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Vacuumless cosmic strings in Brans-Dicke theory

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Abstract

The gravitational fields of vacuumless global and gauge strings have been studied in Brans-Dicke theory under the weak field assumption of the field equations. It has been shown that both global and gauge string can have repulsive as well as attractive gravitational effect in Brans-Dicke theory which is not so in General Relativity.

1 Introduction

Spontaneous symmetry breaking in the gauge field theories may give rise to some topologically trapped regions of a false vacuum, namely domain walls, cosmic strings or monopoles, depending on the dimension of the region [1]. In cosmology, these defects have been put forward as possible source for the density perturbations which seeded the galaxy formation [2].

A typical symmetry breaking lagrangian is of the form

$$\mathcal{L} = \frac{1}{2}\partial_\mu \phi^a \partial^\mu \phi^a - V(f), \tag{1.1}$$

Where ϕ^a is a set of scalar fields, $a = 1, 2, ...N$, $f = (\phi^a\phi^a)^{1/2}$ and $V(f)$ has a minimum at a non zero value of f. The model has $O(N)$ symmetry and domain walls, strings,

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monopoles are formed for $a = 1, 2, 3$ respectively. One has to add gauge field in the above lagrangian and should replace ∂_μ by a gauge covariant derivative, to study the structure of gauge defects.

It has been recently suggested by Cho and Vilenkin [3] that topological defects can also be formed in the models where $V(f)$ is maximum at $f = 0$, and it decreases monotonically to zero for $f \to \infty$ without having any minima. For example,

$$V(f) = \lambda M^{4+n}(M^n + f^n)^{-1}, \tag{1.2}$$

where M, λ and n are positive constants. This type of potential can arise in nonperturbative superstring models [4]. Such potential having a power law tail for large ϕ has also been considered by authors [5] inorder to reconcile the low dynamical estimates of the mean mass density with negligibly small scale curvature which is preferred by inflation. In recent years, potential of this type has been discussed in so called "*quintessence*" models of inflation [6]. Defects arising in these models are termed as Vacuumless. In a recent paper, Cho and Vilenkin have studied the gravitational fields of such vacuumless defects in General Relativity[GR] [7]

At sufficient high energy scales it seems likely that gravity is not given by the Einstein's action, but becomes modified by the superstring terms. In the low energy limit of this string theory, one recovers Einstein's gravity along with a scalar dilaton field which is non minimally coupled to the gravity [8]. On the other hand, scalar tensor theories, such as Brans-Dicke theory(BD) [9], which is compatible with the Mach's principle, have been considerably revived in the recent years. It was shown by La and Steinhardt [10] that because of the interaction of the BD scalar field with the Higgs type sector, the exponential inflation in Guth's model [11] could be slowed down to power law one and the graceful exit in the inflation is thus completed via bubble nucleation. Although dilaton gravity and BD theory arise from entirely different motivations, it can be shown that the formar is a special case of the latter at least formally [12]. As we have mentioned earlier that these vacuumless defects may be formed in the supersymmetric phase transition in the early universe. So it may be relevant to study how these defects interact with BD dilaton field which arises in the low energy superstring theories. Another motivation for studying gravitational properties of defects in BD theory is that only defects we can hope to observe now are those formed after or near the end of inflation, and the formation of such superheavy defects is relatively easy to arrange in Brans-Dicke type theory [13].

In this work we have studied the gravitational fields of vacuumless global and gauge strings in BD theory under the weak field approximation of the field equations. The paper is organised as follows: in section 2 we have briefly outlined the work of Cho and Vilenkin for the vacuumless string in GR. In section 3 we have given the solutions of spacetimes for global and gauge vacuumless string in BD theory under the weak field approximation. The paper ends with a conclusion in section 4.

2 A brief review of vacuumless string in GR

In this section we review the earlier work of Cho and Vilenkin [7]. For global vacuumless string the flat spacetime solution for $f(r)$ is given by

$$f(r) = aM(r/\delta)^{2/(n+2)}, \tag{2.1}$$

where $\delta = \lambda^{-1/2}M^{-1}$ is the core radius of the string; r is the distance from the string axis and $a = (n+2)^{2/(n+2)}(n+4)^{-1/(n+2)}$. The solution (2.1) applies for

$$\delta << r << R, \tag{2.1a}$$

where R is cut off radius determined by the nearest string.

For gauge vacuumless strings, which have magnetic flux localized within a thin tube inside the core, the scalar field outside the core is given by

$$f(r) = aLn(r/\delta) + b. \tag{2.2}$$

But here a and b are sensitive to cut off distance R:

$$a \sim M(R/\delta)^{2/(n+2)}[Ln(R/\delta)]^{-(n+1)/(n+2)}, \quad b \sim aLn(R/\delta). \tag{2.3}$$

For a vacuumless string the spacetime is static, cylindrically symmetric and also has a symmetry with respect to Lorentz boost along the string axis. One can write the corresponding line element as

$$ds^2 = B(r)(-dt^2 + dz^2) + dr^2 + C(r)d\theta^2. \tag{2.4}$$

The general energy momentum tensor for the vacuumless string is given by

$$T_t^t = T_z^z = \frac{f'^2}{2} + \frac{f^2(1-\alpha)^2}{2C} + \frac{\alpha'^2}{2e^2C} + V(f) \tag{2.5a}$$

$$T_r^r = -\frac{f'^2}{2} + \frac{f^2(1-\alpha)^2}{2C} - \frac{\alpha'^2}{2e^2C} + V(f) \tag{2.5b}$$

$$T_\theta^\theta = \frac{f'^2}{2} - \frac{f^2(1-\alpha)^2}{2C} - \frac{\alpha'^2}{2e^2C} + V(f) \tag{2.5c}$$

Where string ansatz for the gauge field is $A_\theta(r) = -\frac{\alpha(r)}{er}$. The T_ν^μ's with $\alpha = 0$ are that for global string.

Under the weak field approximation one can write

$$B(r) = 1 + \beta(r), \;\; C(r) = r^2(1 + \gamma(r)), \qquad 2.5d)$$

where $\beta, \gamma << 1$.For global vacuumless string, one can use the flat space approximation for $f(r)$ in (2.1) for $r >> \delta$ and the form of $V(f)$ given in (1.2). Then the solution for the spacetime under weak field approximation is given by [7]

$$ds^2 = (1 + 2\Phi)(-dt^2 + dz^2) + dr^2 + (1 + m\Phi)d\theta^2 \tag{2.6}$$

where $\Phi = -KGM^2(r/\delta)^{4/(n+2)}$, $K = \pi(n+2)^2/2a^n$ and m is an arbitary constant. The linearized approximation is valid for $\Phi(r) << 1$ and from (2.1) this is equivalent to $f(r) << m_p$ where $m_p = 1/\sqrt{G}$ is the Planck mass.

For gauge vacuumless string the energy momentum tensor with $f(r)$ given in (2.2) can be approximated as [7]

$$T_t^t = T_z^z = T_\theta^\theta = -T_r^r = \frac{f'^2}{2} = \frac{a^2}{2r^2}. \tag{2.8}$$

This form of the energy momentum is valid for

$$r << R/Ln^{1/2}(R/\delta), \tag{2.8a}$$

where R is the cut off radius determined by the nearest string [7]. The complete solution of the line element under weak field approximation is given by

$$ds^2 = (1 + 2\Phi)(-dt^2 + dz^2) + dr^2 + (1 - 4\Phi)d\theta^2 \tag{2.9}$$

where $\Phi = 2\pi Ga^2Ln(r/\delta)$ and a is given by (2.3).

3 Vacuumless string in BD theory

The field equation in the BD theory are written in the form

$$G_{\mu\nu} = \frac{T_{\mu\nu}}{\phi} + \frac{\omega}{\phi^2}(\phi_{,\mu}\phi_{,\nu} - \frac{1}{2}g_{\mu\nu}\phi_{,\alpha}\phi^{,\alpha}) + \frac{1}{\phi}(\phi_{,\mu;\nu} - g_{\mu\nu}\Box\phi), \tag{3.1}$$

$$\Box\phi = \frac{8\pi T}{2\omega + 3}, \tag{3.2}$$

Where ϕ is the scalar field, ω is the BD parameter and T denotes the trace of the energy momentum tenosr T^μ_ν [9]. In the weak field approximation in BD theory one can assume $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$ where $|h_{\mu\nu}| << 1$ and $\phi(r) = \phi_0 + \epsilon(r)$ with $|\epsilon/\phi_0| << 1$ where $1/\phi_0 = G_0 = \frac{(2\omega+3)}{(2\omega+4)}G$.

It has been shown recently by Barros and Romero [14], that in the weak field approximation the solutions of the BD equations are related to the solutions of linearized equations in GR with the same T^μ_ν in the follwing way: if $g^{gr}_{\mu\nu}(G, x)$ is a known solution of the Einstein's equation in the weak field approximation for a given T^μ_ν, then the BD solution corresponding to the same T^μ_ν, in the weak field approximation, is given by

$$g^{bd}_{\mu\nu}(x) = [1 - G_0\epsilon(x)]g^{gr}_{\mu\nu}(G_0, x) \tag{3.2}$$

where $\epsilon(x)$ must satisfy

$$\Box\epsilon(x) = \frac{8\pi T}{(2\omega + 3)}, \tag{3.3}$$

and G is replaced by G_0 defined earlier in this section. Hence, to get the spacetime for vacuumless global and gauge string one has to solve the equation (3.3) with T^μ_ν given in section 2.

3.1 Global String

Using equations (2.4) and (2.5) with $\alpha = 0$, one can calculate $T = T^\mu_\nu$ in equation (3.3) as

$$T = 2f'^2 + \frac{f^2}{C} + 4V(f). \tag{3.4}$$

Now under weak field approximation $C = r^2(1 + \gamma(r))$ where $\gamma(r) << 1$. Hence (3.4) becomes

$$T = 2f'^2 + \frac{f^2}{r^2} + 4V(f)$$

which on substitution of (1.2) and (2.1) with $r >> \delta$ becomes

$$T = Ar^{\frac{-2n}{(n+2)}} \tag{3.5}$$

where $A = a^2 M^2 (\frac{1}{\delta})^{4/(n+2)}[\frac{8}{(n+2)^2} + 1 + \frac{4}{a^{n+2}}]$. Putting (3.5) in (3.3) and using (2.4) and (2.5d), we get

$$\epsilon^{''} + \frac{2\epsilon^{'}}{r} = \frac{8\pi A}{2\omega + 3} r^{\frac{-2n}{(n+2)}}$$

which on integration yields

$$\epsilon = (\frac{8\pi}{2\omega + 3}) C (\frac{r}{\delta})^{\frac{4}{n+2}} - \frac{D}{2\omega + 3} (\frac{r}{\delta})^{-3}, \tag{3.6}$$

where D is an arbitary integration constant and $C = a^2 M^2 [\frac{8}{(n+2)^2} + 1 + \frac{4}{a^{n+2}}] \frac{(n+2)^2}{4(n+6)}$. Hence the complete line element for a global vacuumless string in BD theory under weak field approximation for $r >> \delta$ is given by

$$ds^2 = [1 - G_0 \epsilon(r)] ds^2_{cv}(G_0) \tag{3.7}$$

where $\epsilon(r)$ is given in equation (3.6) and $ds^2_{cv}(G_0)$ is the corrsponding line element obtained by Cho and Vilenkin [7] in GR with G is replaced by G_0 defined earlier in this section. One should keep in mind that r is bounded by equation (2.1a) for the form of $f(r)$ given in equation (2.1) to be valid.

To have an idea of the motion of the particles, one can calculate the radial acceleration vector $\dot{v}^1$ of a particle that remains stationary (i.e., $v^1 = v^2 = v^3 = 0$) in the field of the string. Now, $\dot{v}^1 = v^1_{;0} v^0 = v^0 \Gamma^1_{00} v^0$. Hence using the line element (3.7) one can calculate $\dot{v}^1$ which becomes

$$\dot{v}^1 = \frac{1}{2}(1 - G_0\epsilon)^{-2}(1 + 2\Phi)^{-1}[2\Phi^{'}(1 - G_0\epsilon) - G_0(1 + 2\Phi)\epsilon^{'}]. \tag{3.8a}$$

Now as $G_0\epsilon << 1$ and also $\Phi << 1$ for the linearized approximation to be valid one can approximate equation (3.8) to write

$$\dot{v}^1 = \Phi^{'} - \frac{G_0}{2}\epsilon^{'}. \tag{3.8b}$$

For $\epsilon = constant$ that is in GR the acceleration vector is always -ve and the gravitational force is repulsive. But for $\epsilon^{'} \neq 0$ one can check that $\dot{v}^1$ is -ve or +ve depending on the arbitary constant D. For example, for $n = 2$ in GR the repulsive force is independent of radial distance. But in this case for $n = 2$

$$\dot{v}^1 = -\frac{KG_0M^2}{\delta} - \frac{4\pi cG_0}{(2\omega+3)\delta} - \frac{3DG_0}{2\delta(2\omega+3)}(\frac{r}{\delta})^{-4},$$

and in this case one can see that the gravitational force varies with radial distance and with -ve D one can have a attractive gravitational force as well , as r is bounded by equation (2.1a) for the linearized approximation as mentioned in section 2. So in BD theory the vacuumless global string can have both repulsive and attractive gravitational effect.

3.2 Gauge String

For gauge string using equation (2.8), equation (3.3) becomes

$$\epsilon^{''} + \frac{2\epsilon^{'}}{r} = \frac{a^2}{r^2(2\omega+3)}$$

which on integration yields

$$\epsilon = \frac{a^2}{(2\omega+3)}Ln(r/\delta) - \frac{P}{(2\omega+3)r} \tag{3.9}$$

where P is an arbitary integration constant and we have put another integration constant equal to $\frac{-a^2Ln\delta}{(2\omega+3)}$. Hence the metric for a gauge vacuumless string in BD theory under weak field approximation can be written according to equation (3.2) as

$$ds^2 = [1 - G_0\epsilon]ds^2_{cv}(G_0) \tag{3.91}$$

where $ds^2_{cv}(G_0)$ is the metric given in equation (2.9) with G is replaced by G_0. Here also r is bounded by equation (2.8a). One can calculate the acceleration vector $\dot{v}^1$ for a particle remaining stationary with respect to the string. Assuming $|\epsilon G_0| << 1$ and $\Phi << 1$ one can again approximate the acceleration vector as

$$\dot{v}^1 = \Phi^{'} - \frac{G_0}{2}\epsilon^{'},$$

Which becomes

$$\dot{v}^1 = \frac{G_0}{r^2}[2\pi a^2 r - \frac{a^2 r}{2(2\omega+3)} - \frac{P}{2(2\omega+3)}]. \tag{3.92}$$

Now as r is bounded by (2.8a) for the form of the energy momentum tensor (2.8) to be valid one can have repulsive as well as attractive gravitational effect for $-veP$ and $+veP$ respectively. But this is not so in GR as for $\omega \to \infty$ $\epsilon^{'}$ vanishes.

4 Conclusion

Recently there have been claims that the universe must possess a not yet identified component usually called *quintessence* matter or Q matter, besides its normal content of matter and radiation. These claims have been prompted at the realization that the clustered matter component can be at most one third of the critical density. That is why there must be some additional nonclustered component if the critical density predicted by the inflationary models is to be achieved. Examples of Q matter are fundamental fields or macroscopic objects and network of vacuumless strings may be one such good examples as scalar field with potential like (1.2) can act as quintessence models [15]. In this paper we have examined the gravitational field of vacuumless strings in the BD theory under the weak field approximation of the field equations. In doing so, we have followed the method of Barros and Romero [14] which has been suggested recently. For both global and gauge vacuumless strings, the spacetimes are conformally related to that obtained earlier by Cho and Vilenkin in GR [7]. Both the spacetimes reduces to the corresponding GR solutions for $\omega \to \infty$ limit. It has been shown that both the global and gauge string can have attractive as well as repulsive gravitational effect on a test particle freely moving in its spacetime which is not so in GR where the global string has only repulsive and gauge string has the attractive gravitational effect. As the trajectories of the light rays, which are given by the null geodesics, the only change involved in BD theory is the replacement of G by an new ω dependent "effective" gravitational constant $G_0 = (\frac{(2\omega+3)}{(2\omega+4)})G$ and for ω to be consistent with solar system experiment and observation, $\omega \sim 500$ [16], this means that photons travelling in the spacetime will experience a decrease of gravitational constant as $G_0 \sim 0.999G$. Therefore, it follows that the distortion of the isotropy of the CMBR due to the gravitational field of the vacuumless strings in BD theory may be calculated directly from the results obtained in GR. A detail analysis of the full nonlinear Einstein's equations will certainly give more insight to the problem and for that a detail numerical calculation should be done which will be the aim of our future study.

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