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Monopole induced baryon number violation due to weak anomaly

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ABSTRACT

Magnetic monopoles may catalyze baryon number violation due to weak 't Hooft anomaly, even if the boundary conditions at the monopole core conserve baryon number. We show, by analyzing a simple toy model, that this effect is unsuppressed by any power of the monopole size, weak symmetry breaking scale, or coupling constant, provided the radius of the monopole core is smaller than Eg^2/m_W^2, E being the energy of the external fermions, g the coupling constant of the $SU(2)^{weak}$ group, and m_W the $SU(2)^{weak}$ breaking scale. It is argued that this is a general feature of all monopoles (not necessarily 't Hooft-Polyakov monopoles). Possible suppression factors due to the presence of higher generation fermions are discussed.

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I. INTRODUCTION

Rubakov[1] and Callan[2] have proposed that grand unification monopoles may catalyze baryon number violation at the strong interaction rate. In the original model of Rubakov and Callan, where we ignore the $SU(2)^{weak}$ gauge fields, the origin of baryon number violation is the presence of the baryon number violating gauge field configuration inside the monopole core[2,3]. A simple explanation of this phenomenon may be obtained by studying the conservation laws of the full four dimensional field theory[4]. In the case of the lowest charge SU(5) monpole, interacting with one generation of massless fermions, the conservation of electric charge, color isospin, color hypercharge, and the charge associated with the anomaly free global phase transformation of the fermions, uniquely determines the final state for a given initial state[4]. For example, for an initial state of the form $u_{1L}+d_{3R}$, the unique final state is $u^c_{2L}+e^+_R$. Thus, baryon number violating processes are forced on us by the conservation laws of the system. In the case of more than one generation of massless fermions, if we ignore the off-diagonal gauge interactions other than those in the SU(2) subgroup in which the monopole is embedded, the conservation laws still uniquely determine the final state for a given initial state, and force us to baryon number violating processes.

All the discussion so far uses the structure of the underlying grand unification gauge group. A close examination of the conserved charges shows that the conservation of these charges automatically guarantees the conservation of the weak hypercharge[3,5]. It was pointed out by Goldhaber[6] and Schellekens[7] that we can turn the argument around, and demand the conservation of the color isospin, color hypercharge, electromagnetic charge and the weak hypercharge, to deduce that monopoles catalyze baryon number violation with a cross-section unsuppressed by any power of the monopole size. This may seem surprising at first, since the argument does not involve the underlying structure of the grand unified theory. But the point is that in the presence of the weak Z^0 field, which is responsible for the conservation of the weak hypercharge, the baryon number becomes anomalous[8]. This gives a new source of baryon number violation in the monopole fermion scattering. The possibility of monopole induced baryon number violation through weak anomaly was first noted by Wilczek[9]. The importance of imposing the conservation of weak hypercharge in the monopole fermion interaction was first pointed out by Grossman et. al.[10].

This analysis, however, assumes that weak interaction symmetry is unbroken. One would naively expect that due to the spontaneous breakdown of the weak SU(2) symmetry, any effect caused by the SU(2) anomaly should be suppressed by some power of E/m_w (if not $\exp(-m_w/E)$), where E is the energy of the external particles and m_W is the scale of the weak symmetry breaking. In this paper we shall show that this is not the case, even in the case of broken $SU(2)^{weak}$, monopole induced baryon number violation due to weak anomaly is not suppressed by any power of E/m_W, provided the radius of the monopole core is smaller than $g^2E/m_W{}^2$, g being the gauge coupling constant. We also show that there is a subtlety involved in applying the conservation of the Z^0 charge to predict baryon number violation[6,7]. One may argue that for a monopole radius small compared to $m_W{}^{-1}$, we may assume the Z^0 field to be effectively massless, and hence the Coulomb energy associated with this gauge field will force the conservation of the Z^0 charge carried by the fermions, and consequently enforce baryon number non-conservation. However, the fermion fields are not the only fields that carry the Z^0 charge, the higgs field also carries Z^0 charge. In the region $r \ll m_W{}^{-1}$ the higgs field is effectively massless, and the Z^0 charge may be transfered from the fermion field to the higgs field, which is then absorbed by the vacuum at a scale of order $m_W{}^{-1}$. Thus the Z^0 charge carried by the fermions need not be conserved separately, and the conservation laws no longer force us to baryon number violation. This is precisely what happens when the monopole radius r_0 is small compared to $m_W{}^{-1}$, but large compared to $g^2E/m_W{}^2$. The baryon number is conserved in the scattering, whereas the Z^0 charge is transfered to the vacuum through the higgs field, if the boundary condition on the fields at r_0 are baryon number conserving.

We illustrate our result with the help of a simple toy model, which we introduce in Sec.II. In Sec.III we study the dynamics of this model and show that a charge S_3, which is conserved by the boundary conditions, but is anomalous due to the presence of a gauge interaction, is violated in the monopole fermion interaction at the strong interaction rate, even if the gauge field, responsible for its violation, acquires a large mass m_w by higgs mechanism. We first give a heuristic argument which shows why the violation of S_3 need not be suppressed by any power of m_w. We then bosonize the theory, and derive the effective Lagrangian of the system in the presence of the spontaneously broken gauge interactions (Eq.(3.21) of the text). The effective Lagrangian in the case where the gauge symmetry is spontaneously broken is very different from that in the case of unbroken gauge interactions, even at a distance $r \ll m_w^{-1}$ from the monopole core. However, we find that for a monopole radius r_0 small compared to g^2E/m_w^2, the scattering of external solitons (which represent fermions) from the monopole core conserves the charge S_b associated with the gauge symmetry, but violates the anomalous charge S_3, as in the case of unbroken gauge symmetry. We also check the consistency of our result with all the conservation laws. In Sec.IV, we calculate S_3 violating condensates in our model, and show that they are indeed unsuppressed by any power of m_w. We also point out a subtlety involved in the calculation of such condensates for finite monopole radius. We summarize our results in Sec.V, and discuss possible suppression factors that may be present in the baryon number violating processes due to higher generation fermions. In appendix A we give the details of the bosonization procedure which gives us the effective Lagrangian in our model. In appendix B, we derive some results which are used in Sec.III to derive an upper bound on certain terms in the effective Hamiltonian.

II. THE MODEL

We shall illustrate our result with the help of a simple toy model, which we shall introduce in this section. Let us consider an SU(2) monopole interacting with two Dirac doublet of fermions, $\begin{pmatrix}\psi_{1\uparrow}\\ \psi_{1\downarrow}\end{pmatrix}$ and $\begin{pmatrix}\psi_{2\uparrow}\\ \psi_{2\downarrow}\end{pmatrix}$. As was shown by Callan[2], in the J=0 partial wave, the system is described by an effective two dimensional bosonized field theory with four fields Φ_i, Q_i ($i=1,2$) and their conjugate momenta Π_i, P_i, with the Hamiltonian,

$$H = \int_{r_0}^{\infty} \Big[\frac{1}{2} \sum_{i=1}^{2} \left(P_i^2 + \Pi_i^2 + (\Phi_i')^2 + (Q_i')^2 \right) + \frac{e^2}{32\pi^2 r^2} \left(\Phi_1 + \Phi_2 + Q_1 + Q_2 \right)^2 \Big] dr \tag{2.1}$$

with the boundary conditions

$$\Phi_i = Q_i \quad \text{at } r = r_0, \qquad \Phi_i' + Q_i' = 0 \quad \text{at } r = r_0 \tag{2.2}$$

r_0 being the radius of the monopole core.

We may express the various components of the fermion fields $\psi_{i\uparrow}$, $\psi_{i\downarrow}$ in terms of the boson fields. But more important for us is the expression for various fermionic charges in terms of the fields Φ_i, Q_i. These have been given in appendix A. The conserved charges of the system are[3],

$$S_i = \sum_{\eta=\uparrow,\downarrow} \int \bar{\psi}_{i\eta} \gamma^0 \psi_{i\eta} \, d^3x = \frac{1}{\sqrt{\pi}} \int_{r_0}^{\infty} (\Phi_i' - Q_i') \, dr$$

$$i = 1, 2$$

$$S_3 = \sum_{\eta=\uparrow,\downarrow} \int \left(\overline{\Psi}_{1\eta} \gamma^0 \gamma^3 \Psi_{1\eta} - \overline{\Psi}_{2\eta} \gamma^0 \gamma^3 \Psi_{2\eta} \right) d^3x = \frac{1}{\sqrt{\pi}} \int_{r_0}^{\infty} \left(\dot{\Phi}_1 + \dot{Q}_1 - \dot{\Phi}_2 - \dot{Q}_2 \right) dr$$

$$S_4 = \sum_{i=1,2} \int \left(\overline{\Psi}_{i\uparrow} \gamma^0 \Psi_{i\uparrow} - \overline{\Psi}_{i\downarrow} \gamma^0 \Psi_{i\downarrow} \right) d^3x = \frac{1}{\sqrt{\pi}} \int_{r_0}^{\infty} \left(\Phi_1' + Q_1' + \Phi_2' + Q_2' \right) dr$$

$$(2\text{-}3)$$

where the conservation of S_4 is guaranteed only in the $r_0 \to 0$ limit, in which case we obtain the dynamical boundary condition

$$\Phi_1+\Phi_2+Q_1+Q_2=0 \text{ at } r=r_0 \qquad (2\text{-}4)$$

by requiring the finiteness of energy.

We shall now introduce an extra gauge interaction in the theory, which makes the charge S_3 anomalous. This may be done by coupling the new gauge field b_μ to the fermionic current,

$$g(J_b{}^\mu)_f = g\left(\overline{\Psi}_{1\uparrow} \gamma^\mu \Psi_{1\uparrow} - \overline{\Psi}_{1\downarrow} \gamma^\mu \Psi_{1\downarrow} - \overline{\Psi}_{2\uparrow} \gamma^\mu \Psi_{2\uparrow} + \overline{\Psi}_{2\downarrow} \gamma^\mu \Psi_{2\downarrow} \right)$$

$$(2\text{-}5)$$

This gauge interaction may be assumed to be coming from a bigger gauge group in which the monopole SU(2) subgroup is embedded. The charge S_3 is thus the analog of baryon number in the case of a grand unification monopole where the baryon number is conserved by the boundary condition at the monopole core, but is violated due to anomaly. Such monopoles have been discussed by Dawson and Schellekens[11].

We shall study the violation of S_3 in the monopole-fermion interaction when the new gauge symmetry is broken by the higgs mechanism.

There is one apparent qualitative difference between the charge S_3 in the present model, and the baryon number in the real world. The charge S_3 is chiral, and the gauge field b, which makes it anomalous, couples to the vector current. In the real world, the baryon number is vector-like, whereas the Z^0 field, responsible for the anomaly in the baryon number, couples to the chiral current. This, however, is only an apparent difference. To see this, let us define new fermion fields in the present model,

$$\begin{aligned} \tilde{\psi}_{1\eta} &= \psi_{1\eta R} + \psi_{2\eta L} \\ & \qquad\qquad\qquad \eta = \uparrow \text{ or } \downarrow \qquad\qquad (2.6) \\ \tilde{\psi}_{2\eta} &= \psi_{2\eta R} + \psi_{1\eta L} \end{aligned}$$

where L and R denote positive and negative helicities respectively. The charge S_3, expressed in terms of the new fermionic fields, is vector-like, whereas the current J_b^μ, expressed in terms of these new fields, become chiral. Hence there is no basic qualitative difference between the case analyzed here, and the physical case.

In our discussion so far, we have concentrated on the grand unification monopoles only. We shall argue in Sec.V that our conclusion is valid also for monopoles which do not originate from grand unified theories, e.g. Kaluza-Klein monopoles[12]. For these monopoles the boundary conditions at the core have been argued to be baryon number conserving[13], and hence weak anomaly is the only source of baryon number violation.

If the gauge symmetry associated with the field b is unbroken, we may choose to work in the $b_0=0$ gauge. The part of the Lagrangian involving the b field, and its coupling to the fermionic fields is then given by,

$$\int 4\pi r^2 \, dr \left[\frac{1}{2} \dot{b}_r^2 - g b_r \left(\overline{\psi}_{1\uparrow} \hat{r}\cdot\vec{\gamma}\, \psi_{1\uparrow} - \overline{\psi}_{1\downarrow} \hat{r}\cdot\vec{\gamma}\, \psi_{1\downarrow} \right.\right.$$

$$\left.\left. - \overline{\psi}_{2\uparrow} \hat{r}\cdot\vec{\gamma}\, \psi_{2\uparrow} + \overline{\psi}_{2\downarrow} \hat{r}\cdot\vec{\gamma}\, \psi_{2\downarrow} \right) \right] \tag{2.7}$$

which, when expressed in terms of the two dimensional boson field theory, becomes,

$$\int dr \left[2\pi r^2 \dot{b}_r^2 + \frac{g}{\sqrt{\pi}} b_r \left(\dot{\Phi}_1 + \dot{Q}_1 - \dot{\Phi}_2 - \dot{Q}_2 \right) \right] \tag{2.8}$$

$\dot{b}_r$ may now be eliminated by using the equations of motion, giving a term in the effective Hamiltonian of the form,

$$\int_{r_0}^{\infty} \frac{g^2}{8\pi^2 r^2} \left(\Phi_1 - \Phi_2 + Q_1 - Q_2 \right)^2 dr \tag{2.9}$$

This new interaction gives us a new dynamical boundary condition,

$$\left(\Phi_1 + Q_1 - \Phi_2 - Q_2 \right) \simeq 0 \quad \text{at} \quad r = r_0 \tag{2.10}$$

giving rise to a new conserved charge,

$$S_b = \int d^3x \, (\bar{\psi}_{1\uparrow} \gamma^0 \psi_{1\uparrow} - \bar{\psi}_{2\uparrow} \gamma^0 \psi_{2\uparrow} - \bar{\psi}_{1\downarrow} \gamma^0 \psi_{1\downarrow} + \bar{\psi}_{2\downarrow} \gamma^0 \psi_{2\downarrow})$$

$$= \frac{1}{\sqrt{\pi}} \int_{r_0}^{\infty} dz \, (\Phi_1' + \alpha_1' - \Phi_2' - \alpha_2') \tag{2.11}$$

On the other hand, S_3 fails to commute with the new term (2.9), and is no longer conserved. This is the effect of anomaly. The conservation laws of S_1, S_2, S_b and S_4 uniquely determine the final state for a given initial state, and in some of the processes, (e.g. $\psi_{1\uparrow L}+\psi_{2\downarrow R}\rightarrow\psi_{1\uparrow R}+\psi_{2\downarrow L}$), S_3 is necessarily violated.

We now want to study a model, in which the gauge symmetry associated with the field b is spontaneously broken at a scale m_w. We take m_w to be large compared to the external energies, but small compared to the inverse size of the monopole. In order to do so, we introduce a higgs field χ, with coupling in the Lagrangian,

$$(\mathcal{D}_\mu \chi)^\dagger (\mathcal{D}_\mu \chi) - \lambda (|\chi|^2 - a^2)^2 \tag{2.12}$$

where,

$$\mathcal{D}_\mu \chi = (\partial_\mu - i g b_\mu) \chi \tag{2.13}$$

χ acquires a vev of magnitude a, thus breaking the U(1) symmetry associated with the b field. Defining shifted fields,

$$\chi = a + (\sigma_1 + i\sigma_2)/\sqrt{2} \tag{2.14}$$

the Lagrangian involving the b and χ fields become,

$$\frac{1}{2}(\partial_\mu \sigma_1)^2 + \frac{1}{2}(\partial_\mu \sigma_2)^2 + \frac{1}{2} m_w^2 b_\mu b^\mu - m_w b_\mu \partial^\mu \sigma_2$$
$$- \frac{1}{4} F_{\mu\nu} F^{\mu\nu} + g\, b_\mu (J_b^\mu)_f + \text{Cubic and quartic terms} \tag{2.15}$$

$$m_w = \sqrt{2}\, g a \tag{2.16}$$

The cubic terms involve higher powers of the coupling constant, and we shall ignore them in the rest of our discussion. The truncated Lagrangian is still invariant under the gauge transformation $b_\mu \to b_\mu + \partial_\mu \Lambda$, $\sigma_2 \to \sigma_2 + m_w \Lambda$, hence we believe this is a consistent approximation.

We choose to work in the gauge $\partial_0 b_0 + m_w \sigma_2 = 0$. Formally, we add a gauge fixing term $-C^2/2$ to the Lagrangian, where,

$$C = \alpha^{-1/2} (\partial_0 b_0 + m_w \sigma_2) + \alpha^{1/2} \sum_{k=1}^{3} \partial_k b^k \tag{2.17}$$

and take the limit $\alpha \to 0$. In principle, we also need to add the ghost term, but we ignore it here, since it couples only to the σ_1 field. In the approximation where we neglect all the cubic couplings, σ_1 decouples from the rest of the fields.

We now restrict all the fields to the J=0 partial wave (J denotes the total angular momentum), and bosonize the fermionic part of the Lagrangian. A detailed prescription for bosonization has been given in appendix A, hence here we just sketch the derivation. We express the fermionic current $(J^{\mu}_{b})_{f}$ in terms of the boson fields, using Eqs.(A.8), and define the field ϕ as,

$$\varphi = (\Phi_1 + Q_1 - \Phi_2 - Q_2)/2 \tag{2.18}$$

We may then express (2.15) in terms of the fields σ_1, σ_2, b_μ and ϕ. Of these, σ_1 decouples from the rest of the Lagrangian. σ_2 may be eliminated easily by using the equations of motion, which, in the $\alpha \to 0$ limit becomes,

$$\sigma_2 = - m_w^{-1} \partial_0 b_0 + O(\alpha) \tag{2.19}$$

The part of the Lagrangian, involving the fields b_0, $b_r (= (\hat{r})^i b^i)$ and ϕ is then given by,

$$\begin{aligned} \int_{r_0}^{\infty} dr \, [\, & 2\pi r^2 \{ \dot{b}_r^2 - 2\dot{b}_0^2 + (b_0')^2 - m_w^2 b_r^2 + m_w^2 b_0^2 \\ & + m_w^{-2} (\ddot{b}_0)^2 - m_w^{-2} (\dot{b}_0')^2 \} + \tfrac{1}{2} (\dot{\varphi}^2 - \varphi'^2) \\ & - \frac{2g}{\sqrt{\pi}} (b_0' \varphi + \dot{b}_r \varphi)] \end{aligned} \tag{2.20}$$

where prime and dot denote derivatives with respect r and t respectively. The terms involving $(\ddot{b}_0)^2$ and $(\dot{b}_0')^2$ come from the $(\partial_\mu\sigma_2)(\partial^\mu\sigma_2)$ term, after substituting the value of σ_2 from (2.19).

In the next section, we shall study the scattering of ϕ solitons in the model whose dynamics is described by the Lagrangian (2.20). The dynamics of the other three linearly independent combinations of Φ_1, Φ_2, Q_1 and Q_2 are not affected by the b field coupling, and is identical to the original model[3]. In the original model, an incident ϕ soliton of the form shown in Fig.1(a) would scatter into a soliton of the form shown in Fig.1(b). This scattering conserves the S_3 charge, but violates the S_b charge. When we introduce a massless b field, we get an extra term (2.9) in the Hamiltonian, which forces an incident soliton of the form Fig.1(a) to scatter into a soliton of the form Fig.1(c). This conserves the S_b charge, and violates the S_3 charge. The question we want to study in the next section is which of the two states shown in Fig.1(b) and Fig.1(c) represent the correct final state for an initial state of the form Fig.1(a), when the gauge symmetry associated with the b field is spontaneously broken.

III. MONOPOLE-FERMION INTERACTION

A. A heuristic argument

In this subsection we shall give an heuristic argument about why the violation of the charge S_3 in this model need not be suppressed by any power of m_W. If $J_3{}^{\mu}$ is the current associated with the charge S_3,

$$J_3{}^{\mu} = \sum_{\eta=\uparrow,\downarrow} \left(\bar{\Psi}_{1\eta}\, \gamma^{\mu}\gamma^{5}\, \Psi_{1\eta} - \bar{\Psi}_{2\eta}\, \gamma^{\mu}\gamma^{5}\, \Psi_{2\eta} \right) \tag{3.1}$$

then the anomaly equation for $J_3{}^{\mu}$ is,

$$\partial_{\mu} J_3{}^{\mu} = \frac{ge}{2\pi^2}\, \epsilon^{\mu\nu\rho\sigma}\, \partial_{\mu} b_{\nu}\, F^{(M)}_{\rho\sigma} \tag{3.2}$$

where $F^{(M)}_{\rho\sigma}$ is the magnetic field of the monopole. Substituting the value of the magnetic field, and using the spherical symmetry of J^{μ} and b_{ν}, we get,

$$\partial_{\mu} J_3{}^{\mu} = -\frac{g}{\pi^2 r^2} \left(\dot{b}_r + b_0' \right) \tag{3.3}$$

Integrating over space, we get,

$$\frac{dS_3}{dt} = 4\pi r_0^2 J_{3r}(r_0) + \frac{4}{\pi} g\, b_0(r_0) - \frac{4}{\pi} g \int_{r_0}^{\infty} \dot{b}_r\, dr \tag{3.4}$$

The boundary conditions ensure that $J_{3r}|_{r=r_0}$ vanishes. If we also assume that $b_0|_{r=r_0}=0$, then the only source of S_3 violation is the last term in Eq. (3.4). If E be the energy of the external soliton, then the scattering time is of order E^{-1}, since this is the width of the soliton. Hence, in order to produce a change in S_3 of order unity, we must have,

$$g \int_{r_0}^{\infty} \bar{b}_r \, dr \gtrsim E \tag{3.5}$$

We shall now show that there exists field configurations satisfying (3.5) and with finite action, provided,

$$r_0 \lesssim g^2 E / m_w^2 \tag{3.6}$$

To see this, let us consider a field configuration, for which,

$$b_r \simeq C\, r^{\alpha} \qquad (\alpha < -\tfrac{3}{2}) \tag{3.7}$$

for $r \ll E^{-1}$. The total action for such a field configuration for a time of order E^{-1} is of order,

$$E^{-1} m_w^2 C^2 r_0^{2\alpha+3} \tag{3.8}$$

the major contribution to the action coming from the $m_w^2 b_\mu b^\mu$ term. The finiteness of the action then requires,

$$C^2 r_0^{2\alpha} \lesssim E\, r_0^{-3} m_w^{-2} \tag{3.9}$$

The left hand side of (3.5), on the other hand, is of order,

$$g E C r_0^{\alpha+1} \lesssim g E^{3/2} r_0^{-1/2} m_W^{-1} \qquad (3.10)$$

using the constraint (3.9). As a result, the inequality (3.5) may be satisfied if (3.6) is satisfied, and the charge S_3 is violated by order unity in the monopole fermion scattering, without any suppression factor.

If $b_0(r_0)$ does not vanish, then Eq.(3.4) tells us that we have a new source of S_3 violation. But the contribution to $\dot{S}_3$, coming from the last term of (3.4) is independent of any boundary conditions, and hence we may expect that the violation of S_3 in the monopole-fermion scattering may take place at the strong interaction rate for any boundary condition on b_0, so long as (3.6) is satisfied.

The same conclusion may be obtained by considering the conservation of the S_b charge. As we have discussed, if we demand the conservation of the charge $S_b=\int d^3x\ (J_b{}^0)_f$, then the violation of S_3 is a necessary consequence of the conservation laws. However, the higgs field also couples to the gauge field, and, in general, we expect the sum of the b charge carried by the fermionic fields and the higgs field to be conserved, not each of them separately. Outside the core, there are no interactions which can transfer the b charge from the fermionic field to the higgs field, and vice versa. Thus, if, for some reason, the b charge carried by the higgs field is prevented to flow into or out of the monopole core, the b charge carried by the fermion and the higgs fields are conserved separately. To see when this happens, let us note that in a time of order E^{-1}, the total flow of b charge carried by the χ field into the monopole core is of order $r^2 (J_{br})_\chi E^{-1}$, where,

$$(J_{br})_\chi = i\{\chi^\dagger \partial_\mu \chi - (\partial_\mu \chi)^\dagger \chi\} (\hat{r})^\mu$$
$$\simeq \sqrt{2}\, a\, \partial_r \sigma_2 + 2g a^2 b_r + \text{quadratic terms} \qquad (3.11)$$

is the contribution of the χ field to the radial current of b charge.

Let, during the monopole-fermion scattering, σ_2 be of order Dr^β, D and β being constants with $\beta<-(1/2)$. Then the finiteness of the $\int(\partial_r \sigma_2)^2$ dr dt term of the action for a time of order E^{-1} requires that,

$$E^{-1} D^2 \beta^2 \int_{r_0}^{\infty} r^{2\beta-2} r^2 \, dr$$
$$\sim D^2 \beta^2 E^{-1} (r_0)^{2\beta+1} \lesssim 1 \qquad (3.12)$$

Thus,

$$E^{-1} \{ r^2 \sqrt{2}\, a\, \partial_r \sigma_2 \}|_{r_0} \sim D\beta \, \frac{r_0^{\beta+1} m_W}{gE}$$
$$\lesssim \frac{m_W}{g} \sqrt{\frac{r_0}{E}} \qquad (3.13)$$

from (3.12). Hence the contribution from the σ_2 term in (3.11) to $[r^2 (J_{br})_\chi E^{-1}]_{r=r_0}$ is small if the right hand side of (3.13) is small, i.e. if the condition (3.6) is satisfied.

Similarly, assuming a form (3.7) for b_r, we see that,

$$E^{-1}\,(r^2\, 2g\, a^2\, b_r)\big|_{r_0} \sim \frac{m_w^{\,2}}{gE}\, c\, r_0^{\,2+\alpha}$$

$$\lesssim \frac{m_w}{g}\sqrt{\frac{r_0}{E}} \qquad (3.14)$$

from the inequality (3.9). This is again small if (3.6) is satisfied.

Thus we see that if the constraint (3.6) is satisfied, then $r^2(J_{br})_\chi E^{-1}$ is small, and so, as argued before, the b charge carried by the fermion field must be conserved separately. The conservation of the S_b charge then forces us to S_3 violating processes.

B. Monopole-soliton scattering

In this section we shall study the monopole-soliton scattering in the model introduced in Sec.II. We start with the Lagrangian (2.20). The part of the Lagrangian, quadratic in b_0, may be written as,

$$\int dr\, dt\, \Big[\, \frac{4\pi r^2}{2}\, \big\{\, b_0\,(m_w^{\,2}+\partial_0^{\,2})\, b_0$$

$$+\, m_w^{-2}\, b_0\,(m_w^{\,2}+\partial_0^{\,2})\,\partial_0^{\,2} b_0 + m_w^{-2}\, b_0'\,(m_w^{\,2}+\partial_0^{\,2})\, b_0' \Big]$$

$$(3.15)$$

after doing some integration by parts in the time variable. If we take the Fourier transform in the time variable, the above expression is proportional to $(m_w{}^2-\omega^2)$. Since the energy E of the external fermions is small compared to m_w, and we expect only modes of frequency $\lesssim$ E to be relevant in the calculation of the Green's function involving such external quarks, we may replace $m_w{}^2+\partial_0{}^2$ by $m_w{}^2$ in (3.15). We shall show the self-consistency of this approximation in Sec.IV. Hence the effective action may be approximated as,

$$\int dr\, dt\, \Big[2\pi r^2 \{ \dot{b}_r{}^2 - m_w{}^2 b_r{}^2 - \dot{b}_0{}^2 + m_w{}^2 b_0{}^2 + (b_0')^2 \} + \tfrac{1}{2} (\dot{\varphi}^2 - \varphi'^2) + \frac{2g}{\sqrt{\pi}} (b_0 \varphi' + b_r \dot{\varphi}) \Big] \tag{3.16}$$

The equations of motion are given by,

$$\ddot{b}_r + m_w{}^2 b_r = \frac{2g}{4\pi\sqrt{\pi}\, r^2}\, \dot{\varphi} \tag{3.17}$$

$$-\ddot{b}_0 - m_w{}^2 b_0 + \frac{1}{r^2} \frac{\partial}{\partial r}\left(r^2 \frac{\partial b_0}{\partial r} \right) = \frac{2g}{4\pi\sqrt{\pi}\, r^2}\, \varphi' \tag{3.18}$$

$$\ddot{\varphi} - \varphi'' = -\frac{2g}{\sqrt{\pi}} (\dot{b}_r + b_0') \tag{3.19}$$

One of the nice features of the gauge we have choosen is that the equations of motion of b_0 and b_r decouple from each other. We may now, in principle, solve the equations (3.17) and (3.18) for b_r and b_0 in terms of ϕ, and substitute in

(3.16) to get the effective action in terms of the field ϕ. Eq.(3.9) may be easily solved if we ignore modes of frequency much larger than E. In that case, the $\ddot{b}_r$ term on the left hand side of the equation may be neglected, and we get,

$$b_r = \frac{2g}{4\pi\sqrt{\pi}\, m_W^2 r^2}\,\dot{\varphi} \tag{3.20}$$

Let us, for the time being, freeze the b_0 field, and study the effect of the b_r field on the equations of motion of ϕ. Substituting (3.20) in (3.16), we get the effective Lagrangian as a function of ϕ,

$$\int dr \left[\frac{1}{2}\dot{\varphi}^2 \left(1 + \frac{g^2}{\pi^2 m_W^2 r^2} \right) - \frac{1}{2}(\varphi')^2 \right] \tag{3.21}$$

where we have neglected the $\dot{b}_r^{\,2}$ term compared to the $m_W^{\,2} b_r^{\,2}$ term. The corresponding Hamiltonian is given by,

$$H = \int_{r_0}^{\ell} dr \left[\frac{1}{2}\left(1 + \frac{g^2}{\pi^2 m_W^2 r^2} \right)^{-1} P^2 + \frac{1}{2}(\varphi')^2 \right] \tag{3.22}$$

where,

$$P = \left(1 + \frac{g^2}{\pi^2 m_W^2 r^2} \right)\dot{\varphi} \tag{3.23}$$

is the momentum conjugate to ϕ. If E is the energy of the external particle, conservation of total energy implies that,

$$\frac{g^2}{m_w^2 r_0} (\dot{\varphi}(r_0))^2 \lesssim E \qquad (3.24)$$

Hence, if,

$$r_0 << g^2 E / m_w^2 \qquad (3.25)$$

eq. (3.24) implies that $\dot{\phi}(r_0)$ is small compared to E. Hence $\phi(r_0)$ cannot change appreciably during the scattering, and we get a dynamical boundary condition,

$$\varphi(r_0) = \text{constant} \qquad (3.26)$$

This boundary condition has the same effect as the effective boundary condition (2.10). In the monopole-soliton scattering, for an incoming soliton of the form Fig.1(a), the final state has the form of Fig.1(c). Hence the charge S_b is conserved in the monopole-fermion scattering, whereas the charge S_3 is necessarily violated.

Next, we shall consider the effect of the b_0 field. We again neglect the $\dot{b}_0^2$ term compared to the $m_w^2 b_0^2$ term. The solution to (3.18) may then be written as,

$$b_0 = -\frac{g}{2\pi\sqrt{\pi}} \int G(r, r') \varphi'(r') \, dr' \qquad (3.27)$$

where $G(r,r')$ is the Green's function satisfying the equation,

$$-\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial}{\partial r}G(r,r')\right)+m_w^2\,G(r,r')=\frac{1}{r^2}\,\delta(r-r') \tag{3.28}$$

subject to the boundary condition,

$$G(r,r')\to 0 \quad \text{as} \quad r\to\infty \tag{3.29}$$

and a boundary condition at r_0, which will be discussed shortly.

Before proceeding further, we shall express the various currents, that are relevant for our discussion, in terms of the fields b_r, b_0 and ϕ. The first of these is the contribution from the Higgs scalar χ to the gauge invariant current that couples to b_μ:

$$\begin{aligned}(J_{b\mu})_\chi &= i\left\{\chi^\dagger \overleftrightarrow{\partial_\mu}\chi - 2ig\, b_\mu \chi^\dagger\chi\right\}\\ &= -\sqrt{2}\,a\,\partial_\mu\sigma_2 + 2g\,a^2 b_\mu + \text{quadratic terms}\end{aligned} \tag{3.30}$$

When expressed in the gauge $\partial_0 b_0+m_w\sigma_2=0$, (3.30) reduces to,

$$(J_b^{\mu})_\chi = \frac{1}{g}\left(\partial^\mu \dot{b}_0 + m_w^{2}\, b^\mu\right) \tag{3.31}$$

The fermionic current that couples to the b field, denoted by $(J_b{}^\mu)_f$, is given by,

$$\left.\begin{aligned} (J_b{}^0)_f &= 2\varphi'/4\pi\sqrt{\pi}\, r^2 \\ (J_{b\,r})_f &= -2\dot{\varphi}/4\pi\sqrt{\pi}\, r^2 \end{aligned}\right\} \qquad (3.32)$$

where,

$$(J_{b\,r})_f = \sum_{i=1}^{3} (\hat{r})^i (J_b{}^i)_f \qquad (3.33)$$

The current $J_3{}^\mu$, associated with the charge S_3, is given by,

$$\left.\begin{aligned} J_3{}^0 &= 2\dot{\varphi}/(4\pi\sqrt{\pi}\, r^2) \\ J_{3\,r} &= -2\varphi'/(4\pi\sqrt{\pi}\, r^2) \end{aligned}\right\} \qquad (3.34)$$

Also important is the anomaly free current $\tilde{J}_3{}^\mu$, which is obtained by adding to $J_3{}^\mu$ a gauge non-invariant current $J_a{}^\mu$ involving the gauge fields,

$$\tilde{J}_3{}^\mu = J_3{}^\mu + J_a{}^\mu \qquad (3.35)$$

$$\left.\begin{aligned} J_a{}^0 &= g\, b_r/(\pi^2 r^2) \\ J_{a\,r} &= -g\, b_0/(\pi^2 r^2) \end{aligned}\right\} \qquad (3.36)$$

We may verify the relation $\partial_\mu \tilde{J}_3^{\ \mu}=0$ by using Eq. (3.19) and (3.34)-(3.36). The same result is obtained from the anomaly equation (3.3).

The boundary condition on the field b_0 is determined by the internal dynamics of the monopole core. Since the purpose of this paper is to show that the charge S_3 is violated under the most stringent conditions, we shall try to minimize the non-conservation of S_3 due to boundary effects. Such a choice of boundary condition is given by $b_0=0$, since this sets the second term on the right hand side of (3.4) to be zero. The first term $J_{3r}(r_0) \sim \phi'(r_0)/4\pi r_0^{\,2}$ is already zero due to the boundary condition of ϕ. The only source of violation of the S_3 charge is then the last term of (3.4).

With the boundary condition $b_0=0$, the Green's function $G(r,r')$ is given by,

$$\frac{1}{2 m_w r r'} \Big[\Theta(r-r')\, e^{-m_w(r-r_0)} \left\{ e^{m_w(r'-r_0)} - e^{-m_w(r'-r_0)} \right\} + \Theta(r'-r)\, e^{-m_w(r'-r_0)} \left\{ e^{m_w(r-r_0)} - e^{-m_w(r-r_0)} \right\} \quad (3.37)$$

Thus,

$$b_0 = -\frac{g}{2\pi\sqrt{\pi}} \frac{1}{2 m_w r} \Big[e^{-m_w(r-r_0)} \int_{r_0}^{r} \frac{e^{m_w(r'-r_0)} - e^{-m_w(r'-r_0)}}{r'} \varphi'(r') + \left\{ e^{m_w(r-r_0)} - e^{-m_w(r-r_0)} \right\} \int_{r}^{\infty} \frac{e^{-m_w(r'-r_0)}}{r'} \varphi'(r')\, dr' \Big] \quad (3.38)$$

Total contribution to the action from the terms,

$$\int \left\{ \frac{4\pi x_0^2}{2} \left(-\dot{b}_0^2 + m_w^2 b_0^2 + (b_0')^2 \right) + \frac{2g}{\sqrt{\pi}} b_0 \phi' \right\} dx\, dt \tag{3.39}$$

in (3.16) may then be written as,

$$-\int \frac{4\pi x_0^2}{2} \left(m_w^2 b_0^2 + (b_0')^2 \right) dx\, dt \tag{3.40}$$

with b_0 given by (3.38). In deriving (3.40), we have neglected the $\dot{b}_0^2$ term compared to the $m_w^2 b_0^2$ term. Since the expression for b_0 involves only the spatial derivatives of ϕ, but no time derivatives, the corresponding contribution to the effective Hamiltonian involving the ϕ field is given by,

$$\begin{aligned} &\int 2\pi x_0^2\, dx \left(m_w^2 b_0^2 + (b_0')^2 \right) \\ &= \frac{g^2}{2\pi^2} \int_{x_0}^{\infty} dx \int_{x_0}^{\infty} dx'\, G(x, x')\, \phi'(x)\, \phi'(x') \end{aligned} \tag{3.41}$$

We have shown in appendix A that the contribution from these terms is negligible compared to the energy of the external soliton, so long as E is small compared to m_w. Hence we do not expect these terms to affect the dynamics of the system. The full effective Hamiltonian of the system is then given by the effective Hamiltonian (3.22), obtained by integrating the b_r field. As we have already seen, for

$r_0 << g^2E/m_w^2$, the scattering process conserves the S_b charge, and violate the S_3 charge. In the real world, this means that in the monopole fermion interaction, the Z^0 charge carried by the fermions is conserved, whereas the baryon number is violated.

In the region,

$$m_w^{-1} >> r_0 >> \frac{g^2 E}{m_w^2} \qquad (3.42)$$

the contribution from the term in the Hamiltonian,

$$\int (g^2 \dot{\phi}^2 / \pi^2 m_w^2 r^2) \; dr \qquad (3.43)$$

coming from b_r integration, is small compared to E even if $\dot{\phi} \sim E$ at $r = r_0$. Hence the effect of this term is small, and we may expect the scattering to be identical to the free field case. An incoming soliton of the form of Fig.1(a) will be scattered back as Fig.1(b). This process conserves the S_3 charge and violates the S_b charge coupled to the gauge field b.

C. On the conservation of the S_b charge

The above result is somewhat surprising, since, for $r_0 << m_w^{-1}$, we expect an effective Coulomb energy barrier for depositing any S_b charge inside the monopole core, and hence it is expected to be conserved in the scattering. This puzzle, however, disappears, when we consider the fact that fermions are not the only fields that couple to the b field, the higgs field also couples to the b field. Thus the total conserved current is the sum of $(J_b{}^{\mu})_f$ and $(J_b{}^{\mu})_\chi$ in Eqs.(3.31) and (3.32) respectively, but not each of them individually. To see that the total b charge is indeed conserved in the scattering, let us note that,

$$(J_{br})_{total} = (J_{br})_\chi + (J_{br})_f$$

$$= \frac{1}{g}\left(-\partial_r \dot{b}_r + m_w{}^2 b_r\right) - \frac{2g}{4\pi\sqrt{\pi}\, r^2} \simeq -\frac{1}{g}\left(\omega' + \ddot{b}_r\right) \tag{3.44}$$

using Eq.(3.17). The condition for finiteness of the term,

$$\int (b_r')^2\, 4\pi r^2\, dr \tag{3.45}$$

in the effective Hamiltonian (3.41) implies that,

$$(r^2 b_r')\big|_{r=r_0} \lesssim \sqrt{E r_0} \tag{3.46}$$

Similarly Eq.(3.20) gives,

$$(r^2 \dot{b}_r)\big|_{r=r_0} = \frac{2g}{4\pi\sqrt{\pi}}\,\frac{\dot{\phi}(r_0)}{m_w{}^2} \sim \frac{E}{m_w}\sqrt{E r_0} \tag{3.47}$$

from (3.24). Hence $(J_{br})_{total}$ vanishes at $r=r_0$ for $r_0 \ll E^{-1}$.

Thus the total charge coupled to the b field is conserved in the monopole fermion interaction. However, neither $r^2(J_{br})_f$ nor $r^2(J_{br})_\chi$ vanishes separately at $r=r_0$ for $g^2E/m_w{}^2 \ll r_0 \ll m_w{}^{-1}$, and hence the charge $S_b = \int 4\pi r^2 (J_b{}^0)_f\, dr$ may not be conserved during the scattering.

At this point let us note that we could choose the boundary condition on b_0 in such a way that $(J_{br})_\chi$ vanishes at r_0. This is achieved if,

$$(J_{br})_\chi = \frac{1}{g}(-\partial_r \dot{b}_0 + m_w{}^2 b_r) \simeq \frac{1}{g}\left(-\partial_r \dot{b}_0 + \frac{2g\dot{\varphi}}{4\pi\sqrt{\pi}\, r^2}\right) = 0$$

$$\text{at} \quad r = r_0 \tag{3.48}$$

or,

$$b_0'(r_0) \simeq \frac{2g\,\varphi(r_0)}{4\pi\sqrt{\pi}\; r_0{}^2} \tag{3.49}$$

With this new boundary condition, the solution for b_0 in terms of ϕ may be obtained by adding to (3.38) the solution of the homogeneous equation:

$$\frac{-2g\varphi(r_0)}{4\pi\sqrt{\pi}} \quad \frac{e^{-m_w r}}{r} \left(1 + m_w r_0\right)^{-1} \tag{3.50}$$

Finiteness of the term (3.45) in the Hamiltonian will then imply,

$$\varphi(r_0) \lesssim \sqrt{E r_0}\,/\,g \tag{3.51}$$

and hence the S_b charge is conserved in the monopole-fermion interaction, while the S_3 charge is necessarily violated, even if the monopole radius lies in the region (3.42).

IV. CALCULATION OF S_3 VIOLATING CONDENSATES:

In this section we shall calculate S_3 violating condensates in the model described by the Lagrangian (3.21). There are several purposes for doing this calculation, which are as follows:

i) We want to show that S_3 violating condensates are indeed unsuppressed by any power of m_W.

ii) We want to show that the modes of oscillations of ϕ with frequency much larger than the external energy E do not give a significant contribution to the fermionic Green's function, hence this calculation is at least a self-consistent one.

iii) There are some subtleties involved in calculating the S_3 violating condensates for a finite monopole radius, which we shall illustrate below.

During this calculation we shall follow the convention of Ref.14. Instead of calculating Green's functions involving the external physical fermions, we shall, for simplicity, calculate a condensate involving solitons in the field ϕ, thus ignoring the degrees of freedom corresponding to the three other linear combinations of Φ_1, Φ_2, Q_1 and Q_2. This may be done without any loss of generality, since these other degrees of freedom do not carry any S_3 charge, and hence are irrelevant in the discussion of S_3 violation.

The creation operators for the ingoing and the outgoing solitons, shown in Fig.1(a) and (c) are given by[15],

$$\psi_{in}(x,t) = \sqrt{\frac{\mu c}{2\pi}}\; N_\mu \left\{ e^{i\sqrt{\pi}\left(\varphi(x,t) + \int_{x_0}^{x} \dot{\varphi}(s,t)\, ds\right)} \right\} \tag{4.1}$$

$$\psi_{out}(x,t) = \sqrt{\frac{\mu c}{2\pi}}\; N_\mu \left\{ e^{-i\sqrt{\pi}\left(\varphi(x,t) - \int_{x_0}^{x} \dot{\varphi}(s,t)\, ds\right)} \right\} \tag{4.2}$$

where N_μ denotes normal ordering with respect to the mass μ, and c is a constant of order unity.[F1] We are interested in computing the Green's function,

$$\langle 0 |\, \psi_{out}^{\dagger}(x',t)\; \psi_{in}(x,t)\, | 0 \rangle \tag{4.3}$$

This may be calculated by using the following identities,

$$N_\mu \left(e^{i\int J(x)\,\varphi(x)\, d^2x} \right)$$

$$= e^{i\int J(x)\,\varphi(x)\, d^2x}\; e^{\frac{1}{2}\int d^2x\, d^2y\; J(x)\, \Delta_+(x,y,\mu)\, J(y)} \tag{4.4}$$

$$e^{A}\, e^{B} = e^{A+B}\, e^{\frac{1}{2}[A,B]} \tag{4.5}$$

if [A, B] is a c number.

$$\langle 0 |\, e^{i\int J(x)\,\varphi(x)\, d^2x}\, | 0 \rangle$$

$$= e^{-\frac{1}{2}\int d^2x\, d^2y\; J(x)\, \Delta(x,y)\, J(y)} \tag{4.6}$$

where $\Delta_0(x,y)$ is a two point Wightman function for a free scalar field of mass μ, satisfying the boundary condition $\Phi'(r_0)=0$, and $\Delta(x,y)$ is the two point Wightman function for the field ϕ in the Lagrangian (3.21). They are given by, respectively,

$$\Delta_0(r,t,r',t') = \int \frac{d\omega}{4\pi\sqrt{\omega^2+\mu^2}}\, 4\cos\omega(r-r_0)\cos\omega(r'-r_0)\, e^{-i\sqrt{\omega^2+\mu^2}(t-t')} \tag{4.7}$$

$$\Delta(r,t,r',t') = \int \frac{d\omega}{4\pi\omega}\, f_\omega(r)\, f_\omega(r')\, e^{-i\omega(t-t')} \tag{4.8}$$

where $f_\omega(r)$ is the solution of the equation,

$$\frac{d^2 f_\omega(r)}{dr^2} + \omega^2\left(1 + \frac{g^2}{\pi^2 m_w^2 r^2}\right) f_\omega(r) = 0 \tag{4.9}$$

with the boundary condition,

$$\frac{d}{dr} f_\omega(r) = 0 \quad \text{at} \quad r = r_0 \tag{4.10}$$

and the normalization,

$$\int_{r_0}^{\infty} \left(1 + \frac{g^2}{m_w^2 \pi^2 r^2}\right) f_\omega(r)\, f_{\omega'}(r)\, dr = 2\pi\,\delta(\omega-\omega') \tag{4.11}$$

$$\int_0^\infty d\omega \; f_\omega(r)\, f_\omega(r') = 2\pi\, \delta(r-r') \left(1 + \frac{g^2}{m_w^2 \pi^2 r^2}\right)^{-1} \tag{4.12}$$

The solution to Eq.(4.9) may be written as,

$$f_\omega(r) = \sqrt{\omega r}\, \{ a(\omega)\, J_{\nu(\omega)}(\omega r) + b(\omega)\, Y_{\nu(\omega)}(\omega r) \} \tag{4.13}$$

where,

$$\nu(\omega) = \sqrt{\frac{1}{4} - \frac{\omega^2 g^2}{\pi^2 m_w^2}} \tag{4.14}$$

Boundary condition (4.10) gives,

$$a(\omega)/b(\omega)$$

$$= -\left[\frac{d}{dr}\left(\sqrt{\omega r}\; Y_{\nu(\omega)}(\omega r)\right) \Big/ \frac{d}{dr}\left(\sqrt{\omega r}\; J_{\nu(\omega)}(\omega r)\right)\right]_{r=r_0}$$

$$\simeq \frac{\omega g^2}{2\pi^2 m_w^2 r_0} \tag{4.15}$$

in the $r_0 \to 0$ limit. Thus, for,

$$\omega g^2 / m_w^2 r_0 \gg 1 \tag{4.16}$$

$a(\omega)$ is much larger than $b(\omega)$. Hence we may set $b(\omega)$ to be zero. The normalization condition determines $a(\omega)$ to be

$\sqrt{2\pi}$, and hence, for $r \gg r_0$,

$$f_\omega(r) \simeq \sqrt{2\pi\omega r}\;\; J_{\nu(\omega)}(\omega r) \tag{4.17}$$

if (4.16) is satisfied. Note that for ω of the order of the external energies, (4.16) is identical to the constraint (3.25) derived from purely classical analysis.

Using Eqs. (4.4)-(4.6), we may express (4.3) as,

$$\begin{aligned}
&\frac{\mu c}{2\pi} \exp\Big[-\frac{\pi}{2}\Big\{ \Delta(r,t,r,t) - \Delta_0(r,t,r,t) \\
&+ \int_{r_0}^{r} ds \,\big\{ \partial_t \Delta(s,t,r,t') - \partial_t \Delta_0(s,t,r,t') \big\}\Big|_{t=t'} \\
&+ \int_{r_0}^{r} ds' \,\big(\partial_{t'} \Delta(r,t,s,t') - \partial_{t'} \Delta_0(r,t,s,t') \big)\Big|_{t=t'} \\
&+ \int_{r_0}^{r} ds \int_{r_0}^{r} ds' \;\partial_t \partial_{t'} \big(\Delta(s,t,s',t') - \Delta_0(s,t,s',t') \big)\Big|_{t=t'} \\
&+ \Delta(r',t,r',t) - \Delta_0(r',t,r',t) \\
&- \int_{r_0}^{r'} ds \,\big\{ \partial_t \Delta(s,t,r',t') - \partial_t \Delta_0(s,t,r',t') \big\}\Big|_{t=t'} \\
&- \int_{r_0}^{r'} ds' \,\big(\partial_{t'} \Delta(r',t,s',t') - \partial_{t'} \Delta_0(r',t,s',t') \big)\Big|_{t=t'} \\
&+ \int_{r_0}^{r'} ds \int_{r_0}^{r'} ds' \;\partial_t \partial_{t'} \big(\Delta(s,t,s',t') - \Delta_0(s,t,s',t') \big) \\
&+ 2\,\Delta(r,t,r',t') - 2\int_{r_0}^{r'} ds' \;\partial_{t'} \Delta(r,t,s',t')\Big|_{t'=t} \\
&+ 2\int_{r_0}^{r} ds \;\partial_t \Delta(s,t,r',t')\Big|_{t=t'} \\
&- 2\int_{r_0}^{r} ds \int_{r_0}^{r'} ds' \;\partial_t \partial_{t'} \Delta(s,t,s',t')\Big|_{t=t'} \Big\}\Big]
\end{aligned} \tag{4.18}$$

The above expression may be evaluated by using Eqs. (4.7), (4.8) and (4.17), in the $r_0 \to 0$ limit. The final expression is an integral over ω, the integrand being a finite function of ω, r and r'. Thus any divergence in the exponent must appear from the ω integral. Although the individual terms have logarithmic divergence from the large ω region, all such divergences cancel when we sum over all the terms. For example, the ultraviolet divergence in $\Delta(r,t,r,t)$ is cancelled by that of $\Delta_0(r,t,r,t)$. The integral then receives contribution only from the region $\omega \lesssim r^{-1}, r'^{-1}$. There is also some infrared divergences in the integral coming from the $\Delta_0(r,t,r,t)$ and $\Delta_0(r',t',r',t')$ terms, which are regulated by μ. The net divergent contribution in the exponent is then given by,

$$\frac{\pi}{2} \cdot \frac{2}{\pi} \ln(\mu^{-1}) \tag{4.19}$$

which, when exponentiated, cancels the explicit factor of μ outside the exponential. Thus the final result is finite, and is unsuppressed by any power of m_W. This analysis also shows that the significant contribution to the condensate comes only from the modes of oscillation with frequency of order r^{-1} or r'^{-1}, i.e. of the order of the energies of the external particles, which is taken to be much less than m_W. Hence this approximation is a self-consistent approximation.

Next we shall demonstrate the subtlety involved in calculating Green's function for finite r_0. As can be seen from Eq. (4.15), if for fixed r_0, we consider the modes with frequency $\omega << m_w{}^2 r_0/g^2$, then $b(\omega)$ is much larger than $a(\omega)$. Hence we may set $a(\omega)$ to be zero, and $b(\omega)$ to be $\sqrt{2\pi}$, using the normalization condition (4.11), and get,

$$f_\omega(r) \simeq \sqrt{2\pi\omega r}\; Y_{\nu(\omega)}(\omega r)$$

$$\text{for} \quad \omega << m_w{}^2 r_0/g^2 \tag{4.20}$$

As $\omega \to 0$, $\nu(\omega) \to 1/2$ and $f_\omega(r)$ approaches $2\cos\omega r$. Thus the term,

$$\Delta(r,t,r',t) = \int_0^\infty \frac{d\omega}{4\pi\omega}\; f_\omega(r)\, f_\omega(r') \tag{4.21}$$

has an infrared divergence from the region $\omega << m_w{}^2 r_0/g^2$. As a result, the Green's function (4.3), given by expression (4.18) vanishes identically.

The origin of this divergences may be understood as follows. The effective Lagrangian (3.21) has a conserved charge,

$$\tilde{S}_3 = \int_{r_0}^{\infty} \left(1 + \frac{g^2}{\pi^2 m_w{}^2 r^2}\right) \dot{\varphi}(r,t)\; dr \tag{4.22}$$

as opposed to the charge

$$S_3 = \int_{r_0}^{\infty} \dot{\varphi}(r,t)\; dr \tag{4.23}$$

which is not conserved. $\tilde{S}_3$ denotes the total gauge non-invariant conserved charge $\int \tilde{J}_3^{\,0}\, d^3x$, as can be verified by using Eqs.(3.34)-(3.36), and (3.20). By computing the commutator of ψ_{in} and ψ_{out} with S_3 and $\tilde{S}_3$, we see that $\psi_{in}(r,t)$ carries one unit of S_3 and $\tilde{S}_3$ charge, whereas $\psi_{out}(r,t)$ carries -1 unit of S_3 and $\tilde{S}_3$ charge in the limit $g^2/m_w^{\,2}r^2 \ll 1$. Thus the Green's function (4.3), besides violating S_3, also violates $\tilde{S}_3$ charge. Since $\tilde{S}_3$ is a conserved charge, the Green's function vanishes identically.

In order to gain an insight into the problem, we look back into the classical scattering of the soliton from the core. There, an incoming soliton of the form Fig.1(a) scatters back into a soliton of the form Fig.1(c). This apparently violates both S_3 and $\tilde{S}_3$ charge. But actually, the scattered soliton leaves behind a $\dot{\phi}$ field of small amplitude, so that the total $\tilde{S}_3$ charge is conserved in the scattering process. To see how this may be achieved, let us consider a field configuration near the core with $\dot{\phi} \sim cE(Er)^{\alpha}$ $(\alpha<1)$, where E is the energy of the scattered soliton. For such a configuration,

$$\tilde{S}_3 - S_3 \sim \frac{g^2}{\pi^2 m_w^{\,2}}\, c\, E^{\alpha+1}\, r_c^{\,\alpha-1} \qquad (4.24)$$

$$S_3 \sim c\, E^{\alpha+1}\, r_c^{\,\alpha+1} \sim \frac{\pi^2 m_w^{\,2}}{g^2}\, r_c^{\,2} (\tilde{S}_3 - S_3) \qquad (4.25)$$

whereas the total energy stored in the field is of order,

$$\int \left(1 + \frac{g^2}{m_w^2 \pi^2 r^2}\right) c^2 E^{2+2\alpha} r^{2\alpha-2}\, dr$$

$$\sim c^2 E^{2+2\alpha} \frac{r_0^{2\alpha-1}}{2\alpha-1} + \frac{g^2}{m_w^2 \pi^2} c^2 E^{2+2\alpha} \frac{r_0^{2\alpha-3}}{2\alpha-3}$$

$$\sim \frac{\pi^2 m_w^2}{g^2} r_0 (\tilde{S}_3 - S_3)^2 \qquad (4\text{-}26)$$

Thus, for $r_0 << g^2 E/m_w^2$, there exist classical field configurations around the monopole core, which carry a net $\tilde{S}_3$ charge of order unity, but negligible S_3 charge and negligible energy. As a result, in the monopole-soliton scattering the S_3 charge is violated, whereas the $\tilde{S}_3$ charge is exactly conserved.

Thus, we see that in order to get a non-zero value of the S_3 violating Green's function, we must somehow include these soft modes in the final state. The situation is analogous to the case of four dimentional quantum electrodynamics, where the S-matrix elements involving charged particles in the initial and the final state vanish identically due to the exponentiation of the infra-red divergences. One way to get rid of the infra-red divergences is to sum over soft photon emissions in the final state[16]. However, there is another way of removing infra-red divergences in QED, using coherent state formalism[17]. This formalism is better suited for our purpose. Instead of working with the operators ψ_{in} and ψ_{out}

defined in Eqs. (4.1) and (4.2) respectively, we construct operators $\tilde{\psi}_{in}$ and $\tilde{\psi}_{out}$ as,

$$\tilde{\psi}_{in}(z,t) = e^{i\sqrt{\pi}\int_{z}^{\infty} g(s)\phi(s)\,ds}\,\psi_{in}(z,t) \qquad (4.27)$$

$$\tilde{\psi}_{out}(z,t) = e^{-i\sqrt{\pi}\int_{z}^{\infty} g(s)\phi(s)\,ds}\,\psi_{out}(z,t) \qquad (4.28)$$

where the function g(s) satisfies,

$$\int_{z_0}^{\infty} g(s)\,ds = 1 \qquad (4.29)$$

$$\int_{z_0}^{\infty} g(s)\left(1+\frac{g^2}{\pi^2 m_w^2 s^2}\right)^{-1} ds = C \qquad (4.30)$$

which may be satisfied by taking g(s) to be peaked at small value of s, and small for $s \gtrsim m_w/g$. We must mention at this point that the choice of $\tilde{\psi}_{in}$ and $\tilde{\psi}_{out}$ are not unique. For example, we could have choosen $\tilde{\psi}_{in}$ to be ψ_{in}, and replaced g(s) by 2g(s) in the definition of $\tilde{\psi}_{out}$. The operators $\tilde{\psi}_{in}(r,t)$ and $\tilde{\psi}_{out}(r,t)$ create fermion fields at the point r, together with a coherent $\dot{\phi}$ field, given by,

$$\pm\; g(s)\left(1+\frac{g^2}{\pi^2 m_w^2 s^2}\right)^{-1} \qquad (4.31)$$

where + and − corresponds to $\tilde{\psi}_{in}$ and $\tilde{\psi}_{out}$ respectively. Eqs.(4.29) and (4.30) then tells us that the total $\tilde{S}_3$ charge carried by $\tilde{\psi}_{in}$ and $\tilde{\psi}_{out}$ are zero, whereas the total S_3 charge carried by $\tilde{\psi}_{in}$ and $\tilde{\psi}_{out}$ are the same as those carried by ψ_{in} and ψ_{out} respectively. This may be easily verified by calculating the commutators of S_3 and $\tilde{S}_3$ with $\tilde{\psi}_{in}$ and $\tilde{\psi}_{out}$. $g(s)$ must be choosen in such a way that the total energy stored in the ϕ field for the field configuration given in (4.31) is small compared to E.

We may now easily demonstrate that the Green's function

$$\langle 0|\, \tilde{\psi}_{out}^{\dagger}(r',t)\; \tilde{\psi}_{in}(r,t)\, |0\rangle \qquad (4.32)$$

is free from any infrared divergence, and is finite. For example, the infrared divergent term $\Delta(r,t,r',t)$ in the exponent is now replaced by,

$$\int_{r_0}^{\infty} ds \int_{r_0}^{\infty} ds' \,\{\delta(r-s)+g(s)\}\{\delta(r'-s)+g(s)\}\,\Delta(s,t,s',t$$

$$= \int_0^{\infty} \frac{d\omega}{4\pi\omega}\,\{f_\omega(r) + \int_{r_0}^{\infty} g(s)\, f_\omega(s)\, ds\}$$

$$\times \{f_\omega(r') + \int_{r_0}^{\infty} g(s)\, f_\omega(s)\, ds\} \qquad (4.33)$$

We shall now study the infrared and ultraviolet divergences of this integral. First, note that for $\omega \ll m_w^2 r_0/g^2$, we may replace $f_\omega(r)$, $f_\omega(r')$ and $f_\omega(s)$ in (4.33) by 2, if,

$$r^{-1},\ (r')^{-1} >> (m_w^{2} r_0)/g^2 \qquad (4.34)$$

and the contribution to g(s) from the region $s \gtrsim (m_w^2 r_0/g^2)^{-1}$ can be neglected. Then the integrand is proportional to,

$$\left\{ 2 + 2\int_{r_0}^{\infty} g(s)\, ds \right\}^2 \simeq 0 \qquad (4.35)$$

using Eq. (4.29). In the region $m_w^2 r_0/g^2 << \omega < r^{-1}$, r'^{-1}, the integrand may be written as,

$$2 \sin \omega r + 2\int_{r_0}^{\infty} \sin \omega s \; g(s)\, ds \qquad (4.36)$$

and we get a finite contribution to the integral (4.33), as can be seen by using Eq. (4.29), and by assuming that the contribution to g(s) from the region $s \gtrsim r^{-1}$, r'^{-1} is negligible. Hence (4.33) does not have any infra-red divergence.

Next we have to show that the contribution to (4.33) from the region of integration $\omega >> r^{-1}$, r'^{-1} is negligible. For this, we focus our attention on the integral,

$$\int_0^{\infty} \frac{d\omega}{4\pi\omega} \left\{ \int_{r_0}^{\infty} g(s)\, f_\omega(s)\, ds \right\}^2 \qquad (4.37)$$

We shall now show that it is possible to choose g(s) so that contribution to the above integral from the region $\omega \gg r^{-1}, r'^{-1}$ is negligible. One such choice for g(s) is,

$$g(s) = -\frac{N}{s_0} \frac{(s-r_0)^2}{s^2} e^{-(s-r_0)^2/2s_0^2} \tag{4.38}$$

where N is a normalization constant of order unity, and s_0 is some length small compared to g/m_W. g(s) given in (4.38) satisfies equations (4.29) and (4.30) approximately. The term $(s-r_0)^2/s^2$ in g(s) guarantees that g(s) and its derivative vanishes at $r=r_0$. This is choosen to avoid spurious divergences in the ω integral from sharp cut-offs of g(s) at the boundary.

We shall first consider the region $E \ll \omega \ll m_W/g$. In this region $f_\omega(s)$ is of order ωs, and hence,

$$\int f_\omega(s)\, g(s)\, ds \sim \omega s_0 \tag{4.39}$$

Thus the contribution to (4.37) from this region of ω integration is of order,

$$s_0^2 \int_E^{m_W/g} \omega\, d\omega \sim m_W^2 s_0^2/g^2 \ll 1$$
$$\text{for} \quad s_0 \ll g/m_W \tag{4.40}$$

In the region $\omega \gg m_W/g$, $f_\omega(s)$ may be shown to be proportional to,

$$\sqrt{\frac{\pi m_w}{g}}\; s^{1/2}\; \sin\left(\frac{\omega g}{\pi m_w}\ln s + \theta(\omega)\right) \tag{4.41}$$

for $s \ll m_w^{-1} g$. Here $\theta(\omega)$ is a phase angle to be determined from the boundary condition at r_0. Using (4.38) and (4.41) we may show that,

$$\int f_\omega(s)\, g(s)\, ds$$

$$\lesssim \sqrt{\frac{\pi m_w}{g}}\; s_0^{1/2}\; \frac{\pi m_w}{\omega g} \tag{4.42}$$

and hence the contribution to (4.37) from this region of integration is of order,

$$\frac{\pi^3 m_w^3}{g^3}\, s_0 \int_{m_w/g}^{\infty} \frac{d\omega}{\omega^3} \sim \frac{m_w s_0}{g} \ll 1 . \tag{4.43}$$

Similarly, one can also show that the contribution to the integral from the region $\omega \sim m_w/g$ is also small. It may be easily seen that the contribution to the integral (4.33) from the cross terms is also negligible from the $\omega \gg E$ region. Hence the integral (4.33) is both, ultraviolet, and infrared finite. Besides showing the finiteness of the Green's function (4.32), this analysis again proves the self-consistency of the model, since the final result is insensitive to the high frequency modes.

Finally, we shall comment on the case,

$$\frac{g^2}{m_w^2 r} << r_0 << m_w^{-1} \tag{4.44}$$

Here r^{-1} is of the order of the energy of the external soliton. In this case, in the region $r^{-1} << \omega << (m_w^2 r_0/g^2)$, $f_\omega(s)$ in (4.33) may be replaced by 2, assuming the form (4.38) for $g(s)$. $f_\omega(r)$, on the other hand, may be approximated by $2\cos\omega r$. Thus the contribution to (4.33) from this region may be written as,

$$\int_{r^{-1}}^{(m_w^2 r_0/g^2)} \frac{d\omega}{4\pi\omega}\, 2\,(\cos\omega r - 1)(\cos\omega r' - 1)$$

$$\sim \frac{1}{2\pi} \ln\left(\frac{m_w^2 r_0 r}{g^2}\right) \tag{4.45}$$

One may easily check that this contribution appears with a negative sign in the exponent, hence in the region (4.44), S_3 violating condensates are suppressed by powers of $g^2/(m_w^2 r_0 r) \sim (g^2 E/m_w^2 r_0)$, as expected from the classical analysis.

V. SUMMARY AND DISCUSSIONS

The lesson that we learn from the analysis of the previous sections is that for small enough monopole radius, the gauge charge S_b in our model is conserved in the monopole fermion interaction, whereas the anomalous charge S_3 is necessarily violated. When extrapolated to the case of the real world, this means that in the monopole fermion scattering the weak hypercharge is conserved, and the baryon number is violated. For those monopoles, whose magnetic charge coincides with the lowest charge SU(5) monopole, the baryon number violation is a necessary consequence of the conservation of the weak hypercharge. We however expect the phenomenon of anomaly induced baryon number violation to be present for more general class of monopoles. In the presence of any magnetic monopole, whose magnetic field has an electromagnetic component, the baryon number becomes anomalous through a triangle diagram with one vertex coupled to the magnetic field of the monopole, one vertex to the weak Z^0 field, and the third vertex to the baryon number current. Thus around these monopoles, we expect the presence of baryon number violating condensates, unsuppressed by any power of m_W^{-1}, r_0, or coupling constant.

Although our model was based on 't Hooft-Polyakov monopoles, the results are sensitive to the internal structure of the monopole core only through the boundary condition on the fields at the core radius. We have choosen the most pessimistic boundary condition from the point of view of the non-conservation of the charge S_3, since the boundary conditions ensure that all contribution to $\dot{S}_3$ from the boundary terms vanish. Thus we may expect the baryon number violation due to weak anomaly to be a general effect, even for non-grand-unified monopoles (e.g. Kaluza-Klein monopoles[12]), so long as the internal dynamics of the monopole core may be summarized by boundary conditions on the fields at the core radius r_0, and r_0 is small compared to $(m_w^2/E\ \alpha_{weak})^{-1}$. Here r_0 is defined to be a length scale such that outside the radius r_0, the monopole magnetic field coincides with that of a pure Dirac monopole with appropriate magnetic charge. This effect is particularly interesting for Kaluza-Klein monopoles, since it has been argued recently[13] that such monopoles do not catalyze baryon number violation due to boundary conditions[F2].

In the presence of more than one generation of massless fermions, we still get baryon number violating condensates that are not suppressed by any power of weak scale, coupling constant, or the monopole radius. But the precise nature of the condensates will depend on the effective boundary conditions on the fermionic fields. In some cases, the condensates may carry more than one unit of baryon number, and hence may not contribute to the proton decay amplitude, although they may contribute to the decay of heavy nuclei. This happens if, for example, the boundary conditions conserve the baryon number carried by each generation seperately, then the difference between the baryon number carried by particles of generation i and that of generation j is anomaly free, and will be conserved. Also in this case the baryon number violating condensates necessarily involve heavy quarks[6], which may affect the potentiality of such condensates to catalyze nucleon decay, possibly through a mixing angle suppression[7], or even by some power of m_W, if the heavy quarks appear as intermediate states in the scattering, and eventually decay to light quarks through W boson exchange[6]. These suppression factors, however, come from purely kinematic reasons, (for example, if all the quarks and leptons were light enough for the proton to decay into them, such suppression factors would be absent), and does not affect the main conclusion of the paper, that there exists baryon number violating condensates around the monopole up to the strong interaction length scale, unsuppressed by any power of the coupling constant, monopole radius, or weak scale.

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APPENDIX A

In this appendix we shall give the details of the bosonization procedure for our model. As can be seen from Eqs. (2.5), (2.15) and (2.19) of the text, and the restriction of the fields to J=0 partial wave, the effective action of the fermion-gauge field-higgs system is given by,

$$
\begin{aligned}
\int 4\pi r^2\, dr\, dt \Big[& \tfrac{1}{2} \{ \dot b_z^2 - \dot b_0^2 + (b_0')^2 - m_w^2 b_z^2 \\
& + m_w^2 b_0^2 + m_w^{-2} (\dot b_z')^2 - m_w^{-2} (b_z')^2 \} \\
& + i \sum_{j=1}^{2} \sum_{\eta=\uparrow\downarrow} \bar\Psi_{j\eta} \not\partial \Psi_{j\eta} \\
& + \frac{g}{2} \{ b_0 (\bar\Psi_{1\uparrow} \gamma^0 \Psi_{1\uparrow} - \bar\Psi_{1\downarrow} \gamma^0 \Psi_{1\downarrow} - \bar\Psi_{2\uparrow} \gamma^0 \Psi_{2\uparrow} + \bar\Psi_{2\downarrow} \gamma^0 \Psi_{2\downarrow}) \\
& - b_z (\bar\Psi_{1\uparrow} \hat z \cdot \vec\gamma \Psi_{1\uparrow} - \bar\Psi_{1\downarrow} \hat z \cdot \vec\gamma \Psi_{1\downarrow} - \bar\Psi_{2\uparrow} \hat z \cdot \vec\gamma \Psi_{2\uparrow} + \bar\Psi_{2\downarrow} \hat z \cdot \vec\gamma \Psi_{2\downarrow}) \} \Big]
\end{aligned}
\tag{A.1}
$$

where we have omitted the σ_1 field, since it decouples from the rest of the fields. Also, for simplicity, we have ignored the unbroken gauge interaction responsible for the Coulomb term in (2.1), this may be treated in the same way as in Ref.2. We now define locally gauge invariant fermion fields following Ref. 14,

$$
\Psi_N(r,t) = e^{i \frac{g}{2} \int^{r} b_z(r',t)\, dr'} \Psi(r,t) \tag{A.2}
$$

where $\xi=+1$ for $\psi_{1\uparrow}$ and $\psi_{2\downarrow}$ and $\xi=-1$ for $\psi_{1\downarrow}$ and $\psi_{2\uparrow}$[15]. In terms of these fields, the fermionic part of the Lagrangian density may be written as,

$$i\sum_{j=1}^{2}\left(\bar{\psi}_{Nj\uparrow}\,\partial\!\!\!/\,\psi_{Nj\uparrow}+\bar{\psi}_{Nj\downarrow}\,\partial\!\!\!/\,\psi_{Nj\downarrow}\right)$$

$$+g\left(b_0+\int_{r_0}^{r} b_r\,dr\right)\left(\bar{\psi}_{N1\uparrow}\gamma^0\psi_{N1\uparrow}-\bar{\psi}_{N1\downarrow}\gamma^0\psi_{N1\downarrow}\right.$$

$$\left.-\bar{\psi}_{N2\uparrow}\gamma^0\psi_{N2\uparrow}+\bar{\psi}_{N2\downarrow}\gamma^0\psi_{N2\downarrow}\right) \qquad (A.3)$$

Let us denote the part of the Lagrangian density given in (A.3) by $L_f(\psi_N, b)$, and the rest of the Lagrangian density in (A.1), which involves the b fields only, by $L_g(b)$. A Green's function involving the fields ψ_N is then given by,

$$\int [db]\,[d\psi_N]\,[d\bar{\psi}_N]\; f(\psi_N)\; e^{i\int\{\mathcal{L}_g(b)+\mathcal{L}_f(\psi_N,b)\}4\pi r^2\,dr\,dt} \qquad (A.4)$$

where $f(\psi_N)$ denotes the product of the ψ_N and $\bar{\psi}_N$ fields whose vev is of interest. (A.4) may be written as,

$$\int [db]\; e^{i\int \mathcal{L}_g(b)\,4\pi r^2\,dr\,dt}\; G(b) \qquad (A.5)$$

where,

$$G(b)=\int [d\psi_N]\,[d\bar{\psi}_N]\; f(\psi_N)\; e^{i\int \mathcal{L}_f(\psi_N,b)\,4\pi r^2\,dr\,dt} \qquad (A.6)$$

i.e. $G(b)$ is the Green's function involving the ψ fields with the Lagrangian L_F^{F3}. In calculating $G(b)$, we must take the fields b to be a fixed background field.

In order to calculate $G(b)$, we first find the Hamiltonian of the fermionic system described by the Lagrangian L_F, for a fixed background field b. This is given by,

$$H = \int 4\pi r^2 dr \Big[\sum_{j=1}^{2} \bar{\psi}_{Nj} \left(-i(\hat{r})^i \gamma^i\right) \partial_r \psi_{Nj}$$

$$- g\Big(\int_{r_0}^{r} b_r\, dr + b_0\Big) \big(\bar{\psi}_{N1\uparrow} \gamma^0 \psi_{N1\uparrow} - \bar{\psi}_{N1\downarrow} \gamma^0 \psi_{N1\downarrow}$$

$$- \bar{\psi}_{N2\uparrow} \gamma^0 \psi_{N2\uparrow} + \bar{\psi}_{N2\downarrow} \gamma^0 \psi_{N2\downarrow} \big)$$

$$\equiv H_{free} + H_{int.} \qquad \text{(A.7)}$$

We may now bosonize the above Hamiltonian exactly as in Ref.2. We introduce four boson fields Φ_1, Φ_2, Q_1, Q_2. In the interaction picture, each of the fields ψ_N may be expressed as a function of these boson fields and their time derivatives, which we write as $\psi_N(\Phi_i, Q_i, \dot{\Phi}_i, \dot{Q}_i)$. The various fermionic currents are given by,

$$\bar{\psi}_{Ni\uparrow} \gamma^0 \psi_{Ni\uparrow} = \Phi_i' / (4\pi\sqrt{\pi}\, r^2)$$

$$\bar{\psi}_{Ni\downarrow} \gamma^0 \psi_{Ni\downarrow} = -Q_i' / (4\pi\sqrt{\pi}\, r^2)$$

$$\bar{\psi}_{Ni\uparrow} \gamma^0 \gamma^5 \psi_{Ni\uparrow} = \dot{\Phi}_i / (4\pi\sqrt{\pi}\, r^2)$$

$$\bar{\psi}_{Ni\downarrow} \gamma^0 \gamma^5 \psi_{Ni\downarrow} = \dot{Q}_i / (4\pi\sqrt{\pi}\, r^2)$$

$$\sum_{k=1}^{3} \overline{\psi}_{Ni\uparrow} (\hat{r})^k \gamma^k \psi_{Ni\uparrow} = -\Phi_i/(4\pi\sqrt{\pi}\, r^2)$$

$$\sum_{k=1}^{3} \overline{\psi}_{Ni\downarrow} (\hat{r})^k \gamma^k \psi_{Ni\downarrow} = Q_i/(4\pi\sqrt{\pi}\, r^2)$$

$$\sum_{k=1}^{3} \overline{\psi}_{Ni\uparrow} (\hat{r})^k \gamma^k \gamma^5 \psi_{Ni\downarrow} = -\Phi_i'/(4\pi\sqrt{\pi}\, r^2)$$

$$\sum_{k=1}^{3} \overline{\psi}_{Ni\downarrow} (\hat{r})^k \gamma^k \gamma^5 \psi_{Ni\downarrow} = -Q_i'/(4\pi\sqrt{\pi}\, r^2) \qquad (A.8)$$

The effective boson Hamiltonian in the interaction picture is the sum of a free Hamiltonian and an interaction Hamiltonian, which we write as $(H_{free})_{IP}$ and $(H_{int.})_{IP}$ respectively. These are given by,

$$(H_{free})_{IP} = \frac{1}{2}\int_{r_0}^{\infty} dr \sum_{i=1}^{2} (\dot{\Phi}_i^2 + \dot{Q}_i^2 + \Phi_i'^2 + Q_i'^2) \qquad (A.9)$$

$$(H_{int.})_{IP} = -g\int_{r_0}^{\infty} dr \,[\,(b_0 + \int_{r_0}^{r} b_2(r',t)\, dr')(\Phi_1' - Q_1' - \Phi_2' - Q_2')/\sqrt{\pi}$$

$$(A.10)$$

The boundary conditions on the boson fields are given by Eq.(2.2). Let Π_i and P_i be the momenta conjugate to Φ_i and Q_i in the interaction picture. Equations of motion give,

$$\Pi_i = \dot{\Phi}_i \qquad P_i = \dot{Q}_i \qquad (A.11)$$

Let U be the unitary operator which takes various operators from the interaction picture to the Heisenberg picture. If the subscript H denotes operators in the Heisenberg picture, we have,

$$U \Phi_\lambda U^{-1} = \Phi_{\lambda H} \qquad U Q_\lambda U^{-1} = Q_{\lambda H}$$

$$U \Pi_\lambda U^{-1} = \Pi_{\lambda H} \qquad U P_\lambda U^{-1} = P_{\lambda H} \tag{A.12}$$

Thus,

$$(H)_H = U (H)_{IP} U^{-1}$$

$$= \frac{1}{2} \int_{r_0}^{\infty} dr \sum_{\lambda=1}^{2} \left(\Pi_{\lambda H}^2 + P_{\lambda H}^2 + \Phi'^2_{\lambda H} + Q'^2_{\lambda H} \right)$$

$$- \frac{g}{\sqrt{\pi}} \int_{r_0}^{\infty} dr \left(b_0 + \int_{r_0}^{r} b_z(r',t)\, dr' \right) \left(\Phi'_{1H} + Q'_{1H} - \Phi'_{2H} - Q'_{2H} \right) \tag{A.13}$$

The b fields remain unchanged under this transformation, since they are just c number functions. The field ψ_N in this picture is given by,

$$U \psi_N (\Pi_\lambda, P_\lambda, \Phi_\lambda, Q_\lambda) U^{-1}$$

$$= \psi_N (\Pi_{\lambda H}, P_{\lambda H}, \Phi_{\lambda H}, Q_{\lambda H})$$

$$= \psi_N (\dot{\Phi}_{\lambda H}, \dot{Q}_{\lambda H}, \Phi_{\lambda H}, Q_{\lambda H}) \tag{A.14}$$

since the equations of motion give,

$$\dot{\Phi}_{\lambda H} = \partial (H)_H / \partial \Pi_{\lambda H} = \Pi_{\lambda H} \qquad \dot{Q}_{\lambda H} = \partial (H)_H / \partial P_{\lambda H} = P_{\lambda H} \tag{A.15}$$

We may now go from the Hamiltonian to the Lagrangian picture. Dropping the subscript H from the fields, we get the Lagrangian density as,

$$\mathcal{L}_f(\Phi_\lambda, Q_\lambda, b) = \Big[\frac{1}{2}\sum_{\lambda=1}^{2}\{\dot{\Phi}_\lambda^2 + \dot{Q}_\lambda^2 - \Phi_\lambda'^2 - Q_\lambda'^2\}$$

$$+ \frac{g}{\sqrt{\pi}}(b_x' + \dot{b}_x)(\Phi_1 + Q_1 - \Phi_2 - Q_2) \qquad (A.16)$$

the b field still being treated as a classsical background field. In deriving (A.16), we have done an integration by parts. The Green's function G(b) is then given by,

$$G(b) = \int [d\Phi_\lambda][dQ_\lambda]\, e^{i\int \mathcal{L}_f(\Phi_\lambda, Q_\lambda, b)\,dx\,dt}$$

$$\times f(\psi_N(\dot{\Phi}_\lambda, \dot{Q}_\lambda, \Phi_\lambda, Q_\lambda)) \qquad (A.17)$$

Substituting (A.17) in Eq.(A.5), we see that the vacuum expectation value of the operator product $f(\psi_N)$ is given by,

$$\int [db][d\Phi_\lambda][dQ_\lambda]\, \exp\Big(i\int \mathcal{L}_{eff}(b, \Phi_\lambda, Q_\lambda)\,dx\,dt\Big)$$

$$\times f(\psi_N(\dot{\Phi}_\lambda, \dot{Q}_\lambda, \Phi_\lambda, Q_\lambda)) \qquad (A.18)$$

where,

$$\mathcal{L}_{eff}(b, \Phi_i, Q_i) = \frac{4\pi}{g^2}\Big[\dot{b}_r^2 - \dot{b}_0^2 + (b_0')^2 - m_w^2 b_r^2$$
$$+ m_w^2 b_0^2 + m_w^{-2}(\ddot{b}_r)^2 - m_w^{-2}(\dot{b}_r')^2\Big]$$
$$+ \frac{1}{2}\sum_{i=1}^{2}\left(\dot{\Phi}_i^2 + \dot{Q}_i^2 - \Phi_i'^2 - Q_i'^2\right)$$
$$+ \frac{g}{\sqrt{\pi}}\left(b_0' + \dot{b}_r\right)\left(\Phi_1 + Q_1 - \Phi_2 - Q_2\right) \qquad (A.19)$$

This is the effective Lagrangian involving the boson fields Φ_i, Q_i and the fields b_r, b_0. This, together with Eq.(2.18), gives us the effective Lagrangian (2.20).

APPENDIX B

In this appendix we shall estimate a bound on the contribution to the effective Hamiltonian from the b_0 field. The net contribution to the effective Hamiltonian is given by,

$$\frac{1}{2}\int \{(b_0')^2 + m_w^2 b_0^2\} \, 4\pi r^2 \, dr \tag{B.1}$$

where,

$$b_0 = -\frac{g}{2\pi\sqrt{\pi}} \frac{1}{2 m_w r} \Big[e^{-m_w(r-r_0)} \int_{r_0}^{r} \frac{e^{m_w(r'-r_0)} - e^{-m_w(r'-r_0)}}{r'} \varphi'(r') \, dr' + \left(e^{m_w(r-r_0)} - e^{-m_w(r-r_0)}\right) \int_{r}^{\infty} \frac{e^{-m_w(r'-r_0)}}{r'} \varphi'(r') \, dr' \Big] \tag{B.2}$$

Since $\phi'(r')$ vanishes at the origin due to the boundary conditions, it is reasonable to assume that $\phi'(r')$ does not blow up anywhere, and is hence bounded from above by a term of order E.[F4] Then, for $r \lesssim m_w^{-1}$, b_0 is of order,

$$\frac{gE}{m_w r} \, m_w r \, \ln(m_w^{-1}/r) \tag{B.3}$$

For $r \gtrsim m_w^{-1}$, b_0 is of order,

$$\frac{gE}{m_w r} (m_w r)^{-1} \tag{B.4}$$

Thus the contribution to the term $\int 2\pi r^2 m_W{}^2 b_0{}^2\, dr$ is of order,

$$C\, m_w{}^2 E^2 g^2 \int_{r_0}^{m_w^{-1}} r^2 \ln^2(m_w^{-1}/r)\, dr + D\, \frac{g^2 E^2}{m_w{}^2} \int_{m_w^{-1}}^{\infty} \frac{dr}{r^2}$$

$$\sim g^2 E^2 / m_w \qquad (B.5)$$

C and D being two constants of order unity.

Next, we must estimate the contribution from the b_0' term. From Eq. (B.2) we get,

$$b_0' = -\frac{b_0}{r} - \frac{g}{2\pi\sqrt{\pi}}\, \frac{1}{2 m_w r} \Big[-m_w\, e^{-m_w(r-r_0)} \times \int_{r_0}^{r} \frac{e^{m_w(r'-r_0)} - e^{-m_w(r'-r_0)}}{r'}\, \varphi'(r')\, dr' + m_w \left(e^{m_w(r-r_0)} + e^{-m_w(r-r_0)} \right) \int_{r}^{\infty} \frac{e^{-m_w(r'-r_0)}}{r'}\, \varphi'(r')\, dr' \Big] \qquad (B.6)$$

For $r \lesssim m_W{}^{-1}$, we have,

$$b_0' \leq \frac{gE}{2 m_w r}\, m_w \ln(m_w^{-1}/r) \qquad (B.7)$$

For $r \gtrsim m_W{}^{-1}$,

$$b_0' \lesssim gE/(m_w r^2) \qquad (B.8)$$

We see from (B.7) and (B.8) that the contribution from the term $\int 2\pi r^2 \, (b_0')^2 \, dr$ is of order E^2/m_w. Thus we see that the net contribution from (B.1) is of order E^2/m_w, which is small compared to the energy E of the external soliton. Thus we may neglect these terms while discussing the dynamics of the system.

This may also be seen in the following way. Let $f_\omega(r)$ be the mode of frequency ω for the ϕ field. Then, with the Hamiltonian for the ϕ field given by the sum of (3.22) and (3.41), the equation for $f_\omega(r)$ is given by,

$$\omega^2 \left(1 + \frac{g^2}{\pi^2 m_w^2 r^2}\right) f_\omega(r) + f_\omega''(r) = \frac{2g}{\sqrt{\pi}} b_0' \qquad \text{(B.9)}$$

with b_0 given by,

$$-\frac{g}{2\pi\sqrt{\pi}} \int_{r_0}^{\infty} G(r, r') \, f_\omega'(r') \, dr' \qquad \text{(B.10)}$$

Thus,

$$f_\omega'(r) = -\int_{r_0}^{r} \omega^2 \left(1 + \frac{g^2}{\pi^2 m_w^2 r^2}\right) f_\omega(r) + \frac{2g}{\sqrt{\pi}} b_0(r) \qquad \text{(B.11)}$$

using the boundary conditions on $b_0(r)$ and $f_\omega(r)$ at r_0. Let us now choose r in the region $m_w^{-1} \ll r \ll \omega^{-1}$. In this region $b_0(r)$ may be set to zero up to correction terms of order $1/m_w$. Let us define $c = f_\omega(r_0)$. We then get, from (B.11) ,

$$f_\omega'(r) \sim \omega^2 g^2 c / (m_w^2 r_0) \qquad \text{(B.12)}$$

For $r \gg m_W^{-1}$, $f_\omega(r)$ reduces to a linear combination of $\sin\omega r$ and $\cos\omega r$, since the contribution from the b_0' term, as well as the $(\omega g/\pi m_W r)^2 f_\omega$ term drops out from Eq. (B.9). Proper normalization then demands that,

$$|f_\omega(r)|^2 + \omega^{-2}\,|f_\omega'(r)|^2 = 4 \qquad (B.13)$$

$f_\omega(s)$ reduces to $2\sin\omega s$ for $s \gg m_W^{-1}$ if,

$$|\omega f_\omega(r)| << |f_\omega'(r)| \quad \text{for} \quad m_w^{-1} << r << \omega^{-1} \qquad (B.14)$$

whereas it reduces to $2\cos\omega s$ if,

$$|f_\omega'(r)| << |\omega f_\omega(r)| \quad \text{for} \quad m_w^{-1} << r << \omega^{-1} \qquad (B.15)$$

First, let us consider the region,

$$\omega << \pi^2 m_w^2 r_0 / g^2 \qquad (B.16)$$

Then, from (B.12), we see that

$$|f_\omega'(r)| \quad << c\,\omega \qquad (B.17)$$

$f_\omega(r)$, on the other hand, is of order c, since $f_\omega(r_0)$ is of order c. Hence the inequality (B.15) is satisfied, and $f_\omega(s)$ reduces to $2\cos\omega s$ for $s >> m_w^{-1}$.

On the other hand, in the region,

$$\omega >> \pi^2 m_w^2 r_0 / g^2 \tag{B.18}$$

(B.12) gives,

$$|f_\omega'(r)| >> c\omega \tag{B.19}$$

$f_\omega(r)$, on the other hand, is bounded by the maximum of c or $rf_\omega'(r)$, both of which are smaller than $\omega^{-1}f_\omega'(r)$ for $r << \omega^{-1}$. Hence in this case $f_\omega(s)$ reduces to $2\sin\omega s$. It is clear that for $\omega \sim \pi^2 m_w^2 r_0/g^2$, $f_\omega(s)$ reduces to a linear combination of $\sin\omega s$ and $\cos\omega s$.

These forms of $f_\omega(s)$ are identical to the ones obtained in Sec.IV from the Hamiltonian (3.22). Hence we may conclude that the Hamiltonian (3.22) is a good approximation to the full Hamiltonian, and the contribution from the part of the Hamiltonian given in (3.41) may be ignored, so long as our calculation involves only modes of frequency small compared to m_w.

FOOTNOTES

[F1]As was pointed out in Ref.14, these operators create fermion fields at point r, and an equal and opposite S_b charge at the monopole core. This may be avoided by taking the integrals in the exponential in (A.2) from ∞ to r, instead of from r_0 to r. But so long as Green's function under consideration involves products of operators at equal times, and the total S_b charge carried by all the operators in the product is zero, the choice (A.2) for ψ_N gives the same result as the case when we take the integrals from ∞ to r.

[F2]In the case of a Kaluza-Klein monopole, the Dirac equations do not allow the fermions to reach the monopole core. However, it has been argued by Nelson that this peculiar feature is due to the presence of a long range Brans-Dicke scalar field, which, presumably, is cut off at some length scale due to quantum effects. Beyond this radius, we recover the usual monopole-fermion dynamics. All our analysis may then be reproduced, taking the monopole radius r_0 to be the scale at which the long range scalar field is cut off.

[F3]This identification is correct up to a normalization factor, which cancels at the end.

[F4]This condition is satisfied, for example, by the modes of frequency $\lesssim$ E for the effective Hamiltonian (3.22) of the text.

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FIGURE CAPTIONS

Fig.1. (a) An incoming ϕ soliton.

(b) The outgoing ϕ soliton when S_3 is conserved.

(c) The outgoing ϕ soliton when S_b is conserved.

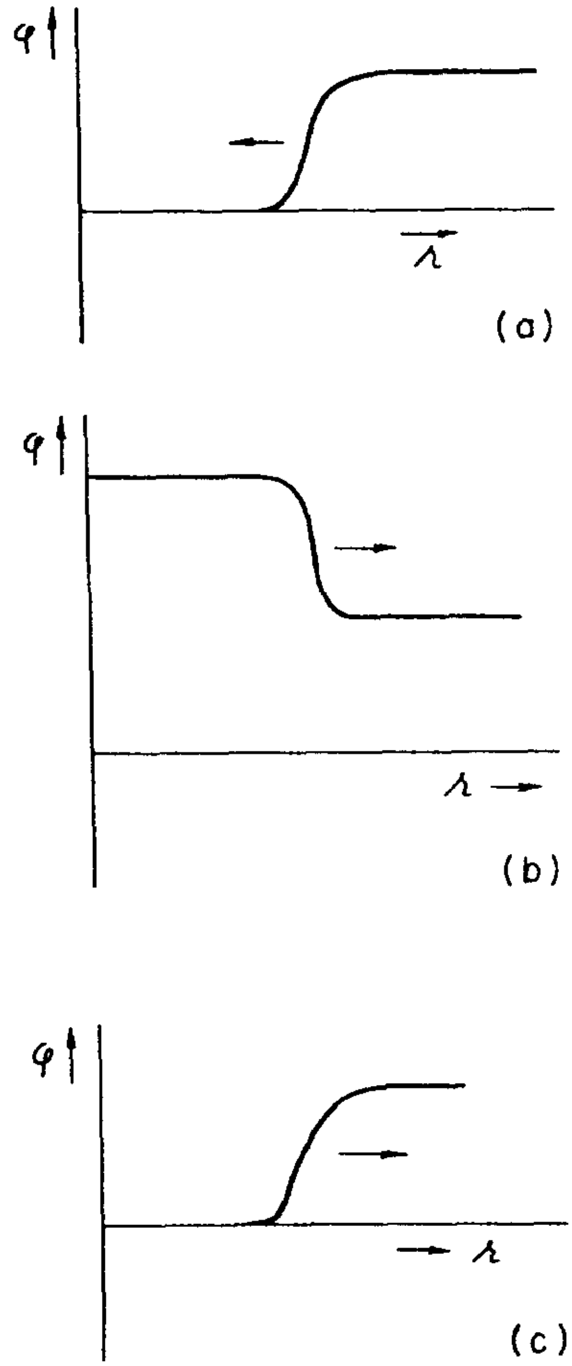

FIG. 1