

MIT-CTP-2240  
 TIFR-TH-93-38  
 hep-th/9309027  
 August 1993

# A NOTE ON GAUGE TRANSFORMATIONS IN BATALIN-VILKOVISKY THEORY

ASHOKE SEN<sup>\*</sup>

*Tata Institute of Fundamental Research  
 Homi Bhabha Road, Bombay 400005, India*

and

BARTON ZWIEBACH<sup>†</sup>

*Center for Theoretical Physics  
 LNS and Department of Physics  
 MIT, Cambridge, MA 02139, U.S.A.*

## ABSTRACT

We give a generally covariant description, in the sense of symplectic geometry, of gauge transformations in Batalin-Vilkovisky quantization. Gauge transformations exist not only at the classical level, but also at the quantum level, where they leave the action-weighted measure  $d\mu_S \equiv d\mu e^{2S/\hbar}$  invariant. The quantum gauge transformations and their Lie algebra are  $\hbar$ -deformations of the classical gauge transformation and their Lie algebra. The corresponding Lie brackets  $[ , ]^q$ , and  $[ , ]^c$ , are constructed in terms of the symplectic structure and the measure  $d\mu_S$ . We discuss closed string field theory as an application.

---

<sup>\*</sup> E-mail address: sen@tifrvax.tifr.res.in, sen@tifrvax.bitnet

<sup>†</sup> E-mail address: zwiebach@irene.mit.edu, zwiebach@mitlns.bitnet.

Supported in part by D.O.E. contract DE-AC02-76ER03069.

Introduction In the antibracket, or Batalin-Vilkovisky (BV) formalism, the master action has long been known to determine not only the BRST transformations but also the gauge transformations. Indeed, as explained in the original paper of Batalin and Vilkovisky [ 1], and elaborated upon in a recent monograph (Ref.[ 2], §17.4.2), the gauge transformations of a field (or antifield)  $\Phi^i$  are

$$\delta\Phi^i = (\omega^{ij}\partial_j\partial_k^r S)\Lambda^k, \quad (1)$$

where  $\omega$  is the symplectic form,  $S$  is the classical master action, and  $\Lambda^k$  are field/antifield independent parameters of local gauge transformations with statistics  $(-)^{k+1}$ .<sup>‡</sup> This result, however, is not completely general. In addition to leaving only the classical master action invariant, the above formula is not covariant under a change of basis; it requires the use of Darboux coordinates which make the components  $\omega^{ij}$  of the symplectic form constant. More seriously, when  $\Lambda^k$  is field/antifield dependent, the above transformations do not generally leave the symplectic form invariant, and therefore they do not qualify as true gauge transformations or true invariances (we recall that in BV quantization the physics is determined by the action *and* the symplectic structure). This is in contrast to ordinary gauge theory where gauge parameters can be chosen to be field dependent, in addition to being spacetime dependent. If the  $\Lambda^k$ 's are field/antifield independent we find

$$\delta\Phi^i = \omega^{ij}\partial_j\left((\partial_k^r S)\Lambda^k\right) = \{ \Phi^i, (\partial_k^r S)\Lambda^k \} \quad (2)$$

showing that the gauge transformations are canonical transformations, and are generated by the hamiltonian  $K = (\partial_k^r S)\Lambda^k$ . This form of the gauge transformations has been widely used since most field theories have been formulated using Darboux coordinates.

---

<sup>‡</sup>  $\partial_j$  and  $\partial_j^r$  stand for  $\frac{\partial^l}{\partial\Phi^j}$  and  $\frac{\partial^r}{\partial\Phi^j}$  respectively, where the superscripts  $l$  and  $r$  denote left and right derivatives.

The antibracket formalism has been recently formulated covariantly in the sense of symplectic geometry. While such covariant description was known for the classical part of the formalism, the quantum part required the introduction of extra geometrical structure [ 3]. In the covariant formalism we need not use Darboux coordinates, and should be able to give the form of the gauge transformations when the  $\omega^{ij}$ 's are not constants. The relevant formula was given in [ 4]

$$\delta' \Phi^i = (\omega^{ij} \partial_j \partial_k^r S) \Lambda^k + \frac{1}{2} (\partial_k^r \omega^{ij}) \Lambda^k \partial_j^r S, \quad (3)$$

giving a variation of the form  $\delta' S = \frac{1}{2} (\partial_k^r \{S, S\}) \Lambda^k$ , which vanishes on account of the classical master equation. While (3) gives an invariance of the classical master action, it does not necessarily leave the antibracket invariant. We do not have, therefore, a parametrization of the allowed gauge transformations. Moreover, with second order partial derivatives, and partial derivatives of the components of the symplectic form, Eqn.(3) is noncovariant.

In this paper we shall (i) write down the classical gauge transformations and their Lie algebra (with the associated Lie bracket  $[ , ]^c$ ) in the general case when the gauge transformation parameters may be field dependent, and, (ii) generalize this to the full quantum theory, where we find a Lie algebra of quantum gauge transformations (with the associated Lie bracket  $[ , ]^q$ ). The proper geometrical interpretation of the gauge parameters  $\Lambda^k$  is seen to be that of hamiltonian vectors arising from some hamiltonian  $\Lambda$ . In our picture, the gauge parameters are taken to be the hamiltonians, which are necessarily field/antifield dependent functions. Standard gauge transformations arise from hamiltonians linear in the fields (or antifields); by including all possible field/antifield dependent hamiltonians we are naturally led to Lie algebras. The construction is fully covariant and involves only the symplectic structure and the action weighted measure  $d\mu_S \equiv d\mu e^{2S/\hbar}$ , left invariant by the quantum gauge transformations. We were led to consider this measure and the notion of quantum gauge transformations in our study of background independence of quantum closed string field theory [ 5].

A path integral measure In the covariant description of the antibracket formalism a measure  $d\mu$  in the space of field/antifield configurations is necessary. This measure is used to define the operator  $\Delta_{d\mu}$ . If  $d\mu = f(\Phi) \prod_i d\Phi^i$ , then  $\Delta_{d\mu}A \equiv \frac{1}{2f}(-1)^i \partial_i (f \omega^{ij} \partial_j A)$ . For any *arbitrary* measure  $d\mu$  the following identities hold [3,6,7]

$$\Delta_{\rho d\mu}A = \Delta_{d\mu}A + \frac{1}{2} \{ \ln \rho, A \} . \quad (4)$$

$$\Delta_{d\mu}\{A, B\} = \{\Delta_{d\mu}A, B\} + (-)^{A+1}\{A, \Delta_{d\mu}B\} . \quad (5)$$

Other identities we will use are the exchange property  $\{A, B\} = (-)^{AB+A+B}\{B, A\}$ , and the Jacobi identity  $(-)^{(A+1)(C+1)}\{A, \{B, C\}\} + \text{cyclic} = 0$ .

A volume element  $d\mu$  is consistent if  $\Delta_{d\mu}^2 = 0$ . Assume we have a consistent volume element  $d\mu$ , and consider the measure  $d\mu_S \equiv d\mu e^{2S/\hbar}$ . The associated delta operator, making use of (4), is found to be

$$\Delta_{d\mu_S} = \Delta_{d\mu} + \frac{1}{\hbar} \{S, \cdot\} . \quad (6)$$

Following [3] one can then show that  $\Delta_{d\mu_S}^2 = \frac{1}{\hbar^2} \{ \hbar \Delta_{d\mu} S + \frac{1}{2} \{S, S\}, \cdot \}$ , which indicates that  $\Delta_{d\mu_S}^2$  is a linear operator, in fact, a hamiltonian vector. This equation also shows that  $d\mu_S$  is a consistent measure ( $\Delta_{d\mu_S}^2 = 0$ ) if  $d\mu$  is consistent, and, in addition,  $S$  satisfies the quantum master equation:  $\frac{1}{2}\{S, S\} + \hbar \Delta_{d\mu} S = 0$ . The operator  $\Delta_{d\mu_S}$  coincides with the operator  $\sigma$  discussed in [2], here we have only pointed out its geometrical interpretation as the delta operator of the particularly relevant measure  $d\mu_S$ . The fact that the master equation can be encoded in the consistency condition for a measure was noticed in [8] and is implicit in [3]. We observe now that the measure  $d\mu_S$  is a rather fundamental object in the BV formalism. There is no need that this measure should be written in terms of another consistent measure  $d\mu$  and a nontrivial function  $S$  satisfying the quantum master equation. To argue this we must look at the definition of observables.

The observables in a theory are defined by  $\langle A \rangle \equiv \int_L d\lambda_S A$ , where  $L$  denotes a Lagrangian submanifold, defined by the condition that at any point  $p \in L$ , for any two tangent vectors  $e_i, e_j \in T_p L$ , we have  $\omega(e_i, e_j) = 0$ . The measure  $d\lambda_S \equiv d\lambda e^{S/\hbar}$ , can be defined directly in terms of  $d\mu_S$  using the same prescription that gives us  $d\lambda$  in terms of  $d\mu$  [ 3]. Let  $p \in L$ , and  $(e_1, \dots, e_n)$  be a basis of  $T_p L$ . One then defines

$$d\lambda_S(e_1, \dots, e_n) \equiv [d\mu_S(e_1, \dots, e_n, f^1, \dots, f^n)]^{1/2} , \quad (7)$$

where the vectors  $f^i$  are any set of tangent vectors of the full manifold at  $p$  satisfying  $\omega(e_i, f^j) = \delta_i^j$ . This condition fixes the vectors  $f^j$  up to the transformation  $f^j \rightarrow f^j + C^{ji} e_i$ . The right hand side of (7), however, is invariant under this transformation, since it corresponds to a transformation of the complete basis  $(\{e_i\}; \{f^j\})$  by a matrix of unit superdeterminant. Eqn.(7) gives us the path integral measure of the gauge fixed theory in terms of  $\omega$  and  $d\mu_S$ . Finally, in order for  $\langle A \rangle$  to be independent of the choice of lagrangian submanifold,  $A$  must be a function of fields/antifields satisfying  $\hbar \Delta_{d\mu} A + \{S, A\} = 0 \rightarrow \Delta_{d\mu_S} A = 0$  (by Eqn.(6)). This condition defines physical operators in terms of  $d\mu_S$  and  $\omega$ .

Classical gauge transformations If the gauge transformations are to be symplectic they must be generated by a hamiltonian function. We should therefore have

$$\delta\Phi^i = \{\Phi^i, K\} , \quad (8)$$

for some suitable *odd* function  $K$ . The symmetries of the BV action are generated by  $K$ 's for which  $\delta S = \{S, K\} = 0$ . Since this is the condition for  $K$  to be a classical observable, to every local observable, we can associate a local gauge symmetry of the classical theory. A special class of solutions are given by the trivial observables

$$K = \{S, \Lambda\} , \quad (9)$$

since  $\{S, K\} = 0$ , by virtue of the Jacobi identity and the classical master equation. Here  $\Lambda$  is an even function of  $\Phi^i$ . As we will see, standard gauge transformations

arise from trivial observables  $K$ . We will discuss later, in the context of string theory, why nontrivial observables do not lead generically to standard gauge transformations.

Let us now see how we recover the gauge transformations in Eqn.(1). In a Darboux frame,  $\Lambda = \Phi^i \omega_{ik} \Lambda^k$ , with  $\Lambda^k$  constants, leads to  $K = (\partial_k^r S) \Lambda^k$ . Back in Eqn.(8) we then recover the original form of the gauge transformations. Note that, by construction,  $\Lambda^k$  is the hamiltonian vector associated to the hamiltonian  $\Lambda$ . We have therefore found that the original parameters of gauge transformations have the geometrical interpretation of hamiltonian vectors. It is also instructive to understand the way the transformations given in (3) fit into this description. Such transformations are easily rewritten as

$$\delta' \Phi^i = \{ \Phi^i, (\partial_k^r S) \Lambda^k \} - \omega^{ij} (\partial_j \Lambda^k) (\partial_k^r S) + \frac{1}{2} (\partial_k^r \omega^{ij}) \Lambda^k \partial_j^r S, \quad (10)$$

We now recall that a transformation of the type  $\delta_t \Phi^i = (\partial_j^r S) \mu^{ij}$  leaves  $S$  invariant in a trivial fashion if  $\mu^{ij} = (-)^{ij+1} \mu^{ji}$ , although it does not necessarily leave  $\omega$  invariant. One can verify that the second and third term in the above equation *do not* correspond in general to trivial transformations. However, if  $\Lambda^k$  is hamiltonian ( $\Lambda^k = \omega^{kj} \partial_j \Lambda$ ) they do. In this case we can prove that  $\delta' \Phi^i = \{ \Phi^i, (\partial_k^r S) \Lambda^k \} + (\partial_j^r S) \mu^{ij}$ , with  $\mu^{ij} = \frac{1}{2} (-)^i [\omega^{jk} \partial_k \Lambda^i + (-)^{(i+1)(j+1)} \omega^{ik} \partial_k \Lambda^j]$ . Therefore, when  $\Lambda^k$  is hamiltonian the  $\delta'$  transformations differ from gauge transformations by trivial transformations. Note, however, that the  $\delta'$  transformation still cannot be regarded as a genuine symmetry of the BV theory since it does not represent in general a canonical transformation.

Let us now compute the algebra of gauge transformations. Let  $\widehat{\delta}_K$  denote the canonical transformation generated by the odd function  $K$ :  $\widehat{\delta}_K f = \{f, K\}$ . Given that  $[\widehat{\delta}_{K_2}, \widehat{\delta}_{K_1}] = \widehat{\delta}_{\{K_1, K_2\}}$ , canonical transformations generated by observables form a Lie algebra, since we have  $\{S, \{K_1, K_2\}\} = 0$ , whenever  $K_1$  and

---

\* We use the Jacobi identity as well as the identities in Appendix B of [7].

$K_2$  are observables. While any local observable in the theory generates a symmetry, not much can be said about the algebra of non-trivial observables unless we know which specific theory we are studying. Hence we shall focus our attention on the Lie subalgebra whose elements are the canonical transformations generated by trivial observables  $K = \{S, \Lambda\}$ . Let  $\delta_\Lambda^c = \widehat{\delta}_{\{S, \Lambda\}}$  denote the classical gauge transformations induced by  $\Lambda$ , then  $[\delta_{\Lambda_2}^c, \delta_{\Lambda_1}^c] = [\widehat{\delta}_{\{S, \Lambda_2\}}, \widehat{\delta}_{\{S, \Lambda_1\}}] = \widehat{\delta}_{\{\{S, \Lambda_1\}, \{S, \Lambda_2\}\}}$ . Moreover, the Jacobi identity and the master equation imply that  $\{\{S, \Lambda_1\}, \{S, \Lambda_2\}\} = \frac{1}{2}\{S, \{\Lambda_1, \{S, \Lambda_2\}\}\} - \frac{1}{2}\{S, \{\Lambda_2, \{S, \Lambda_1\}\}\}$ . As a consequence we have that

$$[\delta_{\Lambda_2}^c, \delta_{\Lambda_1}^c] = \delta_{[\Lambda_1, \Lambda_2]^c}^c, \quad [\Lambda_1, \Lambda_2]^c \equiv \frac{1}{2}\{\Lambda_1, \{S, \Lambda_2\}\} - \frac{1}{2}\{\Lambda_2, \{S, \Lambda_1\}\}. \quad (11)$$

In contrast with the standard description of gauge transformations via generating sets of transformations which do not give a Lie algebra, and close only up to trivial symmetries (see [2]), our description of gauge transformations arising from hamiltonian functions  $\Lambda$  gives directly a closed Lie algebra. The commutator of two gauge transformations is a gauge transformation of the same type. The usual gauge transformations, arising from  $\Lambda$ 's that are only linear in fields or antifields, as expected, do not close among themselves in general. They close when supplemented with  $\Lambda$ 's having additional field/antifield dependence. The trivial identity  $[[\delta_{\Lambda_3}^c, \delta_{\Lambda_2}^c], \delta_{\Lambda_1}^c] + \text{cyclic} = 0$ , implies that  $\delta_{([\Lambda_1, [\Lambda_2, \Lambda_3]^c]^c + \text{cyclic})} = 0$ , and, as a consequence  $([\Lambda_1, [\Lambda_2, \Lambda_3]^c]^c + \text{cyclic})$  must be either zero or a gauge parameter that generates no gauge transformation. The latter is true, the Jacobi identity for  $[\ , ]^c$ , calculated using (11) gives a result of the form  $\{S, \chi\}$ , which, as a gauge parameter, generates no gauge transformation.

The above observations indicate that  $[\ , ]^c$  leads to a *strict Lie bracket* in the space of even functions  $\Lambda$  modulo functions of the form  $\{S, \chi\}$ . Let us denote the equivalence relation by  $\approx$ , that is  $\Lambda \approx \Lambda + \{S, \chi\}$ . Indeed, the definition given above implies that  $[\Lambda, \{S, \chi\}]^c \approx 0$ , and therefore the bracket depends only on the equivalence class of the gauge parameters. This bracket defines the Lie algebra of classical gauge transformations.

Quantum gauge transformations. The main difficulty in understanding the notion of gauge transformations at the quantum level was due to the apparent lack of a suitable invariant object. In Darboux coordinates, the gauge transformations given in (1) do not leave invariant the quantum master action, nor the measure  $d\mu e^{S/\hbar}$ , as one could naively hope. This is easily verified using the result that the variation of *any* measure  $d\mu$  under a canonical transformation generated by  $K$  is given by  $\widehat{\delta}_K d\mu = 2d\mu \cdot \Delta_{d\mu} K$  (see [7], Eqn.(3.26)). Since we are taking  $K = \{S, \Lambda\}$  with  $\Delta_{d\mu} \Lambda = 0$  ( $\Lambda$  is linear in fields), this result, with the help of eqs.(4), (5), and the Jacobi identity, leads immediately to  $\widehat{\delta}_K d\mu e^{S/\hbar} = 2d\mu e^{S/\hbar} \cdot \{\Delta_{d\mu} S + \frac{1}{4\hbar} \{S, S\}, \Lambda\}$ . If instead of a factor  $\frac{1}{4}$  multiplying the  $\{S, S\}$  term, we would have a  $\frac{1}{2}$ , the master equation would imply invariance. This means that the measure  $d\mu_S = d\mu e^{2S/\hbar}$  introduced earlier is actually invariant under the gauge transformations of Eqn.(1).

We can now easily generalize the result to arbitrary coordinate systems, and field dependent gauge transformation parameters. The variation of the measure  $d\mu_S$  under a canonical transformation generated by  $K$  is given by  $\widehat{\delta}_K d\mu_S = 2d\mu_S \Delta_{d\mu_S} K$ , and therefore the condition of invariance is simply  $\Delta_{d\mu_S} K = 0$ , i.e.  $K$  must be an observable in the full quantum theory. A special class of solutions, representing trivial observables, is given by,

$$K \equiv \hbar \Delta_{d\mu_S} \Lambda = \hbar \Delta_{d\mu} \Lambda + \{S, \Lambda\}, \quad (12)$$

where invariance follows due to the nilpotency of  $\Delta_{d\mu_S}$ . These gauge transformations close under commutation. Defining the quantum gauge transformation  $\delta_\Lambda^q \equiv \widehat{\delta}_{\hbar \Delta_{d\mu_S} \Lambda}$ , we find

$$[\delta_{\Lambda_2}^q, \delta_{\Lambda_1}^q] = \delta_{[\Lambda_1, \Lambda_2]^q}^q, \quad [\Lambda_1, \Lambda_2]^q \equiv \frac{1}{2} \{ \Lambda_1, \hbar \Delta_{d\mu_S} \Lambda_2 \} - \frac{1}{2} \{ \Lambda_2, \hbar \Delta_{d\mu_S} \Lambda_1 \}. \quad (13)$$

The bracket  $[\ , ]^q$  differs from its classical counterpart  $[\ , ]^c$ , given in Eqn.(11), by terms of order  $\hbar$ . In exact analogy to the classical case, we have that  $([\Lambda_1, [\Lambda_2, \Lambda_3]^q]^q + \text{cyclic})$  is of the form  $\Delta_{d\mu_S} \chi$ , which, as a gauge parameter, generates no gauge



transformation. Therefore  $[ , ]^q$  leads to a *strict Lie bracket* in the space of even functions  $\Lambda$  modulo functions of the form  $\Delta_{d\mu_S}\chi$ . We can check that  $[\Lambda, \Delta_{d\mu_S}\chi]^q \approx 0$ , and therefore the bracket depends only on the equivalence class  $\Lambda \approx \Lambda + \Delta_{d\mu_S}\chi$  of the gauge parameters. This bracket defines the Lie algebra of quantum gauge transformations.

Example I: Scalar Field Theory We first illustrate our ideas with the help of the simplest theory, namely the theory of a free scalar field  $\phi$  in  $D$  dimensions described by the action  $S = \int d^D x (\partial_\mu \phi \partial^\mu \phi - V(\phi))$ . In this case the classical theory does not possess any gauge invariance in the usual sense. The BV formulation of the theory involves the field  $\phi$  and its anti-field  $\phi^*$ , and the BV master action coincides with the classical action  $S$ . As a result, any local function  $K(\phi)$ , independent of antifields, corresponds to a local observable ( $\Delta_{d\mu_S} K(\phi) = 0$ ), and hence generates a gauge symmetry.\* The resulting transformations are

$$\delta\phi = \{\phi, K(\phi)\} = 0, \quad \delta\phi^* = \{\phi^*, K(\phi)\} = -\frac{\delta K(\phi)}{\delta\phi} \quad (14)$$

This can easily be seen to be a symmetry of the theory leaving both the action  $S$  (which is independent of  $\phi^*$ ), and the measure  $d\phi d\phi^*$  separately invariant. This, of course, need not be the case for general  $K(\phi, \phi^*) = \hbar \Delta_{d\mu_S} \Lambda(\phi, \phi^*)$ .

Example II: Gauge Transformations in Closed String Field Theory. The closed string field theory master action is given by [ 9]

$$S = \sum_{g=0}^{\infty} \hbar^g \sum_{\substack{N=2 \text{ for } g=0 \\ N=1 \text{ for } g \geq 1}}^{\infty} \frac{1}{N!} \langle V^{(g,N)} | \Psi \rangle_1 \cdots | \Psi \rangle_N. \quad (15)$$

The corresponding measure is  $d\mu = \prod_i d\psi^i$  (for notation, see refs.[ 9,10,5].) Let us study gauge transformations generated by observables of the form  $K = \hbar \Delta_{d\mu_S} \Lambda$ .

---

\* Note that a general  $K$  of this form cannot be written as  $\Delta_{d\mu_S} \Lambda$  for a *local*  $\Lambda$ .

The most general form of  $\Lambda$  is given by,

$$\Lambda = \sum_{g=0}^{\infty} \hbar^g \sum_{N=1}^{\infty} \frac{1}{N!} {}_{1\dots N}\langle \Lambda^{(g,N)} | \Psi \rangle_1 \cdots | \Psi \rangle_N. \quad (16)$$

Separating out the contribution of the terms involving  $\langle V^{(0,2)} | = \langle \omega_{12} | Q^{(2)}$ , we get,

$$\begin{aligned} K &= \hbar \Delta_{d\mu_S} \Lambda \equiv \sum_{g,N \geq 0} \hbar^g \frac{1}{N!} \langle K^{(g,N)} | \Psi \rangle_1 \cdots | \Psi \rangle_N \\ &= - \sum_{g \geq 0, N \geq 1} \hbar^g \frac{1}{N!} \langle \Lambda^{(g,N)} | \sum_{i=1}^N Q^{(i)} | \Psi \rangle_1 \cdots | \Psi \rangle_N \\ &\quad - \sum_{g,N \geq 0} \hbar^g \sum_{g_1=0}^g \sum_{m=1}^{N-1 \text{ for } g_1=g, N+1 \text{ for } g_1 < g} \frac{1}{(N-m+1)!(m-1)!} {}_{1\dots} \langle V^{(g-g_1, N-m+2)} | \otimes {}_{1'\dots} \langle \Lambda^{(g_1, m)} | \mathcal{S}_{11'} \rangle \\ &\quad \quad \quad \cdot | \Psi \rangle_2 \cdots | \Psi \rangle_{N-m+2} | \Psi \rangle_{2'} \cdots | \Psi \rangle_{m'} \\ &\quad - \frac{1}{2} \sum_{g \geq 1, N \geq 0} \hbar^g \frac{1}{N!} \langle \Lambda^{(g-1, N+2)} | \mathcal{S}_{12} \rangle | \Psi \rangle_3 \cdots | \Psi \rangle_{N+2}, \end{aligned} \quad (17)$$

where  $|\mathcal{S}\rangle$  denotes the sewing ket [10]. This generates the gauge transformation:

$$\delta_{\Lambda}^g | \Psi \rangle_e = \sum_{g,N \geq 0} \hbar^g \frac{1}{N!} \langle K^{(g, N+1)} | \mathcal{S}_{0e} \rangle | \Psi \rangle_1 \cdots | \Psi \rangle_N. \quad (18)$$

In particular, choosing a  $\Lambda$  for which only  $\langle \Lambda^{(0,1)} | \equiv \langle \Lambda |$  is non-zero, we get

$$\begin{aligned} \delta_{\Lambda}^g | \Psi \rangle_e &= - {}_0 \langle \Lambda | Q^{(0)} | \mathcal{S}_{0e} \rangle - \sum_{g=0}^{\infty} \hbar^g \sum_{\substack{N \geq 1 (g=0) \\ N \geq 0 (g>0)}}^{\infty} \frac{1}{N!} \langle V^{(g, N+2)} | {}_0 \langle \Lambda | \mathcal{S}_{01} \rangle | \mathcal{S}_{2e} \rangle | \Psi \rangle_3 \cdots | \Psi \rangle_{N+2} \\ &= Q | \Lambda \rangle_e + \sum_{g=0}^{\infty} \hbar^g \sum_{\substack{N \geq 1 (g=0) \\ N \geq 0 (g>0)}}^{\infty} \frac{1}{N!} \langle V^{(g, N+2)} | \mathcal{S}_{1e} \rangle | \Lambda \rangle_2 | \Psi \rangle_3 \cdots | \Psi \rangle_{N+2}, \end{aligned} \quad (19)$$

where  $|\Lambda\rangle_e \equiv {}_0 \langle \Lambda | \mathcal{S}_{0e} \rangle$ . Note that the  $|\Psi\rangle$  independent term of the gauge transformation receives contribution from two-punctured surfaces of all genera. This

means that the notion of an unbroken gauge symmetry at the classical level differs from the corresponding notion at the quantum level. If we truncate to  $g = 0$  we recover the standard gauge transformations of classical closed string field theory.

Are there gauge transformations generated by nontrivial observables? To answer this we need to see if there are local<sup>†</sup> observables in string field theory which are not of the form  $\Delta_{d\mu_S}\Lambda$ . It might perhaps be possible to construct nontrivial local observables that are quadratic and higher orders in the fields. They would generate transformations  $\delta|\Psi\rangle$  that are linear and higher orders in  $|\Psi\rangle$ , but do not contain any  $|\Psi\rangle$  independent piece. Although these would give rise to local symmetries of the theory, they would not be gauge symmetries in the conventional sense of the term. If we want a symmetry transformation that contains a field independent piece, we need a  $K$  that is linear in the string field  $|\Psi\rangle$ . We shall now argue that in string field theory there is no *local* observable that is linear in  $|\Psi\rangle$  and is not of the form  $\Delta_{d\mu_S}\Lambda$ . The intuition is simple, such transformations would correspond at the linearized level to shifts of the type  $|\Psi\rangle \rightarrow |\Psi\rangle + |H^Q\rangle$ , where  $|H^Q\rangle$  is an element of the cohomology of  $Q$ . Such transformations leave the kinetic term of the string field action invariant, but certainly do not qualify as gauge transformations.

To prove this it is enough to take  $K = \langle K|\Psi\rangle$  linear in  $|\Psi\rangle$ , and analyze the  $\Psi$  independent terms in the equation  $\Delta_{d\mu_S}K = 0$ . This gives,  $Q^{(e)}{}_0\langle K|\mathcal{S}_{0e}\rangle = 0$ , and shows that  ${}_0\langle K|\mathcal{S}_{0e}\rangle$  must be a BRST invariant operator. Furthermore, since we want to exclude solutions of the form  $\Delta_{d\mu_S}\Lambda$ , we must also require that  ${}_0\langle K|\mathcal{S}_{0e}\rangle$  be a non-trivial member of the BRST cohomology. But in string theory we know that non-trivial members of the BRST cohomology are found only for certain specific values of momentum  $k^\mu$ , satisfying mass-shell constraints of the form  $k^2 = m^2$ ,

---

<sup>†</sup> Here by local expressions, we mean terms that when expressed in momentum space, the integrands have well defined Taylor series expansion in momenta about the point where all momenta vanish. In position space these terms will be represented by a series containing higher derivative terms. In this limited sense the string field theory lagrangian and the gauge transformations are local, since they contain integration over subspaces of the moduli space which do not include any degeneration point.

where  $m$  is the mass of one of the particles in the spectrum of string theory. Thus, when expressed as a momentum space integral,  $\langle K|\Psi\rangle$  will contain a factor of  $\delta(k^2 - m^2)$  in the integrand, and hence it does not correspond to a *local* observable in the theory.

In this example we have found the quantum gauge transformations of closed string field theory. An obvious question is whether our formalism can help us understand the gauge structure of string theory. Since the present approach deals with Lie algebras, it may provide an alternative or complementary approach to current studies based on homotopy Lie algebras [ 11]. It remains to be seen if the Lie brackets  $[ , ]^c$  and  $[ , ]^q$  define manageable structures in string theory. Another interesting project would be to isolate from the string algebra the Lie subalgebra representing coordinate transformations in string theory [ 12].

## REFERENCES

1. I. A. Batalin and G. A. Vilkovisky, Phys. Rev. **D28** (1983) 2567.
2. M. Henneaux and C. Teitelboim, ‘Quantization of Gauge Systems’, Princeton University Press, Princeton, New Jersey, 1992.
3. A. Schwarz, ‘Geometry of Batalin-Vilkovisky quantization’, UC Davis preprint, hep-th/9205088, July 1992.
4. E. Witten, ‘On background independent open-string field theory’, Phys. Rev. **D46** (1992) 5467, hep-th/9208027.
5. A. Sen and B. Zwiebach, ‘Quantum background independence of closed string field theory’, MIT preprint, CTP#2244, to appear.
6. E. Witten, ‘A note on the antibracket formalism’, Mod. Phys. Lett. **A5** (1990) 487.
7. H. Hata and B. Zwiebach, ‘Developing the covariant Batalin-Vilkovisky approach to string theory’, MIT-CTP-2184, to appear in Annals of Physics, hep-th/9301097.
8. E. Getzler, ‘Batalin-Vilkovisky algebras and two-dimensional topological field theories’, MIT Math preprint, hep-th/9212043.
9. B. Zwiebach, ‘Closed string field theory: Quantum action and the Batalin-Vilkovisky master equation’, Nucl. Phys. **B390** (1993) 33, hep-th/9206084.

10. A. Sen and B. Zwiebach, ‘Local background independence of classical closed string field theory’, MIT preprint, CTP#2222, submitted to Nucl. Phys. B, hep-th/9307088.
11. E. Witten and B. Zwiebach, ‘Algebraic structures and differential geometry in 2D string theory’, Nucl. Phys. **B377** (1992) 55. hep-th/9201056.  
E. Verlinde, ‘The master equation of 2D string theory’ IASSNS-HEP-92/5, to appear in Nucl. Phys. B. hep-th/9202021.  
T. Kimura, J. Stasheff and A. Voronov, ‘On operad structures of moduli spaces and string theory’, hep-th/9307114.
12. D. Ghoshal and A. Sen, ‘Gauge and general coordinate invariance in non-polynomial closed string field theory’, Nucl. Phys. **B380** (1992) 103.