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Baryon number violation induced by the monopoles of the Pati-Salam model

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ABSTRACT

We study the possible baryon number violating effects induced by the monopoles that are formed due to the spontaneous breakdown of the Pati-Salam group ($SU(2)_L \times SU(2)_R \times SU(4)$). This effect is due to the weak 't Hooft anomaly. Although the baryon number violating condensates involving only the first and the second generation fermions are suppressed by powers of mixing angles, we show that all the relevant mixing angles may be large, while still being consistent with the smallness of the Kobayashi-Maskawa mixing angles. Hence the baryon number violating effects caused by such monopoles need not be suppressed.

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It has been realized for some time that grand unification monopoles of the 't Hooft-Polyakov type may catalyze baryon number violation at the strong interaction rate[1)-2)]. In the original treatment of Rubakov[1)] and Callan[2)], the origin of this effect was the baryon number violating boundary conditions at the monopole core, which, in turn, is due to the classical gauge field configuration inside the monopole core, in whose presence the baryon number ceases to be a conserved quantum number. This mechanism becomes inoperative in models where the elementary gauge bosons of the theory do not mediate baryon number violation. The Pati-Salam model[3)], based on the $SU(2)_L \times SU(2)_R \times SU(4)_{PS}$ gauge group provides an example of such a system.

There is, however, a completely different mechanism which may cause baryon number violation in such a system[4)-6)]. This is the effect of weak 't Hooft anomaly[7)]. In the presence of the $SU(2)_L$ gauge interactions, the baryon number becomes anomalous. It is this effect which is responsible for baryon number violation induced by monopoles of the Pati-Salam model. It was shown[6)] that for sufficiently small monopole radius, the baryon number violating condensates induced by weak anomaly are unsuppressed by any power of the monopole radius, coupling constant or the $SU(2)_L$ breaking scale.

In the absence of any mixing between various generations, these condensates contain quark-lepton fields from all the three generations. As a result, these condensates necessarily carry three units of baryon number. Also, if we want to get rid of the heavy quarks and leptons from the condensate, the result is suppressed by some mixing angles.

In this paper, we shall specifically study the baryon number violating effects caused by the double strength monopole of the Pati-Salam model[8]. Such monopoles are of interest, since it has been argued recently[9] that they may be abundant in the present universe. We shall show that although the baryon number violating condensates involving first and second generation fermions are suppressed by some powers of 'mixing angles', these angles are unrelated to the standard Kobayashi-Maskawa mixing angles. In particular, all the relevant mixing angles appearing in the calculation of the baryon number violating condensates can be made to be of order unity, while keeping the Kobayashi-Maskawa mixing angles small. Hence for a favorable choice of the parameters of the theory, the baryon number violating condensates involving first and second generation fermions need not be suppressed by any small number at all.

Under the $SU(2)_L \times SU(2)_R \times SU(4)_{PS}$ gauge transformation, the left and the right handed fermions transform in the representation (2,1,4) and (1,2,4) respectively. For example, the left handed fermions may be classified as,

$$\begin{pmatrix} u_{1L}^{(i)} & u_{2L}^{(i)} & u_{3L}^{(i)} & \nu_{eL}^{(i)} \\ d_{1L}^{(i)} & d_{2L}^{(i)} & d_{3L}^{(i)} & e_L^{-(i)} \end{pmatrix} \qquad (1)$$

where i(=1,2,3) is the generation index. The $SU(2)_L$ group mixes various rows, and the SU(4) group mixes various columns. The right handed fermions may be classified similarly, with the $SU(2)_R$ group connecting the different rows rather than the $SU(2)_L$ group. Let us assume that the gauge group is broken to $SU(2)_L \times U(1) \times SU(3)_{color}$ at a large scale ($>10^{10}$GeV), which is broken to $U(1)_{em} \times SU(3)_{color}$ at about 100 GeV. The electric charge operator is given in terms of various generators of the group as,

$$Q_{e.m} = I_{3L} + I_{3R} + \left(\frac{2}{3}\right)^{1/2} F_{15} \qquad (2)$$

where I_{3L} and I_{3R} are the diagonal generators of the $SU(2)_L$ and the $SU(2)_R$ subgroups, and F_{15} is the properly normalized diagonal generator of SU(4) with its diagonal entries proportional to (1,1,1,-3).

Let us consider an SU(2) subgroup generated by,

$$\vec{I} = \vec{I}_L + \vec{I}_R + \vec{I}_{(34)} \qquad (3)$$

where $\vec{I}_L$ and $\vec{I}_R$ are the generators of $SU(2)_L$ and $SU(2)_R$ respectively, and $\vec{I}_{(34)}$ is the generator of the SU(2) subgroup spanning the 3-4 subspace of SU(4). The SU(2) subgroup generated by $\vec{I}$ breaks down to the U(1) group generated by,

$$I_3 = Q_{em} - \frac{1}{6} Y_{color} \qquad (4)$$

at the scale of breaking of $SU(2)_R \times SU(4)_{PS}$. The double strength monopole is constructed by embedding a standard 't Hooft-Polyakov monopole in this subgroup. The fields,

$$\begin{pmatrix} u_1^{(i)} \\ d_1^{(i)} \end{pmatrix}_{L,R} \quad \text{and} \quad \begin{pmatrix} u_2^{(i)} \\ d_2^{(i)} \end{pmatrix}_{L,R}$$

transform as doublets under this SU(2) subgroup, whereas the fields,

$$\begin{pmatrix} u_3^{(i)} \\ \frac{1}{\sqrt{2}}\left(d_3^{(i)} + \nu_e^{(i)}\right) \\ e^{-(i)} \end{pmatrix}_{L,R}$$

transform as triplets. All other fields are singlets under this gauge group. In studying the interaction of these fermions with monopoles, we may treat

$$\begin{pmatrix} u_3^{(i)} \\ e^{-(i)} \end{pmatrix}_{L,R}$$

as another doublet, while the field $(d_3^{(i)} + \nu_e^{(i)})/\sqrt{2}$ decouples from the system[10)-12)].

We may now analyze the system by studying its conservation laws[13)]. The case with one generation of fermions has been discussed by Schellekens[11)], who found that the baryon number violating condensates in this model are identical to those in the case of the lowest charge monopole of SU(5). Hence we directly analyze the system with three generations of fermions. There are eighteen global U(1) symmetries in this model, corresponding to the independent phase transformations of each left and right handed doublets. In general, these charges are anomalous. We may calculate the anomaly in a charge S_i by calculating triangle diagrams, with the current associated with the charge S_i at one vertex, the magnetic field of the monopole at the second vertex, and the current corresponding to any arbitrary linear combination of electric charge, color isospin, color hypercharge and weak isospin at the third vertex. We find that the following independent linear combinations of the charges are anomaly free,

$$
\begin{aligned}
S_i &= N_{u_1^{(i)}} + N_{d_1^{(i)}} - N_{u_2^{(i)}} - N_{d_2^{(i)}}\,, && i=1,2,3 \\
S_{i+3} &= 2\left(N_{u_2^{(i)}} + N_{d_2^{(i)}}\right) - \left(N_{u_3^{(i)}} + N_{e^{-(i)}}\right), && i=1,2,3 \\
S_{i+6} &= N_{u_{1L}^{(i)}} + N_{d_{1L}^{(i)}} - N_{u_{2L}^{(i)}} - N_{d_{2L}^{(i)}}\,, && i=1,2,3 \\
S_{i+8} &= N_{u_1^{(i)}} + N_{d_1^{(i)}} - N_{u_1^{(1)}} - N_{d_1^{(1)}}\,, && i=2,3 \\
S_{i+10} &= N_{u_{1L}^{(i)}} + N_{d_{1L}^{(i)}} - N_{u_{1L}^{(1)}} - N_{d_{1L}^{(1)}}\,, && i=2,3 \\
S_{i+12} &= N_{u_{3L}^{(i)}} + N_{e_L^{-(i)}} - N_{u_{3L}^{(1)}} - N_{e_L^{-(1)}} && i=2,3
\end{aligned}
\tag{5}
$$

Besides these fifteen conservation laws associated with the global symmetries, the total electric charge, color isospin, color hypercharge and weak isospin must be conserved. Only three of them are independent of the S_i's listed above,

$$G_1 = \sum_{i=1}^{3} \left\{ \frac{2}{3} \left(N_{u_1^{(i)}} + N_{u_2^{(i)}} + N_{u_3^{(i)}} \right) - \frac{1}{3} \left(N_{d_1^{(i)}} + N_{d_2^{(i)}} \right) - N_{e^{-(i)}} \right\}$$

$$G_2 = \sum_{i=1}^{3} \left(N_{u_1^{(i)}} + N_{d_1^{(i)}} + N_{u_2^{(i)}} + N_{d_2^{(i)}} - 2 N_{u_3^{(i)}} \right)$$

$$G_3 = \sum_{i=1}^{3} \left(N_{u_{1L}^{(i)}} - N_{d_{1L}^{(i)}} + N_{u_{2L}^{(i)}} - N_{d_{2L}^{(i)}} + N_{u_{3L}^{(i)}} - N_{e_L^{-(i)}} \right) \qquad (6)$$

All the S_i's and G_i's must vanish for any operator which gets a non-vanishing vacuum expectation value (v.e.v.) around the monopole. This gives,

$$N_{u_1^{(i)}} + N_{d_1^{(i)}} = N_{u_2^{(i)}} + N_{d_2^{(i)}} = \frac{1}{2} \left(N_{u_3^{(i)}} + N_{e^{-(i)}} \right) = N$$

$$i = 1, 2, 3 \qquad (7)$$

$$\sum_{j=1}^{3} N_{u_3^{(j)}} = \sum_{j=1}^{3} N_{e^{-(j)}} = 3N \qquad (8)$$

If N vanishes, then using Eqs. (7) and (8) we see that,

$$N_{u_1^{(i)}} + N_{d_1^{(i)}} = N_{u_2^{(i)}} + N_{d_2^{(i)}} = \sum_{j=1}^{3} N_{u_3^{(j)}} = \sum_{j=1}^{3} N_{e^{-(j)}} = 0 \qquad (9)$$

which shows that the total baryon number carried by such an operator must vanish. For arbitrary N, the operator carries a baryon number of,

$$\frac{1}{3}\sum_{i=1}^{3}\left(N_{u_1^{(i)}}+N_{u_2^{(i)}}+N_{d_1^{(i)}}+N_{d_2^{(i)}}+N_{u_3^{(i)}}\right)=3N \qquad (10)$$

which shows that the lowest dimensional operator with a non-zero baryon number carries three units of baryon number. One such operator, which satisfies all the conservation laws, is given by,

$$\prod_{i=1}^{3}\left(u_{1R}^{(i)}\; d_{2R}^{(i)}\; u_{3R}^{(i)}\; e_R^{-(i)}\right) \qquad (11)$$

The fields appearing in (11) are the gauge group eigenstates and involve fields from all generations. Naively, we would expect that when expressed in terms of the mass eigenstates, these operators would have components which involve fields from only the first generation, or from the first and the second generations. However, as was shown in Ref.6, if the v.e.v. of this operator satisfies the $(SU(3))^6$ symmetry, which corresponds to independent rotations of the six doublets in the generation space, the operators of the form given in (11) with non-vanishing v.e.v. must involve fermions from all three generations, even when we express the operators in terms of the mass eigenstates[f1]. Hence starting from an operator of the form given in (11), we cannot get rid of the third generation fermions.

However, there are other baryon number violating operators which acquire a non-zero vev in the vicinity of the monopole. One such operator is,

$$u^{(1)}_{1R}\, u^{(1)}_{2R}\, u^{(1)}_{3R}\, u^{(1)}_{3R}\, u^{(2)}_{1R}\, d^{(2)}_{2R}\, u^{(2)}_{3R}\, e^{-(2)}_{R}\, d^{(3)}_{1R}\, d^{(3)}_{2R}\, e^{-(3)}_{R}\, e^{-(3)}_{R} \tag{12}$$

If we express gauge group eigenstates in terms of the mass eigenstates, then the above operator will have components which involve particles from the first and the second generations only. To see this, let us consider the case of extreme mixing in the right handed sector, where the gauge group eigenstates are expressed in terms of the mass eigenstates u, c, t, d, s, b, e^-, μ^-, τ^-, ν_e, ν_μ and ν_τ as,

$$\begin{pmatrix} u_1 & u_2 & u_3 & \nu_e \\ d_1 & d_2 & d_3 & e^- \end{pmatrix}_L , \quad \begin{pmatrix} c_1 & c_2 & c_3 & \nu_\mu \\ s_1 & s_2 & s_3 & \mu^- \end{pmatrix}_L$$

$$\begin{pmatrix} t_1 & t_2 & t_3 & \nu_\tau \\ b_1 & b_2 & b_3 & \tau^- \end{pmatrix}_L , \quad \begin{pmatrix} u_1 & u_2 & u_3 & \nu_e \\ b_1 & b_2 & b_3 & \tau^- \end{pmatrix}_R$$

$$\begin{pmatrix} c_1 & c_2 & c_3 & \nu_\mu \\ s_1 & s_2 & s_3 & \mu^- \end{pmatrix}_R , \quad \begin{pmatrix} t_1 & t_2 & t_3 & \nu_\tau \\ d_1 & d_2 & d_3 & e^- \end{pmatrix}_R \tag{13}$$

where we have used the convention described in Eq.(1) to display the gauge group eigenstates. By choosing the proper Yukawa couplings and the v.e.v. of the Higgs fields, it is possible to arrange that the fermions in the upper and the lower row get their masses from different terms, and that the fermion mass terms are really diagonal when expressed in terms of the fields u, c, t etc. Expressed in terms of the mass eigenstates, the operator (12) is given by,

$$u_{1R}\, u_{2R}\, u_{3R}\, u_{3R}\, c_{1R}\, s_{2R}\, c_{3R}\, \mu^-_R\, d_{1R}\, d_{2R}\, e^-_R\, e^-_R \tag{14}$$

which involve first and second generation fermions only. Note that all the standard Kobayashi-Maskawa (KM) mixing angles vanish with our choice of the mixing, since for the left handed fermions the mass eigenstates are identical to the gauge group eigenstates. This shows that the smallness of the KM angles does not give any upper bound on the operator (14) in the general case. The mixing angles required for calculating the v.e.v. of (14) may be made as large as we want and at the same time, the standard KM angles may be kept as small as we want.

The operator given in (14) involves two c and one s quarks. Hence the final state of a $\Delta B=3$ decay of a nucleus must contain two $\bar{c}$ and one $\bar{s}$ quarks. Since the rest mass of the three protons is about 3 GeV, we only need a small amount of center of mass energy to produce such particles in the final state. For non-relativistic monopoles the monopole-nucleus system does not have enough center of mass energy to produce two $\bar{c}$'s in the final state. In this case one of the $\bar{c}$'s must decay into lighter particles via W boson exchange. This makes the baryon decay cross-section to be of the order of the weak cross section rather than the strong cross section. However, it was argued by Goldhaber[4] that if the monopole is able to capture protons, then the effective decay rate, as observed in a laboratory experiment, will be the same, irrespective of whether the intrinsic decay rate is strong or weak. (In this case, however, the monopole must capture at least three nucleons before it can induce baryon decay).

Finally, we must mention that within the present approximation there is no way to get rid of the second generation fermions in a dimension eighteen operator. This may be explicitly verified by writing down all the baryon number violating condensates consistent with the conservation laws, and using the $(SU(3))^6$ symmetry in the generation space. We may avoid this conclusion if the fermion mass terms play a non-trivial role in determining these condensates[6], but we do not discuss it here, since the solution of the monopole-fermion system in the presence of massive fermion is still unknown.

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FOOTNOTES

[f1]This is because the operator

$$\langle u_{1R}^{(i_1)} d_{2R}^{(j_1)} u_{3R}^{(k_1)} e_R^{-(\ell_1)} u_{1R}^{(i_2)} d_{2R}^{(j_2)} u_{3R}^{(k_2)} e_R^{-(\ell_2)} u_{1R}^{(i_3)} d_{2R}^{(j_3)} u_{3R}^{(k_3)} e_R^{-(\ell_3)} \rangle$$

is proportional to

$$\epsilon_{i_1 i_2 i_3}$$

in the limit of massless fermions. Here for simplicity we have ignored dependences on the j_i, k_i and ℓ_i indices. If the mass eigenstates $\tilde{u}^{(i)}$ of the u type quarks are related to the gauge group eigenstates $u^{(i)}$ as,

$$\tilde{u}^{(i)} = \sum_{i'=1}^{3} V_{ii'}\, u^{(i')}$$

we get,

$$\langle \tilde{u}_{1R}^{(i_1)} d_{2R}^{(j_1)} u_{3R}^{(k_1)} e_R^{-(\ell_1)} \tilde{u}_{1R}^{(i_2)} d_{2R}^{(j_2)} u_{3R}^{(k_2)} e_R^{-(\ell_2)} \tilde{u}_{1R}^{(i_3)} d_{2R}^{(j_3)} u_{3R}^{(k_3)} e_R^{-(\ell_3)} \rangle$$

$$\propto \epsilon_{i_1' i_2' i_3'}\; V_{i_1 i_1'}\; V_{i_2 i_2'}\; V_{i_3 i_3'}$$

which vanishes unless i_1, i_2 and i_3 are different.

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