SLAC – PUB – 3908
March 1986
T

SUPERSPACE ANALYSIS OF LOCAL LORENTZ AND GAUGE ANOMALIES IN THE HETEROTIC STRING THEORY*

ASHOKE SEN

Stanford Linear Accelerator Center
Stanford University, Stanford, California, 94305

ABSTRACT

Local Lorentz and gauge anomalies in the sigma model describing the propagation of the heterotic string in an arbitrary background field is discussed in the superspace formulation. An expression for these anomalies is written down in terms of the superfields, and is shown to cancel against an anomalous variation of the antisymmetric tensor field. World sheet supersymmetry is manifest throughout this analysis.

Submitted to *Physics Letters B*

⋆ Work supported by the Department of Energy, contract DE – AC03 – 76SF00515.

In a previous paper[1] we analyzed local gauge and Lorentz anomalies in the σ-model describing the propagation of the heterotic string in an arbitrary background field. The analysis was done in the component field language, and it was shown that the gauge and Lorentz invariance of the sigma model† could be restored by adding suitable counterterms to the Lagrangian. Although these counterterms are not supersymmetric by themselves, it was shown that to one loop order the full effective action remains supersymmetric.

In this paper we shall repeat the analysis using superfield formulation[2]. Although superfield formulation has been used extensively to study anomalies in supersymmetric gauge theories[3], to our knowledge it has not been directly applied to study σ-model anomalies[4]. In this formulation, we always maintain explicit supersymmetry. As we shall show, there is a supersymmetric extension of the local gauge or Lorentz anomaly in the sigma model. This term may be expressed in terms of superfields, and hence may be regarded as gauge (or Lorentz) variation of some manifestly supersymmetric non-local effective action. This action differs from the standard expression for the anomalous action, written in terms of the component fields, by a local counterterm, which is precisely the counterterm which was added by hand in Ref.[1] in order to restore local gauge and Lorentz invariance, as well as supersymmetry invariance. We shall also show that this anomalous variation of the effective action may be cancelled completely by redefining the transformation laws of the antisymmetric tensor field under local gauge and Lorentz transformations[5].

The dynamical fields in the first quantized heterotic string in the light cone gauge are the eight bosonic coordinates X^i, eight left-handed Majorana-Weyl fermions λ^i, and thirty two right handed Majorana-Weyl fermions ψ^s. Let us introduce thirty two auxiliary scalar fields F^s, an anticommuting parameter θ

† Here by local gauge and Lorentz transformations we mean certain transformations on the parameters of the σ-model, as given in Eq.(5). Invariance under these transformations implies that the theory remains unchanged if we change the parameters of the σ-model as in Eq.(5).

and define the superfields,

$$\Phi^i = X^i + \theta\lambda^i$$
$$\Lambda^s = \psi^s + \theta F^s \tag{01}$$

If τ, σ denote the variables labelling the string world-sheet, we define,

$$\partial_\pm = \frac{1}{\sqrt{2}}(\partial_\tau \pm \partial_\sigma)$$
$$D = (\partial_\theta + i\theta\partial_-) \tag{02}$$

The action for the heterotic string in an arbitrary background metric $G_{ij}(x)$, antisymmetric tensor field $B_{ij}(x)$ and gauge field $A_i^M(x)$ is then given by[5–7]

$$S = \frac{1}{2\pi\alpha'}\int d\sigma d\tau d\theta\Big[-i\big(G_{ij}(\Phi) + B_{ij}(\Phi)\big)D\Phi^i\partial_+\Phi^j$$
$$-\Lambda^s\big(i\delta_{st}D + A_i^M(\Phi)(T^M)_{st}D\Phi^i\big)\Lambda^t\Big] \tag{03}$$

Since we are working in flat two dimensional space-time, the action does not depend on the background value of the dilaton field Φ^D. In this theory, such a background value of the dilaton simply corresponds to the freedom of adding a term proportional to $(\partial_\alpha\partial_\beta - \delta_{\alpha\beta}\partial^2)\Phi^D$ to the stress tensor, as shown in [8].

Expanding (3) in terms of the component fields and eliminating the auxiliary fields F^s by their equations of motion, we may recover the component field Lagrangian for the heterotic string theory in arbitrary background field[5–7]. Let us introduce the vielbein fields $e_i^a(x)$ satisfying,

$$G_{ij}(x) = e_i^a(x)e_j^a(x) \tag{04}$$

and define local gauge and Lorentz transformations on the background fields as,

$$\delta A_i^M(x) = \partial_i\Theta^M(x) + f^{MNP}A_i^N(x)\Theta^P(x)$$
$$\delta e_i^a(x) = \Theta^{ab}(x)e_i^b(x) \tag{05}$$

where f^{MNP} is the structure constant of the gauge group, and Θ^M and Θ^{ab} are the gauge and Lorentz transformation parameters respectively. The classical action (3) is invariant under these transformations provided we transform the fields ψ and F accordingly. Quantum mechanically, however, these transformations are anomalous. If we carry out the calculation of the effective action using superspace perturbation theory[2], the final result will be expressed in terms of superfields, and hence will be explicitly supersymmetric. As a result, any variation of the effective action under local gauge and Lorentz transformations defined in Eq.(5) must also be expressed as a function of the superfields. We propose the following form of the variation of the effective action under these transformations:

$$\begin{aligned}\delta S_{eff} = -\frac{i}{8\pi}\int d\tau d\sigma d\theta \Big[& D\Theta^M(\Phi)\partial_+\Phi^i A_i^M(\Phi) - A_i^M(\Phi)D\Phi^i\partial_+\Theta^M(\Phi) \\ & - D\Theta^{ab}(\Phi)\omega_i^{ab}(\Phi)\partial_+\Phi^i + \omega_i^{ab}(\Phi)D\Phi^i\partial_+\Theta^{ab}(\Phi)\Big]\end{aligned} \tag{06}$$

where ω is the spin connection constructed from the vielbeins e_i^a. To show that this is indeed the correct expression, we expand it in terms of the component fields. For simplicity we shall display only the part involving Θ^M. The result is,

$$\begin{aligned}\delta S_{eff} = \frac{1}{8\pi}\int d\tau d\sigma \Big[& \epsilon^{\alpha\beta}\partial_\alpha\Theta^M(X)A_i^M(X)\partial_\beta X^i \\ & - i(\partial_i A_j^M\partial_k\Theta^M + \partial_j A_k^M\partial_i\Theta^M + \partial_k A_i^M\partial_j\Theta^M)\lambda^i\lambda^j\partial_+X^k\Big]\end{aligned} \tag{07}$$

The first term in (7) is the standard expression for the gauge anomaly, whereas the rest of the terms may be written as,

$$\delta_{gauge}[\frac{i}{16\pi}\Omega_3(A)_{ijk}\lambda^i\lambda^j\partial_+X^k] \tag{08}$$

where,

$$\Omega_3(A)_{ijk} = \tfrac{1}{2}[A^M_{[i}F^M_{jk]} + \tfrac{1}{3}f^{MNP}A^M_{[i}A^N_jA^P_{k]}] \tag{09}$$

is the Chern-Simons three form. Thus the expression for the gauge anomaly given in (6) takes the standard form plus the gauge variation of a local counterterm given in (8). Note that this is precisely the term which had to be added by hand in Ref.[1] in order to restore local gauge symmetry and supersymmetry.*

A similar analysis shows that (6) also gives the standard component field expression for Lorentz anomaly, up to the addition of a local counterterm like (8), with A replaced by ω. We should again mention that a similar counterterm had to be added by hand in Ref.[1] in order to restore local Lorentz symmetry.

With the form (6) for the total gauge and Lorentz anomaly, it is easy to show that this anomaly may be cancelled by redefining the transformation laws of the antisymmetric tensor field B_{ij} as,

$$\delta B_{ij} = -\tfrac{1}{4}\alpha'(\partial_{[i}\Theta^M A^M_{j]} - \partial_{[i}\Theta^{ab}\omega^{ab}_{j]}) \tag{010}$$

Indeed, the variation of the classical action (3) under this transformation law of B is given by,

$$\frac{i}{8\pi}\int d\sigma d\tau d\theta \Big[\partial_{[i}\Theta^M(\Phi)A^M_{j]}(\Phi) - \partial_{[i}\Theta^{ab}(\Phi)\omega^{ab}_{j]}(\Phi)\Big] D\Phi^i\partial_+\Phi^j \tag{011}$$

which exactly cancels δS_{eff} given in (6).

We shall conclude by discussing the possible ambiguity in defining the anomaly given in (6). If T^M_i and T^{ab}_i denote arbitrary gauge and Lorentz covariant vector fields, the expression for anomaly may be modified by changing A^M_i to $A^M_i + T^M_i$ and ω^{ab}_i to $\omega^{ab}_i + T^{ab}_i$ [1,9]. Consequently the transformation laws of the field B_{ij} must also be modified. Modifying the expression for anomaly in this way is equivalent to adding a local term to the effective action of the form,

$$-\frac{i}{8\pi}\int d\sigma d\tau d\theta \Big[T^M_{[i}(\Phi)A^M_{j]}(\Phi) - T^{ab}_{[i}(\Phi)\omega^{ab}_{j]}(\Phi)\Big] D\Phi^j\partial_+\Phi^i \tag{012}$$

This term, however, may be absorbed into the definition of the antisymmetric

---

* The coefficient in front of Ω_3 in Eq.(8) is different from that given in Ref.[1] by a factor of 2 due to a different choice of normalization of the fields λ^i.

tensor field B_{ij}. If we define,

$$B'_{ij} = B_{ij} + \frac{\alpha'}{4}(A^M_{[i}T^M_{j]} - \omega^{ab}_{[i}T^{ab}_{j]}) \tag{013}$$

then the new action, expressed as a function of G_{ij}, B'_{ij} and A^M_i will look the same as the original action.

In fact, since Eq.(3) describes the most general renormalizable lagrangian invariant under $N = \frac{1}{2}$ supersymmetry up to a wave-function renormalization of the superfields Λ^s, any supersymmetric local counterterm added to the lagrangian may be absorbed into the definition of various coupling constants G_{ij}, B_{ij} and A^M_i of the σ-model. Hence there is no ambiguity in defining the theory, and we may try to obtain unambiguous answers to various related questions, e.g. the criteria for the vanishing of the two (or more) loop β-functions, the criteria for the model to have extended (2,0) supersymmetry, etc. This will resolve the ambiguity recently discussed by Strominger[10], and tell us if the manifolds with torsion, recently discussed in Refs.[10, 11] are indeed consistent manifolds for string compactification. It will also be interesting to see if the analysis given in this paper can be applied to the analysis of anomalies in the Green-Schwarz version of the heterotic string coupled to background fields[12].

ACKNOWLEDGEMENTS

I wish to thank S. J. Gates for useful discussions.

REFERENCES

1. A. Sen, Phys. Lett. 166B (1986) 300.

2. See, for example, 'Superspace', by S. J. Gates, M. T. Grisaru, M. Rocek and W. Siegel, Benjamin Publishing Co. (1983).

3. O. Piguet, K. Sibold and M. Schweda, Nucl. Phys. B174, (1980) 183; O. Piguet and K. Sibold, Nucl. Phys. B247 (1984), 484; S. Ferrara, L. Girardello, O. Piguet and R. Stora, Phys. Lett. 157B (1985) 179; N. K. Nielsen, Nucl. Phys. B244 (1984) 499; E. Guadagnini, K. Konishi and M. Mintchev, Phys. Lett. 157B (1985) 37; M. Pernici and M. Riva, preprint ITP-SB-85-10; K. Harada and K. Shituya, Tohoka preprint TH-85-280; R. Garreis, M. Schobl and J. Wess, Karlsruhe preprint, RATHEP-85-4; G. Girardi, R. Grimm and R. Stora, Phys. Lett. 156B (1985) 203; L. Bonora, P. Pasti and M. Tonin, Phys. Lett. 156B (1985) 341, Nucl. Phys. B261 (1985) 241, preprint DFPD 21/85; L. N. Mcarthur and H. Osborn, preprint DAMTP-85-27; H. Itoyama, V. P. Nair and H. C. Ren, IAS preprint (1985), preprint FERMILAB-PUB-85-139.

4. P. Nelson and G. Moore, Phys. Rev. Lett. 53 (1984) 1519; preprint HUTP-84/A076; E. Cohen and C. Gomez, Nucl. Phys. B254 (1985) 235; P. Di Vecchia, S. Ferrara and L. Girardello, Phys. Lett. 151B (1985) 199; A. Manohar, G. Moore and P. Nelson, Phys. Lett. 152B (1985) 68; L. Alvarez-Gaume and P. Ginsparg, preprint HUTP-85/A015; J. Bagger, D. Nemeschansky and S. Yankielowicz, SLAC-PUB-3588.

5. C. Hull and E. Witten, Phys. Lett. 160B (1985) 398. See also R. Nepomechie, Univ. of Washington, Seattle, preprint No. 40048-28 P5.

6. A. Sen, Phys. Rev. D32 (1985) 2102, Phys. Rev. Lett. 55 (1985) 1846; C. G. Callan, D. Friedan, E. Martinec and M. Perry, Nucl. Phys. B262 (1985) 593.

7. E. Bergshoeff, E. Sezgin and H. Nishino, Phys. Lett. 166B (1986) 141; R.

Brooks, F. Muhammad and S. Gates, Maryland preprint; M. Evans and B. Ovrut, Rockefeller Univ. preprint No. RU/86/149.

8. C. Lovelace, Rutgers preprint, RU-85-51; T. Banks, D. Nemeschansky and A. Sen, SLAC-PUB-3885.
9. C. Hull, DAMTP preprint (1985).
10. A. Strominger, preprint NSF-ITP-86-16.
11. E. Witten, Princeton preprint (1985); C. Hull, DAMTP preprint (1985).
12. E. Witten, princeton preprint, 1985; J. Atick, A. Dhar and B. Ratra, SLAC-PUB-3840.