralised locally when each orientifold plane has four D-branes on top of it. In this case the gauge group associated with each orientifold plane is again $U(4)$, which we shall denote by $U(4)_u$. This can be broken to $SU(2)_u \times SU(2)'_u$ by pulling the seven branes away from the orientifold planes in pairs, which can be interpreted as due to the vev of two hypermultiplets in the **6** representation of $SU(4)_u$. Again a special case of this, when the four D-branes coincide, but are away from the orientifold plane, is $Sp(4)_u$.

4. Finally there are open string states starting on a seven brane parallel to the u plane and ending on a seven brane parallel to the v plane. At the $(U(4)_v)^4 \times (U(4)_u)^4$ point these hypermultiplets transform in the $(4,4)$ representation for each pair of $(U(4)_v, U(4)_u)$. By giving vev to these hypermultiplets it is possible to break the gauge group completely, but we shall not discuss this branch of the vacuum configuration in much detail.

[4]Actually, if we take into account possible v dependence of ϕ_v, we see that the projection (2.6) only requires a pair of seven branes to be joined togeter at $v = 0$. But if we want the seven branes to be parallel, this requires that they must coincide for all v.

In order to simplify our analysis, we shall focus on the physics of only one orientifold plane (together with four D-branes) parallel to the u plane intersecting only one orientifold plane (together with four D-branes) parallel to the v plane. This is in the spirit of the analysis of ref.[5] and captures much of the essential physics of the problem. Such a configuration is obtained if we do not compactify the type IIB theory on T^4 but simply mod out the type IIB theory in (9+1) dimensional flat space-time by the $Z_2 \times Z_2$ symmetry generated by g and h, and put appropriate D-branes to neutralise all RR charges globally. In this theory the maximal gauge symmetry group in the classical limit is $U(4)_v \times U(4)_u$. This can be broken to $SU(2)_v \times SU(2)'_v \times SU(2)_u \times SU(2)'_u$ by expectation values of complex scalar fields in the **6** representation of both $SU(4)$'s. Four complex parameters m_1, m_2, m'_1, m'_2 characterise the vacuum expectation values of these fields. In the special case $m_1 = m_2$, $SU(2)_v \times SU(2)'_v$ gets enhanced to $Sp(4)_v$. Similarly for $m'_1 = m'_2$, $SU(2)_u \times SU(2)'_u$ gets enhanced to $Sp(4)_u$.

Besides these four complex parameters, we need a few others to completely characterise the vacuum. One of them is the asymptotic value τ of the axion-dilaton field:

$$\lambda = \phi + ie^{-\Phi/2} \,, \tag{2.10}$$

where ϕ denotes the RR scalar field of the type IIB theory and Φ denotes the dilaton field. Thus

$$\tau = \lim_{\substack{u\to\infty \\ v\to\infty}} \lambda(u, v) \,. \tag{2.11}$$

We set the asymptotic metric to be $\eta_{\mu\nu}$ by using the freedom of general coordinate transformation in the (9+1) dimensional theory; thus we do not get any extra parameter from the metric. The only other parameters (besides vev of hypermultiplets in the (4,4) representation of $U(4)_v \times U(4)_u$ which we are setting to zero) are the ones associated with the vev of massless closed string fields originating in the sector twisted by gh. These correspond to the blow up modes of the orbifold singularilty at the intersection of the two orientifold planes, which we shall take to be the point ($u = 0$, $v = 0$). However, as was shown in ref.[15], due to anomaly effects these modes acquire mass at one loop order if the unbroken gauge group at the classical level contains at least one $U(1)$ factor. Thus we expect these modes to be present only if both the $U(1)$ factors are broken at the classical level by Higgs mechanism. In particular if $m_1 = m_2 = 0$ or if $m'_1 = m'_2 = 0$, then these modes should be absent.

3 Non-perturbative Description of the Model

3.1 The Problem

So far we have described the model in the weak coupling limit. We now want to give a non-perturbative description of the background for this orientifold. For this we focus our attention on the variation of the dilaton-axion modulus field λ as a function of the coordinates u and v. In the classical limit $\lambda \to i\infty$ near the Dirichlet seven branes, and to $-i\infty$ near the orientifold planes[5]. This however cannot be the true in the full quantum theory since it follows from the definition (2.10) of λ that the imaginary part of λ must be positive. Thus quantum effects must modify λ. The question that we shall be asking is: what is the fully quantum corrected $\lambda(u, v)$? This function must satisfy the following two requirements:

- $Im(\lambda)$ must be positive everywhere in the complex u, v plane.
- λ must be single valued in the u, v plane up to an SL(2,Z) transformation of the form:

$$\lambda \to \frac{p\lambda + q}{r\lambda + s}, \qquad p, q, r, s \in Z, \qquad ps - qr = 1. \tag{3.1}$$

This means that for every point in the complex (u, v) plane we have a torus whose modular parameter is given by λ. This torus is described by an equation of the form:

$$y^2 = x^3 + f(u, v)x + g(u, v)\,, \tag{3.2}$$

with λ being related to f and g through the relation:

$$j(\lambda) = \frac{4.(24f)^3}{4f^3 + 27g^2}\,. \tag{3.3}$$

$j(\lambda)$ is the modular function with a simple pole at $\lambda = i\infty$. Thus in order to determine λ we need to determine the functions $f(u, v)$ and $g(u, v)$. Note that $j(\lambda)$ blows up at the zeroes of

$$\Delta(u, v) = 4f^3 + 27g^2\,. \tag{3.4}$$

The locations of the zeroes of Δ can be identified as the locations of seven branes in the full non-perturbative background.

One of the guiding principles in our attempt at determining f and g will be the fact that as $u \to \infty$ at fixed v, the influence of the orientifold plane and the D-branes parallel to the v plane, situated near $u = 0$, must disappear and the answer should reduce to the known answer for a configuration of a single orientifold seven plane and four Dirichlet seven branes parallel to the u-plane[5]. This gives for large u

$$\begin{aligned} f(u,v) &\simeq \phi^2(u) f_{SW}(v; m_1', m_2', m_1', m_2', \tau) \\ g(u,v) &\simeq \phi^3(u) g_{SW}(v; m_1', m_2', m_1', m_2', \tau) \end{aligned} \tag{3.5}$$

where $\phi(u)$ is an arbitrary function of u and f_{SW} and g_{SW} are the functions that appear in describing the Seiberg-Witten curve[16] for the N=2 supersymmetric SU(2) gauge theory with four hypermultiplets in the fundamental representation[5]. Similarly for large v we must have

$$\begin{aligned} f(u,v) &\simeq \widetilde{\phi}^2(v) f_{SW}(u; m_1, m_2, m_1, m_2, \tau) \\ g(u,v) &\simeq \widetilde{\phi}^3(v) g_{SW}(u; m_1, m_2, m_1, m_2, \tau) \end{aligned} \tag{3.6}$$

where $\widetilde{\phi}$ is an arbitrary function. Since f_{SW} and g_{SW} are polynomials of degree two and three respectively, this motivates us to look for polynomial solutions for f and g, with f quadratic in both u and v and g cubic in both u and v. In this case the vacuum described by eq.(3.2) appears to be very similar to an F-theory compactification of type IIB theory[17, 18], but there are some differences which we shall point out later.

The problem of finding f and g also has an interpretation in terms of non-perturbative dynamics on a three brane world-volume theory in the spirit of refs.[9, 20, 21]. For this let us consider probing this configuration by a three brane lying in the 0123 plane. The world-volume theory on the three brane has an $N = 1$ space-time supersymmetry, with the three brane coordinates u, v and $x^4 + ix^5$ serving as scalar components of chiral superfields U, V and Φ respectively. Of these fields Φ decouples from the rest of the dynamics, so we shall focus on U and V. Besides these chiral superfields, for generic u and v the three brane world-volume theory also has a U(1) gauge multiplet. If W_α denotes the chiral superfield representing the gauge field strength, then the low energy effective field theory contains a gauge kinetic term of the form:

$$\int d^4x \int d^2\theta \, \lambda(U,V) W_\alpha W^\alpha \, + \, c.c. \, , \tag{3.7}$$

where the function $\lambda(u,v)$ is the same function that we have been trying to determine. Thus the problem that we are trying to address can also be formulated as the problem of determining the effective gauge coupling on the three brane world-volume theory.

The classical limit of this three brane world volume theory can be analysed along the lines of ref.[8]. For example, in the limit $u \to 0$, the U(1) gauge symmetry on the three brane world volume theory gets enhanced to SU(2), with the open string states streched between the three brane and its image under $\mathcal{I}_{67} \cdot (-1)^{F_L} \cdot \Omega$ becoming massless. Let us denote this group by $SU(2)_1$. On the other hand, for $v \to 0$ the same U(1) gauge group is enhanced to another SU(2), whose massless charged gauge bosons correspond to open string states stretched between the three brane and its image under $\mathcal{I}_{89} \cdot (-1)^{F_L} \cdot \Omega$. This is clearly a different SU(2) group, at least in this classical limit. Let us denote this by $SU(2)_2$. In particular in the limit $u \to 0$, $v \to 0$ we shall have the same U(1) group shared by two different SU(2) groups! As shown in ref.[8], the unbroken gauge symmetry in this limit is $SU(2) \times SU(2)$. Later we shall see how non-perturbative effects modify this picture.

3.2 The Solution

3.2.1 $U(4)_v \times U(4)_u$ point

This point in the moduli space is the easiest to describe, since the RR charge is neutralised locally, and hence we expect $\lambda(u,v)$ to be a constant independent of u and v. From eq.(3.3) we see that this requires f^3/g^2 to be a constant. Since f and g are assumed to be polynomials of degree 2 and 3 respectively in u and v, the most general solution is

$$f(u,v) = \alpha u^2 v^2, \qquad g(u,v) = \beta u^3 v^3, \tag{3.8}$$

where we have used the freedom of shifting u and v by constants to bring the zeroes of f and g to $u = v = 0$. The constant value of λ, which we shall denote by τ, is given by the equation:

$$j(\tau) = \frac{4.(24\alpha)^3}{4\alpha^3 + 27\beta^2} \,. \tag{3.9}$$

One of the constants α and β can be absorbed in a rescaling of the form

$$f \to k^2 f, \qquad g \to k^3 g \,, \tag{3.10}$$

for any constant k. This leaves the ratio f^3/g^2 fixed. Thus this vacuum is characterised by only one complex parameter τ as expected.

Note that the background described by (3.8) looks very similar to an F-theory background with a pair of intersecting D_4 singularities. However there are some differences.

First of all, as we move around the origin in the u (v) plane, there is an $SO(8)_u$ ($SO(8)_v$) monodromy given by the $SO(8)$ matrix $\mathcal{M}$ defined in (2.4). This breaks the gauge group to $U(4)_u$ ($U(4)_v$). In the language of ref.[18, 19] this would correspond to an inner automorphism of $SO(8)$, which is normally taken to be absent in conventional F-theory background. Furthermore, in the conventional F-theory vacuum, an intersection of two D_4 singularities produces a collapsed two cycle, and hence a tensionless string associated with a three brane wrapped around the two cycle[18]. In the present case the collapsed two cycle is associated with the Z_2 orbifold singularity obtained by modding out R^4 by the group element gh. However, as was shown by Aspinwall[22], in the conformal field theory orbifold we have half unit of the $B_{\mu\nu}$ flux through the collapsed two cycle, so that the Kahler class associated with the two cycle, instead of vanishing, is purely imaginary. For type IIA theory, this prevents the masses of two branes wrapped on the two cycle from vanishing. By T-duality we expact the same mechanism to prevent the tension of the three brane wrapped on the two cycle to vanish for conformal field theory orbifolds.

3.2.2 $SU(2)_v \times SU(2)'_v \times U(4)_u$ Gauge Group

In the classical limit this point is obtained by pulling the four seven branes parallel to the v plane away from the orientifold plane at $u = 0$, keeping the seven brane positions parallel to the u plane intact. (Here we are following the convention introduced in the previous section, according to which the subscript u (v) labels the gauge fields living on the branes parallel to the u (v) plane, *i.e.* transverse to the v (u) plane.) Since the RR charge associated with branes transverse to the v plane is still locally neutralised, we expect λ to be independent of v. Thus the v dependence of f and g should still be of the form v^2 and v^3 respectively. The u dependence of f and g can then be determined by going to the large v limit, in which case the f and g must satisfy the boundary condition (3.6). This gives, for all v,

$$
\begin{aligned}
f(u,v) &= v^2 f_{SW}(u; m_1, m_2, m_1, m_2, \tau) \\
g(u,v) &= v^3 g_{SW}(u; m_1, m_2, m_1, m_2, \tau)
\end{aligned} \tag{3.11}
$$

From eq.(3.4) we get

$$
\Delta(u,v) = v^6 (4 f_{SW}(u)^3 + 27 g_{SW}(u)^2) \equiv v^6 \Delta_{SW}(u; m_1, m_2, m_1, m_2, \tau)\,. \tag{3.12}
$$

From the analysis of ref.[16] we know that for $m_1 = m_3$ and $m_2 = m_4$, Δ_{SW} has two second order zeroes and two first order zeroes, without f and g vanishing at those points. Locally this would correspond to a $U(2)_v \times U(2)'_v$ non-abelian gauge symmetry living on the branes parallel to the v plane. (In the orientifold description, these would be generated by the $SO(8)$ generators commuting with the Higgs vev given by eqs.(2.7), (2.9).) The $SO(8)_v$ monodromy (2.4) around the origin in the v plane breaks this to $SU(2)_v \times SU(2)'_v$. Note that since in the v-plane all the zeroes of Δ are at $v = 0$, there is no further monodromy. For $m_1 = m_2$, Δ_{SW} has a fourth order zero and two first order zeroes, signalling a local $U(4)_v$ gauge group. Again this is broken to $Sp(4)_v$ by the $SO(8)_v$ monodromy (2.4).

The D_4 singularity at $v = 0$, signalled by the zeroes of order two, three and six in f, g and Δ respectively, implies that locally there is an $SO(8)_u$ gauge symmetry living on the $v = 0$ plane. This breaks to $U(4)_u$ by the $SO(8)_u$ monodromy (2.4) at $u = \infty$. However, since there are now several singularities in the u plane, we need to ensure that monodromy around these singularities does not break the $U(4)_u$ gauge group any further. Otherwise there will be a discrepancy between the perturbative and non-perturbative description of the model signalling that the non-perturbative description that we are proposing is not correct. In particular, since now there is non-trivial SL(2,Z) monodromy in the u plane, these will induce triality automorphisms in $SO(8)_u$[16, 5], and can break $U(4)_u$ further unless these automorphisms commute with $U(4)_u$[18]. We shall now show that this does not happen. Since the $U(1)$ factor of $U(4)_u$ is broken in any case by anomaly effects, we shall focus on the $SU(4)_u$ subgroup of $U(4)_u$.

To analyse these monodromies, let us consider the generators of SL(2,Z) transformation:

$$S = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}, \qquad T = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}. \tag{3.13}$$

As was shown in [16], S and T induce the following triality transformations on the three $SO(8)$ representations 8_v, 8_s and 8_c:

$$\begin{aligned} T &: \; 8_s \leftrightarrow 8_c, \qquad 8_v \to 8_v \\ S &: \; 8_s \leftrightarrow 8_v, \qquad 8_c \to 8_c \end{aligned} \tag{3.14}$$

Around the double zeroes of Δ in the u-plane we get an SL(2,Z) monodromy conjugate to T^2, which, from (3.14), can be seen to have trivial $SO(8)$ monodromy.[5] Thus we need

[5]This reflects a consistency check for the model. If the orientifold model allowed a deformation that separates all the four seven branes parallel to the v plane away from each other, then it would be impossible to keep $SU(4)_u$ unbroken under this deformation since there will be non-trivial $SU(4)_u$ monodromy around these seven branes.

to focus our attention on the monodromy around the single zeroes of Δ_{SW} in the u plane. Since the product of these two monodromies must equal (2.4) which has been already taken into account, we only need to focus on the monodromy around one of the single zeroes of Δ. Let us focus on the singularity that corresponds to a massless monopole[16]. The SL(2,Z) monodromy around this point is given by STS^{-1}. From eq.(3.14) we see that this corresponds to SO(8) automorphism:

$$8_v \leftrightarrow 8_c, \qquad 8_s \rightarrow 8_s \, . \tag{3.15}$$

Let us now study the decomposition of these representations under $SU(4)_u \subset SO(8)_u$, embedded as described in eqs.(2.4), (2.5). It is as follows:

$$\begin{array}{rcl} 8_v & \rightarrow & 4 + \bar{4} \\ 8_c & \rightarrow & 4 + \bar{4} \\ 8_s & \rightarrow & 6 + 2 \, . \end{array} \tag{3.16}$$

Thus we see that the monodromy around the single zero of Δ commutes with the $SU(4)_u$ group and does not break it any further.

Another way of interpreting this result is as follows. The monodromy (3.15) breaks $SO(8)_u$ to $SO(7)_u$[18]. Thus the final unbroken gauge group should be the intersection of $SO(7)_u$ and $SU(4)_u$. What the above analysis shows is that it is consistent to embedd the $SO(7)_u$ in $SO(8)_u$ in such a way that it contains the $SU(4)_u \equiv SO(6)_u$ subgroup of $SO(8)_u$.

3.2.3 $Sp(4)_v \times Sp(4)_u$ Gauge Group

In the classical limit this corresponds to pulling the four seven branes parallel to the u plane away from the orientifold plane $v = 0$ keeping them together, and at the same time pulling the four seven branes parallel to the v plane away from the orientifold plane $u = 0$ keeping them together. Locally there is a $U(4)_u$ ($U(4)_v$) gauge group living on the seven branes parallel to the u (v) plane which is broken to $Sp(4)_u$ ($Sp(4)_v$) by the monodromy (2.4) in the u (v) plane at $u = \infty$ ($v = \infty$). In order to get these gauge groups in the non-perturbative description we must choose f and g in such a way that there is an A_3 singularity parallel to the u plane and an A_3 singularity parallel to the v plane. This corresponds to a fourth order zero of Δ at $u = u_0$ and a fourth order zero at $v = v_0$ for some u_0, v_0. Thus we need a Δ of the form:

$$\Delta(u,v) = (u-u_0)^4(v-v_0)^4\delta(u,v)\,, \tag{3.17}$$

where δ is a polynomial of degree two in u and v. Thus the question now is: is it possible to choose f and g so that Δ defined in eq.(3.4) has the form (3.17)?

At the first sight it would seem unlikely that such f and g exist. Δ is a polynomial of degree six in both u and v. A generic polynomial of this kind is labelled by 49 parameters. On the other hand the right hand side of (3.17) is labelled by 11 parameters, two from u_0 and v_0, and 9 from $\delta(u,v)$. Thus if we start with a generic f and g, requiring Δ to be of the form (3.17) would give $49-11=38$ constraints on the coefficients appearing in f and g. Since the total number of parameters appearing in f and g is $9+16=25$, we get a set of 38 equations for 25 parameters. This is a hightly overdetermined system of equations!

Nevertheless it turns out that there does exist a family of solutions to this system of equations. As has already been stated earlier, the main guide for obtaining these solutions is the use of boundary conditions (3.5), (3.6) with $m_1=m_2$, $m'_1=m'_2$. The relevant f_{SW} and g_{SW} appearing in these boundary conditions are given by

$$\begin{aligned} f_{SW}(u;m,m,m,m,\tau) &= c^2m^4\tilde{f}_{SW}\Big(\frac{u}{m^2c}-\frac{b-3}{2};\tau\Big) \\ g_{SW}(u;m,m,m,m,\tau) &= c^3m^6\tilde{g}_{SW}\Big(\frac{u}{m^2c}-\frac{b-3}{2};\tau\Big)\,, \end{aligned} \tag{3.18}$$

where

$$b = -\frac{3\vartheta_2^4(\tau)}{\vartheta_1^4(\tau)+\vartheta_3^4(\tau)}, \qquad c = -\frac{1}{3}(\vartheta_1^4(\tau)+\vartheta_3^4(\tau))\,, \tag{3.19}$$

$$\begin{aligned} \tilde{f}_{SW}(\tilde{u};\tau) &= -\frac{1}{3}b^4+2b^2\tilde{u}-\frac{1}{4}(3+b^2)\tilde{u}^2 \\ \tilde{g}_{SW}(\tilde{u};\tau) &= \frac{1}{108}(-b^2+3\tilde{u})(8b^4-48b^2\tilde{u}+9(b^2-1)\tilde{u}^2)\,. \end{aligned} \tag{3.20}$$

ϑ_i are the Jacobi theta functions. In extracting f_{SW} and g_{SW} from ref.[16] we have used the rescaling freedom (3.10).

The general solution for $f(u,v)$ and $g(u,v)$ (up to rescaling of f and g of the form(3.10)) satisfying these boundary conditions and giving a Δ of the form (3.17) is

$$f(u,v) \;=\; m^4(m')^4c^4\tilde{f}\Big(\frac{u}{m^2c}-\frac{b-3}{2},\,\frac{v}{(m')^2c}-\frac{b-3}{2};\tau\Big)$$

$$g(u,v) = m^6(m')^6c^6\widetilde{g}\Big(\frac{u}{m^2c}-\frac{b-3}{2}, \frac{v}{(m')^2c}-\frac{b-3}{2};\tau\Big), \tag{3.21}$$

where

$$\begin{aligned}\widetilde{f}(\widetilde{u},\widetilde{v};\tau) &= \frac{1}{12}[-\alpha^2-4\alpha b^2(\widetilde{u}+\widetilde{v})-4b^4(\widetilde{u}^2+\widetilde{v}^2)+(12\alpha-8b^4)\widetilde{u}\widetilde{v} \\ &\quad +24b^2\widetilde{u}\widetilde{v}(\widetilde{u}+\widetilde{v})-(9+3b^2)\widetilde{u}^2\widetilde{v}^2] \\ \widetilde{g}(\widetilde{u},\widetilde{v};\tau) &= \frac{1}{216}[-\alpha-2b^2(\widetilde{u}+\widetilde{v})+6\widetilde{u}\widetilde{v}] \\ &\quad \times[2\alpha^2+8\alpha b^2(\widetilde{u}+\widetilde{v})+8b^4(\widetilde{u}^2+\widetilde{v}^2)+(16b^4-24\alpha)\widetilde{u}\widetilde{v} \\ &\quad -48b^2\widetilde{u}\widetilde{v}(\widetilde{u}+\widetilde{v})+9(b^2-1)\widetilde{u}^2\widetilde{v}^2].\end{aligned} \tag{3.22}$$

Here α is an arbitrary complex parameter whose significance will be explained later.

It is easy to verify that f and g defined this way satisfy the boundary conditions (3.5), (3.6) with $m_1 = m_2$ and $m'_1 = m'_2$. Also Δ computed from f and g given in eq.(3.21) is given by

$$\begin{aligned}\Delta(u,v) &= -\frac{1}{16}b^2(9-b^2)^2\Big(u-\frac{1}{2}(b-3)\,c\,m^2\Big)^4\Big(v-\frac{1}{2}(b-3)\,c\,(m')^2\Big)^4 \\ &\quad \Big[uv-c\Big(\frac{b+3}{2}+\sqrt{9-b^2}\Big)(u(m')^2+vm^2) \\ &\quad +\Big\{\frac{1}{4}(b-3)^2+(3+\sqrt{9-b^2})(b-3)-\frac{\alpha}{2b^2}(3+\sqrt{9-b^2}\Big\}m^2(m')^2c^2\Big] \\ &\quad \Big[uv-c\Big(\frac{b+3}{2}-\sqrt{9-b^2}\Big)(u(m')^2+vm^2) \\ &\quad +\Big\{\frac{1}{4}(b-3)^2+(3-\sqrt{9-b^2})(b-3)-\frac{\alpha}{2b^2}(3-\sqrt{9-b^2}\Big\}m^2(m')^2c^2\Big]\end{aligned} \tag{3.23}$$

This shows that Δ does indeed have the form of (3.17) with u_0 and v_0 given by $(b-3)cm^2/2$ and $(b-3)c(m')^2/2$ respectively.

Let us first examine the existence of $Sp(4)_v \times Sp(4)_u$ gauge group by examining the singularity structure of this configuration. In terms of coordinates $\widetilde{u}$, $\widetilde{v}$ defines as

$$\widetilde{u} = \frac{u}{cm^2}-\frac{1}{2}(b-3), \qquad \widetilde{v} = \frac{v}{c(m')^2}-\frac{1}{2}(b-3), \tag{3.24}$$

Δ has a fourth order zero at $\widetilde{v}=0$. f and g can be rewritten in this coordinate system as

$$f(u,v) = -3m^4(m')^4c^4(h_1^2(\widetilde{u})+h_1(\widetilde{u})h_2(\widetilde{u})\widetilde{v}+h_3(\widetilde{u})\widetilde{v}^2)$$

$$
\begin{aligned}
g(u,v) &= m^6(m')^6c^6\Big(2h_1^3(\widetilde{u}) + 3h_1^2(\widetilde{u})h_2(\widetilde{u})\widetilde{v} + 3(h_3(\widetilde{u}) + \frac{1}{4}h_2^2(\widetilde{u}))h_1(\widetilde{u})\widetilde{v}^2 \\
&\quad +(\frac{3}{2}h_3(\widetilde{u}) - \frac{1}{8}h_2^2(\widetilde{u}))h_2(\widetilde{u})\widetilde{v}^3\Big)\,, \qquad (3.25)
\end{aligned}
$$

where,

$$
\begin{aligned}
h_1(\widetilde{u}) &= -\frac{1}{6}(\alpha + 2b^2\widetilde{u}) \\
h_2(\widetilde{u}) &= \frac{2}{3}(-b^2 + 3\widetilde{u}) \\
h_3(\widetilde{u}) &= \frac{1}{9}b^4 - \frac{2}{3}b^2\widetilde{u} + \frac{1}{12}(b^2+3)\widetilde{u}^2\,. \qquad (3.26)
\end{aligned}
$$

Comparing with the results of ref.[18] we see that this corresponds to a non-split A_3 singularity. (A split A_3 singularity will require h_1 to have only double zeroes in the $\widetilde{u}$ plane.) Thus the gauge group living on the $\widetilde{v} = 0$ plane is $Sp(4)_u$ as expected. An identical analysis shows that the gauge group living on the $\widetilde{u} = 0$ plane is also $Sp(4)_v$ as expected.

Next we discuss the interpretation of the complex parameter α. From eqs.(3.21), (3.22) it is clear that in the limit $m \to 0$ or $m' \to 0$ this parameter disappears from the expressions for $f(u,v)$ and $g(u,v)$. In other words if either $U(4)_u$ or $U(4)_v$ is unbroken at the classical level, then we do not have the deformation of the non-perturbative background labelled by α, whereas if both $U(4)$'s are broken then this deformation of the background is present. This is precisely what we expect for the deformation associated with the massless closed string state from the twisted sector. When either of the $U(1)$ factors is present in the classical theory, then due to one loop anomaly this $U(1)$ gauge field becomes massive by absorbing these twisted sector closed string states; whereas if both $U(1)$'s are broken by Higgs mechanism at the classical level, then these twisted sector closed string states remain massless and act as moduli field[15]. Furthermore, from eq.(3.22) we see that the deformation associated with α does not affect the form of f and g for large u or large v, again as is expected of a blow up mode localised near the orbifold point. Thus it is very likely that the parameter α is related to the deformations associated with the twisted sector closed string states (the blow up modes of the orbifold singularity) although we do not have a direct proof of this statement.

The geometry of seven brane configurations described by this non-perturbative background can be studied by examining the zeroes of Δ. From eq.(3.23) we see that there are four coincident seven branes at $u=(b-3)m^2c/2$ and four coincident seven branes at $v=(b-3)(m')^2c/2$. This is identical to the configuration of seven branes found in the classical limit. Also, in the $\tau\to i\infty$ limit,

$$b\to(-3)\,,\qquad\qquad c\to-(1/3)\,, \tag{3.27}$$

as can be seen from (3.19). Thus the locations of these coincident seven branes approach $u\simeq m^2$ and $v\simeq(m')^2$, exactly as expected from the orientifold description.

More interesting is the configuration of seven branes coming from the second factor of Δ. From eq.(3.23) we see that we have two more seven branes situated on the surfaces:

$$\begin{aligned}&\Big[uv-c\Big(\frac{b+3}{2}\pm\sqrt{9-b^2}\Big)(u(m')^2+vm^2)+\Big\{\frac{1}{4}(b-3)^2+(3\pm\sqrt{9-b^2})(b-3)\\&-\frac{\alpha}{2b^2}(3\pm\sqrt{9-b^2}\Big\}m^2(m')^2c^2\Big]=0\,.\end{aligned} \tag{3.28}$$

For large v, this equation reduces to

$$u\simeq\Big(\frac{b+3}{2}\pm\sqrt{9-b^2}\Big)cm^2\,. \tag{3.29}$$

These two surfaces simply represent the two seven branes into which an orientifold plane parallel to the v plane would split in the absence of the projection $(-1)^{F_L}\cdot\Omega\cdot\mathcal{I}_{89}$. Similarly for large u eq.(3.28) becomes

$$v\simeq\Big(\frac{b+3}{2}\pm\sqrt{9-b^2}\Big)c(m')^2\,. \tag{3.30}$$

These two surfaces represent the two seven branes into which an orientifold plane parallel to the u plane would split in the absence of the projection $(-1)^{F_L}\cdot\Omega\cdot\mathcal{I}_{67}$.

The phenomenon of the orientifold plane splitting into two seven branes under non-perturbative quantum corrections is not new[5]. What is new is the phenomenon that the two seven branes into which the orientifold plane parallel to the u plane splits smoothly join the two seven branes into which the orientifold plane parallel to the v plane splits. Thus at the end we get only two seven branes instead of two pairs of intersecting seven branes.[6]

These two seven branes intersect at the points

$$\begin{aligned}&\Big(u=\frac{b-3}{2}cm^2,\ v=\Big(\frac{b-3}{2}-\frac{\alpha}{2b^2}\Big)c(m')^2\Big)\,,\\&\Big(u=\Big(\frac{b-3}{2}-\frac{\alpha}{2b^2}\Big)cm^2,\ v=\frac{b-3}{2}(cm')^2\Big)\,.\end{aligned} \tag{3.31}$$

[6]Of course at special values of α given by $\alpha=-2b^2(3\pm\sqrt{9-b^2})$ one of these two seven branes degenerates into two intersecting seven branes.

The physical interpretation of these intersection points is as follows. Since at $u = (b-3)cm^2/2$ there are four coincident seven branes, $\lambda(u,v) \to i\infty$ on this plane. Thus the phenomenon of the splitting of the orientifold plane at $v = 0$ must disappear for $u = (b-3)cm^2/2$. and the two seven branes into which the orientifold plane splits must meet on this plane. Similarly the two seven branes into which the $u = 0$ orientifold plane splits must meet at $v = (b-3)c(m')^2/2$.

We can now use these results to extract information about the non-perturbative dynamics on the corresponding three brane world-volume theory. We shall concentrate on the infrared dynamics on the three brane world volume theory when it is close to one of the two seven branes given in eq.(3.28). In this case the corresponding infrared dynamics is governed by an N=2 supersymmetric theory on the world volume with a U(1) vector multiplet and a charged hypermultiplet representing open string states stretched between the three brane and the seven brane[23]. If we move the three brane along such a seven brane to $u \to \infty$, then the massless charged hypermultiplet that we get can be interpreted as the monopole / dyon state associated with the $SU(2)_2$ gauge group on the three brane world volume introduced at the end of subsection 3.1. On the other hand we can move the three brane along the same seven brane to $v \to \infty$ without hitting any singularity, and in this case the same massless hypermultiplet can be interpreted as the monopole / dyon associated with the $SU(2)_1$ gauge group introduced at the end of subsection 3.1. Thus we see that once non-perturbative effects are taken into account, an $SU(2)_1$ monopole can be continuously transformed into an $SU(2)_2$ monopole and vice versa, although classically they correspond to two distinct gauge groups.

In the weak coupling limit (3.27) both the seven branes given in (3.28) coincide and are described by the equation:

$$uv - \Big(1 + \frac{1}{54}\alpha\Big) m^2 (m')^2 = 0\,. \tag{3.32}$$

This reflects the fact the phenomenon of the splitting of the orientifold plane disappears in the classical limit. However for generic value of α, (3.32) still describes a smooth surface instead of a pair of intersecting orientifold planes as in ref.[6]. Only for $\alpha = -54$ (3.32) takes the form

$$uv = 0, \tag{3.33}$$

describing a pair of intersecting orientifold planes. Thus the orbifold limit in weak coupling must correspond to $\alpha = -54$. A generic α corresponds to a blown up version of the orbifold singularity, and hence we do not expect a singular intersection at $u = v = 0$. This reconfirms our interpretation of α as the vev of twisted sector closed string states.

3.2.4 $SU(2)_v \times SU(2)'_v \times Sp(4)_u$ Gauge Group

In the classical limit, this corresponds to pulling apart the four seven branes parallel to the v plane into two pairs of seven branes situated at $u = m_1^2$ and $u = m_2^2$. Since enhanced SU(2) gauge symmetry requires Δ to have zeroes of order two, we are looking for $f(u, v)$ and $g(u, v)$ such that Δ defined in eq.(3.4) takes the form

$$\Delta(u, v) = (u - u_1)^2 (u - u_2)^2 (v - v_0)^4 \widehat{\delta}(u, v)\,, \qquad (3.34)$$

where u_1, u_2 and v_0 are constants, and $\widehat{\delta}$ is a polynomial of degree (2,2) in u and v. The most general f and g satisfying (3.34) and the required asymptotic behavior at infinity is given by

$$\begin{aligned} f(u, v) &= m_1^2 m_2^2 (m')^4 c^4 \widetilde{f}(\widetilde{u}, \widetilde{v}) \\ g(u, v) &= m_1^3 m_2^3 (m')^6 c^6 \widetilde{g}(\widetilde{u}, \widetilde{v})\,, \end{aligned} \qquad (3.35)$$

where b, c are as defined in eq.(3.19), and,

$$\widetilde{u} = \frac{u}{m_1 m_2 c} - \frac{1}{2}(b-3)\eta, \qquad \widetilde{v} = \frac{v}{(m')^2 c} - \frac{1}{2}(b-3)\,, \qquad (3.36)$$

$$\eta = \frac{1}{2}\Big(\frac{m_1}{m_2} + \frac{m_2}{m_1}\Big)\,, \qquad (3.37)$$

$$\begin{aligned} \widetilde{f}(\widetilde{u}, \widetilde{v}) &= -3(h_1^2(\widetilde{u}) + h_1(\widetilde{u}) h_2(\widetilde{u}) \widetilde{v} + h_3(\widetilde{u}) \widetilde{v}^2) \\ \widetilde{g}(\widetilde{u}, \widetilde{v}) &= \Big(2h_1^3(\widetilde{u}) + 3h_1^2(\widetilde{u}) h_2(\widetilde{u}) \widetilde{v} + 3(h_3(\widetilde{u}) + \frac{1}{4} h_2^2(\widetilde{u})) h_1(\widetilde{u}) \widetilde{v}^2 \\ &\quad + (\frac{3}{2} h_3(\widetilde{u}) - \frac{1}{8} h_2^2(\widetilde{u})) h_2(\widetilde{u}) \widetilde{v}^3\Big)\,, \end{aligned} \qquad (3.38)$$

where,

$$\begin{aligned} h_1(\widetilde{u}) &= -\frac{1}{6}(\alpha + 2b^2 \widetilde{u}) \\ h_2(\widetilde{u}) &= \frac{2}{3}(-b^2 \eta + 3\widetilde{u}) \\ h_3(\widetilde{u}) &= \frac{1}{36} b^4 (3 + \eta^2) - \frac{3}{4} b^2 (1 - \eta^2) - \frac{2}{3} b^2 \eta \widetilde{u} + \frac{1}{12}(b^2 + 3) \widetilde{u}^2\,. \end{aligned} \qquad (3.39)$$

$\Delta(u,v)$ calculated from f and g given in eqs.(3.35)-(3.39) is given by

$$\begin{aligned}\Delta(u,v) &= -\frac{1}{16}b^2(9-b^2)^2 m_1^6 m_2^6 (m')^{12} c^{12} (\widetilde{u}^2 + b^2(1-\eta^2))^2 \widetilde{v}^4 \\ &\quad \Big[\widetilde{u}^2\widetilde{v}^2 - 6\widetilde{u}\widetilde{v}(\widetilde{u}+\eta\widetilde{v}) + b^2(\widetilde{u}^2+\widetilde{v}^2) + 9(\eta^2-1)\widetilde{v}^2 \\ &\quad +(2b^2\eta - \frac{3\alpha}{b^2})\widetilde{u}\widetilde{v} + \alpha(\widetilde{u}+\eta\widetilde{v}) + \frac{\alpha^2}{4b^2}\Big]\,. \end{aligned} \tag{3.40}$$

From this we see that Δ has two second order zeroes at

$$u = \Big(\frac{b-3}{2}\eta \pm b\sqrt{\eta^2-1}\Big) m_1 m_2 c\,, \tag{3.41}$$

signalling the presence of $SU(2)_v \times SU(2)'_v$ gauge symmetry group. It also has a fourth order zero at

$$v = \frac{b-3}{2}(m')^2 c\,. \tag{3.42}$$

From the structure of $f(u,v)$ and $g(u,v)$ given in eqs.(3.35)-(3.39) we see that this corresponds to a non-split A_3 singularity and hence $Sp(4)_u$ gauge group.

Finally we note that the last factor of Δ given in eq.(3.40) does not factorise into two factors as in eq.(3.23). This shows that the pair of seven branes into which the intersecting pair of orientifold planes split now further join together smoothly to give one single seven brane.

3.2.5 $SU(2)_v \times SU(2)'_v \times SU(2)_u \times SU(2)'_u$ Gauge Group

We could further split the four seven branes parallel to the u plane into two pairs to obtain the $SU(2)_v \times SU(2)'_v \times SU(2)_u \times SU(2)'_u$ model. This model will be characterised by four complex parameters m_1, m_2, m'_1 and m'_2 labelling the positions of the four seven brane pairs in the classical limit $-$ two parallel to the v-plane and two parallel to the u-plane $-$ besides the parameters τ and α. We now need to look for f and g such that Δ defined in eq.(3.4) takes the form:

$$\Delta(u,v) = (u-u_1)^2(u-u_2)^2(v-v_1)^2(v-v_2)^2\widetilde{\delta}(u,v)\,, \tag{3.43}$$

where u_i and v_i are arbitrary constants and $\widetilde{\delta}$ is a polynomial of degree two in u and v. A family of solutions for f and g satisfying this criteria is given by

$$f(u,v) \;=\; m_1^2 m_2^2 (m'_1)^2 (m'_2)^2 c^4 \widetilde{f}(\widetilde{u},\widetilde{v})$$

$$g(u,v) = m_1^3 m_2^3 (m_1')^3 (m_2')^3 c^6 \widetilde{g}(\widetilde{u},\widetilde{v})\,, \tag{3.44}$$

where,

$$\begin{aligned}
\widetilde{f}(\widetilde{u},\widetilde{v}) &= \frac{1}{12}(-\alpha^2 + 27b^4((\eta')^2-1)(\eta^2-1) - 3b^6((\eta')^2-1)(\eta^2-1) - 4\alpha b^2\eta'\widetilde{u} \\
&+27b^2(1-(\eta')^2)\widetilde{u}^2 - b^4(3+(\eta')^2)\widetilde{u}^2 - 4\alpha b^2\eta\widetilde{v} + 12\alpha\widetilde{u}\widetilde{v} - 8b^4\eta'\eta\widetilde{u}\widetilde{v} \\
&+24b^2\eta'\widetilde{u}^2\widetilde{v} + 27b^2(1-\eta^2)\widetilde{v}^2 - b^4(3+\eta^2)\widetilde{v}^2 + 24b^2\eta\widetilde{u}\widetilde{v}^2 - 9\widetilde{u}^2\widetilde{v}^2 - 3b^2\widetilde{u}^2\widetilde{v}^2) \\
\\
\widetilde{g}(\widetilde{u},\widetilde{v}) &= \frac{1}{216}(-\alpha - 2b^2\eta'\widetilde{u} - 2b^2\eta\widetilde{v} + 6\widetilde{u}\widetilde{v}) \\
&[2\alpha^2 - 81b^4((\eta')^2-1)(\eta^2-1) + 9b^6((\eta')^2-1)(\eta^2-1) + 8\alpha b^2\eta'\widetilde{u} \\
&+b^4(9-(\eta')^2)\widetilde{u}^2 + 81b^2((\eta')^2-1)\widetilde{u}^2 + 8\alpha b^2\eta\widetilde{v} - 24\alpha\widetilde{u}\widetilde{v} + 16b^4\eta'\eta\widetilde{u}\widetilde{v} \\
&-48b^2\eta'\widetilde{u}^2\widetilde{v} + 81b^2(\eta^2-1)\widetilde{v}^2 + b^4(9-\eta^2)\widetilde{v}^2 - 48b^2\eta\widetilde{u}\widetilde{v}^2 + 9(b^2-1)\widetilde{u}^2\widetilde{v}^2]\,,
\end{aligned} \tag{3.45}$$

$$\eta = \frac{1}{2}\Big(\frac{m_1}{m_2} + \frac{m_2}{m_1}\Big)\,, \qquad \eta' = \frac{1}{2}\Big(\frac{m_1'}{m_2'} + \frac{m_2'}{m_1'}\Big)\,, \tag{3.46}$$

and,

$$\widetilde{u} = \frac{u}{m_1 m_2 c} - \frac{1}{2}(b-3)\eta, \qquad \widetilde{v} = \frac{v}{m_1' m_2' c} - \frac{1}{2}(b-3)\eta'\,. \tag{3.47}$$

Δ computed from this f and g is given by,

$$\begin{aligned}
\Delta &= \frac{1}{64}m_1^6 m_2^6 (m_1')^6 (m_2')^6 c^{12}(-9+b^2)^2(b^2 - b^2\eta^2 + \widetilde{u}^2)^2(b^2 - b^2(\eta')^2 + \widetilde{v}^2)^2 \\
&[-\alpha^2 + 4b^4(9-b^2)(1-\eta^2)(1-(\eta')^2) - 4\alpha b^2\eta'\widetilde{u} \\
&+36b^2(1-(\eta')^2)\widetilde{u}^2 - 4b^4\widetilde{u}^2 - 4\alpha b^2\eta\widetilde{v} + 12\alpha\widetilde{u}\widetilde{v} - 8b^4\eta'\eta\widetilde{u}\widetilde{v} + 24b^2\eta'\widetilde{u}^2\widetilde{v} \\
&+36b^2(1-\eta^2)\widetilde{v}^2 - 4b^4\widetilde{v}^2 + 24b^2\eta\widetilde{u}\widetilde{v}^2 - 4b^2\widetilde{u}^2\widetilde{v}^2]\,.
\end{aligned} \tag{3.48}$$

The geometry of the seven brane configurations representing the split orientifold plane does not have any new feature that was not already present in the previous examples. Note that in this expression if we take one of the m_i's or one of the m_i''s to zero, the α dependence drops out from f and g. In this case, in the classical limit we recover an $U(2)$ gauge group[6]. The presence of the $U(1)$ factor implies that the twisted sector closed string states would become massive due to anomaly effects. This is again consistent with the interpretation of α as the blow up mode.

3.2.6 More General Deformations

We can further deform the model by switching on the vev of the hypermultiplets in the (4,4) representation of $U(4)_v \times U(4)_u$. Since these hypermultiplets are localised at the intersection of the seven branes in the classical limit, we expect that the vev of these hypermultiplets should not change the asymptotic form of f and g for large u or large v. Thus switching on vev of these hypermultiplets should correspond to addition of terms in f and g of the form:

$$\begin{aligned}\delta f(u,v) &= \sum_{m,n=0}^{1} a_{mn} u^m v^n \,, \\ \delta g(u,v) &= \sum_{m,n=0}^{2} b_{mn} u^m v^n \,,\end{aligned} \tag{3.49}$$

This will generically break the gauge group completely, and hence must describe a configuration where the seven branes, which previously existed only in pairs, are separated from each other. Naively it might sound like a violation of the condition (2.9) and hence of (2.6), but this is not so. To see how this contradiction is avoided, let us note that given any $f(u,v)$ and $g(u,v)$ which are of degree (2,2) and (3,3) respectively, we can express them as

$$\begin{aligned}f(u,v) &= f_{SW}(u; m_1(v), m_2(v), m_3(v), m_4(v), \tau(v)) \,, \\ g(u,v) &= g_{SW}(u; m_1(v), m_2(v), m_3(v), m_4(v), \tau(v)) \,,\end{aligned} \tag{3.50}$$

after suitable v dependent shift of u, and v dependent rescaling of f and g that keeps $\lambda(u,v)$ invariant. Here m_i are in general functions of v. Eq.(2.6) then implies that

$$m_1(z) = m_3(-z), \qquad\qquad m_2(z) = m_4(-z) \,, \tag{3.51}$$

where $z = \sqrt{v}$. Thus there is no need for m_1 and m_3 (or m_2 and m_4) to be equal as long as they are allowed to vary with v.

To see how eq.(3.50) is realised in practice, let us consider perturbing the $SU(2)_v \times SU(2)'_v \times SU(2)_u \times SU(2)'_u$ model by adding terms of the form (3.49) to f and g. This will, in general, split the double zeroes of Δ given in (3.43). Let us focus on the zeroes of Δ near $u = u_1$. After addition of (3.49) to f and g, Δ near $u = u_1$ for large v takes the form:

$$\Delta \simeq C v^6 (u - u_1)^2 - \epsilon v^5 \,. \tag{3.52}$$

Here C and ϵ are constants, with ϵ associated with the hypermultiplet vev. The crucial point is that due to the nature of the form (3.49) of δf and δg, the coefficient of ϵ for large v is of order v^5 and not of order v^6. The zeroes of Δ now get shifted to

$$u \simeq u_1 \pm \sqrt{\epsilon / C v} \,, \tag{3.53}$$

for large v. In the weak coupling limit these may be identified to $m_1^2(v)$ and $m_3^2(v)$ respectively. Thus we see that under $v \to e^{2\pi i} v$, $m_1^2(v)$ and $m_3^2(v)$ get interchanged as required. Similar reasoning shows that other m_i^2 and $(m_i')^2$'s also satisfy the required monodromy in the u / v plane.

One can also consider special subspaces of the full moduli space where some of the diagonal $SU(2)$ subgroups are unbroken. Consider for example the hypermultiplet transforming in the $(2,2)$ representation of $SU(2)_u \times SU(2)_v$. In this case we can switch on the vev of the component of the hypermultiplet that is singlet under the diagonal subgroup of the two $SU(2)$'s, thereby breaking $SU(2)_u \times SU(2)_v$ to this diagonal $SU(2)$ subgroup. The non-perturbative description of this class of vacua will be provided by choosing f and g such that a pair of coincident seven branes parallel to the v plane smoothly join a pair of coincident seven branes parallel to the u plane, thus giving just one pair of coincident seven branes. Such configurations have been discussed in the context of F-theory in [18].

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