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MACROSCOPIC CHARGED HETEROTIC STRING

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Abstract

Classical solutions of equations of motion in low energy effective field theory, describing fundamental charged heterotic string, are found. These solutions automatically carry an electric current equal to the charge per unit length, and hence are accompanied by both, electric and magnetic fields. Force between two parallel strings vanish due to cancellation between electric and magnetic forces, and also between graviton, dilaton, and antisymmetric tensor field induced forces. Multi-string solutions describing configuration of parallel strings are also found. Finally, the solutions are shown to possess partially broken space-time supersymmetry.

1. Introduction

In the study of string theory, much of the recent attention has been focussed on the construction of classical solutions in string theory. The motivation for such study is manifold. First of all, in order to study non-perturbative string theory, one must include, in addition to the standard Fock space states, the soliton states in the spectrum. In particular, some of the underlying 'symmetries' of string theory may become manifest only after including these solitonic states in the spectrum. Secondly, since string theory is expected to provide us with a finite consistent theory of quantum gravity, new insights into phenomena like black hole evaporation in quantum gravity might be gained by studying evaporation of black holes in string theory. Finally, the macroscopic soliton like solutions in string theory might have been produced in the early universe, and may have observable consequences and astrophysical implications.

In refs.[1][2] a specific classical solution in string theory was constructed which represents fields around a macroscopic heterotic string[3]. It was found that the force between two such parallel strings vanish, and classical solutions of low energy effective field theory equations of motion were constructed which represent multi-string solutions. Finally, it was shown that this solution has partially broken space-time supersymmetry. Using space-time supersymmetry, a Bogomol'nyi bound for the mass per unit length of the string was found, and the solution representing fundamental string source was shown to saturate this bound. Later, it was also realized[4] that these solutions may be regarded as the extremal limit of black string solutions in heterotic string theory, where the horizon approaches the singularity.

Other aspects of macroscopic string-like solutions in string theory have been discussed in ref.[5]. Related work on the construction of other supersymmetric solutions of string theory equations of motion has been done in refs.[6][7]. Other multi-soliton solutions in string theory have been constructed in ref.[8].

In heterotic string theory, besides the string coordinates X^μ, there are other degrees of freedom. In particular, it contains 16 internal coordinates $Y^{(I)}$ which are constrained to be left moving on the world sheet, and are responsible for giving rise to gauge fields in the spectrum of the theory. Thus one would expect that if one considers a string source, where the world-sheet momenta conjugate to the coordinates $Y^{(I)}$ are non-vanishing, then it would represent a macroscopic string carrying a finite amount of charge per unit length. As we shall see later, due to the constraint that the coordinates $Y^{(I)}$ are left moving, such a string also carries an electric current equal to the charge per unit length of the string.† In this paper, we shall construct explicit solutions of the classical equations of motion of the low energy effective field theory, which represent such charged strings. The method that we shall be using is the method of twisting[9]-[13] which is a class of transformations, that act on a classical solution to generate new inequivalent classical solutions. In particular, in ref.[12] a class of such transformations were found, which, acting on an electrically neutral solution, generates an electrically charged solution. We shall show that the same transformations, when applied to the solution of ref.[2], generate new solutions representing charged macroscopic string. Furthermore, most of the features of the charge neutral solution, e.g. no force between parallel strings, explicit construction of multi-string solutions, space-time supersymmetry, and fundamental strings as extremal black strings also hold for the charge carrying solution.

The plan of the paper is as follows. In sect.2 we shall apply the method of ref.[12] to generate charged single string solution from the charge neutral solutions of ref.[2], and calculate the various charges and effective energy momentum tensor associated with the solution. In sect.3 we shall perform the same transformations on the multi-string solutions of ref.[2] and construct a multi-charged-string solution. This solution, however, is characterized by the fact that all the strings carry the same amount of charge per unit length. By slightly modifying this solution we construct multi-string solutions where different strings carry different amount of charge per unit length. In sect.4 we show that the elementary charged string solution corresponds to the extremal limit of a charged black string solution. We start from a black string solution carrying no electric or anti-symmetric tensor gauge field charge, and generate from this a black string solution carrying both these types of charges by using the transformation of ref.[12]. We then take a specific limit (in which

† Superconducting nature of these strings was discussed in ref.[3].

the horizon approaches the singularity) of this solution and show that the solution reduces to that representing a fundamental charged heterotic string. In sect.5 we discuss supersymmetry transformation properties of the solution, and show that it is invariant under half of the supersymmetry transformations of the theory. We first give a general argument showing that the twisting procedure that generates charged solutions from the uncharged ones commute with space-time supersymmetry transformations. Hence the space-time supersymmetry of the original solution of ref.[2] implies that the transformed solution will also be invariant under space-time supersymmetry. We then also explicitly verify that for the charged string solution, the supersymmetry transform of all the fermionic fields in the theory vanish. We conclude in sect.6 with a summary of our results.

2. Single String Solution

We shall consider heterotic string compactified to D dimensions. Let us assume that during the process of compactification p of the original 16 $U(1)$ gauge symmetries remain unbroken. The interaction between the massless modes of the string theory and that between the massless modes and the degrees of freedom of the fundamental string, are described by the following low energy effective Lagrangian:

$$
\begin{aligned}
S = & -\int d^D x \sqrt{-G} e^{-\Phi} (-R + \frac{1}{12} H_{\mu\nu\rho} H^{\mu\nu\rho} - G^{\mu\nu} \partial_\mu \Phi \partial_\nu \Phi + \frac{1}{8} F^{(I)}_{\mu\nu} F^{(I)\mu\nu}) \\
& - \frac{\mu}{2} \int d\sigma d\tau (\sqrt{-\gamma} \gamma^{mn} \partial_m X^\mu \partial_n X^\nu G_{\mu\nu}(X) + \epsilon^{mn} \partial_m X^\mu \partial_n X^\nu B_{\mu\nu}(X) \\
& + C A^{(I)}_\mu (X) \epsilon^{mn} \partial_m Y^{(I)} \partial_n X^\mu)
\end{aligned}
\tag{2.1}
$$

Here $G_{\mu\nu}$ is the metric, R is the scalar curvature, $F^{(I)}_{\mu\nu} = \partial_\mu A^{(I)}_\nu - \partial_\nu A^{(I)}_\mu$ is the field strength corresponding to the $U(1)$ gauge fields $A^{(I)}_\mu$ $(1 \le I \le p)$, Φ is the dilaton field,

$$
H_{\mu\nu\rho} = \partial_\mu B_{\nu\rho} + \text{ cyclic permutations} - (\Omega_3(A))_{\mu\nu\rho}
\tag{2.2}
$$

where $B_{\mu\nu}$ is the antisymmetric tensor gauge field, and,

$$
(\Omega_3(A))_{\mu\nu\rho} = \frac{1}{4} (A^{(I)}_\mu F^{(I)}_{\nu\rho} + \text{ cyclic permutations})
\tag{2.3}
$$

is the gauge Chern Simons term. The metric used here is the σ-model metric, and is related to the Einstein metric G_E by the relation

$$G_{\mu\nu} = exp(2\Phi/(D-2))G_{E\mu\nu} \tag{2.4}$$

The variables X^μ denote the coordinates of the string, σ, τ are the world sheet coordinates, and γ_{mn} is the world sheet metric. μ is the string tension and C is a constant $\propto \mu^{-1/2}$ which is adjusted in such a way that in the low energy effective action involving the massless fields, the gauge fields appear with the normalization given in eq.(2.1). $Y^{(I)}$ denote the internal coordinates of the string responsible for gauge symmetry, and satisfy,

$$(\sqrt{-\gamma}\gamma^{mn} - \epsilon^{mn})\partial_n Y^{(I)} = 0 \tag{2.5}$$

where ϵ^{mn} is the tensor,

$$\epsilon^{\sigma\tau} = -\epsilon^{\tau\sigma} = 1, \quad \epsilon^{\tau\tau} = \epsilon^{\sigma\sigma} = 0 \tag{2.6}$$

We shall choose a gauge in which,

$$(\sqrt{-\gamma})^{-1}\gamma_{mn} = a\eta_{mn} + bP_{mn} \tag{2.7}$$

where a and b are constants to be specified later, η is the Minkowski metric, and,

$$P = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} \tag{2.8}$$

The first row/column corresponds to σ, the second row/column corresponds to τ. In this gauge, eq.(2.5) takes the form:

$$(\partial_\tau - \partial_\sigma)Y^{(I)} = 0 \tag{2.9}$$

Note that in writing down the above effective action, we have ignored the non-Abelian components of the gauge fields, as well as any other massless fields that might arise during compactification from 10 to D dimensions. Similarly, we have ignored the world sheet fermionic degrees of freedom of the string which carry space-time Lorentz index. Since we shall look for solutions of the equations of motion where these degrees of freedom are not excited, this provides a consistent truncation for our purpose. We have chosen our mass scale such that $2\kappa^2 = 1$.

The equations of motion for the fields $G_{\mu\nu}$, $B_{\mu\nu}$, Φ and A_μ, derived from the action

(2.1) are given by,*

$$
\begin{aligned}
&\sqrt{-G}e^{-\Phi}G^{\mu\xi}G^{\nu\eta}[R_{\mu\nu}-\frac{1}{2}RG_{\mu\nu}+D_\mu D_\nu\Phi-G_{\mu\nu}D^\rho D_\rho\Phi+\frac{1}{2}G_{\mu\nu}D_\rho\Phi D^\rho\Phi\\
&-\frac{1}{4}(H_{\mu\rho\tau}H_\nu{}^{\rho\tau}-\frac{1}{6}H_{\rho\tau\sigma}H^{\rho\tau\sigma}G_{\mu\nu})-\frac{1}{4}(F_{\mu\rho}F_\nu{}^\rho-\frac{1}{4}G_{\mu\nu}F_{\rho\tau}F^{\rho\tau})]\\
=&-\frac{\mu}{2}\int d\sigma d\tau\sqrt{-\gamma}\gamma^{mn}\partial_m X^\xi\partial_n X^\eta\delta^{(D)}(x-X(\sigma,\tau))\\
&\sqrt{-G}D_\rho(e^{-\Phi}H^{\mu\nu\rho})=\mu\int d\sigma d\tau\epsilon^{mn}\partial_m X^\mu\partial_n X^\nu\delta^{(D)}(x-X(\sigma,\tau))\\
&R-D_\mu\Phi D^\mu\Phi-\frac{1}{12}H^2-\frac{1}{8}F^2+2D^\mu D_\mu\Phi=0\\
&\sqrt{-G}\Big(D_\mu(e^{-\Phi}F^{(I)\mu\nu})+\frac{1}{2}e^{-\Phi}H_{\rho\mu}{}^{\nu}F^{(I)\rho\mu}\Big)\\
=&C\mu\int d\sigma d\tau\epsilon^{mn}\partial_m Y^{(I)}\partial_n X^\nu\delta^{(D)}(x-X(\sigma,\tau))
\end{aligned}
\tag{2.10}
$$

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* Actually, there is a subtlety in deriving the $A^{(I)}_\nu$ equation. The left hand side of the $A^{(I)}_\nu$ equation, as derived by varying the low energy effective action with respect to $A^{(I)}_\nu$, contains an extra term $-\frac{1}{2}A^{(I)}_\rho D_\mu(e^{-\Phi}H^{\mu\nu\rho})$, which, in turn, is proportional to the left hand side of the $B_{\mu\nu}$ equation. This term is gauge dependent, which is seen by manifest appearance of $A^{(I)}_\rho$ in the equation. This apparent inconsistency may be removed as follows. Since this extra term comes from the variation of the Chern-Simons term $\Omega_3(A)$ in the effective action, which, in turn, arises from the two loop β function[14], we need to include the one loop effective action describing the interaction between *world-sheet fields* and $A^{(I)}_\mu$ in order to get a consistent set of equations of motion. This effective action is proportional to[14],

$$
\int d\sigma d\tau A^{(I)}_\mu\partial_m X^\mu[\sqrt{-\gamma}\gamma^{mn}-(\sqrt{-\gamma}\gamma^{mm'}+\epsilon^{mm'})\frac{\partial_{m'}\partial_{n'}}{\partial^2}(\sqrt{-\gamma}\gamma^{nn'}+\epsilon^{nn'})]A^{(I)}_\nu\partial_n X^\nu
$$

If we include this extra term in the expression for the action S given in eq.(2.1), and write down the equation of motion for $A^{(I)}_\nu$, the right hand side of the equation receives an extra term. This term exactly cancels the gauge non-invariant term on the left hand side by the $B_{\mu\nu}$ equation of motion if the background satisfies the condition $\partial_\mu A^{(I)}_\nu\partial_m X^\mu\partial_n X^\nu=0$. Since we shall restrict to backgrounds satisfying this condition, we can use eq.(2.10) for studying classical solutions of the equations of motion in the presence of fundamental string background.

A solution of these equations of motion in the presence of a string source of the form:

$$X^0=\tau,\qquad X^{D-1}=\sigma,\qquad X^\mu=0 \text{ for } 1\le\mu\le D-2,\qquad Y^{(I)}=0,\qquad (\sqrt{-\gamma})^{-1}\gamma_{mn}=\eta_{mn} \tag{2.11}$$

was obtained in ref.[2]. This solution is given by,

$$\begin{aligned} ds^2 =& e^E\{-dt^2+(dx^{D-1})^2\}+\sum_{i=1}^{D-2}dx^i dx^i \\ B_{(D-1)t} =& 1-e^E, \quad \Phi=E, \quad A_\mu^{(I)}=0 \end{aligned} \tag{2.12}$$

where,

$$e^{-E}=1+MG^{(D-2)}(\vec{r}) \tag{2.13}$$

$\vec{r}$ denotes the $D-2$ dimensional vector $(x^1,\ldots x^{D-2})$, and $G^{(D-2)}$ is the $D-2$ dimensional Green's function, given by,

$$\begin{aligned} G^{(D-2)}(\vec{r}) =& \frac{1}{(D-4)\omega_{D-3}r^{D-4}} \quad \text{for } D>4 \\ =& -\frac{1}{2\pi}\ln r \quad \text{for } D=4 \end{aligned} \tag{2.14}$$

where ω_{D-3} is the volume of a unit $D-3$ sphere. M is a constant which can be identified to μ by looking at the source terms in the equation of motion.

In this case there is no source for the fields $A_\mu^{(I)}$, and as a result the solution represents fields around a charge neutral heterotic string. We shall now discuss the more general case when the coordinate $Y^{(I)}$ has the form,

$$\gamma^{\tau m}\partial_m Y^{(I)}=p^{(I)} \tag{2.15}$$

where $p^{(I)}$ are a set of constants. This gives rise to a source for the gauge field, and hence, in general, represents a charged string. Note, also, that due to the constraint given in eq.(2.9), the string acts as a source for both, the A_0 and the A_{D-1} fields. As a result, the string is accompanied by both, electric and magnetic fields, and acts as a source of charge density, as well as electric current. For convenience of presentation, we shall construct a solution for which only $p^{(1)}$ is non-zero. A general solution for which all the $p^{(I)}$'s are non-zero may easily be found by rotating the final solution in the p dimensional space spanned by the indices I.

The technique that we shall use in constructing a classical solution of the equations of motion in the presence of such a string source is based on the method of twisting discussed in refs.[12], hence we shall first briefly review part of the results of ref.[12] which is relevant for our study. Let $\{G_{\mu\nu}, B_{\mu\nu}, \Phi, A_\mu^{(1)}\}$ denote a solution of the equations of motion (2.9) which is independent of d of the D coordinates, including the time coordinate $t \equiv x^0$. Let x^α denote these d coordinates, and x^i denote the rest of the coordinates. We shall further assume that the components $G_{i\alpha}$ and $B_{i\alpha}$ vanish, so that both, the metric and the antisymmetric tensor field, have block diagonal form. Let us regard $G_{\alpha\beta}$ and $B_{\alpha\beta}$ as $d \times d$ matrices, and $A_\alpha^{(1)}$ as a d dimensional column vector, with the last row/column corresponding to the time coordinate. Let $\eta_{\alpha\beta}$ denote the usual Minkowski metric. We now define,

$$K_{\alpha\beta} = -B_{\alpha\beta} - G_{\alpha\beta} - \frac{1}{4} A_\alpha^{(1)} A_\beta^{(1)} \tag{2.16}$$

and the $(2d+1) \times (2d+1)$ matrix,

$$M = \begin{pmatrix} (K^T - \eta) G^{-1} (K - \eta) & (K^T - \eta) G^{-1} (K + \eta) & -(K^T - \eta) G^{-1} A \\ (K^T + \eta) G^{-1} (K - \eta) & (K^T + \eta) G^{-1} (K + \eta) & -(K^T + \eta) G^{-1} A \\ -A^T G^{-1} (K - \eta) & -A^T G^{-1} (K + \eta) & A^T G^{-1} A \end{pmatrix} \tag{2.17}$$

where T denotes transposition of a matrix. The result of ref.[12] then says that $\{G'_{\mu\nu}, B'_{\mu\nu}, \Phi', A_\mu^{(1)\prime}\}$ also describe a solution of the classical equations of motion in a region of space where the right hand sides of eqs.(2.10) are zero (i.e. in the absence of source terms) if the primed variables are related to the unprimed ones through the relations,

$$M' = \Omega M \Omega^T, \quad \Phi' - \ln\det G' = \Phi - \ln\det G, \quad G'_{ij} = G_{ij}, \quad B'_{ij} = B_{ij} \tag{2.18}$$

where Ω is a matrix of the form

$$\Omega = \begin{pmatrix} S & \\ & R \end{pmatrix} \tag{2.19}$$

S and R being arbitrary $O(d-1,1)$ and $O(d,1)$ matrices which preserve the matrices η and $\begin{pmatrix} \eta & \\ & 1 \end{pmatrix}$ respectively.

The solution given in eq.(2.12) is independent of the coordinates t and x^{D-1}, and is block diagonal. Thus we may generate new solutions from it by making the transformations given in eq.(2.18) with $d = 2$. We choose,

$$\Omega = \begin{pmatrix} I_3 & & \\ & \cosh\alpha & \sinh\alpha \\ & \sinh\alpha & \cosh\alpha \end{pmatrix} \tag{2.20}$$

where I_n is the $n \times n$ identity matrix, and α is an arbitrary number. Applying this transformation on the solution given in eq.(2.12) we get a new solution given by,

$$\begin{aligned} ds^2 =& \frac{1}{\cosh^2\frac{\alpha}{2}e^{-E} - \sinh^2\frac{\alpha}{2}}(-dt^2 + (dx^{D-1})^2) \\ &+ \frac{\sinh^2\frac{\alpha}{2}(e^{-E}-1)}{(\cosh^2\frac{\alpha}{2}e^{-E} - \sinh^2\frac{\alpha}{2})^2}(dt + dx^{D-1})^2 + \sum_{i=1}^{D-2} dx^i dx^i \\ B_{(D-1)t} =& \frac{\cosh^2\frac{\alpha}{2}(e^{-E}-1)}{\cosh^2\frac{\alpha}{2}e^{-E} - \sinh^2\frac{\alpha}{2}} \\ A^{(1)}_{D-1} =& A^{(1)}_t = \frac{\sinh\alpha(e^{-E}-1)}{\cosh^2\frac{\alpha}{2}e^{-E} - \sinh^2\frac{\alpha}{2}} \\ \Phi =& -\ln(\cosh^2\frac{\alpha}{2}e^{-E} - \sinh^2\frac{\alpha}{2}) \end{aligned} \tag{2.21}$$

Using eq.(2.13) the solution may be written as,

$$
\begin{aligned}
ds^2 =& \frac{1}{1+NG^{(D-2)}(\vec{r})}(-dt^2+(dx^{D-1})^2) + \frac{q^2 G^{(D-2)}(\vec{r})}{4N(1+NG^{(D-2)}(\vec{r}))^2}(dt+dx^{D-1})^2 \\
&+\sum_{i=1}^{D-2} dx^i dx^i \\
B_{(D-1)t} =& \frac{NG^{(D-2)}(\vec{r})}{1+NG^{(D-2)}(\vec{r})} \\
A^{(1)}_{D-1} =& A^{(1)}_t = \frac{qG^{(D-2)}(\vec{r})}{1+NG^{(D-2)}(\vec{r})} \\
\Phi =& -\ln(1+NG^{(D-2)}(\vec{r}))
\end{aligned}
\tag{2.22}
$$

where

$$
N = M\cosh^2\frac{\alpha}{2}, \qquad q = M\sinh\alpha \tag{2.23}
$$

The solution is not invariant under a boost in the x^{D-1} direction, but such a boost corresponds to changing the parameter q. Note that the solution becomes singular as $\vec{r}\to 0$, showing that there are possible source terms at $\vec{r}=0$. These source terms may be calculated by explicitly evaluating the δ function singularities on the left hand side of eqs.(2.10), and can be shown to be consistent with the following string configuration:

$$
X^0 = \tau, \qquad X^{D-1} = \sigma, \qquad \gamma^{\tau m}\partial_m Y^{(1)} = p^{(1)} \tag{2.24}
$$

provided we make the identification,

$$
\mu = N, \qquad p^{(1)} = -\frac{q}{\mu C} \tag{2.25}
$$

Here γ is the world sheet metric induced by the target space metric, and satisfies,

$$
(\sqrt{-\gamma})^{-1}\gamma_{mn} = \eta_{mn} + \frac{q^2}{4N^2}P_{mn} \tag{2.26}
$$

The various field strength tensors may be calculated from this solution and we get the following results,

$$\begin{aligned} F^{(1)}_{r(D-1)} =& F^{(1)}_{rt} = -\frac{q}{r^{D-3}\omega_{D-3}(1+NG^{(D-2)}(\vec{r}))^2} \\ H_{r(D-1)t} =& -\frac{N}{r^{D-3}\omega_{D-3}(1+NG^{(D-2)}(\vec{r}))^2} \end{aligned} \tag{2.27}$$

Note that $\Omega_3(A)$ vanishes everywhere for the specific solution we have constructed. For $D > 4$ the Einstein metric defined in eq.(2.4) is asymptotically flat, and the electric charge Q, the electric current J and the axionic charge Z associated with the solution may be defined in terms of the asymptotic behavior of the field strengths in the $r \to \infty$ limit as follows:

$$\begin{aligned} \frac{1}{2\sqrt{2}} F^{(1)}_{rt} \simeq& -\frac{Q}{r^{D-3}\omega_{D-3}} \\ \frac{1}{2\sqrt{2}} F^{(1)}_{r(D-1)} \simeq& -\frac{J}{r^{D-3}\omega_{D-3}} \\ H_{r(D-1)t} \simeq& -\frac{Z}{r^{D-3}\omega_{D-3}} \end{aligned} \tag{2.28}$$

(The factor of $2\sqrt{2}$ in the definition of the electric charge and electric current has been introduced so that our normalization matches that of refs.[4].) Eqs.(2.27) and (2.28) give,

$$Q = \frac{q}{2\sqrt{2}}, \qquad J = \frac{q}{2\sqrt{2}}, \qquad Z = N \tag{2.29}$$

Thus the string carries an electric current equal to its electric charge per unit length, as expected from the general arguments given before.

For $D > 4$, the energy momentum tensor associated with the solution may be calculated by the procedure used in ref.[2]. We first compute the Einstein metric using eq.(2.4), and define,

$$h_{\mu\nu} = G_{E\mu\nu} - \eta_{\mu\nu} \tag{2.30}$$

From this we can compute the linearised Ricci tensor as,

$$R^{(1)}_{\mu\nu} = \frac{1}{2}\Big(\frac{\partial^2 h^\rho_\mu}{\partial x^\rho \partial x^\nu} + \frac{\partial^2 h^\rho_\nu}{\partial x^\rho \partial x^\mu} - \frac{\partial^2 h^\rho_\rho}{\partial x^\mu \partial x^\nu} - \frac{\partial^2 h_{\mu\nu}}{\partial x^\rho \partial x_\rho} \Big) \tag{2.31}$$

where the indices are raised or lowered by the metric $\eta_{\mu\nu}$. The total energy momentum tensor $\Theta_{\mu\nu}$ is then defined as[2],

$$\Theta_{\mu\nu} = 2(R^{(1)}_{\mu\nu} - \frac{1}{2}R^{(1)\rho}{}_{\rho}\eta_{\mu\nu}) \tag{2.32}$$

The effective two dimensional energy momentum tensor $T_{\alpha\beta}$ associated with the string may then be obtained as,

$$\begin{aligned} T_{\alpha\beta} &= \int d^{D-2}x\Theta_{\alpha\beta}(x) \\ &= \int_{S^{(D-3)}} \Big[-\frac{\partial h_{\alpha\beta}}{\partial x^l} - \eta_{\alpha\beta}\{\frac{\partial h_{kl}}{\partial x^k} + \frac{\partial h_{tt}}{\partial x^l} - \frac{\partial h_{(D-1)(D-1)}}{\partial x^l} - \frac{\partial h_{kk}}{\partial x^l}\}\Big] n^l r^{D-3} d\Omega_{D-3} \end{aligned} \tag{2.33}$$

where we have performed an integration by parts. Here $S^{(D-3)}$ denotes the surface at $r = \infty$, and n^i is a vector normal to the surface. Evaluating the left hand side of eq.(2.33) for the present solution, we get,

$$T_{\alpha\beta} = N(-\eta_{\alpha\beta} + \frac{q^2}{4N^2}P_{\alpha\beta}) \tag{2.34}$$

where the matrix P has been defined in eq.(2.8).

3. Multi-string Solutions

In this section we shall construct explicit solutions which represent multiple parallel strings at rest. In order to show that such a solution is at all possible, we shall first show that in the field of the single string given in eq.(2.22), a test string parallel to the original string does not encounter any force. The test string is given in the following gauge:

$$X^0 = \tau, \qquad X^{(D-1)} = \sigma, \qquad (\sqrt{-\gamma})^{-1}\gamma_{mn} = a\eta_{mn} + bP_{mn} \tag{3.1}$$

where a and b are two constants whose value we shall not need for our analysis. The equations of motion for the transverse coordinates X^i derived from the action given in eq.(2.1) then takes the form:

$$\begin{aligned} \sqrt{-\gamma}\gamma^{mn}\partial_m\partial_n X^i = &- \sqrt{-\gamma}\gamma^{mn}\Gamma^i_{\nu\rho}\partial_m X^\nu \partial_n X^\rho + \frac{1}{2}H^i{}_{\nu\rho}\partial_m X^\nu \partial_n X^\rho \epsilon^{mn} \\ &+ CG^{ij}F^{(I)}_{j\mu}\epsilon^{mn}\partial_m Y^{(I)}\partial_n X^\mu \end{aligned} \tag{3.2}$$

We now start with the following configuration of the test string at a given time,

$$\partial_\tau X^i = \partial_\sigma X^i = 0, \qquad (\partial_\tau - \partial_\sigma) Y^{(I)} = 0 \tag{3.3}$$

Eq.(3.2) then gives,

$$\frac{\partial^2 X^i}{\partial \tau^2} = 0 \tag{3.4}$$

showing that the test string does not encounter a force in the transverse direction.

Let us now turn to the problem of explicitly constructing the multi-string solution. As a first step, we take the multi string solution of ref.[2] and transform it by the transformation given in eq.(2.18). The final result has the form given in eq.(2.21), except that e^{-E} is now given by,

$$e^{-E} = 1 + \sum_l M G^{(D-2)}(\vec{r} - \vec{r}_l) \tag{3.5}$$

where $\vec{r}_l$ are the locations of the strings. Substituting this into eq.(2.21) we get the following multi-string solution:

$$\begin{aligned}
ds^2 =& \frac{1}{1 + N \sum_l G^{(D-2)}(\vec{r} - \vec{r}_l)} (-dt^2 + (dx^{D-1})^2) \\
&+ \frac{q^2 \sum_l G^{(D-2)}(\vec{r} - \vec{r}_l)}{4N(1 + N \sum_l G^{(D-2)}(\vec{r} - \vec{r}_l))^2} (dt + dx^{D-1})^2 + \sum_{i=1}^{D-2} dx^i dx^i \\
B_{(D-1)t} =& \frac{N \sum_l G^{(D-2)}(\vec{r} - \vec{r}_l)}{1 + N \sum_l G^{(D-2)}(\vec{r} - \vec{r}_l)} \\
A^{(1)}_{D-1} =& A^{(1)}_t = \frac{q \sum_l G^{(D-2)}(\vec{r} - \vec{r}_l)}{1 + N \sum_l G^{(D-2)}(\vec{r} - \vec{r}_l)} \\
\Phi =& -\ln(1 + N \sum_l G^{(D-2)}(\vec{r} - \vec{r}_l))
\end{aligned} \tag{3.6}$$

Although this represents a charged multi-string solution, we would like to construct more general multi-string solutions where the different strings carry independent $U(1)$ charges

$Q_l^{(I)}$. Examining eq.(3.6), we take the following ansatz for such a multi-string solution:

$$
\begin{aligned}
ds^2 =& \frac{1}{1+N\sum_l G^{(D-2)}(\vec{r}-\vec{r}_l)}(-dt^2+(dx^{D-1})^2) \\
&+ g(\vec{r})(dt+dx^{D-1})^2 + \sum_{i=1}^{D-2} dx^i dx^i \\
B_{(D-1)t} =& \frac{N\sum_l G^{(D-2)}(\vec{r}-\vec{r}_l)}{1+N\sum_l G^{(D-2)}(\vec{r}-\vec{r}_l)} \\
A^{(I)}_{D-1} =& A^{(I)}_t = \frac{\sum_l q_l^{(I)} G^{(D-2)}(\vec{r}-\vec{r}_l)}{1+N\sum_l G^{(D-2)}(\vec{r}-\vec{r}_l)} \\
\Phi =& -\ln(1+N\sum_l G^{(D-2)}(\vec{r}-\vec{r}_l))
\end{aligned}
\tag{3.7}
$$

where $g(\vec{r})$ is a function to be determined. With this ansatz, the equations of motion for $B_{\mu\nu}$, Φ and $A^{(I)}_\mu$ are satisfied identically. The equation of motion for the metric gives rise to the following differential equation for the function $g(\vec{r})$:

$$
\begin{aligned}
&\sum_{k=1}^{D-2} \partial_k\partial_k \Big\{ g(\vec{r})\big(1+\sum_l NG^{(D-2)}(\vec{r}-\vec{r}_l)\big)\Big\} \\
=& -\frac{1}{2}\big(1+\sum_l NG^{(D-2)}(\vec{r}-\vec{r}_l)\big)\sum_{I=1}^{16}\sum_{k=1}^{D-2}\Big(\partial_k\Big(\frac{\sum_l q_l^{(I)} G^{(D-2)}(\vec{r}-\vec{r}_l)}{1+\sum_l NG^{(D-2)}(\vec{r}-\vec{r}_l)}\Big)\Big)^2
\end{aligned}
\tag{3.8}
$$

A solution to the above equation is given by,

$$
g(\vec{r}) = \frac{1}{4}\Big[\frac{\sum_{l,I}(q_l^{(I)})^2 G^{(D-2)}(\vec{r}-\vec{r}_l)}{N\big(1+\sum_l NG^{(D-2)}(\vec{r}-\vec{r}_l)\big)} - \frac{\sum_I \big(\sum_l q_l^{(I)} G^{(D-2)}(\vec{r}-\vec{r}_l)\big)^2}{\big(1+\sum_l NG^{(D-2)}(\vec{r}-\vec{r}_l)\big)^2}\Big]
\tag{3.9}
$$

Eqs.(3.7) and (3.9) gives the general multi-string solution.

4. Fundamental Strings as Extremal Black Holes

In ref.[4] it was shown that the charge neutral fundamental string solutions of ref.[2] can be regarded as extremal limit of black string solution. In this section we shall show that even the charged fundamental string can be regarded as extremal limit of charged black string solutions. In order to construct the charged black string solutions whose extremal limit are these fundamental strings, we start with the black string solution in $D(>4)$ dimensions without any electric, magnetic, or antisymmetric tensor gauge field charge. The solution is given by,

$$\begin{aligned} ds^2 =& -(1-\frac{r_+^{D-4}}{r^{D-4}})dt^2 + \frac{dr^2}{1-\frac{r_+^{D-4}}{r^{D-4}}} + r^2 d\Omega_{D-3}^2 + (dx^{D-1})^2 \\ \Phi =& 0, \qquad A_\mu^{(I)} = 0, \qquad B_{\mu\nu} = 0 \end{aligned} \tag{4.1}$$

where $d\Omega_{D-3}$ denotes the line element of a $D-3$ sphere. The solution is independent of the coordinates t and x^{D-1}. We now perform the transformation given in eq.(2.18) with the following choice of the matrix Ω:

$$\Omega = \begin{pmatrix} I_2 & & & \\ & \cosh\alpha_2 & \sinh\alpha_2 & \\ & \sinh\alpha_2 & \cosh\alpha_2 & \\ & & & 1 \end{pmatrix} \begin{pmatrix} I_3 & & \\ & \cosh\alpha_1 & \sinh\alpha_1 \\ & \sinh\alpha_1 & \cosh\alpha_1 \end{pmatrix} \tag{4.2}$$

The transformed solution in the $D=5$ case was explicitly worked out in ref.[12]. The solution for general D can be obtained from the results of ref.[12] in a straightforward manner, and we get,

$$\begin{aligned} ds^2 =& -\frac{1}{4(r^{D-4}-r_0^{D-4})^2}(4r^{D-4}(r^{D-4}-r_+^{D-4}) - \beta^2 r_+^{2D-8})dt^2 + (dx^{D-1})^2 \\ & + \beta\frac{r_+^{D-4}}{r^{D-4}-r_0^{D-4}} dx^{D-1}dt + \frac{dr^2}{1-\frac{r_+^{D-4}}{r^{D-4}}} + r^2 d\Omega_{D-3}^2 \\ B_{t(D-1)} =& \beta\frac{r_+^{D-4}}{2(r^{D-4}-r_0^{D-4})} \\ A_t^{(1)} =& \gamma\frac{r_+^{D-4}}{r^{D-4}-r_0^{D-4}} \\ A_\mu^{(1)} =& 0 \qquad \text{for } 1 \le \mu \le D-1 \\ \Phi =& -\ln(1-\frac{r_0^{D-4}}{r^{D-4}}) \end{aligned} \tag{4.3}$$

where,

$$\begin{aligned} \gamma =& \sinh\alpha_1 \\ \beta =& \cosh\alpha_1 \sinh\alpha_2 \\ (r_0)^{D-4} =& \frac{1}{2}(r_+)^{D-4}(1-\sqrt{1+\beta^2+\gamma^2}) \end{aligned} \tag{4.4}$$

The above solution describes a black string with horizon at $r = r_+$, and carry charges associated with the antisymmetric tensor gauge field $B_{\mu\nu}$ and $U(1)$ gauge field $A^{(1)}_\mu$ proportional to $\beta(r_+)^{D-4}$ and $\gamma(r_+)^{D-4}$ respectively. $r_+ \to 0$ corresponds to the extremal limit of the black string, since in this limit the horizon approaches the singular point.

We shall now show that the extremal limit of the black hole corresponds to the fundamental black string solution that we have constructed in the previous section. This is done in two stages. First we boost the solution in the x^{D-1} direction by an angle η. Then we take the limit $r_+ \to 0$, $\beta \to -\infty$, $\gamma \to \infty$, $\eta \to \infty$, with the following combinations kept fixed:

$$C_1 = \frac{1}{2}|\beta|(r_+)^{D-4}, \qquad C_2 = \gamma \cosh\eta (r_+)^{D-4}, \qquad \gamma^2 = 2|\beta| \tag{4.5}$$

The resulting solution is of the form:

$$\begin{aligned} ds^2 =& \frac{1}{1+\frac{C_1}{r^{D-4}}}(-dt^2 + (dx^{D-1})^2) + \frac{C_2^2}{4C_1}\frac{1}{r^{D-4}(1+\frac{C_1}{r^{D-4}})^2}(dt+dx^{D-1})^2 \\ &+ (dr^2 + r^2 d\Omega^2_{D-3}) \\ B_{(D-1)t} =& \frac{C_1}{r^{D-4}+C_1} \\ A^{(1)}_t =& A^{(1)}_{D-1} = \frac{C_2}{r^{D-4}+C_1} \\ \Phi =& -\ln(1+\frac{C_1}{r^{D-4}}) \end{aligned} \tag{4.6}$$

This is identical to the solution given in eq.(2.22) with the identification,

$$C_1 = \frac{N}{(D-4)\omega_{D-3}}, \qquad C_2 = \frac{q}{(D-4)\omega_{D-3}} \tag{4.7}$$

5. Space-time Supersymmetry

The original solution constructed in ref.[2] had a partially broken space-time supersymmetry. Thus it is natural to ask if the solution given in eq.(2.22) also possesses such a supersymmetry. We shall first give a general argument showing that the transformation given in eqs.(2.18), (2.20) commutes with the space-time supersymmetry transformation, hence if the original transformation is space-time supersymmetric, so must be the transformed solution. We shall then verify explicitly that the transformed solution is indeed space-time supersymmetric.

Let us first recall the string field theoretic origin of the 'symmetry' transformation given in eq.(2.18), with Ω given in eq.(2.20)[12]. In string field theory, a general off-shell string field configuration corresponds to a state in the combined matter-ghost superconformal field theory. Restricting to string field configurations that are independent of the coordinates x^0 and x^{D-1}, and has only abelian gauge field configurations, correspond to restricting to conformal field theory states carrying zero momentum in the x^0, x^{D-1} and $Y^{(I)}$ directions. In particular, the dependence of the corresponding vertex operators on the world sheet fields X^0 and $Y^{(1)}$ is through powers of $\partial^n X^0$, $\bar\partial^m X^0$, and $\bar\partial^l Y^{(1)}$. The correlation functions involving these vertex operators are invariant under the transformation,

$$\partial^m X^0 \to \partial^m X^0, \qquad \begin{pmatrix} \bar\partial^m X^0 \\ \bar\partial^m Y^{(1)} \end{pmatrix} \to \begin{pmatrix} \cosh\alpha & \sinh\alpha \\ \sinh\alpha & \cosh\alpha \end{pmatrix} \begin{pmatrix} \bar\partial^m X^0 \\ \bar\partial^m Y^{(1)} \end{pmatrix} \qquad \forall\, m \tag{5.1}$$

Since the interaction vertices of string field theory are constructed in terms of the correlation functions in the conformal field theory, this symmetry of the conformal field theory correlation functions gives rise to a 'symmetry' of string field theory action for this restricted class of field configutrations. This is the origin of the 'symmetry' given in eqs.(2.18), (2.20) in the effective field theory.

Since so far we do not have a consistent closed heterotic string field theory (although we do have such a theory involving only the Neveu-Schwarz states[15]) we do not know precisely how the space-time supersymmetry operator will look like in the string field theory. However, from the general analysis[16] it is clear that space-time supersymmetry will act only on the holomorphic part of the vertex operators representing a general off-shell string field configuration. Since the symmetry transformation given in eq.(5.1) acts on the anti-holomorphic part of the vertex operators, these two symmetry transformations commute. As a result, (5.1), or, equivalently, (2.18) with Ω given in eq.(2.20), will transform a supersymmetric solution to a supersymmetric solution.

To verify explicitly that the solution given in eq.(2.22) (and also in eqs.(3.6), (3.7)) has partial space-time supersymmetry, we need to show the existence of a supersymmetry transformation parameter ξ such that variations of the gravitino field ψ_μ, dilatino field λ, and the gaugino field $\chi^{(I)}$ vanish. The corresponding constraints on ξ can be expressed as[6]

$$\begin{aligned} \delta\psi_\mu = 0 &\rightarrow D_\mu \xi - \frac{1}{8} H_{\mu\nu\rho} \Gamma^{\nu\rho} \xi = 0 \\ \delta\lambda = 0 &\rightarrow (\not{\partial}\Phi)\xi - \frac{1}{6} H_{\mu\nu\rho} \Gamma^{\mu\nu\rho} \xi = 0 \\ \delta\chi^{(I)} = 0 &\rightarrow F^{(I)}_{\mu\nu} \Gamma^{\mu\nu} \xi = 0 \end{aligned} \tag{5.2}$$

where $\Gamma^{\mu_1\cdots\mu_n}$ denote the antisymmetrized product of the γ-matrices. (Note that the fields and the supersymmetry transformation parameters in our convention are related to those used in ref.[2] by a set of field redefinitions.) These equations may be satisfied by choosing,

$$\xi = e^{\frac{\Phi}{4}} \xi_0 \tag{5.3}$$

where ξ_0 is a constant spinor, satisfying,

$$\sqrt{-\det \begin{pmatrix} G_{tt} & G_{t(D-1)} \\ G_{t(D-1)} & G_{(D-1)(D-1)} \end{pmatrix}} (\Gamma^t \Gamma^{(D-1)} - \Gamma^{(D-1)} \Gamma^t) \xi_0 = 2\xi_0 \tag{5.4}$$

This shows that the solutions are invariant under half of the space-time supersymmetry generators, since the supersymmetry, instead of being generated by an arbitrary spinor, is generated by a spinor satisfying the constraint given in eq.(5.4).

6. Conclusion

In this paper we have constructed solutions of the classical equations of motion of low energy effective field theory describing single and multiple charged heterotic strings parallel to each other. These solutions are characterized by the novel feature that a charged string always carries a current equal to its charge per unit length, and hence is accompanied by both electric and magnetic fields. We have also shown that the charged string solution may be regarded as the extremal limit of a charged black string solution. Finally, these solutions were shown to be invariant under half of the space-time supersymmetry generators of the theory. It will be interesting to investigate the cosmological and astrophysical implications of these charge and current carrying strings.

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