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HETEROTIC STRING THEORY ON CALABI-YAU MANIFOLDS IN THE GREEN-SCHWARZ FORMALISM*

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ABSTRACT

The σ-model describing the propagation of the heterotic string in a Calabi-Yau background is written down in the Green-Schwarz formulation. This model has manifest N=1 space-time supersymmetry, however only the SO(2) subgroup of the four dimensional Lorentz group is realized linearly. The criteria for the existence of the full SO(3,1) Lorentz symmetry in this model are studied. These requirements turn out to be identical to those obtained by demanding the existence of space-time supersymmetry in the Neveu-Schwarz-Ramond formulation of the theory, where the SO(3,1) Lorentz symmetry is manifest, but space-time supersymmetry is not. The analysis is easily generalized to more general backgrounds, which give rise to (2,0) superconformal field theories on the world sheet. Finally it is shown that if the requirements which give unbroken N=1 space-time supersymmetry and full SO(3,1) Lorentz invariance are satisfied, then the unbroken gauge group after the compactification of the $E_8 \times E_8$ heterotic string theory on Calabi-Yau spaces is $E_8 \times E_6$.

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1. INTRODUCTION

Current interest in string theories has its origin at the discovery of anomaly cancellation,[1] subsequent discovery of the heterotic string theory,[2] and the semi-realistic compactification of this theory on Calabi-Yau manifolds[3] and orbifolds.[4–7] The original motivation for compactifying string theories on Calabi-Yau manifolds was to get an effective four dimensional field theory with unbroken space-time supersymmetry. The analysis was carried out in the zero slope limit of the string theory. With certain simplifying assumptions it was shown that the requirement of unbroken space-time supersymmetry forces the background metric to be Ricci-flat and Kahler, and the gauge connection to be identified with the spin connection. As an added bonus, it was also found that such background fields satisfy the equations of motion of the string theory. This was shown by using the equivalence between the equations of motion of the string theory and conformal invariance of the σ-model describing the propagation of the string in background fields,[3,8–15] and using the fact that supersymmetric σ-models on manifolds with Ricci-flat Kahler metric have vanishing β-function, at least to low orders in the perturbation theory.[16] Since then more general compactification schemes with unbroken supersymmetry have been proposed,[17–19] and it has also been argued, based on an analysis of the effective four dimensional field theory, that these compactification schemes are solutions of classical string field equations, and also give unbroken space-time supersymmetry, to all orders in the σ-model perturbation theory.[17] For compactification on Calabi-Yau manifolds these arguments have also been shown to be valid non-perturbatively[20]. Other applications of σ-models to string theories have been discussed in Refs.[21-32].

Recent calculations have shown, however, that the β-function for supersymmetric σ-models on Ricci-flat Kahler background receives non-vanishing contribution at the four loop order.[33–35] It was then shown[20,36] that it is always possible to find a (non Ricci-flat) Kahler metric on a Calabi-Yau manifold which gives vanishing β-function for the σ-model to all orders in the perturbation theory, and hence is a solution of the string field equations. One may then ask the question as to whether such a background also gives unbroken space-time supersymmetry to all orders in the σ-model perturbation theory. Although general arguments[17,20] indicate that the answer is affirmative, it will be nice to see it directly in the σ-model. This is the question that we shall address in this paper.

The free heterotic string may be formulated either in the Neveu-Schwarz-Ramond (NSR) formulation,[37] or in the Green-Schwarz (GS) formulation.[38] In NSR formulation the two dimensional fermions transform in the vector representation of the ten dimensional Lorentz group, whereas in the GS formulation the two dimensional fermions transform in the spinor representation of the Lorentz group. The NSR model can be quantized by maintaining the full SO(9,1) Lorentz symmetry manifest, but not the space-time supersymmetry. In fact, the space-time supersymmetry charge in the covariant formulation of the NSR model has been constructed only recently.[39,40] The GS model, on the other hand, has been quantized only in the light-cone gauge, so that only the SO(8) subgroup of the SO(9,1) Lorentz group is realized linearly. The space-time supersymmetry, however, is manifest in this formulation.

Both, the NSR and the GS string has been successfully coupled to the massless background fields.[3,8–10,30–32] However, most of the calculations beyond the tree level of the σ-model have been done in the NSR formulation for general background field configurations. To understand the reason for this we compare the NSR and GS model in the light-cone gauge. Taking the background metric $G_{ij}(X)$, the antisymmetric tensor field $B_{ij}(X)$ and the gauge field $A_i^M(X)$ to lie in the transverse directions only, we write down the action for the NSR and the GS strings,

$$\begin{aligned} S_{NSR} = \frac{1}{2\pi\alpha'}\int d^2\xi \frac{i}{2}\Big[&\lambda^a\partial_-\lambda^a + \lambda^a\lambda^b(\omega_i^{ab}(X) - S_i^{ab}(X))\partial_- X^i \\ &+ \tfrac{1}{2}F_{ab}^M(X)\psi^\ell(T^M)_{\ell k}\psi^k\lambda^a\lambda^b\Big] + \ldots \end{aligned} \tag{1.1}$$

$$S_{GS} = \frac{1}{2\pi\alpha'}\int d^2\xi \frac{i}{2}\Big[S^\alpha\partial_- S^\alpha + \frac{i}{2}S^\alpha(\Sigma^{ab})_{\alpha\beta}S^\beta(\omega_i^{ab}(X) - S_i^{ab}(X))\partial_- X^i$$
$$+ \frac{i}{4}F_{ab}^M(X)\psi^\ell(T^M)_{\ell k}\psi^k S^\alpha(\Sigma^{ab})_{\alpha\beta}S^\beta\Big] + ... \tag{1.2}$$

In the above equations α' is the inverse string tension, λ^a and S^α are two dimensional right-handed Majorana-Weyl fermions transforming in the vector and spinor representations of the SO(8) Lorentz group respectively, w_i^{ab} is the spin connection constructed from the metric G_{ij} (a,b are tangent space indices), S_i^{ab} is the curl of the anti-symmetric tensor field B_{ij}, X^i are the eight bosonic fields, F_{ab}^M is the field strength constructed from the gauge field A_i^M, ψ^ℓ are the thirty two left-handed Majorana-Weyl fermions transforming in the 32 or (16,1)+(1,16) representation of the gauge groups SO(32) or SO(16)×SO(16), T^M are the generators of the gauge group, and Σ^{ab} denote generators of SO(8) in the spinor representation. The ... denotes terms which are identical for the two formulations. Formally we can show the equivalence of the two theories[41–46] by writing down a functional integral involving the fields X^i, ψ^ℓ and λ^a or S^α, and then for fixed X and ψ, showing the equivalence between the currents $\lambda^a\lambda^b$ and $\frac{i}{2}S^\alpha(\Sigma^{ab})_{\alpha\beta}S^\beta$ using the triality properties of SO(8). However we must remember that a σ-model is specified not only by the background fields, but also the renormalization scheme used for describing the quantum theory. Hence before we say that the two theories are equivalent, we must specify the renormalization scheme for each of the theories.

In the NSR formulation there is a natural symmetry, namely the (1,0) supersymmetry,[3,9,10,47–49] which must be respected by the renormalization scheme that we use. In the covariant formulation this symmetry is required to show the decoupling of the negative normed states, whereas in the light-cone gauge this symmetry is required to show the existence of the residual Lorentz symmetry (e.g. for compactification on a six dimensional manifold the (1,0) superconformal algebra is required to show the existence of the SO(3,1) Lorentz algebra.) The GS σ-model, on the other hand, has no such obvious symmetry.

As a result the number of independent dimension two operators in the theory is larger than the number of independent background fields ($G_{ij}(X)$, $B_{ij}(X)$ and $A_i^M(X)$), and the description of the theory becomes ambiguous. Presumably, there is a specific renormalization scheme for which the GS model becomes equivalent to the corresponding NSR model with (1,0) supersymmetric renormalization scheme, but there is no simple way to find out what this renormalization scheme is. Due to this reason, most of the study of string propagation in background fields have been restricted to the NSR model beyond the tree level of the σ-model. This has been the the case for the analysis of the conformal invariance of the σ-model,[9,10] as well as the analysis of space-time supersymmetry.[39,50,51]*

In this paper we follow a different approach. We start by writing down the σ-model describing the heterotic string theory on a Calabi-Yau manifold in the GS formalism. We then show that there is a specific renormalization scheme in which the model has N=1 supersymmetry in four dimensions, to all orders in the σ-model perturbation theory, however, only the SO(2) subgroup of the SO(3,1) Lorentz group is manifest in this formalism. In this scheme the σ-model reduces to the sum of a free field theory, and a (2,2) supersymmetric conformally invariant theory on a Calabi-Yau manifold. We then try to construct the full SO(3,1) Lorentz generators of the model, which satisfy standard commutation relations among themselves, as well as the supersymmetry and translation generators. It turns out that the existence of these generators is guaranteed if there exists a certain unitary operator in this two dimensional superconformal field theory, satisfying specific commutation relations with the super-Virasoro generators of the theory. In this case the complete set of the SO(3,1) Lorentz generators may be constructed in terms of the super-Virasoro generators of the theory and this unitary operator U, and may be shown to satisfy the correct commutation relations.

* The GS σ-model has been used in Refs.[3,18,19] to analyze the criteria for space-time supersymmetry in a given background field by looking at the classical symmetries of the σ-model.

This condition also turns out to be equivalent to the existence of a scalar field ϕ conjugate to the U(1) part of the (2,0) superconformal current of the non-linear σ-model such that this current may be expressed as a derivative of this scalar field. More precisely, if $H(z)$ is the U(1) current, then it should be expressible as $-i\sqrt{3}\partial_z\phi$, where $e^{\sqrt{3}i\phi}$ is a local operator of the theory. Using the known operator product of the U(1) current with the other components of the superconformal current we may calculate the operator product of ϕ with these currents, and also with itself. The generators of the SO(3,1) Lorentz group may now be written in terms of this scalar field and other super-conformal currents, and their commutators may be calculated explicitly from the known operator product expansions. These operator product expansions, being consequences of the (2,0) superconformal algebra, are the same as in the free field theory. Since there is no anomaly in the Lorentz algebra in the free field case, it follows that there is no anomaly in the Lorentz algebra in the interacting theory either. This analysis is easily generalized to more general background fields, which give rise to (2,0) superconformal field theories on the world sheet.

We may compare this result with the corresponding analysis in the NSR formulation where the full SO(3,1) Lorentz symmetry is manifest, but the space-time supersymmetry is not. This theory may also be described as the sum of a free field theory and a (2,2) supersymmetric conformal field theory on a Calabi-Yau manifold, although the basic fermionic variables of the theory transform differently under the Lorentz group compared to the GS formulation. In this case the space-time supersymmetry generators of the theory may be constructed in terms of the field ϕ if it exists, and may be shown to satisfy the correct commutation relations with the other symmetry generators of the theory. Thus from both the formalisms the criteria for getting a Lorentz invariant, space-time supersymmetric theory seems to be that the U(1) part of the superconformal current may be expressed as the derivative of a scalar field satisfying the criteria mentioned above. These results also indicate that the GS (with the renormalization scheme that we are using) and NSR formulations of the theory are equivalent only if the above condition is satisfied.

Our analysis may also be applied to determine the unbroken gauge symmetry group of the theory after compactification. In the compactification of the $E_8 \times E_8$ heterotic string theory on a Calabi-Yau manifold only the $E_8 \times SO(10) \times U(1)$ part of the symmetry group is manifest in the two dimensional theory. The construction of the extra symmetry generators which convert $\mathrm{SO}(10) \times \mathrm{U}(1)$ to the full E_6 group is very similar to the construction of the supersymmetry generators in the NSR formalism, the only difference being that we need to use the (0,2) part of the superconformal algebra. The left-right symmetry of the model implies that if the left handed U(1) current can be written as the derivative of a scalar field satisfying certain constraints, so can be the right-handed U(1) current which is a part of the (0,2) superconformal algebra. The extra gauge generators may then be constructed in terms of this new scalar field $\tilde{\phi}$, and may be shown to satisfy the correct commutation relations with the rest of the generators.

In Sec.II of the paper we review the proof of Lorentz invariance of the heterotic string in flat background in the GS formalism. We also recast the proof in a form so that it is easily generalizable to the case of non-trivial background fields. In Sec.III we show how to construct the Lorentz and supersymmetry generators for the heterotic string on a Calabi-Yau background in the GS formalism, assuming the existence of the field ϕ mentioned before. The construction is also generalized to more general models which have (2,0) supersymmetry on the world-sheet. In Sec.IV we analyze the unbroken gauge symmetry for the heterotic string compactified on a Calabi-Yau background, and find that it is $E_8 \times E_6$, again assuming the existence of the scalar field $\tilde{\phi}$. We summarize our result in Sec.V. We also give a heuristic argument in this section showing that the heterotic string theory compactified on a Calabi-Yau manifold indeed satisfies the criteria for unbroken supersymmetry and full $E_8 \times E_6$ gauge symmetry.

2. FREE HETEROTIC STRING IN THE GREEN-SCHWARZ FORMALISM

In this section we shall analyze the space-time supersymmetry and Lorentz invariance of the free heterotic string in the GS formalism. Although this analysis has been carried out in the past, we recast the analysis in a form which may be easily generalized in the presence of background fields. Also, even though the theory has full SO(9,1) Lorentz invariance and space-time supersymmetry, we shall focus our attention on an SO(3,1) subgroup of the Lorentz group, and a four dimensional N=1 supersymmetry. These are the subgroups which are expected to remain unbroken upon compactification of the theory on a Calabi-Yau manifold.

Although classically there exists a covariant formulation of the free GS string,[52,53] it has been quantized only in the light-cone gauge, where only the SO(8) subgroup of the full SO(9,1) Lorentz symmetry is manifest. In this gauge the string theory is described in terms of a two dimensional free field theory. The basic variables of the theory are eight bosonic fields X^i transforming in the vector(8_v) representation of SO(8), eight right-handed Majorana-Weyl fermions S^α transforming in the spinor(8_s) representation of SO(8), and thirty two left-handed Majorana-Weyl fermions ψ^ℓ which are singlets of the Lorentz group, but transform in the (16,1)+(1,16) representation of the SO(16)×SO(16) subgroup of the $E_8 \times E_8$ gauge group. (For definiteness we are considering the $E_8 \times E_8$ heterotic string theory). The action for the theory is given by,[2]

$$S = \frac{1}{2\pi\alpha'}\int d^2\xi(\partial_+ X^i \partial_- X^i + \frac{i}{2}S^\alpha\partial_- S^\alpha + \frac{i}{2}\psi^\ell\partial_+\psi^\ell) \tag{2.1}$$

where (ξ^0, ξ^1) denote the usual string variables τ and σ respectively, and

$$\begin{aligned} \xi^\pm &= \frac{1}{\sqrt{2}}(\xi^0 \pm \xi^1) \\ \partial_\pm &= \frac{1}{\sqrt{2}}(\partial_0 \pm \partial_1) \end{aligned} \tag{2.2}$$

We shall now identify the first six directions (i=1,...6) as 'internal dimensions'. The seventh and eighth directions, as well as the light-cone directions $X^{\pm}$ will be identified with ordinary Minkowski space. As a result only the SO(2) subgroup of the SO(3,1) Lorentz group will be manifest in this formalism. The internal Lorentz group SO(6) has SU(3)×U(1) as its subgroup. Under this subgroup X^7 and X^8 are neutral, whereas X^i (i=1,...6) transform as,

$$3(2) + \bar{3}(-2)$$

where the U(1) charge has been denoted in the bracket. Thus X^i (i=1,...6) may be split into three complex coordinates X^r and their complex conjugates ($X^{\bar{r}}$) which transform in the 3 and $\bar{3}$ representation of the SU(3) group respectively. Also we define,

$$\begin{aligned} X^{\theta} &= \frac{1}{\sqrt{2}}(X^7 + iX^8) \\ X^{\bar{\theta}} &= \frac{1}{\sqrt{2}}(X^7 - iX^8) \end{aligned} \tag{2.3}$$

X^{θ} and $X^{\bar{\theta}}$ carry charges 1 and -1 respectively under the SO(2) subgroup of the four dimensional Lorentz group, whereas X^r, $X^{\bar{r}}$ are neutral under this group. In the following discussion SO(2) will always denote the subgroup of the SO(3,1) Lorentz group, and U(1) will denote the subgroup of the 'internal' SO(6) group.

Let us now turn to the spinor representation. Under the SU(3)×U(1)×SO(2) group, the spinor representation decomposes as,

$$3(-1,\tfrac{1}{2}) + \bar{3}(1,-\tfrac{1}{2}) + 1(3,\tfrac{1}{2}) + 1(-3,-\tfrac{1}{2})$$

where the U(1)×SO(2) charges have been denoted in the bracket. We shall call these fields S^r, $S^{\bar{r}}$, S^{θ} and $S^{\bar{\theta}}$ respectively. We have used the same set of indices to denote the X's and the S's since they transform in the same way under the 'internal' SU(3) group. Note however that the X and the S fields do not carry the same U(1)×SO(2) charges.

Finally let us turn to the left handed fermions ψ^ℓ. We pick up an SO(6) subgroup of SO(16)×SO(16) such that ψ^ℓ (ℓ=7,...32) are singlets under this group, and ψ^ℓ (ℓ=1,...6) transform in the vector representation of SO(6). Under an SU(3) subgroup of SO(6) these fields transform as $(3+\bar{3})$. Let us denote them by ψ^r and $\psi^{\bar{r}}$ respectively. We have used the same index r for ψ^r and X^r, in anticipation of the identification of the background spin and gauge connection for compactification on a Calabi-Yau manifold.

In terms of the new fields we may write the action given in (2.1) as,

$$S = S_0 + S_1 \tag{2.4}$$

$$S_0 = \frac{1}{\pi}\int d^2\xi(\partial_+ X^\theta \partial_- X^{\bar{\theta}} + \partial_+ X^{\bar{\theta}} \partial_- X^\theta + iS^{\bar{\theta}}\partial_- S^\theta + \frac{i}{2}\sum_{\ell=7}^{32}\psi^\ell\partial_+\psi^\ell) \tag{2.5}$$

$$S_1 = \sum_{r=1}^{3}\frac{1}{\pi}\int d^2\xi(\partial_+ X^r \partial_- X^{\bar{r}} + \partial_+ X^{\bar{r}} \partial_- X^r + iS^{\bar{r}}\partial_- S^r + i\psi^{\bar{r}}\partial_+\psi^r) \tag{2.6}$$

where we have set α' to be $\frac{1}{2}$. The reason for splitting up the action in this form is that when we introduce background fields, the fields involved in the action S_0 wil remain free fields, while those involved in the action S_1 will become interacting. Thus in forming the space-time supersymmetry and the Lorentz generators of the theory we shall try to avoid using the fields X^r, S^r, ψ^r and their complex conjugates explicitly. Instead we shall use only the symmetry generators of the theory which are expected to remain symmetries of the theory even in the presence of the background fields.

The symmetries which turn out to be useful for this purpose are the (2,0) superconformal symmetries which the action S_1 posseses. The (2,0) supersymmetry changes X^r to S^r and $X^{\bar{r}}$ to $S^{\bar{r}}$ respectively. Although the action S_1 describes a left-right symmetric model and hence describes a (2,2) superconformal field theory, we shall not need to use the left-handed superconformal generators for our analysis. The useful generators are L_n, $\tilde{L}_n$, $G_n^{\pm}$ and H_n, satisfying the (anti-)commutation relations:

$$
\begin{aligned}
&[\tilde{L}_m, \tilde{L}_n] = (m-n)\tilde{L}_{m+n} + \frac{c}{8}(m^3 - m)\delta_{m,-n} \\
&[\tilde{L}_m, L_n] = [\tilde{L}_m, G_n^{\pm}] = [\tilde{L}_m, H_n] = 0 \\
&[L_m, L_n] = (m-n)L_{m+n} + \frac{c}{8}(m^3 - m)\delta_{m,-n} \\
&\{G_m^+, G_n^-\} = L_{m+n} + \tfrac{1}{2}(m-n)H_{m+n} + \frac{c}{4}(m^2 - \tfrac{1}{4})\delta_{m,-n} \\
&[H_m, G_n^+] = G_{m+n}^+ \\
&[H_m, G_n^-] = -G_{m+n}^- \\
&[L_m, H_n] = -nH_{m+n} \\
&[L_m, G_n^+] = (\frac{m}{2} - n)G_{m+n}^+ \\
&[L_m, G_n^-] = (\frac{m}{2} - n)G_{m+n}^- \\
&[H_m, H_n] = \frac{c}{2}m\delta_{m,-n} \\
&\{G_m^+, G_n^+\} = \{G_m^-, G_n^-\} = 0
\end{aligned}
\tag{2.7}
$$

where,

$$
c = 6 \tag{2.8}
$$

In order to express the superconformal generators in terms of the fundamental fields of the theory, we use the mode expansion,

$$
\begin{aligned}
X^r &= x^r + p^r\tau + \frac{i}{2}\sum_{n\neq 0}\frac{1}{n}[\alpha_n^r e^{-2in(\tau+\sigma)} + \tilde{\alpha}_n^r e^{-2in(\tau-\sigma)}] \\
S^r &= \sum_{n=-\infty}^{\infty} S_n^r e^{-2in(\tau+\sigma)} \\
\psi^r &= \sum_{n=-\infty}^{\infty} \psi_n^r e^{-2in(\tau-\sigma)}
\end{aligned}
\tag{2.9}
$$

with similar expressions for $X^{\bar{r}}$, $S^{\bar{r}}$, $\psi^{\bar{r}}$. Note that ψ^r may satisfy either periodic or anti-periodic boundary conditions in σ, here we have taken it to be periodic for definiteness. The various oscillators in (2.9) satisfy the (anti)commutation relations:

$$\begin{aligned}
&[x^r, p^{\bar{s}}] = [x^{\bar{r}}, p^s] = i\delta^{rs} \\
&[\alpha^r_m, \alpha^{\bar{s}}_n] = [\tilde{\alpha}^r_m, \tilde{\alpha}^{\bar{s}}_n] = m\delta_{m,-n}\delta^{rs}, \quad m \neq 0 \\
&\{S^r_m, S^{\bar{s}}_n\} = \delta_{m,-n}\delta^{rs} \\
&\{\psi^r_m, \psi^{\bar{s}}_n\} = \delta_{m,-n}\delta^{rs}
\end{aligned} \tag{2.10}$$

All other (anti-)commutators vanish. In terms of these oscillators the generators of the superconformal algebra may be expressed as,

$$\begin{aligned}
\tilde{L}_m &=: \tilde{\alpha}^r_{m-n}\tilde{\alpha}^{\bar{r}}_n : + (n - \frac{m}{2}) : \psi^{\bar{r}}_{m-n}\psi^r_n : + \frac{c}{16}\delta_{m,0} \\
L_m &=: \alpha^r_{m-n}\alpha^{\bar{r}}_n : + (n - \frac{m}{2}) : S^{\bar{r}}_{m-n}S^r_n : + \frac{c}{16}\delta_{m,0} \\
G^+_m &= S^{\bar{r}}_{m-n}\alpha^r_n \\
G^-_m &= S^r_{m-n}\alpha^{\bar{r}}_n \\
H_m &=: S^{\bar{r}}_{m-n}S^r_n :
\end{aligned} \tag{2.11}$$

where sum over repeated indices is implied, and,

$$\alpha^r_0 = \tilde{\alpha}^r_0 = \tfrac{1}{2}p^r \tag{2.12}$$

Using Eqs.(2.10) and (2.11) we may explicitly verify the commutation relations (2.7).

It will be nice if we can express the Lorentz and supersymmetry generators of the theory in terms of the generators of the superconformal algebra. It turns out, however, that we need two more sets of operators involving the fields X^r and S^r for this purpose. They are,

$$\begin{aligned}
Q_p &= \tfrac{1}{2}\epsilon^{\bar{r}\bar{s}\bar{t}}S^r_m S^s_n \alpha^t_{-m-n+p} \\
\bar{Q}_p &= \tfrac{1}{2}\epsilon^{rst}S^{\bar{r}}_m S^{\bar{s}}_n \alpha^{\bar{t}}_{-m-n+p}
\end{aligned} \tag{2.13}$$

These operators may be expressed as,

$$Q_p = \{G_0^+, P_p\}$$
$$\bar{Q}_p = \{G_0^-, \bar{P}_p\} \tag{2.14}$$

where,

$$P_p = \frac{1}{6}\epsilon^{\bar{r}\bar{s}\bar{t}} S_m^r S_n^s S_{-m-n+p}^t$$
$$\bar{P}_p = \frac{1}{6}\epsilon^{rst} S_m^{\bar{r}} S_n^{\bar{s}} S_{-m-n+p}^{\bar{t}} \tag{2.15}$$

Commutation relations of P_n, $\bar{P}_n$ with various generators of the superconformal algebra are,

$$\begin{aligned}
&[\tilde{L}_m, P_n] = [\tilde{L}_m, \bar{P}_n] = 0\\
&[L_m, P_n] = (\frac{m}{2} - n)P_{m+n}\\
&\{G_m^+, P_n\} = Q_{m+n}\\
&\{G_m^-, P_n\} = 0\\
&[H_m, P_n] = -3P_{m+n}\\
&[L_m, \bar{P}_n] = (\frac{m}{2} - n)\bar{P}_{m+n}\\
&\{G_m^+, \bar{P}_n\} = 0\\
&\{G_m^-, \bar{P}_n\} = \bar{Q}_{m+n}\\
&[H_m, \bar{P}_n] = 3\bar{P}_{m+n}
\end{aligned} \tag{2.16}$$

Commutation relations of Q_m, $\bar{Q}_m$ with various generators of the superconformal algebra may be derived using Eqs.(2.14), (2.16) and the Jacobi identity. They are,

$$
\begin{aligned}
&[\tilde{L}_m, Q_n] = [\tilde{L}_m, \bar{Q}_n] = 0 \\
&[L_m, Q_n] = (m-n)Q_{m+n} \\
&[G^+_m, Q_n] = 0 \\
&[G^-_m, Q_n] = (2m-n)P_{m+n} \\
&[H_m, Q_n] = -2Q_{m+n} \\
&[L_m, \bar{Q}_n] = (m-n)\bar{Q}_{m+n} \\
&[G^+_m, \bar{Q}_n] = (2m-n)\bar{P}_{m+n} \\
&[G^-_m, \bar{Q}_n] = 0 \\
&[H_m, \bar{Q}_n] = 2\bar{Q}_{m+n}
\end{aligned}
\tag{2.17}
$$

Finally we also need the anticommutator,

$$
\{P_0, \bar{P}_0\} = -(\tfrac{1}{2}H_0^2 + \sum_{m>0} H_{-m}H_m)
\tag{2.18}
$$

from which we can derive,

$$
\begin{aligned}
[Q_0, \bar{Q}_0] =&[\sum_{m>0}(L_{-m}H_m + H_{-m}L_m + G^-_{-m}G^+_m - G^+_{-m}G^-_m) \\
&+ (L_0 - \frac{c}{16})H_0 + \tfrac{1}{2}(G^-_0 G^+_0 - G^+_0 G^-_0)]
\end{aligned}
\tag{2.19}
$$

This completes the list of all the commutation relations that we need to know for our analysis. We now introduce the oscillators in the theory described by S_0:

$$
\begin{aligned}
X^\theta &= x^\theta + p^\theta\tau + \frac{i}{2}\sum_{n\neq 0}\frac{1}{n}[\alpha^\theta_n e^{-2in(\tau+\sigma)} + \tilde{\alpha}^\theta_n e^{-2in(\tau-\sigma)}] \\
S^\theta &= \sum_{n=-\infty}^{\infty} S^\theta_n e^{-2in(\tau+\sigma)} \\
\psi^\ell &= \sum_{n=-\infty}^{\infty} \psi^\ell_n e^{-2in(\tau-\sigma)}
\end{aligned}
\tag{2.20}
$$

$X^{\bar\theta}$ and $S^{\bar\theta}$ have similar expansions. The (anti)commutation relations are,

$$
\begin{aligned}
&[x^\theta, p^{\bar\theta}] = [x^{\bar\theta}, p^\theta] = i \\
&[\alpha^\theta_m, \alpha^{\bar\theta}_n] = [\tilde\alpha^\theta_m, \tilde\alpha^{\bar\theta}_n] = m\delta_{m,-n} \\
&\{S^\theta_m, S^{\bar\theta}_n\} = \delta_{m,-n} \\
&\{\psi^\ell_m, \psi^k_n\} = \delta_{m,-n}\delta^{\ell k}
\end{aligned}
\tag{2.21}
$$

all other (anti)commutators being zero. Since the fields involved in the action S_0 remain free fields even in the presence of background fields, we may use these fields explicitly in the construction of the Lorentz and supersymmetry generators of the theory. It is however convenient to introduce the following operators,

$$
\begin{aligned}
\tilde L^{(0)}_m &=: \tilde\alpha^\theta_{m-n}\tilde\alpha^{\bar\theta}_n : + \frac{1}{2}(n - \frac{m}{2}) : \psi^\ell_{m-n}\psi^\ell_n : \\
L^{(0)}_m &=: \alpha^\theta_{m-n}\alpha^{\bar\theta}_n : + (n - \frac{m}{2}) : S^{\bar\theta}_{m-n} S^\theta_n :
\end{aligned}
\tag{2.22}
$$

Finally we introduce the light-cone coordinates $x^\pm$ and their conjugate variables $p^\mp$, satisfying the commutation relations,

$$
[x^-, p^+] = [x^+, p^-] = i \tag{2.23}
$$

We are now ready to write down the Lorentz generators $J^{\theta\bar\theta}$, $J^{\theta\pm}$ and $J^{\bar\theta\pm}$, as well as the four component supersymmetry generators K^a (a=1,...4). They are given by,

$$
\begin{aligned}
J^{\theta\bar\theta} &= x^\theta p^{\bar\theta} - x^{\bar\theta} p^\theta - i\sum_{n=1}^{\infty}\frac{1}{n}(\alpha^\theta_{-n}\alpha^{\bar\theta}_n - \alpha^{\bar\theta}_{-n}\alpha^\theta_n + \tilde\alpha^\theta_{-n}\tilde\alpha^{\bar\theta}_n - \tilde\alpha^{\bar\theta}_{-n}\tilde\alpha^\theta_n) \\
&\quad - \frac{i}{2}\Big(\sum_{n=1}^{\infty}(S^\theta_{-n}S^{\bar\theta}_n - S^{\bar\theta}_{-n}S^\theta_n) + \tfrac{1}{2}(S^\theta_0 S^{\bar\theta}_0 - S^{\bar\theta}_0 S^\theta_0)\Big) + \frac{i}{2}H_0
\end{aligned}
\tag{2.24}
$$

$$
J^{\theta +} = x^\theta p^+ - x^+ p^\theta \tag{2.25}
$$

$$J^{\bar\theta +} = x^{\bar\theta} p^+ - x^+ p^{\bar\theta} \tag{2.26}$$

$$\begin{aligned} J^{\theta -} &= x^\theta p^- - x^- p^\theta - i(p^+)^{-1} \sum_{n=1}^{\infty} \frac{1}{n} (\alpha^\theta_{-n} L^{(0)}_n - L^{(0)}_{-n} \alpha^\theta_n + \tilde\alpha^\theta_{-n} \tilde L^{(0)}_n - \tilde L^{(0)}_{-n} \tilde\alpha^\theta_n) \\ &- i(p^+)^{-1} \Big[\sum_{n=1}^{\infty} \{ \frac{1}{n} (\alpha^\theta_{-n} L_n - L_{-n} \alpha^\theta_n) + \frac{1}{n} (\tilde\alpha^\theta_{-n} \tilde L_n - \tilde L_{-n} \tilde\alpha^\theta_n) \\ &- \tfrac{1}{2} (\alpha^\theta_{-n} H_n + H_{-n} \alpha^\theta_n) + (G^-_{-n} S^\theta_n - S^\theta_{-n} G^-_n) \} - \tfrac{1}{2} \alpha^\theta_0 H_0 + G^-_0 S^\theta_0 + Q_0 \Big] \end{aligned} \tag{2.27}$$

$$\begin{aligned} J^{\bar\theta -} &= x^{\bar\theta} p^- - x^- p^{\bar\theta} - i(p^+)^{-1} \sum_{n=1}^{\infty} \frac{1}{n} (\alpha^{\bar\theta}_{-n} L^{(0)}_n - L^{(0)}_{-n} \alpha^{\bar\theta}_n + \tilde\alpha^{\bar\theta}_{-n} \tilde L^{(0)}_n - \tilde L^{(0)}_{-n} \tilde\alpha^{\bar\theta}_n) \\ &- i(p^+)^{-1} \Big[\sum_{n=1}^{\infty} \{ \frac{1}{n} (\alpha^{\bar\theta}_{-n} L_n - L_{-n} \alpha^{\bar\theta}_n) + \frac{1}{n} (\tilde\alpha^{\bar\theta}_{-n} \tilde L_n - \tilde L_{-n} \tilde\alpha^{\bar\theta}_n) \\ &+ \tfrac{1}{2} (\alpha^{\bar\theta}_{-n} H_n + H_{-n} \alpha^{\bar\theta}_n) + (G^+_{-n} S^{\bar\theta}_n - S^{\bar\theta}_{-n} G^+_n) \} + \tfrac{1}{2} \alpha^{\bar\theta}_0 H_0 + G^+_0 S^{\bar\theta}_0 + \bar Q_0 \Big] \end{aligned} \tag{2.28}$$

$$K^1 = (p^+)^{\frac{1}{2}} (S^\theta_0 + S^{\bar\theta}_0) \tag{2.29}$$

$$K^2 = i(p^+)^{\frac{1}{2}} (S^{\bar\theta}_0 - S^\theta_0) \tag{2.30}$$

$$K^3 = (p^+)^{-\frac{1}{2}} (G^+_0 + G^-_0 + S^\theta_n \alpha^{\bar\theta}_{-n} + S^{\bar\theta}_n \alpha^\theta_{-n}) \tag{2.31}$$

$$K^4 = i(p^+)^{-\frac{1}{2}} (G^+_0 - G^-_0 + S^\theta_n \alpha^{\bar\theta}_{-n} - S^{\bar\theta}_n \alpha^\theta_{-n}) \tag{2.32}$$

These expressions reduce to the standard ones[2,38] once they are expressed in terms of the component fields. Using the commutation relations (2.7), (2.17), (2.19) and (2.21) one may verify that $J^{\mu\nu}$ and K^a satisfy the standard commutation relations,

$$\begin{aligned}
\{K^a, K^b\} &= 2(\gamma_\mu \gamma^0)^{ab} p^\mu \\
[J^{\mu\nu}, K^a] &= -\frac{i}{4}[\gamma^\mu, \gamma^\nu]^a{}_b K^b \\
[J^{\mu\nu}, J^{\rho\lambda}] &= -i[J^{\mu\lambda}\eta^{\nu\rho} - J^{\nu\lambda}\eta^{\mu\rho} - J^{\mu\rho}\eta^{\nu\lambda} + J^{\nu\rho}\eta^{\mu\lambda}]
\end{aligned} \tag{2.33}$$

if the mass-shell conditions

$$\tfrac{1}{4}p^+p^- - (L_0^{(0)} + L_0 - \frac{c}{16}) = \tfrac{1}{4}p^+p^- - (\tilde{L}_0^{(0)} + \tilde{L}_0 - \frac{c}{16}) = 0 \tag{2.34}$$

are satisfied. Here γ^μ are the four dimensional γ-matrices. This is not a new result. What is new is that the derivation of Eqs.(2.33) does not require the use of the commutation relations between the X^r, S^r, ψ^r, and their complex conjugate fields, instead it only relies on the superconformal algebra (2.7), and the commutation relations involving Q_0, $\bar{Q}_0$.

We shall now show that P_m, $\bar{P}_m$ may be expressed almost entirely in terms of the superconformal generators of the theory,* from which the commutation relations (2.16) and (2.18) follow directly. Since all commutation relations involving Q_m's may be derived from the ones involving P_m's, it follows then that all the commutation relations that we need for proving Lorentz and space-time supersymmetry are consequences of the (2,0) superconformal algebra. In order to illustrate this construction it is convenient to euclideanize the two dimensional space, introduce complex variables $z = e^{2(\tau+i\sigma)}$, $\bar{z} = e^{2(\tau-i\sigma)}$, and define,

$$\begin{aligned}
\hat{S}^{r,\bar{r}}(z) &= z^{-\frac{1}{2}} S^{r,\bar{r}}(z) \\
\hat{\psi}^{r,\bar{r}}(\bar{z}) &= \bar{z}^{-\frac{1}{2}} \psi^{r,\bar{r}}(\bar{z})
\end{aligned} \tag{2.35}$$

---

* This construction has been used by Dixon et. al.[4] for the construction of the spin operators in terms of the superconformal generators on an orbifold.

In terms of these fields the action (2.6) may be written as,

$$\frac{1}{\pi}\int d^2z(\partial_z X^r \partial_{\bar z} X^{\bar r} + \partial_z X^{\bar r}\partial_{\bar z} X^r + i\hat S^{\bar r}\partial_{\bar z}\hat S^r + i\hat\psi^{\bar r}\partial_z\hat\psi^r) \tag{2.36}$$

We now define

$$\begin{aligned} P(z) &\equiv \sum_m P_m z^{-m-\frac{3}{2}} = \hat S^1(z)\hat S^2(z)\hat S^3(z) \\ H(z) &\equiv \sum_m H_m z^{-m-1} = \sum_r \hat S^{\bar r}(z)\hat S^r(z) \end{aligned} \tag{2.37}$$

We shall now show that $P(z)$ may be expressed almost completely in terms of the generators H_m. Let us bosonize the fields $\hat S^r$, $\hat S^{\bar r}$ as,

$$\begin{aligned} \hat S^r &\sim :e^{i\phi^r}: \\ \hat S^{\bar r} &\sim :e^{-i\phi^r}: \end{aligned} \tag{2.38}$$

where ϕ^r is a free right moving scalar field. The U(1) current generated by H may be expressed in terms of the fields ϕ^r as,

$$H(z) = -i\sum_r \partial_z\phi^r \equiv -i\sqrt{3}\partial_z\phi \tag{2.39}$$

where $\phi(=\frac{1}{\sqrt{3}}\sum\phi^r)$ is a free right moving scalar field. Then,

$$\sum_r \phi^r = i(\sum_{m\neq 0}(-\frac{1}{m})H_m z^{-m} + H_o \ln z) + \sum_r \phi_0^r \tag{2.40}$$

where ϕ_0^r denotes the center of mass coordinate of the field ϕ^r. Then,

$$P(z) \sim :e^{i\sum_r\phi^r}: \sim :e^{\sqrt{3}i\phi}: \sim :exp(\sum_{m\neq 0}\frac{1}{m}H_m z^{-m} - H_0\ln z + \sqrt{3}i\phi_0): \tag{2.41}$$

The normal ordering in the above expression is done with respect to the oscillators H_m, or equivalently with respect to the oscillators of the field ϕ. ϕ_0 is the center of mass coordinate of the field ϕ, and is the conjugate variable to the center of mass momentum $\sqrt{\frac{1}{3}}H_0$. We choose the normal ordering prescription in such a way that all the terms involving ϕ_0 is to the left of all the terms involving H_0. Defining the operator,

$$U = exp(\sqrt{3}i\phi_0) \tag{2.42}$$

Eqn. (2.41) may be written as,

$$P(z) \sim U{:}\,exp(\sum_{m\neq 0}\frac{1}{m}H_m z^{-m} - H_0 \ln z){:} \tag{2.43}$$

The operator U satisfies the following commutation relations with the super-Virasoro generators,

$$\begin{aligned} UL_mU^{-1} &= L_m + H_m + \frac{c}{4}\delta_{m,0} \\ UG_m^+U^{-1} &= G_{m+1}^+ \\ UG_m^-U^{-1} &= G_{m-1}^- \\ UH_mU^{-1} &= H_m + \frac{c}{2}\delta_{m,0} \end{aligned} \tag{2.44}$$

Thus $P(z)$ and so all the P_m's are expressed in terms of the oscillators H_m and the operator U. We may now calculate the commutators involving the P_m's in terms of commutators involving the H_m's and U and verify (2.16) and (2.18).

Thus we see that the existence of the (2,0) superconformal algebra with correct central charge (c=6) and the existence of the operator U satisfying the commutation relations (2.44) is sufficient to construct the Lorentz and the supersymmetry generators of the theory. In the next section we shall use this result to discuss the space-time supersymmetry and Lorentz invariance of the heterotic string theory compactified on a Calabi-Yau manifold.

3. HETEROTIC STRING THEORY ON A CALABI-YAU BACKGROUND

In this section we shall discuss Lorentz and supersymmetry invariance for the heterotic string theory compactified on a Calabi-Yau background. We start by writing down the action for the heterotic string theory in the presence of the background metric $G_{ij}(x)$ and gauge field $A_i^M(x)$ in the GS formalism in the light-cone gauge,[3,30–32]

$$S = \frac{1}{\pi}\int d^2\xi[G_{ij}(X)\partial_+X^i\partial_-X^j + \frac{i}{2}S^\alpha\partial_-S^\alpha - \tfrac{1}{4}S^\alpha(\Sigma^{ab})_{\alpha\beta}S^\beta\omega_i^{ab}(X)\partial_-X^i$$
$$+ \frac{i}{2}\psi^\ell\partial_+\psi^\ell + \psi^\ell(T^M)_{\ell k}\psi^k A_i^M(X)\partial_+X^i - \tfrac{1}{8}F_{ab}^M(X)\psi^\ell(T^M)_{\ell k}\psi^k S^\alpha(\Sigma^{ab})_{\alpha\beta}S^\beta] \tag{3.1}$$

where again we have set α' to be $\frac{1}{2}$.

It is known[3,19] that if we identify the gauge connection with the spin connection, and set the background metric to be Ricci-flat and Kahler (e.g. if the holonomy group is SU(3)), then the above action looks identical to that of the NSR model, and has (2,2) supersymmetry. This is due to the fact that under the SU(3) subgroup both the spinor and the vector representations of SO(8) transform in the same way. Hence the term $\frac{i}{2}S^\alpha(\Sigma^{ab})_{\alpha\beta}S^\beta$ looks identical to the term $\lambda^a\lambda^b$ that we would have gotten in the NSR formalism, if we replace the components S^θ, $S^{\bar\theta}$, S^r and $S^{\bar r}$ by λ^θ, $\lambda^{\bar\theta}$, λ^r and $\lambda^{\bar r}$ respectively. The problem, however, is that the background metric, which gives vanishing β-function in the NSR formalism, is not Ricci-flat, and so does not have SU(3) holonomy. The action (3.1) with such a background has extra terms proportional to the U(1) part of the spin connection which does not look like the corresponding terms in the NSR formalism, and hence the model looses (2,2) supersymmetry.

It turns out that there is a simple solution to this problem. It was pointed out in Ref.[36] that although the physical background metric is not Ricci-flat, from the point of view of the two dimensional σ-model we may still take the background metric to be Ricci flat, and compensate for this by adding higher order counterterms in the perturbation theory. This gives rise to the same physical theory as the one formulated on a non-Ricci flat background, only the calculational scheme is somewhat different. The renormalization prescription that we shall use to define the theory given by the action (3.1) is the following. We start from a Ricci flat background for which the theory reduces to a (2,2) supersymmetric model, and then choose the higher order counterterms according to the prescription of Ref.[36], which gives a conformally invariant theory. Alternatively we can start from a non Ricci-flat background so that the classical theory described by the action (1.1) does not have (2,2) supersymmetry, and then choose the higher order counterterms in such a way so as to cancel the unwanted terms, and restore (2,2) supersymmetry, as well as conformal invariance at the quantum level.*

In the presence of a Ricci-flat Kahler metric with gauge connection identified with the spin connection, the action (3.1) reduces to,

$$S = S_0 + S_1' \tag{3.2}$$

where S_0 has been given in (2.5) and,

$$\begin{aligned}S_1' = \frac{1}{\pi}\int d^2\xi \Big[& G_{r\bar{s}}(\partial_+ X^r \partial_- X^{\bar{s}} + \partial_+ X^{\bar{s}} \partial_- X^r) + \frac{i}{2} G_{r\bar{s}}(S^{\bar{s}}\partial_- S^r \\ & + \Gamma_t{}^r{}_u S^{\bar{s}} S^u \partial_- X^t + S^r \partial_- S^{\bar{s}} + \Gamma_{\bar{t}}{}^{\bar{s}}{}_{\bar{u}} S^r S^{\bar{u}} \partial_- X^{\bar{t}} \\ & + \psi^{\bar{s}}\partial_+ \psi^r + \Gamma_t{}^r{}_u \psi^{\bar{s}}\psi^u \partial_+ X^t + \psi^r \partial_+ \psi^{\bar{s}} + \Gamma_{\bar{t}}{}^{\bar{s}}{}_{\bar{u}} \psi^r \psi^{\bar{u}} \partial_+ X^{\bar{t}}) \\ & - R_{r\bar{s}t\bar{u}} S^r S^{\bar{s}} \psi^t \psi^{\bar{u}} \Big]\end{aligned} \tag{3.3}$$

Here Γ is the Christoffel symbol, and R the Riemann tensor constructed from the metric $G_{r\bar{s}}$. (3.3) describes a (2,2) supersymmetric, conformally invariant

* Note that we are not claiming to have proved at this stage that this renormalization scheme for the GS model is equivalent to maintaining (2,2) superconformal invariance in the NSR model. Certainly the supersymmetry in the GS model (which is part of the space-time supersymmetry) is very different from the world sheet supersymmetry in the NSR model. The Lorentz generators look completely different when expressed in terms of the GS and the NSR fermions, even when the background has SU(3) holonomy. We expect, however, that if we are able to construct a Lorentz and supersymmetry invariant string theory using this prescription, then this theory will be identical to the corresponding theory in the NSR formulation.

σ-model with the renormalization prescription given in Ref.[36]. As a result, the σ-model posseses a (2,2) superconformal algebra whose (2,0) part is identical to that given in Eq.(2.7). Although no general proof has been given that the central charge c of this model has the value 6, it has been verified to be so to four loop order.[54] We assume here that it does not get renormalized in higher loop orders in the perturbation theory. If we further assume that there exists an operator U in the theory satisfying the commutation relations given in Eq.(2.44), we have all the necessary ingredients to construct the Lorentz and supersymmetry generators of the theory. We define them exactly as in Eqs.(2.24-2.33) with Q_m and P_m defined through Eqs.(2.14) and (2.43) respectively. Since the superconformal algebra remains the same in the interacting case, so is the algebra of Lorentz and supersymmetry generators, thereby showing that the theory has SO(3,1) Lorentz invariance, as well as N=1 space-time supersymmetry.

The criteria for the existence of the operator U may also be replaced by the existence of a scalar field ϕ such that the U(1) current $H(z)$ may be expressed as in Eq.(2.39), and $P(z) = e^{\sqrt{3}i\phi}$ is a local operator in the theory. As we can see from Eq.(2.40), all the fourier components of ϕ, except ϕ_0 are defined in terms of the oscillators H_m. The operator U essentially defines ϕ_0. Using Eq.(2.39) we may write down the operator product of $\phi(z)$ with itself, as well as all the superconformal currents. To see this let us write the operator product of $H(z)$ with the superconformal currents,

$$\begin{aligned}
T(z)H(w) &\sim \frac{1}{(z-w)^2}H(w) + \frac{1}{z-w}\partial_w H(w) \\
G^+(z)H(w) &\sim \frac{1}{w-z}G^+(z) \\
G^-(z)H(w) &\sim -\frac{1}{w-z}G^-(z) \\
H(z)H(w) &\sim \frac{3}{(z-w)^2}
\end{aligned} \tag{3.4}$$

from which we get,

$$
\begin{aligned}
T(z)\phi(w) &\sim \frac{1}{z-w}\partial_w\phi(w)\\
G^+(z)\phi(w) &\sim \frac{i}{\sqrt{3}}\ln(z-w)G^+(z)\\
G^-(z)\phi(w) &\sim -\frac{i}{\sqrt{3}}\ln(z-w)G^-(z)\\
H(z)\phi(w) &\sim -i\sqrt{3}\frac{1}{w-z}\\
\phi(z)\phi(w) &\sim -\ln(w-z)
\end{aligned}
\tag{3.5}
$$

The operator $P(z)$ may now be expressed in terms of the field $\phi(z)$ using Eqs.(2.41), and its operator product with itself, as well as with various components of the superconformal current may be calculated using Eqs.(3.5). Since these operator products will be identical to the ones in the free field theory, it follows that the commutators (2.16) and (2.18) remain unchanged from the free field result. This, in turn, implies that there is no anomaly in the Lorentz and supersymmetry algebra.

We now compare our result with the result of the analysis in the NSR formalism[39,50,51] . Let us first briefly recall the construction of the supersymmetry generators in the absence of any background fields. The ten Majorana-Weyl fermions $\lambda^1,...\lambda^{10}$ are first grouped into five complex fermions $\lambda^a,\lambda^{\bar{a}}$, (a=1,...5) and then bosonized as,

$$\lambda^a \sim e^{i\xi^a}, \quad \lambda^{\bar{a}} \sim e^{-i\xi^a} \tag{3.6}$$

The supersymmetry generators of the full ten dimensional theory are then given by,

$$e^{-\frac{1}{2}\eta}e^{\frac{i}{2}(\pm\xi^1\pm\xi^2...\pm\xi^5)} \tag{3.7}$$

where η is a bosonized ghost field.

If we identify the first six directions as internal directions, then the generators of a four dimensional N=1 supersymmetry transformation are given by,

$$e^{-\frac{1}{2}\eta} e^{\pm\frac{i}{2}(\xi^1+\xi^2+\xi^3\pm\xi^4\pm\xi^5)} \tag{3.8}$$

Upon compactification, the fields λ^r, $\lambda^{\bar{r}}$ ($1\leq r\leq 3$) becomes interacting, the action for these fields being identical to (3.3) with S^r, $S^{\bar{r}}$ replaced by λ^r, $\lambda^{\bar{r}}$ (or, in the bosonized version, ϕ^r replaced by ξ^r). The problem of constructing the supersymmetry generators in the theory is to find the analog of the operator $e^{\frac{i}{2}(\xi^1+\xi^2+\xi^3)}$ in the interacting theory, which satisfy the same operator product expansion with itself, and the superconformal currents, as in the free theory. As can be easily seen, the term in the exponent is precisely $\frac{\sqrt{3}}{2}i$ times the analog of the field ϕ. We may then construct the supersymmetry generators in the interacting theory by replacing the term $e^{\frac{i}{2}(\xi^1+\xi^2+\xi^3)}$ by $e^{i\frac{\sqrt{3}}{2}\phi}$. Hence the existence of the field ϕ in the (2,2) supersymmetric σ-model on a Calabi-Yau manifold also guarantees that the corresponding NSR formulation of the theory has N=1 space-time supersymmetry.* In this case the supersymmetry current will be local only with respect to those vertex operators which survive the GSO projection.

As can be easily seen, our analysis trivially generalizes to the case where the background fields are such as to give a (2,0) superconformal field theory involving the fields X^r, S^r, ψ^r and their complex conjugates. The Lorentz and supersymmetry generators are constructed in terms of the (2,0) superconformal generators, and the operator U, we never needed to use the (0,2) part of the superconformal algebra that exists on a Calabi-Yau manifold. This part of the algebra, however, will be useful to us in the next section in the discussion of the unbroken gauge symmetries of the model.

* Note that although the interacting part of the theory looks identical in the GS and the NSR formulation of the theory, the basic fermionic variables in the theory are very different. However, the constraint of the existence of the field ϕ satisfying the necessary properties is a constraint on the particular σ-model, irrespective of the interpretation of the fields appearing in the model. Hence the existence of such a field in the theory can be used in both, the GS and the NSR formulation.

4. GAUGE SYMMETRIES

The free heterotic string has gauge group $E_8 \times E_8$. Only the SO(16)×SO(16) part of this gauge group is realized linearly, under which the thirty two fermions tranform in the (16,1)+(1,16) representation.[2] The other generators of the gauge group are constructed in terms of the spin operators of the theory. For simplicity let us focus on one of the E_8 groups. We first combine the sixteen fermions ψ^ℓ into eight complex fermions ψ^a, $\psi^{\bar{a}}$ (a=1,...8), and then bosonize them to get eight left-moving scalar fields χ^a:

$$\psi^a \sim :e^{i\chi^a}: \, , \quad \psi^{\bar{a}} \sim :e^{-i\chi^a}: \tag{4.1}$$

where the fields χ^a are taken to lie on the fundamental torus of SO(16). The spin operators are given by,

$$:e^{i\sum_{a=1}^{8}\alpha_a\chi^a}: \tag{4.2}$$

where α_a is a weight vector in the spinor representation of SO(16). There are 128 such operators, which, combined with the 120 generators of SO(16), gives the 248 generators of the E_8 group.

Let us now identify an SU(3) subgroup of SO(16) under which the first three ψ^a, $\psi^{\bar{a}}$ transform as 3 and $\bar{3}$ respectively, while the other components of ψ remain neutral. We identify ψ^a, $\psi^{\bar{a}}$, (a=1,..3) with ψ^r and $\psi^{\bar{r}}$ appearing in Eq.(2.6). The 128 dimensional spinor representation of SO(16) contains 32 singlets of this SU(3) group. These transform in the 16+$\overline{16}$ representation of the SO(10) group which commutes with this SU(3). The corresponding weight vectors are given by,

$$\begin{aligned} &\alpha_1 = \alpha_2 = \alpha_3 = \tfrac{1}{2} \\ &\alpha_a = \beta_a, \quad (4 \le a \le 8) \end{aligned} \tag{4.3}$$

and,

$$\begin{aligned} &\alpha_1 = \alpha_2 = \alpha_3 = -\tfrac{1}{2} \\ &\alpha_a = \hat{\beta}_a, \quad (4 \le a \le 8) \end{aligned} \tag{4.4}$$

where β_a and $\hat{\beta}_a$ denote weight vectors for 16 and $\overline{16}$ representations of SO(10) respectively. The corresponding spin operators are,

$$: e^{\frac{i}{2}(\chi^1+\chi^2+\chi^3)} e^{i\sum_{a=4}^{8}\beta_a\chi^a} : \tag{4.5}$$

and,

$$: e^{-\frac{i}{2}(\chi^1+\chi^2+\chi^3)} e^{i\sum_{a=4}^{8}\hat{\beta}_a\chi^a} : \tag{4.6}$$

Upon introducing the background fields the fields χ^s (s=1,..3) will become interacting and as a result most of the spin operators given in Eq.(4.2) will cease to be symmetry generators of the theory. However, the operator $\chi^1 + \chi^2 + \chi^3$ may be identified to $\sqrt{3}\tilde{\phi}$, where $\tilde{\phi}$ is the analog of ϕ in the left handed sector. (The existence of ϕ guarantees the existence of $\tilde{\phi}$, due to the left-right symmetry of the model.) We may now generalize (4.5) and (4.6) in the interacting theory as,

$$e^{i\frac{\sqrt{3}}{2}\tilde{\phi}} e^{i\sum_{a=4}^{8}\beta_a\chi^a} \tag{4.7}$$

and,

$$e^{-i\frac{\sqrt{3}}{2}\tilde{\phi}} e^{i\sum_{a=4}^{8}\hat{\beta}_a\chi^a} \tag{4.8}$$

respectively. The operator product of these operators with the conformal currents will be the same as in the free field case, hence they remain symmetry generators of the theory. These are the generators which transform in $16+\overline{16}$ representation of the SO(10) group. Again, these operators will be local only with respect to those vertex operators which survive the GSO projection in the left-handed sector.

We may now count the total number of symmetry generators. There are forty five generators of SO(10) which generate rotations among the ten free fermions $\psi^7, ...\psi^{16}$. There is the U(1) generator $\tilde{H}_0$ itself. Finally there are thirty two spin operators belonging to the $16+\overline{16}$ representation of SO(10). These together give the seventy eight generators of E_6. There are of course sixteen other free fermions

$\psi^{17}, ...\psi^{32}$ with a symmetry group E_8 that is untouched by compactification. Thus the unbroken symmetry group of the theory is $E_8 \times E_6$. Notice that this analysis has been based on the existence of the (2,2) superconformal algebra, and the assumption that the U(1) current may be witten as the derivative of a local field, and does not directly require the knowledge of background gauge field expectation value.

We conclude this section with the following comment. In the NSR formalism, in order to get (2,2) supersymmetric model we had to set the gauge connection to be equal to the spin connection. Then, in order to get vanishing β-function we had to choose a spin connection which does not have SU(3) holonomy, but has U(3) holonomy. One might then think that the gauge connection takes value in a U(3) subgroup of E_8, and hence breaks the E_8 group to SO(10)$\times$U(1). It turns out, however, that the determination of the unbroken gauge group from this analysis is much more subtle. The subtlety lies in an ambiguity introduced by the renormalization prescription. In the NSR model, two different renormalization prescriptions, each maintaining (1,0) supersymmetry, differ from each other by a redefinition of various background fields.[55,56] Let us now compare the following two different renormalization schemes. In one scheme, we carry out the renormalization without imposing any constraints on the background fields at the beginning, and choose the renormalization counterterms in such a way that it maintains (1,0) supersymmetry, and the gauge fields have standard gauge transformation laws. Let us call the background gauge field appearing in this scheme A_{phys}. In the other scheme, we set the gauge connection to be equal to the spin connection at the beginning so that we have a (1,1) supersymmetry, and carry out the renormalization maintaining this symmetry. Let us denote the background gauge field in this scheme by A_{susy}. The two different renormalization schemes may be related to each other[55,56] by a functional relation between A_{phys} and A_{susy}. In other words, the theory at a given value of A_{susy} in the second scheme will be identical to the theory at a different value of A_{phys} in the first scheme, the value being given by a function $A_{phys}(A_{susy})$. (In general the functional relation will involve other background fields as well, we have not displayed them explicitly for convenience.)

Our analysis of conformal invariance tells us that A_{susy} needs to have U(3) holonomy in order to maintain conformal invariance. However, in order to determine the unbroken gauge group we need to know A_{phys}. It is perfectly possible that A_{phys} still has SU(3) holonomy, and hence gives an unbroken E_6 gauge group. One may try to determine the vev of A_{phys} either by direct calculation of the β-function of the (1,0) model, or by calculating the effective action for the heterotic string from the scattering amplitudes, and solving the equations of motion derived from the effective action. Partial results have already been obtained.[57,58]

5. CONCLUSION

In this paper we have analyzed the propagation of the the heterotic string theory on a Calabi-Yau background in the Green-Schwarz formulation. The two dimensional theory describing the string propagation in such a background is the sum of a free field theory and a (2,2) supersymmetric conformally invariant σ-model on the Calabi-Yau manifold. N=1 space-time supersymmetry is manifest in this formalism, however only the SO(2) subgroup of the full SO(3,1) Lorentz group is realized linearly. We find that a sufficient condition for the theory to have full SO(3,1) Lorentz invariance is that the U(1) part of the (2,0) superconformal current of the theory may be expressed as $-i\sqrt{3}\partial_z\phi$ where ϕ is a scalar field satisfying the constraint that $P(z) = e^{\sqrt{3}i\phi}$ is a local operator in the theory. In this case we can explicitly construct all the Lorentz generators in terms of the field ϕ, the superconformal currents and some free fields corresponding to the uncompactified dimensions. The commutators of the Lorentz generators with themselves, as well as with the supersymmetry generators, may be calculated only by using the superconformal algebra and some free field commutators, and may be shown to be free from any anomaly.

The same constraint on the U(1) current may be obtained from an analysis of the Neveu-Schwarz-Ramond formulation of the theory. In this formulation the SO(3,1) Lorentz invariance is manifest, but not the space-time supersymmetry. The two dimensional field theory describing the string propagation in background fields is again the sum of a free field theory and a (2,2) superconformal σ-model on the Calabi-Yau manifold. If the U(1) part of the (2,0) superconformal current satisfies the constraint mentioned above, the space-time supersymmetry generators of the theory may be constructed in terms of the field ϕ and some other free fields, and may be shown to satisfy the correct commutation relation among themselves, as well as with the Lorentz generators.

Thus our analysis determines a sufficient condition for obtaining a Lorentz invariant, space-time supersymmetric theory by string compactification on Calabi-

Yau manifolds. Since only the (2,0) part of the full (2,2) superconformal algebra is relevant for our analysis, our result easily generalizes to more general class of manifolds which give rise to (2,0) superconformal field theories on the world sheet. On the other hand, the (0,2) part of the superconformal algebra is relevant for discussing gauge symmetries of the compactified theory. Since the model is left-right symmetric, the U(1) part of the (0,2) superconformal current may be written as the derivative of a scalar field $\tilde{\phi}$, if the U(1) part of the (2,0) superconformal current satisfies a similar condition. It turns out that the generators of the full $E_8 \times E_6$ group may then be constructed in terms of the field $\tilde{\phi}$, the (0,2) superconformal currents, and some free fields, and may be shown to satisfy the correct commutation relations. Thus the unbroken gauge symmetry in this theory is $E_8 \times E_6$.

It remains to be seen whether superconformal field theories on Calabi-Yau manifolds satisfy the criteria derived in this paper. It may be possible to prove this in general following the line of arguments presented in Ref.[17]. On the other hand this may also be investigated in the perturbation theory. The current $P(z)$, being conserved ($\partial_{\bar{z}} P(z) = 0$), generates a symmetry transformation of the theory, which, to lowest order, is given by,

$$\delta S^{\bar{r}} \propto \epsilon^{\bar{r}\bar{s}\bar{t}} S^{r} S^{t} \tag{5.1}$$

This is a symmetry of the classical σ-model described by the action (3.3) if the background has SU(3) holonomy. In order to guarantee the existence of the current $P(z)$, some generalization of this symmetry must be maintained in the perturbation theory. A particularly interesting question would be to study what happens at the three loop order, since the requirement of the vanishing of the β-function forces us to choose a metric which has U(3) holonomy at order α'^3. (Or, equivalently, forces us to choose a finite counterterm at the three loop order which does not respect the symmetry (5.1)).

A closely related question would be to study how the local U(1) symmetry generated by $H(z)$ is maintained in the perturbation theory. This symmetry suffers from an anomaly proportional to the Ricci tensor at one loop order. Thus, taking the background to be non Ricci-flat produces an apparent anomaly. On the other hand, (2,2) supersymmetry and conformal invariance guarantees that this local U(1) symmetry cannot be anomalous, since it is part of the superconformal algebra. Hence at the four loop order, the U(1) anomaly must receive some new contribution, which cancels the term proportional to the Ricci tensor. Since the z-dependent part of the operator $P(z)$ is constructed solely in terms of the operator $H(z)$, we believe that a detailed understanding of the symmetry generated by H will also throw some light on the symmetry generated by P.

In order to illustrate this point we shall now give a heuristic argument showing that the criteria mentioned above are indeed satisfied by the theories considered here. Let us introduce vielbein fields e^a_r, $e^{\bar{a}}_{\bar{r}}$, and the fields $S^a (= e^a_r S^r)$, $S^{\bar{a}}$, ψ^a and $\psi^{\bar{a}}$. Expressed in terms of these fields, the action (3.3) naturally splits into a free part given by the action (2.6), and an interacting part which does not contain any derivatives of S or ψ. We may now go into the hamiltonian formalism and bosonize the fields S^a, $S^{\bar{a}}$ in the interaction picture as[59]

$$\begin{aligned} \left[S^a\right]_I &\sim \left[: e^{i\phi^a} :\right]_I \\ \left[S^{\bar{a}}\right]_I &\sim \left[: e^{-i\phi^a} :\right]_I \end{aligned} \tag{5.2}$$

where $\left[\phi^a\right]_I$ (a=1,..3) denote free right moving scalar fields, and the subscript I indicates that we are working in the interaction picture. The σ and τ components of the U(1) current H are given by, (we are now working in the usual σ, τ coordinate system with Minkowski signature),

$$\begin{aligned} \left[H_\sigma\right]_I &\sim -i\partial_\sigma \left[\sum_{a=1}^{3} \phi^a\right]_I \\ \left[H_\tau\right]_I &\sim -i\partial_\tau \left[\sum_{a=1}^{3} \phi^a\right]_I = -i\partial_\sigma \left[\sum_{a=1}^{3} \phi^a\right]_I \end{aligned} \tag{5.3}$$

since in the interaction picture ϕ is a free right moving scalar field. If H_{int} is the interaction hamiltonian, and $U(\tau)$ is the matrix,

$$U(\tau) = T\ exp(-i \int_0^\tau H_{int}(\tau')d\tau')$$

where T denotes time ordering. Then the currents H_τ and H_σ are given in the Heisenberg picture as,

$$H_\tau = H_\sigma \sim -i\partial_\sigma\Big(U(\tau)\Big[\sum \phi^a(\tau,\sigma)\Big]_I U^{-1}(\tau)\Big) \tag{5.4}$$

H_τ in Eq.(5.4) may be expressed as,

$$H_\tau \sim -i\partial_\tau\Big(U(\tau)\Big[\sum \phi^a\Big]_I U^{-1}(\tau)\Big) + U(\tau)\Big[H_{int}, \sum \phi^a\Big]_I U^{-1}(\tau) \tag{5.5}$$

Since H describes a conserved current, $\partial_\sigma H_\sigma - \partial_\tau H_\tau = 0$. Using Eq.(5.4) and the equations of motion of $\left[\phi^a\right]_I$ the conservation equation becomes,

$$\partial_\sigma\Big[H_{int}(\tau), \sum \phi^a(\tau,\sigma)\Big]_I = 0 \tag{5.6}$$

In general the commutator $[H_{int}(\tau), \sum \phi^a(\tau,\sigma)]$ will be given by a local operator at (σ,τ). Eq.(5.6) tells us that this operator is independent of σ. The only such operator in the theory is the identity operator. Thus we get,

$$[H_{int}(\tau), \sum \phi^a(\tau,\sigma)] = c \tag{5.7}$$

where c is a constant. Assuming, for simplicity, that c vanishes, we get,

$$H_\tau \sim -i\partial_\tau\Big(U(\tau)\Big[\sum \phi^a\Big]_I U^{-1}(\tau)\Big) \tag{5.8}$$

from Eq.(5.5). This shows that the current H may be expressed as in Eq.(2.39) with ϕ given by, $\frac{1}{\sqrt{3}}U\big[\sum \phi^a\big]_I U^{-1}$. The operator $P(z)$ is given by $\epsilon^{abc}S^aS^bS^c$, as can be seen by re-fermionizing the bosonic variables ϕ^a, and hence is a local operator in the theory. A non-zero c in Eq.(5.7) will give an extra τ-dependent phase in the definition of P.

Notice that in this derivation we have never used the criteria for SU(3) holonomy. However, if we try to calculate H_{int} explicitly, and evaluate the commutators (5.6) or (5.7), we shall get a term proportional to the Ricci tensor. This, of course, is a reflection of the U(1) anomaly. There must be some subtle quantum effects (which may come from the dynamics of the fields X or ψ) which makes the commutator (5.6) vanish, since (2,0) superconformal invariance guarantees a conserved U(1) current. The argument that we have presented here uses only this fact to establish the existence of the field ϕ. Since our argument is based on the (2,0) superconformal invariance only, it can be extended to more general σ-models with (2,0) superconformal invariance.

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