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Remarks on Tachyon Driven Cosmology

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Abstract

We begin by reviewing the results on the decay of unstable D-branes in type II string theory, and the open-closed string duality proposal that arises from these studies. We then apply this proposal to the study of tachyon driven cosmology, namely cosmological solutions describing the decay of unstable space filling D-branes. This naturally gives rise to a time reversal invariant bounce solution with positive spatial curvature. In the absence of a bulk cosmological constant the universe always begins with a big bang and ends in a big crunch. In the presence of a bulk cosmological constant one may get non-singular cosmological solutions for some special range of initial conditions on the tachyon.

Introduction and review of classical dynamics of open string tachyon

Type IIA and IIB string theories contain in their spectrum unstable non-BPS D-branes besides the stable BPS D-branes. These unstable branes are characterized by having a single tachyonic mode of mass$^2 = -\frac{1}{2}$ (in $\alpha' = 1$ unit) living on their world-volume. The tachyon potential $V(T)$, which is even under the change of sign of the tachyon field T, has a local maximum at $T = 0$ and a pair of global minima at $\pm T_{min}$ where the negative contribution from the potential exactly cancels the tension of the D-brane. As a result the configuration where the tachyon potential is at its minimum corresponds to vacuum without any D-branes, and fluctuations of the open string field around these minima do not contain any perturbative open string states in the spectrum[1].

One can study time dependent solutions where the tachyon rolls away from near the top of the potential towards the minimum of the potential. If we consider such a rolling tachyon solution on an unstable Dp-brane located at $x^\alpha = 0$ ($(p+1) \le \alpha \le 9$), where we denote by x^α the coordinates transverse to the D-brane, by x^0 the time coordinate and by x^i ($1 \le i \le p$) the spatial directions tangential to the Dp-brane, then the energy momentum tensor associated with the rolling tachyon solution takes the form[2, 3]:

$$T_{00} = \text{constant}, \quad T_{ij}(x^0) = p(x^0)\, \delta_{ij}\, \delta(\vec{x}_\perp), \quad T_{i0} = T_{0\alpha} = T_{i\alpha} = T_{\alpha\beta} = 0\,. \tag{1}$$

Here $p(x^0)$ is a function of the time coordinate x^0 and is commonly known as the pressure of the system. $\vec{x}_\perp = (x^{p+1}, \ldots x^9)$ denotes the coordinate vector transverse to the Dp-brane. Another important quantity is the dilaton charge density associated with the rolling tachyon configuration, defined as the coupling of the rolling tachyon configuration to the dilaton field at fixed *string metric*. This takes the form:

$$Q(x^0)\, \delta(\vec{x}_\perp)\,. \tag{2}$$

Explicit forms of $p(x^0)$ and $Q(x^0)$ for a given T_{00} is known[2, 3, 4, 5]. To specify them we need to consider two cases separately.

1. First consider the case where the total energy density T_{00} of the system is less than the tension $\widetilde{\mathcal{T}}_p$ of the non-BPS Dp-brane. In this case we can parametrize T_{00} as

$$T_{00} = \widetilde{\mathcal{T}}_p \cos^2(\pi\widetilde{\lambda})\,. \tag{3}$$

   $p(x^0)$ and $Q(x^0)$ for this system are given by[3]

$$\begin{aligned} &p(x^0) = -\widetilde{\mathcal{T}}_p\, f(x^0), \qquad Q(x^0) = \widetilde{\mathcal{T}}_p f(x^0)\,, \\ &f(x^0) = \frac{1}{1 + e^{\sqrt{2}x^0} \sin^2(\widetilde{\lambda}\pi)} + \frac{1}{1 + e^{-\sqrt{2}x^0} \sin^2(\widetilde{\lambda}\pi)} - 1\,. \end{aligned} \tag{4}$$

For any fixed $\widetilde{\lambda}$, $f(x^0) \to 0$ as $x^0 \to \infty$, and hence the pressure and the dilaton charge density vanishes asymptotically. The other imporatnt point to note is that at $\widetilde{\lambda} = \frac{1}{2}$ all components of the stress tensor and the dilaton charge density vanish, indicating that this gives the vacuum without any D-brane.

2. Next we consider the case where the total energy density T_{00} of the system is larger than the tension $\widetilde{\mathcal{T}}_p$ of the non-BPS Dp-brane. In this case we can parametrize T_{00} as

$$T_{00} = \widetilde{\mathcal{T}}_p \cosh^2(\pi\widetilde{\lambda}) \,. \tag{5}$$

$p(x^0)$ and $Q(x^0)$ for this system are given by[3]

$$\begin{aligned} & p(x^0) = -\widetilde{\mathcal{T}}_p\, f(x^0), \qquad Q(x^0) = \widetilde{\mathcal{T}}_p f(x^0)\,, \\ & f(x^0) = \frac{1}{1 + e^{\sqrt{2}x^0}\sinh^2(\widetilde{\lambda}\pi)} + \frac{1}{1 + e^{-\sqrt{2}x^0}\sinh^2(\widetilde{\lambda}\pi)} - 1\,. \end{aligned} \tag{6}$$

Again we see that for any fixed $\widetilde{\lambda}$, $f(x^0) \to 0$ as $x^0 \to \infty$, and hence the pressure and the dilaton charge density vanish asymptotically.

Coupling to closed strings and open-closed string duality conjecture

So far in our discussion we have ignored the coupling of the D-brane to closed strings. Since the rolling tachyon background acts as a time dependent source for various closed string fields, we expect that there will be closed string radiation from the D-brane as the tachyon rolls down towards the minimum of the potential. However, in the weak coupling limit we would expect this effect to be small, since the closed strings couple to the D-brane at order g where g is the string coupling constant. Explicit calculation[6, 7, 8] shows that this is indeed the case for individual closed string modes; the fraction of the D-brane energy carried away by each closed string mode is of order g and falls off exponentially as we go to higher massive closed string states. But due to the rapid growth of closed string density of states at large mass level, the picture changes dramatically when we sum over all the closed string modes. The results of [7, 8] for emission of closed strings from unstable D-branes all of whose spatial world-volume directions lie along a compact torus can be summarized as follows:

1. All the energy of the D-brane is radiated away into closed strings.

2. Most of the emitted energy is carried by closed strings of mass $\sim 1/g$.

3. Typical momentum transverse to the brane, and typical winding charge along the brane of an emitted closed string is of order $1/\sqrt{g}$. This in particular shows that

the typical velocity of a closed string along directions transverse to the brane is of order $\sqrt{g}$.

Given that all the energy of the D-brane is carried away by the closed strings, it would seem that the effect of closed string emission invalidates the open string analysis. However, before we make such a conclusion, let us compare the the properties of the emitted closed strings with those infered from the tree level open string analysis[9]. First of all, tree level open string analysis tells us that the final system has:

$$Q/T_{00} = 0\,. \tag{7}$$

On the other hand by examining the closed string world-sheet action in the background *string metric* $G_{\mu\nu}$, the anti-symmetric tensor field $B_{\mu\nu}$ and the dilaton Φ at zero momentum,

$$S_{world-sheet} = \int d^2z (G_{\mu\nu}(X) + B_{\mu\nu}(X))\partial_z X^\mu \partial_{\bar{z}} X^\nu\,, \tag{8}$$

we see that the closed string world-sheet does not couple to the zero momentum dilaton, provided we take the *string metric*, the anti-symmetric tensor field and the dilaton as independent fields. This shows the final state closed strings carry zero total dilaton charge. Hence the dilaton charge of the final state closed strings agrees with that computed in the open string description.

Next we note that the tree level open string analysis tells us that the final system has:

$$p/T_{00} = 0\,. \tag{9}$$

On the other hand, closed string analysis tells us that the final closed strings have mass m of order $1/g$, momentum $k_\perp$ transverse to the D-brane of order $1/\sqrt{g}$ and winding $w_\parallel$ tangential to the D-brane of order $1/\sqrt{g}$. For such a system the ratio of transverse pressure to the energy density is of order $(k_\perp/m)^2 \sim g$ and the ratio of tangential pressure to the energy density is of order $-(w_\parallel/m)^2 \sim -g$. Since both these ratios vanish in the $g \to 0$ limit, we again see that the pressure of the final state closed strings matches the result computed in the open string description.

Such agreements between open and closed string results also hold for more general cases, *e.g.* in the decay of unstable branes in the presence of electric field. Consider, for example, the decay of a Dp-brane along $x^1, \dots x^p$ plane, with an electric field e along the x^1 axis. In this case the final state is characterized by its energy-momentum tensor $T_{\mu\nu}$, source $S_{\mu\nu}$ for anti-symmetric tensor field $B_{\mu\nu}$ and the dilaton charge Q. One can show that in the $x^0 \to \infty$ limit[4]:

$$T^{00} = |\Pi|\, e^{-1}\,, \quad T^{11} = -|\Pi|\, e\,, \quad S^{01} = \Pi\,, \tag{10}$$

where Π is a parameter labelling the solution. All other components of $T_{\mu\nu}$ and $S_{\mu\nu}$, as well as the dilaton charge vanishes in this limit. It can be shown that these tree level open string results again agree exactly with the properties of the final state closed strings into which the D-brane decays[10].

Since in all these cases the tree level open string results for various properties of the final state agree with the properties of the closed strings produced in the decay of the brane, we are led to conjecture that the tree level open string theory provides a description of the rolling tachyon system which is *dual* to the description in terms of closed string emission[9, 10].[1,2] This is different from the usual duality between closed strings and open strings on stable D-branes where *one loop* open string theory contains information about closed strings. The difference is likely to be due to the fact that in the latter case the perturbative excitations around the stable minimum of the potential are open string excitations, whereas in the former case excitations around the stable minimum of the potential are closed string excitations.

In order to put the tree level open-closed string duality conjecture on a firmer footing, one must show that it arises from a more complete conjecture involving full quantum open string theory on an unstable D-brane. The full conjecture, suggested in [11], takes the following form:

There is a quantum open string field theory (OSFT) that describes the full dynamics of an unstable Dp-brane without an explicit coupling to closed strings. Furthermore, Ehrenfest theorem holds in the weakly coupled OSFT; the classical results correctly describe the evolution of the quantum expectation values.

Note that this conjecture *does not* imply that the quantum open string theory on a given system of unstable D-branes gives a complete description of the *full string theory.* It only states that this open string field theory describes a quantum mechanically consistent subsector of the full string theory, and is fully capable of describing the quantum decay process of an unstable D-brane. One of the consequences of this correspondence is that the notion of naturalness of a solution may differ dramatically in the open and the closed string description. A solution describing the decay of a single (or a few) unstable D-branes may look highly contrived in the closed string description, since a generic deformation of

[1]This correspondence has been checked only for the space averaged values of various quantities, and not for example, for the local distribution of the various charges like the stress tensor, dilaton charge and anti-symmetric tensor field charge. This is due to the fact that we can easily give a gauge invariant definition of the space-averaged quantities since they are measured by coupling to on-shell (zero momentum) closed string states, but it is more difficult to give a gauge invariant definition of local distribution of these charges[11].

[2]The precise mechanism of how the closed strings emerge from open string theory is still not understood. For some ideas see [12, 9].

this background in closed string theory may not be describable by the dynamics of a single D-brane, and may require a large (or even infinite) number of D-branes for its description in open string theory.

Lessons from $c = 1$ string theory

A concrete illustration of the open-closed string duality may be found in the $c = 1$ string theory[13, 14, 11]. The states in this theory are in one to one correspondence with the states of the free fermion field theory, with each fermion moving in an inverted harmonic oscillator potential:

$$V(q) = -\frac{1}{2}q^2 + \frac{1}{g}\,. \tag{11}$$

The ground state of the theory corresponds to all levels below zero being filled and all levels above zero being empty. The closed string states of the theory are related to the bosons obtained by bosonizing the fermion field[15]. On the other hand, the states of a single D0-brane of this theory correspond to a single fermion excited from the fermi level to a level above zero. Thus the quantum dynamics of a single D0-brane is described by an inverted harmonic oscillator Hamiltonian with potential given in (11), with a sharp cut-off on the energy eigenvalue ($E \geq 0$) due to the Pauli exclusion principle. This is a fully consistent quantum system. However, this set of states clearly form only a small subset of all the quantum excitations of the full $c = 1$ string theory. In particular we cannot describe generic closed string excitations involving particle-hole pair creation as a state of this single D0-brane quantum mechanics[11]. This makes the description of a D0-brane state in closed string theory look highly contrived, since this is a highly specialized coherent state of closed strings in which a finely tuned coherent closed string radiation form the D0-brane and decays back into a time reversed version of the initial incoming radiation[7, 8].[3] Any generic deformation of this incoming closed string radiation will give rise to a state that is not describable as single fermion excitations of the $c = 1$ string theory, *i.e.* as a state of a single D0-brane.

The lesson to be taken back from here is that if we are trying to find a closed string description of a D-brane decay process, we may have to forgo our usual notion of naturalness, and look for highly specialized closed string configurations. This does not mean that a generic closed string deformation of such a state is not an allowed state of string theory; but it is not an allowed state of the open string theory describing the D-brane under consideration. We shall now use this insight for describing the gravitational field configuration associated with the decay of a space-filling D-brane.

Tachyon driven cosmology

[3] Such classical solutions have been called S-brane in the literature[22].

The system that we have in mind is a superstring theory compactified on a six dimensional compact manifold so that we have 3 large spatial directions. We shall assume that all the moduli are frozen, and that the universe has a bulk cosmological constant Λ, as in the KKLT model[16]. In this theory we take an unstable D-brane that extends along the three large spatial directions. If for example we are considering type IIB string theory as in [16], we can take an unstable D4-brane wrapped on a one cycle that arises by modding out the Calabi-Yau space by a discrete subgroup.[4] In this case, following [17, 7, 18, 19, 20, 21] we shall model this system by the effective action:

$$\begin{aligned} S &= -\frac{1}{16\pi G}\int d^4x\left[-\sqrt{-\det g}\,R + V(T)\sqrt{-\det(g_{\mu\nu}+\partial_\mu T\partial_\nu T)} + \Lambda\sqrt{-\det g}\right], \\ & \qquad V(T) = V_0/\cosh(T/\sqrt{2})\,, \end{aligned} \tag{12}$$

where $V_0 \sim g^{-1}$ is the mass per unit three volume of the unstable brane and the Newton's constant G is of order g^2. We shall assume that $\Lambda << V_0$. We look for a spatially homogeneous time dependent solution of this equation in the Friedman-Robertson-Walker (FRW) form:

$$\begin{aligned} T &= T(x^0), \\ ds^2 &= -(dx^0)^2 + a(x^0)^2\left(\frac{dr^2}{1-kr^2} + r^2 d\theta^2 + \sin^2\theta d\phi^2\right), \quad k = 0 \quad \text{or} \quad \pm 1\,. \end{aligned} \tag{13}$$

The equations of motion for a and T derived from the action (12) take the form:

$$\begin{aligned} \frac{\ddot a}{a} &= \frac{8\pi G}{3}\left[\Lambda + \frac{V(T)}{\sqrt{1-\dot T^2}} - \frac{3}{2}\frac{\dot T^2 V(T)}{\sqrt{1-\dot T^2}}\right] \\ \ddot T &= -(1-\dot T^2)\left[\frac{V'(T)}{V(T)} + 3\,\dot T\,\frac{\dot a}{a}\right]. \end{aligned} \tag{14}$$

The Friedman equation, obtained by varying the action with respect to g_{00}, takes the form:

$$\left(\frac{\dot a}{a}\right)^2 = -\frac{k}{a^2} + \frac{8\pi G}{3}\left[\frac{V(T)}{\sqrt{1-\dot T^2}} + \Lambda\right]. \tag{15}$$

This is a constraint equation. The time derivative of this equation is automatically satisfied once eqs.(14) is satisfied.

[4] We shall not worry about the relative magnitude of various mass scales and try to justify the use of the action (12) from first principles; instead we shall simply use (12) as our starting point as has been done in all the recent papers on tachyon matter cosmology.

In order to solve these equations we need to know the initial conditions. Since there are two variables T and a, satisfying second order differential equation (14), we might naively expect that four initial conditions, giving the values of T, $\dot{T}$, a and $\dot{a}$ at $x^0 = 0$, are needed to find the solution. However the Friedman equation (15) gives one constraint among the four initial conditions. This leaves us with three independent initial conditions. By choosing the origin of time suitably we can set either T to $\dot{T}$ to zero at $x^0 = 0$, therby eliminating one more initial condition[2, 3]. We shall for definiteness consider the class of solutions where $\dot{T} = 0$ at $x^0 = 0$. In this case we need two initial conditions to determine the solution completely. Without loss of generality we can take them to be the values of T and $\dot{a}$ at $x^0 = 0$.

According to the open-closed duality conjecture, the decay of an unstable D-brane can be described completely by working in the open string field theory. In this theory, if we ignore the effect of massive open string modes, the inequivalent classical solutions describing the time evolution of a homogeneous tachyon field is described by one initial condition[2, 3]. For definiteness we shall take $\dot{T} = 0$ at $x^0 = 0$ and use the value of the tachyon field T at $x^0 = 0$ as the parameter labelling inequivalent solutions. Thus we see that the open string description of the rolling tachyon solution has one less parameter compared to the description in which we use both open and closed string fields (T and $g_{\mu\nu}$) to describe the solution. This is not necessarily a contradiction, since as we have pointed out already, the open string description of the system does not have the capability of describing an arbitrary configuration in string theory, but can describe only a special class of configurations which describe the decay of a D-brane. What this mismatch of numbers indicates is that only a one parameter subspace of the two parameter family of solutions of eqs.(14), (15) corresponds to the decay of a 'pure D-brane' without any additional closed string background. Fortunately in this case it is easy to identify this one parameter family of solutions. For this we note from eq.(4) that the rolling tachyon solution describing the decay of a D-brane in open string theory is time reversal symmetric. Thus the solution of eqs.(14), (15) representing the decay of a pure D-brane must also be time reversal symmetric.[5] This gives the initial conditions on a and T to be:

$$\dot{a} = 0, \quad \dot{T} = 0, \quad T = T_0 \,, \qquad \text{at} \quad x^0 = 0 \,. \tag{16}$$

The Friedman equation now gives:

$$k = 1, \qquad a(x^0 = 0) = \left[\frac{8\pi G}{3} \left(V(T_0) + \Lambda\right)\right]^{-1/2} \equiv (H_0)^{-1} \,. \tag{17}$$

[5] Examples of time reversal symmetric cosmological solutions in the context of matrix model have been considered in [23].

Since $G \sim g^2$ and $V(T_0) \sim g^{-1}$, we see that for small g $H_0 \sim g^{1/2}$.

The analysis of the equations of motion may be carried out as follows. Since initially $\dot{T} = 0$, for sufficiently small T_0 there will be an interval of time during which T remains fixed near T_0. The first equation in (14) and the initial condition (16) shows that during this period a grows as

$$a \simeq \cosh(H_0 x^0) , \tag{18}$$

where H_0 has been defined in (17). In particular if $T_0 = 0$ then T does not evolve, and (18) becomes exact. Since during this evolution the scale factor a grows exponentially, the k/a^2 term in the Friedman equation (15) decays rapidly compared to the energy density in the tachyon field and the bulk cosmological constant Λ as given by the right hand side of this equation.

For any non-zero T_0, however, the tachyon eventually starts evolving. Once T is of order 1, V'/V is negative and of order 1. On the other hand, $\dot{a}/a$, which is of order H_0, is small in the weak coupling limit since H_0 is of order $g^{1/2}$. Thus we expect the $\dot{T}\dot{a}/a$ term in the second equation in (14) to be small in magnitude compared to the $V'(T)/V(T)$ term. As a result the term inside the square bracket in the right hand side of this equation is negative and $\dot{T} = 1$ is an attractive fixed point of this equation[24, 21]. As T evolves and $\dot{T}$ approaches its limiting value 1, the energy density $V(T)/\sqrt{1-\dot{T}^2}$ associated with the tachyon field behaves like pressureless matter[25, 24] and falls off as a^{-3}. Since the curvature term k/a^2 in eq.(15) falls off as a^{-2}, eventually the curvature term dominates over the tachyon matter term proportional to $V(T)/\sqrt{1-\dot{T}^2}$.

The crucial question that determines the subsequent evolution of the universe is: how does the cosmological constant term Λ compare with the curvature term at this cross-over point? If the magnitude of the curvature term is large compared to Λ at this point, then soon after the cross-over, the curvature term overcomes the combined contribution from tachyon matter and Λ and brings the expansion to a halt. After this the universe begins to recollapse, and eventually ends up in a big crunch. By time reversal symmetry such a universe must also have began with a big bang. On the other hand if the curvature term is small compared to the cosmological constant term at the cross-over point, then the right hand side of the Friedman equation never vanishes, and the universe continues to expand. Thus there is no singularity in the future, and hence, by time reversal symmetry, no singularity in the past.

For any given Λ, which of these two scenarios takes place depends on the initial value T_0 of T. Suppose a_f is the value of the scale factor at the end of the initial inflationary period. At this stage the energy density stored in the tachyon field is of the same order as the initial energy density $V_0 \equiv V(0)$. After this the tachyon contribution begins to behave

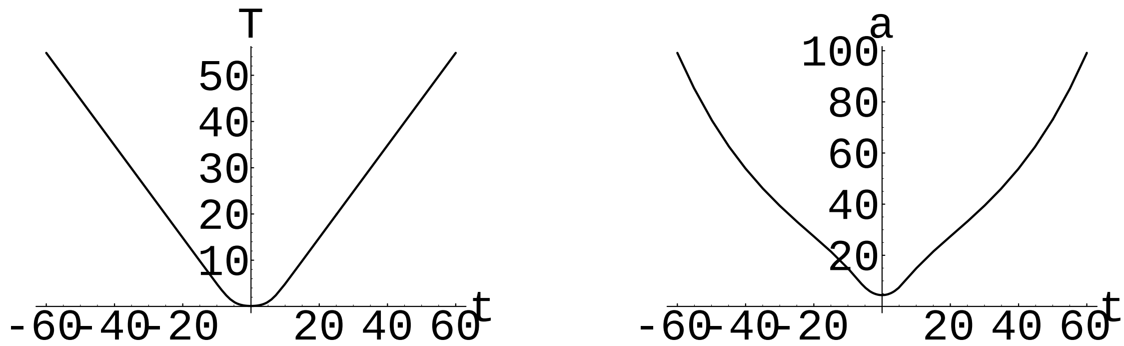

Figure 1: Numerical results for the time evolution of T and a for $8\pi GV(0)/3 = .05$, $8\pi G\Lambda/3 = .001$, $T_0 = .1$. In this case a is always positive and hence the solution is non-singular.

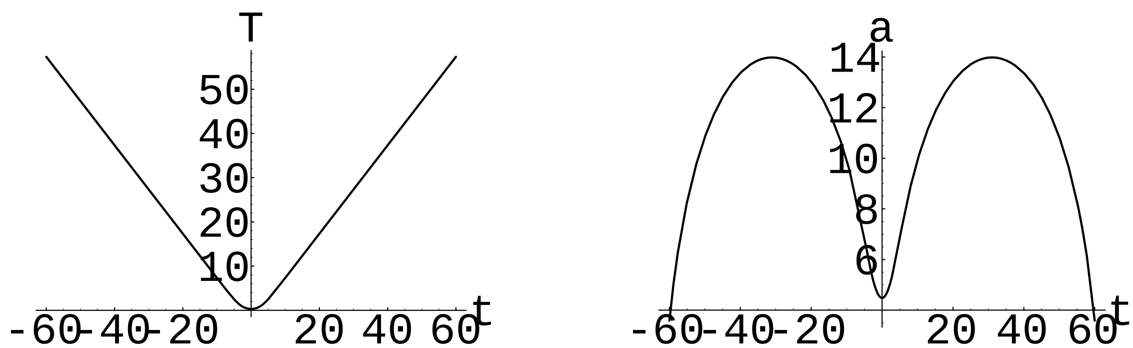

Figure 2: Numerical results for the time evolution of T and a for $8\pi GV(0)/3 = .05$, $8\pi G\Lambda/3 = .001$, $T_0 = .32$. In this case a vanishes at a finite time, making the solution singular.

like matter and its energy density falls off as a^{-3}. Then during further expansion we may estimate the tachyon contribution to the Friedman equation to be of order $V_0 a_f^3/a^3$, and write the Friedman equation as:

$$\left(\frac{\dot{a}}{a}\right)^2 \sim -\frac{1}{a^2} + \frac{8\pi G}{3}\left[\frac{V_0 a_f^3}{a^3} + \Lambda\right] . \tag{19}$$

The value of a where the curvature term becomes comparable to the tachyon contribution is

$$a_{cross} \sim G\, V_0 a_f^3 \,. \tag{20}$$

Thus in order that at this point the Λ term dominates over the curvature term, we need:

$$\Lambda >> G^{-1} \, (a_{cross})^{-2} \sim G^{-3} (V_0)^{-2} (a_f)^{-6} \,. \tag{21}$$

a_f is determined by the initial value T_0 of T. In particular smaller is the value of T_0, larger is the duration of the initial near exponential expansion, and larger is the value of a_f. Since $a_f = \infty$ for $T_0 = 0$, we see that for any Λ there is a critical value T_c such that for all $T_0 < T_c$ we satisfy the inequality (21) and get a non-singular cosmological solution. In order to illustrate this point we have plotted in figs. 1 and 2 the numerical results for the time evolution of T and a for $8\pi G V(0)/3 = .05$, $8\pi G\Lambda/3 = .001$, with two initial conditions on T: $T = .1$ at $x^0 = 0$ and $T = .32$ at $x^0 = 0$. For the first case a is always positive and the solution is non-singular, whereas for the second case a vanishes at some finite time and the solution hits a big-crunch singularity in the future. As is clear from these figures, due to time reversal symmetry of these solutions the existence of the final singularity is correlated with the existence of the initial singularity. Thus a solution that is free from big crunch singularity is also free from big bang singularity, and describes a completely non-singular cosmology. On the other hand, a solution with a big crunch singularity also has a big bang singularity. Thus if we hypothesize that initial conditions in string theory avoid big bang singularity, then it automatically forces the universe to have sufficient inflation so as to avoid a big crunch singularity.

The above example illustrates how open closed duality conjecture can restrict the class of allowed cosmological solutions describing the decay of space-filling branes. We should emphasize again that this does not imply that the solutions outside this class are not allowed, but just that their description in the language of open string field theory is more complicated and probably involves multiple D-branes.

Before we conclude we would like to add some cautionary remarks:

1. We should keep in mind that the tachyon effective action (12) is at best a qualitative description of the system, and even then its range of validity is limited. In particular at late time it always gives the equation of state $p/T_{00} = 0$. However given the interpretation of tachyon matter as a system of massive closed strings at high density[9, 10], we know that as the universe expands and the density falls below the Hagedorn density, the equation of state of the system should go from that of matter to that of radiation, and hence the universe will expand as a radiation dominated universe. However the same argument as given earlier shows that even in this case for sufficiently small T_0 there will always be enough inflation at the beginning so that

by the time the curvature term becomes comparable to the radiation contribution, the cosmological constant Λ takes over and hence the expansion continues.

2. The inflation produced by the solution is not a slow roll inflation, since V''/V is of order unity. Thus this solution cannot be used for a realistic model for inflationary cosmology. Nevertheless such solutions could have played a role in the early history of the universe, *e.g.* during a preinflationary phase.

3. We have not considered the possibility that the time reversal non-invariant classical solutions in tachyon effective field theory coupled to gravity, corresponding to putting arbitrary boundary condition on $\dot{a}$, could also correspond to solutions in open string field theory on a single space-filling D-brane, with complicated boundary conditions on the massive open string fields which do not respect time reversal symmetry. While this is certainly a possibilty, given our experience in $c = 1$ matrix model that a generic closed string deformation cannot be described by an open string field configuration on a single D-brane, this seems unlikely.

4. Our analysis has been entirely classical. One could argue that when quantum corrections are taken into account then non-singular solutions of the type considered here, where the tachyon remains near the top of the potential for a long period, may not be possible due to quantum uncertainty. On the other hand, if we examine the quantum theory of an inverted harmonic oscillator (which approximates the tachyon dynamics in the open string theory near the top of the potential), the spectrum is continuous and there certainly is an eigenstate of the Hamiltonian with exactly zero eigenvalue. If open-closed duality conjecture holds, one could ask what will be the description of this state in the combined open-closed string theory. We hope that the wave-function of such a state will be peaked around the classical trajectory in the (a-T)-plane that we get by solving the coupled equations (14)-(17). An analysis of the quantum system corresponding to the classical equations (14)-(17) might give us some insight into this issue.

Finally we would like to note that in the case of the decay of unstable D-branes in $c = 1$ string theory the correct computation of the closed string radiation from the brane is obtained by using the Hartle-Hawking prescription[7, 8]. We have arrived at the open-closed duality conjecture based on the results of this computation, and applied it to determine the initial condition for cosmological solutions describing the decay of an unstable brane. It will be interesting to explore any possible relation that might exist between this proposal and the Hartle-Hawking prescription[26] for determining the allowed wave-functions of the universe.

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