

Tachyon Condensation and Brane Descent Relations in p -adic String Theory

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Abstract

It has been conjectured that an extremum of the tachyon potential of a bosonic D-brane represents the vacuum without any D-brane, and that various tachyonic lump solutions represent D-branes of lower dimension. We show that the tree level effective action of p -adic string theory, the expression for which is known exactly, provides an explicit realisation of these conjectures.

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Contents

1	Introduction and Summary	2
2	Solitonic q -branes of p -adic string theory	3
3	World-volume theory on the solitonic q -branes	6
4	Comparison with the world-volume theory on a Dirichlet q -brane	8
5	Comments	14

1 Introduction and Summary

The world-volume theory on the D-brane of a bosonic string theory contains a tachyonic mode. It has been conjectured that the tachyon potential has a non-trivial extremum where the potential energy of the tachyon exactly cancels the tension of the D-brane, and that this configuration represents the closed string vacuum without any D-brane[1]. It has been further conjectured that various tachyonic lump solutions on the D-brane world-volume represent D-branes of lower dimensions[2, 1]. These conjectures and their generalisations to superstring theories[3]–[7] have been tested by various methods[2, 6],[8]–[21]. However, since the exact effective action for the tachyon field is not known, there is no direct proof of these conjectures.

In this paper we point out that in the p -adic string theory introduced in [22]–[27] (see [28] for a review) we can explicitly check these conjectures. It should be emphasised that although the p -adic ‘string’ is an exotic object, the spacetime it describes is the familiar one³. In the p -adic open string theory, which in modern language can be regarded as the world-volume theory of a space-filling D-brane, the *exact* classical action of the tachyon field and various solutions of the equations of motion are known[24]. Among the known non-trivial solutions is a translationally invariant solution with the property that it is a local minimum of the potential, and that the propagator of the tachyon field describing fluctuations around this background has no physical pole. Thus this configuration has no physical open string excitations, and is naturally identified with the vacuum without a D-brane. The exact tachyon equation of motion of the p -adic string theory also has classical lump solutions for all codimension ≥ 1 , which approach the vacuum solution

³A different type of p -adic string was considered in [29].

far away from the core of the soliton. If the original open string theory is formulated in $(d - 1, 1)$ dimensional space-time⁴, then such a lump solution of codimension $(d - q - 1)$ describes a solitonic q -brane. We show that the world-volume theory on the solitonic q -brane agrees with the expected world-volume theory on a Dirichlet q -brane in the p -adic string theory, to the extent that we can compare them with the present knowledge. This provides strong evidence that these lump solutions can be identified as lower dimensional D-branes.

The paper is organised as follows. In section 2 we summarise the exact effective action of the tachyon in the p -adic string theory, the known solutions of the equation of motion derived from the action and their properties. In section 3 we analyse the world-volume theory of the solitonic q -brane, and in section 4 we compare this with the world-volume theory of a Dirichlet q -brane. Section 5 contains some comments on further extension of this work, and ends with speculation on its possible application to the study of tachyon condensation in ordinary bosonic string theory.

2 Solitonic q -branes of p -adic string theory

In ref.[22] p -adic string theory was defined as follows. Consider the expressions for various amplitudes in ordinary bosonic open string theory, written as integrals over the boundary of the world-sheet which is the real line \mathbf{R} . Now replace the integrals over \mathbf{R} by integrals over the p -adic field \mathbf{Q}_p with appropriate measure, and the norms of the functions in the integrand by the p -adic norms. These rules were subsequently derived from a local action defined on the “world-sheet” of the p -adic string[30, 28]. Using p -adic analysis, it is possible to compute N tachyon amplitudes at tree-level for all $N \geq 3$.

This leads to an *exact* action for the open string tachyon in d dimensional p -adic string theory. This action is given in ref.[24]

$$\begin{aligned} S &= \int d^d x \mathcal{L} \\ &= \frac{1}{g^2} \frac{p^2}{p-1} \int d^d x \left[-\frac{1}{2} \phi p^{-\frac{1}{2} \square} \phi + \frac{1}{p+1} \phi^{p+1} \right], \end{aligned} \quad (2.1)$$

where \square denotes the d dimensional Laplacian, ϕ is the tachyon field (after a rescaling and

⁴ There is as yet no compelling reason for a critical dimension in p -adic string theory, but the so called adelic formula[23, 28] relating the product of four tachyon amplitudes in p -adic strings for all primes p to that in the bosonic string suggests that they all have the same critical dimension $d = 26$.

a shift), g is the open string coupling constant, and p is an arbitrary prime number. We are using metric with signature $(-, +, + \dots +)$. If we denote by $(2\pi\alpha'_p)^{-1}$ the ‘tension of the p -adic string’ as defined in ref.[30], then our choice of units correspond to[30]

$$\alpha'_p = \frac{\ln p}{2\pi}. \quad (2.2)$$

We have added a constant term to the Lagrangian density \mathcal{L} so that it vanishes at $\phi = 0$. Fig.1 shows the qualitative features of the tachyon potential for different values of p .

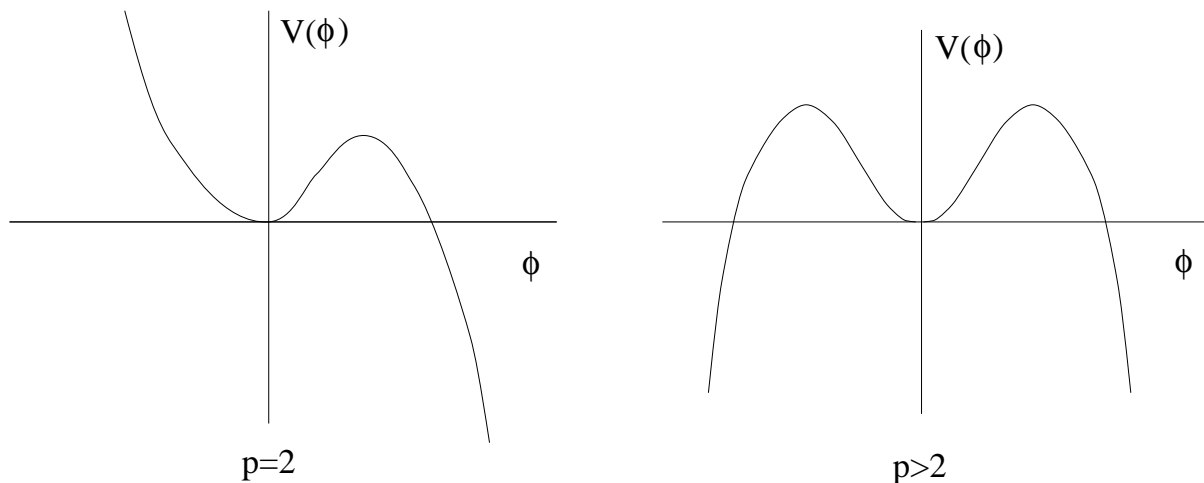


Figure 1: The effective tachyon potential for the p -adic string.

The equation of motion derived from this action is,

$$p^{-\frac{1}{2}\square}\phi = \phi^p. \quad (2.3)$$

Different known solutions of this equation are as follows[24]:

- The configuration $\phi = 1$ is the original vacuum around which we quantised the string⁵. We shall call this the D- $(d-1)$ -brane solution. The energy density associated with this configuration, which can be identified as the tension T_{d-1} of the D- $(d-1)$ -brane configuration, is given by

$$T_{d-1} = -\mathcal{L}(\phi = 1) = \frac{1}{2g^2} \frac{p^2}{p+1}. \quad (2.4)$$

⁵For $p \neq 2$, there is also an equivalent solution corresponding to $\phi = -1$. Since the action is symmetric under $\phi \rightarrow -\phi$, we shall restrict our analysis to solutions with positive ϕ .

- The configuration $\phi = 0$ denotes a configuration around which there is no perturbative physical excitation. We shall identify this with the vacuum configuration. By definition we have taken the energy density of this vacuum to be zero.
- The configuration:

$$\phi(x) = f(x^{q+1})f(x^{q+2}) \dots f(x^{d-1}) \equiv F^{(d-q-1)}(x^{q+1}, \dots, x^{d-1}), \quad (2.5)$$

with

$$f(\eta) \equiv p^{\frac{1}{2(p-1)}} \exp\left(-\frac{1}{2} \frac{p-1}{p \ln p} \eta^2\right), \quad (2.6)$$

describes a soliton solution with energy density localised around the hyperplane $x^{q+1} = \dots = x^{d-1} = 0$. This follows from the identity:

$$p^{-\frac{1}{2}\partial_\eta^2} f(\eta) = (f(\eta))^p. \quad (2.7)$$

We shall call (2.5), with f as in (2.6), the solitonic q -brane solution. Let us denote by $x_\perp = (x^{q+1}, \dots, x^{d-1})$ the coordinates transverse to the brane and by $x_\parallel = (x^0, \dots, x^q)$ those tangential to it. The energy density per unit q -volume of this brane, which can be identified as its tension T_q , is given by

$$T_q = - \int d^{d-q-1} x_\perp \mathcal{L}(\phi = F^{(d-q-1)}(x_\perp)) = \frac{1}{2g_q^2} \frac{p^2}{p+1}, \quad (2.8)$$

where,

$$g_q = g \left[\frac{p^2 - 1}{2\pi p^{2p/(2p-1)} \ln p} \right]^{(d-q-1)/4}. \quad (2.9)$$

From eqs.(2.4),(2.8) and (2.9) we see that the ratio of the tension of a q -brane to a $(q-1)$ -brane is

$$\frac{T_q}{T_{q-1}} = \left[\frac{2\pi p^{\frac{2p}{p-1}} \ln p}{p^2 - 1} \right]^{-\frac{1}{2}} = \frac{\sqrt{p^2 - 1}}{p^{\frac{p}{p-1}}} \frac{1}{2\pi \sqrt{\alpha'_p}}. \quad (2.10)$$

In the above equation we have used dimensional analysis and (2.2) to restore factors of α'_p . Note that the ratio (2.10) is independent of q . This is a feature of the D-branes in ordinary bosonic string theory, and suggests that the solitonic q -branes of p -adic string theory should have interpretation as D-branes. This also suggests that the self-dual radius

R_{sd} of the p -adic string theory, where the tension $2\pi R_{sd}T_q$ of a wrapped q -brane is equal to the tension T_{q-1} of a $(q-1)$ -brane, is given by

$$R_{sd} = \frac{p^{p/(p-1)}}{\sqrt{p^2-1}} \sqrt{\alpha'_p}. \quad (2.11)$$

Note that as $p \rightarrow \infty$, this approaches the formula for the self-dual radius in ordinary bosonic string theory.

3 World-volume theory on the solitonic q -branes

Let us now consider a configuration of the type

$$\phi(x) = F^{(d-q-1)}(x_\perp)\psi(x_\parallel), \quad (3.1)$$

with $F^{(d-q-1)}(x_\perp)$ as defined in (2.5),(2.6). For $\psi = 1$ this describes the solitonic q -brane. Fluctuations of ψ around 1 denote fluctuations of ϕ localised on the soliton; thus $\psi(x_\parallel)$ can be regarded as one of the fields on its world-volume. We shall call this the tachyon field on the solitonic q -brane world-volume⁶. Substituting (3.1) into (2.3) and using (2.7) we get

$$p^{-\frac{1}{2}\square_\parallel}\psi = \psi^p, \quad (3.2)$$

where \square_\parallel denotes the $(q+1)$ dimensional Laplacian involving the world-volume coordinates x_\parallel of the q -brane. The action involving ψ can be obtained by substituting (3.1) into (2.1):

$$\begin{aligned} S_q(\psi) &= S\left(\phi = F^{(d-q-1)}(x_\perp)\psi(x_\parallel)\right) \\ &= \frac{1}{g_q^2} \frac{p^2}{p-1} \int d^{q+1}x_\parallel \left[-\frac{1}{2}\psi p^{-\frac{1}{2}\square_\parallel}\psi + \frac{1}{p+1}\psi^{p+1} \right], \end{aligned} \quad (3.3)$$

where g_q has been defined in eqn.(2.9).

Note that a solution of (3.2) gives an *exact* solution of the full equation of motion (2.3). Thus eq.(3.2) describes the dynamics of the mode ψ on the q -brane world-volume exactly. This does not mean that there are no other modes on the q -brane world-volume; rather what this implies is that it is possible to obtain a consistent truncation of the world-volume theory of the q -brane by setting all the modes except ψ to zero. In terms

⁶In the linearised approximation this tachyonic mode was discussed in ref.[31].

of scattering amplitudes this means that the tree level S-matrix on the q -brane world-volume, involving *only* external tachyon states, can be calculated exactly from the action (3.3).

Of the various other (infinite number of) modes living on the q -brane world-volume are the $(d - q - 1)$ massless modes ξ^i associated with translations of the brane in the $(d - q - 1)$ directions x_\perp transverse to the brane. Inclusion of these modes correspond to deformation of ϕ of the form

$$\phi(x) = F^{(d-q-1)}(x_\perp) \psi(x_\parallel) + \partial_{x_\perp^i} F^{(d-q-1)}(x_\perp) \xi^i(x_\parallel) + \dots . \quad (3.4)$$

Substituting this in eq.(2.3), and comparing the coefficients of the independent functions $(F(x_\perp))^p$ and $(F(x_\perp))^{p-1} \partial_{x_\perp^i} F(x_\perp)$ on both sides, we get the following equations of motion:

$$\begin{aligned} p^{-\frac{1}{2}\square_\parallel} \psi &= \psi^p + \mathcal{O}(\xi^2) \\ p^{-\frac{1}{2}\square_\parallel} \xi^i &= \psi^{p-1} \xi^i + \mathcal{O}(\xi^2). \end{aligned} \quad (3.5)$$

The above equations can be derived from the effective action:

$$\begin{aligned} S_q(\psi, \xi^i) &= \frac{1}{g_q^2} \frac{p^2}{p-1} \int d^{q+1} x_\parallel \left[-\frac{1}{2} \psi p^{-\frac{1}{2}\square_\parallel} \psi + \frac{1}{p+1} \psi^{p+1} \right. \\ &\quad \left. - C \left\{ \frac{1}{2} \xi^i p^{-\frac{1}{2}\square_\parallel} \xi^i - \frac{1}{2} \psi^{p-1} \xi^i \xi^i \right\} + \mathcal{O}(\xi^3) \right]. \end{aligned} \quad (3.6)$$

C is a normalisation constant whose value is not important to this order, as it can be changed by rescaling ξ^i .

$\psi = 1$ corresponds to the solitonic q -brane solution. For computing amplitudes involving the world-volume fields on the q -brane, we define the shifted field σ and rescaled fields χ^i through the relation

$$\psi = 1 + \frac{g_q \sigma}{p}, \quad \xi^i = \frac{g_q}{\sqrt{p} C} \chi^i, \quad (3.7)$$

and expand the action (3.6) in powers of σ and χ^i . This gives

$$\begin{aligned} S_q &= \frac{p}{p-1} \int d^{q+1} x_\parallel \left[-\frac{1}{2} \sigma p^{-\frac{1}{2}\square_\parallel - 1} \sigma + \frac{1}{g_q^2} \frac{p}{p+1} \left(1 + \frac{g_q \sigma}{p} \right)^{p+1} - \frac{\sigma}{g_q} - \frac{p}{2g_q^2} \right. \\ &\quad \left. - \frac{1}{2} \chi^i p^{-\frac{1}{2}\square_\parallel} \chi^i + \frac{1}{2} \left(1 + \frac{g_q \sigma}{p} \right)^{p-1} \chi^i \chi^i + \mathcal{O}(\chi^3) \right]. \end{aligned} \quad (3.8)$$

There is no linear term in σ in action (3.8) reflecting the fact that $\sigma = 0$ is a solution of the equation of motion. The momentum space σ and the χ^i propagators computed from this action are given by:

$$\begin{aligned}\Delta_{\sigma\sigma}(k) &= -i \frac{p-1}{p} \frac{1}{p^{\frac{1}{2}k^2-1} - 1}, \\ \Delta_{\chi^i\chi^j}(k) &= -i \frac{p-1}{p} \frac{1}{p^{\frac{1}{2}k^2} - 1} \delta_{ij}.\end{aligned}\tag{3.9}$$

The residues at the poles in the σ and the χ^i propagators (at $k^2 = 2$ and $k^2 = 0$ respectively) have the same values. This will help us compare the amplitudes involving σ 's and χ^i 's in the external legs.

4 Comparison with the world-volume theory on a Dirichlet q -brane

We shall now compare the world-volume action on the solitonic q -brane with that on the D- q -brane. We are immediately faced with the question whether it is possible to have *Dirichlet* branes in p -adic string theory. Fortunately, a ‘world-sheet’ approach to p -adic string has been developed in ref.[30]. According to these authors, this ‘world-sheet’ is a so called Bruhat-Tits tree — a Bethe lattice with $p+1$ nearest neighbours — the ‘boundary’ of which is the field of p -adic numbers \mathbf{Q}_p . The generalisation of the Polyakov action is the lattice discretisation of the action for free scalar fields corresponding to the target space coordinates. Now one can either choose Neumann or Dirichlet boundary conditions as in the case of ordinary strings. It was shown in [30] that Neumann boundary conditions leads to the tachyon amplitudes postulated in [22, 24].

While this may be the proper way to define D-branes in p -adic string theory, we shall content ourselves with the continuation of the relevant formulæ from ordinary bosonic string theory. Thus, for our purposes the world-volume theory of a p -adic D- q -brane is defined by taking the expressions for various amplitudes for an ordinary D- q -brane, written as integrals over world-sheet coordinates of the appropriate vertex operators, and then replacing the integrals over real line by integrals over the p -adic field, with all the norms appearing in the integrand replaced by p -adic norms as in ref.[24]. In principle one should be able to derive these rules from the world-sheet description in ref.[30].

For amplitudes involving the external tachyons, described by the vertex operators of the type $e^{ik \cdot X_{\parallel}}$ with momentum k restricted to lie along the world-volume of the D- q -brane, the computation of the amplitude is identical to the one described in ref.[24]. Thus, following the analysis there, these S-matrix elements can be obtained from an effective action of the form:

$$\widehat{S}_q(\psi) = \frac{1}{\widehat{g}_q^2} \frac{p^2}{p-1} \int d^{q+1}x_{\parallel} \left[-\frac{1}{2} \psi p^{-\frac{1}{2} \square_{\parallel}} \psi + \frac{1}{p+1} \psi^{p+1} \right], \quad (4.1)$$

where the tachyon field ψ is shifted so that $\psi = 1$ describes the D- q -brane, and \widehat{g}_q denotes the coupling constant which characterises the strength of the interaction in the world-volume theory of the D- q -brane. Comparing this with (3.3) we see that the world-volume actions for the tachyon fields on the solitonic q -brane and the Dirichlet q -brane agree exactly if we choose:

$$\widehat{g}_q = g_q. \quad (4.2)$$

At present there is no independent derivation of \widehat{g}_q in terms of g , and hence we cannot verify eqn.(4.2) independently⁷. But assuming (4.2) to be true, we have a complete agreement between the world-volume theories involving the tachyon fields on the D- q -brane and the solitonic q -brane.

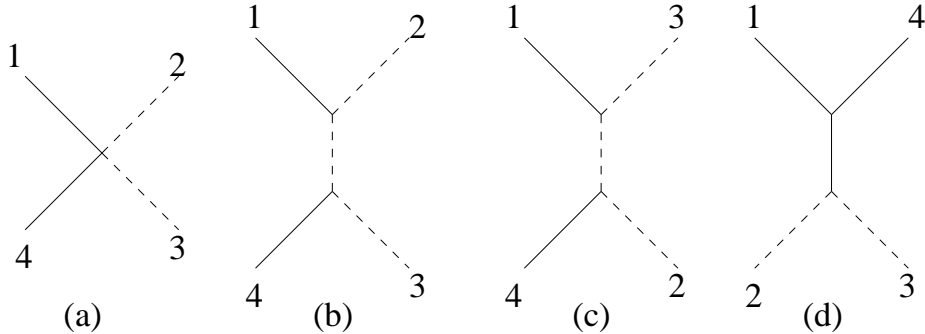


Figure 2: The Feynmann diagrams contributing to the amplitude $\langle \sigma \sigma \chi \chi \rangle$ on the solitonic q -brane. A dashed line denotes the χ propagator and a solid line denotes the σ propagator.

Next we shall compare the amplitude $\langle \sigma \sigma \chi^i \chi^j \rangle$ on the solitonic q -brane and the D- q -brane. First let us compute this on the solitonic q -brane using the action (3.8). The four

⁷This is related to the problem of computing the tension of the D- q -brane independently.

Feynmann diagrams contributing to it have been shown in Fig.2. These can be easily evaluated, and the answer is:

$$A_{ij}(k_1, k_2, k_3, k_4) = \delta_{ij} g_q^2 \left[\frac{p-2}{p} + \frac{p-1}{p} \left\{ \frac{1}{p^{k_1 \cdot k_2 + 1} - 1} + \frac{1}{p^{k_1 \cdot k_3 + 1} - 1} + \frac{1}{p^{k_1 \cdot k_4 + 1} - 1} \right\} \right], \quad (4.3)$$

with the contribution to the four terms in the right hand side of (4.3) coming from the Feynman digrams (a), (b), (c) and (d) respectively in Fig.2. In deriving (4.3) we have used the mass-shell conditions

$$k_1^2 = k_4^2 = 2, \quad k_2^2 = k_3^2 = 0. \quad (4.4)$$

Let us now evaluate the same amplitude on the D- q -brane. The vertex operator associated with the mode χ^i on the D- q -brane is given by⁸ $\partial X_{\perp}^i e^{ik \cdot X_{\parallel}}$. Inserting the χ^i and χ^j vertex operators carrying momenta k_2 and k_3 at 0 and 1 respectively, and the two σ vertex operators carrying momenta k_1 and k_4 at x and ∞ respectively, we can express the amplitude as:

$$\widehat{A}_{ij}(k_1, k_2, k_3, k_4) = \delta_{ij} \widehat{g}_q^2 \int_{\mathbf{Q}_p} dx |x|^{k_1 \cdot k_2} |1-x|^{k_1 \cdot k_3}. \quad (4.5)$$

Here $|\cdot|$ denotes the p -adic norm and integral over x is over the p -adic field. This is precisely the integral evaluated in [24]. Using the identity

$$k_1 \cdot k_2 + k_1 \cdot k_3 + 2 = -k_1 \cdot k_4, \quad (4.6)$$

we can express this amplitude as

$$\widehat{A}_{ij}(k_1, k_2, k_3, k_4) = \delta_{ij} (\widehat{g}_q)^2 \left[\frac{p-2}{p} + \frac{p-1}{p} \left\{ \frac{1}{p^{k_1 \cdot k_2 + 1} - 1} + \frac{1}{p^{k_1 \cdot k_3 + 1} - 1} + \frac{1}{p^{k_1 \cdot k_4 + 1} - 1} \right\} \right]. \quad (4.7)$$

This agrees precisely with eq.(4.3) for $\widehat{g}_q = g_q$.

In fact, it is possible to give a general argument showing that an amplitude with two external χ fields and N external σ fields for arbitrary N , computed from the action (3.8), agrees with the corresponding amplitude on a Dirichlet q -brane. To see this, let us consider the situation where we start with the action (2.1) with g replaced by another coupling constant \bar{g} , and compactify⁹ $(d-q-1)$ directions on circles of radii $1/\sqrt{2}$. Let u^i

⁸The notation ∂X is schematic, as care is needed to define the correct vertex operator that corresponds to the analogous one for ordinary string.

⁹Alternatively, we can work with the uncompactified theory, but just examine those modes of ϕ which carry either 0 or $\pm\sqrt{2}$ units of momentum in $(d-q-1)$ of the directions.

denote the compact coordinates and z^μ the non-compact ones, and consider an expansion of the field ϕ of the form:

$$\phi(x) = \tilde{\psi}(z) + \sqrt{\frac{C}{p}} \sum_{i=1}^{d-q-1} \tilde{\xi}^i(z) \left(\sqrt{2} \cos(\sqrt{2}u^i) \right) + \dots \quad (4.8)$$

We have restricted ϕ to be even under $u^i \rightarrow -u^i$ for each i ; this gives a consistent truncation of the theory at the tree level. The dots stand for higher momentum modes which will not be required for our analysis. Substituting this into (2.1) (with g replaced by \bar{g}) we get the action:

$$\begin{aligned} \frac{1}{\bar{g}^2} \frac{p^2}{p-1} \left(\frac{2\pi}{\sqrt{2}} \right)^{d-q-1} \int d^{q+1}z \left[-\frac{1}{2} \tilde{\psi} p^{-\frac{1}{2}\square_z} \tilde{\psi} + \frac{1}{p+1} \tilde{\psi}^{p+1} \right. \\ \left. - C \left\{ \frac{1}{2} \tilde{\xi}^i p^{-\frac{1}{2}\square_z} \tilde{\xi}^i - \frac{1}{2} \tilde{\psi}^{p-1} \tilde{\xi}^i \tilde{\xi}^i \right\} + \mathcal{O}(\tilde{\xi}^3) + \dots \right], \end{aligned} \quad (4.9)$$

If we identify

$$g_q^2 = \bar{g}^2 \left(\frac{\sqrt{2}}{2\pi} \right)^{d-q-1}, \quad (4.10)$$

this action looks identical to the one in (3.6) with the fields ψ , ξ^i replaced by $\tilde{\psi}$, $\tilde{\xi}^i$ and the identification $x_{\parallel} \sim z$. In particular we can define the analogues of eqs.(3.7)

$$\tilde{\psi} = 1 + \frac{g_q \tilde{\sigma}}{p}, \quad \tilde{\xi}^i = \frac{g_q}{\sqrt{pC}} \tilde{\chi}^i, \quad (4.11)$$

and compute the S-matrix elements around the vacuum $\tilde{\psi} = 1$ by expanding (4.9) in a power series in $\tilde{\sigma}$ and $\tilde{\chi}^i$. Similarity of (4.9) and (3.6) (and hence (3.8)) shows that the S-matrix elements computed from the action (4.9) around the $\psi = 1$ background are identical to those computed from (3.8) around the $\tilde{\psi} = 1$ background. In particular the S-matrix element involving a $\tilde{\chi}^i$, a $\tilde{\chi}^j$, and an arbitrary number of $\tilde{\sigma}$ quanta for (4.9) is identical to the S-matrix element involving χ^i , χ^j and an arbitrary number of σ quanta in (3.8)¹⁰.

On the other hand, the S-matrix elements computed from (4.9) have direct string theory interpretation, as the action is obtained by compactifying a p -adic string theory[33]. In particular the amplitude $\langle \tilde{\chi}^i \tilde{\chi}^j \tilde{\sigma}^N \rangle$ is given in terms of correlation functions of $\tilde{\chi}^i$,

¹⁰Since the similarity of (3.6) and (4.9) holds only to quadratic order in χ ($\tilde{\chi}$), we can only make this claim for two or less external χ ($\tilde{\chi}$) particles.

$\tilde{\chi}^j$ and $N \tilde{\sigma}$ vertex operators on the upper half plane. The vertex operator for $\tilde{\sigma}$ is proportional to $e^{ik \cdot Z}$, whereas that for $\tilde{\chi}^i$ is given by $\sqrt{2} \cos(\sqrt{2}U^i) e^{ik \cdot Z}$. Comparing this with the corresponding computation for the D- q -brane we see that the $\tilde{\sigma}$ vertex operator is identical to the σ vertex operator with X_{\parallel} replaced by Z . The χ^i vertex operator on the D- q -brane, given by $\partial X_{\perp}^i e^{ik \cdot X_{\parallel}}$, looks different from the $\tilde{\chi}^i$ vertex operator even after we identify X_{\parallel} with Z . However if we note that on the boundary of the upper half plane the two point functions $\langle \sqrt{2} \cos(\sqrt{2}U^i(x_1)) \sqrt{2} \cos(\sqrt{2}U^j(x_2)) \rangle$ and $\langle \partial X_{\perp}^i(x_1) \partial X_{\perp}^j(x_2) \rangle$ are identical, both being equal to $\delta_{ij} |x_1 - x_2|^{-2}$, we can conclude that these particular amplitudes in the compactified string theory are indeed identical to those on the D- q -brane.

To summarise, we have shown that the amplitudes $\langle \chi^i \chi^j \sigma^N \rangle$ computed from (3.8) are identical to the corresponding amplitudes in the compactified string theory, which in turn are identical to the corresponding ones on the D- q -brane. This establishes the desired result. In presenting this argument we have not been careful about the overall normalisation factors, but the equality already established for the amplitudes $\langle \sigma^N \rangle$ and $\langle \chi^i \chi^j \sigma \sigma \rangle$ in the two theories guarantees that the overall normalisation factors also agree in the two theories.

This provides strong evidence that the solitonic q -branes of the p -adic string theory should be identified with Dirichlet q -branes.

It will be interesting to systematically extend this comparison to S-matrix elements involving more than two external χ^i states, and also to S-matrix elements involving higher level states. It is not easy to establish this in all generality, however we can consider a subset of the massive modes on the q -brane and show the agreement between the S-matrix elements on the solitonic q -brane and D- q -brane with at most two of these states on the external leg.

We start with the solitonic q -brane, and consider a generalisation of the expansion (3.4):

$$\phi(x) = F^{(d-q-1)}(x_{\perp}) \psi(x_{\parallel}) + \sum_{r=1}^{d-q-1} \sum'_{\{i_1, \dots, i_r\}} \partial_{x_{\perp}^{i_1}} \dots \partial_{x_{\perp}^{i_r}} F^{(d-q-1)}(x_{\perp}) \xi^{i_1 \dots i_r}(x_{\parallel}) + \dots, \quad (4.12)$$

where \sum' above denotes sum over those indices $\{i_1, \dots, i_r\}$ for which no two in the set are equal. In this case

$$\partial_{x_{\perp}^{i_1}} \dots \partial_{x_{\perp}^{i_r}} \left(F^{(d-q-1)}(x_{\perp}) \right)^p = p^r \left(F^{(d-q-1)}(x_{\perp}) \right)^{p-1} \partial_{x_{\perp}^{i_1}} \dots \partial_{x_{\perp}^{i_r}} F^{(d-q-1)}(x_{\perp}). \quad (4.13)$$

Substituting (4.12) into the equation of motion (2.3), and using eq.(4.13) we get

$$\begin{aligned} p^{-\frac{1}{2}\square_{\parallel}}\psi &= \psi^p + \mathcal{O}(\xi^2) \\ p^{-\frac{1}{2}\square_{\parallel}}\xi^{i_1\dots i_r} &= p^{1-r}\psi^{p-1}\xi^{i_1\dots i_r} + \mathcal{O}(\xi^2). \end{aligned} \quad (4.14)$$

The action involving these fields is given by

$$\begin{aligned} S_q(\psi, \xi) &= \frac{1}{g_q^2} \frac{p^2}{p-1} \int d^{q+1}x_{\parallel} \left[-\frac{1}{2}\psi p^{-\frac{1}{2}\square_{\parallel}}\psi + \frac{1}{p+1}\psi^{p+1} \right. \\ &\quad \left. - \sum_{r=1}^{d-q-1} \sum'_{\{i_1, \dots, i_r\}} C_r \left\{ \frac{1}{2}\xi^{i_1\dots i_r} p^{-\frac{1}{2}\square_{\parallel}+r-1}\xi^{i_1\dots i_r} - \frac{1}{2}\psi^{p-1}\xi^{i_1\dots i_r}\xi^{i_1\dots i_r} \right\} + \mathcal{O}(\xi^3) \right]. \end{aligned} \quad (4.15)$$

C_r is a normalisation constant which can be absorbed into the definition of $\xi^{i_1\dots i_r}$. From this we see that the mass-shell constraint for $\xi^{i_1\dots i_r}$, in the $\psi = 1$ background, is

$$k^2 = 2(1-r). \quad (4.16)$$

In order to compare this with the world-volume theory on the D- q -brane, we need to first identify the vertex operator corresponding to the mode $\xi^{i_1\dots i_r}$. We take this to be

$$V_{i_1\dots i_r} = \partial X_{\perp}^{i_1} \dots \partial X_{\perp}^{i_r} e^{ik \cdot X_{\parallel}}. \quad (4.17)$$

This describes a physical state satisfying the same mass shell constraint as eq.(4.16) as long as the indices in the set $\{i_1, \dots, i_r\}$ are all different.

We shall now compare the S-matrix elements computed from the action (4.15) with two or less external ξ -legs to that computed directly in the D- q -brane. For two external tachyons and two external ξ we can do this explicitly and verify that it agrees with the corresponding computation on the D- q -brane. The computation is identical to the one discussed earlier. For arbitrary number of external tachyon legs, one can generalise the argument given for external χ^i -legs. The key ingredient of this argument is that the two point function of $\partial X_{\perp}^{i_1} \dots \partial X_{\perp}^{i_r} e^{ik \cdot X_{\parallel}}$ and $\partial X_{\perp}^{i'_1} \dots \partial X_{\perp}^{i'_r} e^{ik' \cdot X_{\parallel}}$ for the D- q -brane is identical to that between the vertex operators $(\sqrt{2})^r \cos(\sqrt{2}U^{i_1}) \dots \cos(\sqrt{2}U^{i_r}) e^{ik \cdot Z}$ and $(\sqrt{2})^r \cos(\sqrt{2}U^{i'_1}) \dots \cos(\sqrt{2}U^{i'_r}) e^{ik' \cdot Z}$ of the compactified string theory.

5 Comments

- We have shown that one can get a consistent truncation of the world-volume theory on a D- q -brane in p -adic string theory by keeping only the tachyonic mode. Thus by examining the tree level tachyon amplitudes in the world-volume theory we shall not discover the existence of the other modes. This suggests that there may be other (massless and massive) modes living on the world-volume of the space-filling D- $(d - 1)$ -brane as well, inspite of the fact that there are no poles in the tachyon S-matrix elements corresponding to these states. Indeed, ref.[32] attempted to generalise the p -adic string amplitudes to external vector states. If these modes are present they will give rise to new degrees of freedom on the solitonic q -brane, and will have to be taken into account in comparing the world-volume theory on the D- q -brane with that on the solitonic q -brane.
- It will also be of interest to compute the tension of a Dirichlet q -brane in the p -adic string theory independently, and compare with eq.(2.8) describing the tension of a solitonic q -brane. This will require careful analysis of the cylinder amplitude, and a proper understanding of the closed string sector of the theory.
- It has been shown in ref.[24] that it is possible to assign Chan-Paton factors to the open string states of a p -adic string theory. This shows the existence of multiple D- $(d - 1)$ -branes. Furthermore if there are massless gauge fields in the spectrum of open strings in the p -adic string theory, and if there is a T-duality transformation relating the D- $(d - 1)$ -brane to D- q -brane, then by switching on Wilson lines corresponding to the gauge fields followed by a T-duality transformation, we can produce static configuration of D- q -branes separated in space. It will be interesting to examine if the equation of motion (2.3) admit such solutions. Ideas developed in ref.[34] may be useful in this context.
- It is natural to ask if this analysis has any relevance to the ordinary bosonic string theory. Firstly, we would like to point out that even if the tachyon potential in the p -adic string theory is totally unrelated to that in the ordinary bosonic string theory, it can be regarded as a toy model which nicely illustrates the features expected of the full bosonic string field theory action. Besides, there is evidence of close relationship between tachyon amplitudes in the p -adic and ordinary bosonic string theory[23, 30].

Thus one might hope that the full tachyon effective action in bosonic string field theory is related in some way to the tachyon effective action in p -adic string theory.

In this direction, we cannot resist the temptation to point out some apparent similarities between the equation of motion (2.3) and that in the open bosonic string field theory[35]. To lowest order in the level truncation scheme[11], the tachyon equation of motion in open bosonic string field theory may be written as[18]

$$\left[(\alpha' \square + 1) e^{-c \alpha' \square} - 2 \right] \phi = \bar{g} \phi^2, \quad (5.1)$$

where $c = \ln(3^3/4^2)$, \bar{g} is open string coupling constant after suitable normalisation, and ϕ is related to the original tachyon field T by a field redefinition $T = e^{-c \alpha' \square/2} \phi + \bar{g}^{-1}$, so that $\phi = 0$ is the vacuum without any D-brane, and $\phi = -1/\bar{g}$ denotes the D-brane. If we drop the first and the third terms on the left hand side of eq.(5.1) by hand, then this equation, after suitable rescaling of x and ϕ , reduces to eq.(2.3) for $p = 2$. Of course there is no justification for dropping these terms, so we shall not pursue this matter any further; but it is not inconceivable that some exact relation between bosonic string field theory and p -adic string theory will be discovered in the future.

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