

Fundamental Strings in Open String Theory at the Tachyonic Vacuum

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Abstract

We show that the world-volume theory on a D-p-brane at the tachyonic vacuum has solitonic string solutions whose dynamics is governed by the Nambu-Goto action of a string moving in (25+1) dimensional space-time. This provides strong evidence for the conjecture that at this vacuum the full (25+1) dimensional Poincare invariance is restored. We also use this result to argue that the open string field theory at the tachyonic vacuum must contain closed string excitations.

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1 Introduction and Summary

It has been conjectured that the tachyonic vacuum in open bosonic string theory on a D-brane describes the closed string vacuum without D-branes, and that various soliton solutions in this theory describe D-branes of lower dimension[1]. Similar conjectures have also been put forward for superstring theories[2, 3, 4]. Evidence for these conjectures come from both, first[5, 6] and second[7, 8, 9, 10] quantised string theories.

Given that all D-branes can be regarded as solitons in the open string field theory[11], one might wonder if the open string field theory could be used for a non-perturbative formulation of string theory[12]. For this one needs to show that not only the D-branes, but other known objects in string theory, namely the fundamental closed strings and the NS five-branes are also present in this open string field theory. Progress in identifying the fundamental string has been made in refs.[13, 14, 8, 15, 16]. In particular, in [14, 8, 15, 16] it was shown that the effective action[17] describing the dynamics of the D-brane around the tachyonic vacuum admits string-like classical solution whose tension matches that of a fundamental string. It was also established that on a D-25-brane world-volume, the dynamics of these strings is described by that of a Nambu-Goto string moving in $(25 + 1)$ dimensions.

On the world-volume of a D- p -brane embedded in the $(25+1)$ dimensional space-time, the full $(25+1)$ dimensional Poincare invariance is spontaneously broken to the product of $(p+1)$ dimensional Poincare group, and the $(25-p)$ dimensional rotation group. However, if the tachyonic ground state really represents the vacuum without a D-brane, then we expect that in this vacuum the full $(25+1)$ dimensional Poincare invariance should be restored. Thus the dynamics of the string-like solutions should be described by a Nambu-Goto action with $(25+1)$ dimensional target space rather than a $(p+1)$ dimensional target

space. This is what we shall demonstrate in this paper.³ Since the Nambu-Goto action in $(25+1)$ dimensional target space has full $(25+1)$ dimensional Poincare invariance, this result provides a strong support to the conjecture that at the tachyonic vacuum of the D- p brane the full $(25+1)$ dimensional Poincare invariance is restored.⁴

Since the D- p -brane world-volume is $(p + 1)$ dimensional, and the string solution lives on the D- p -brane, it may sound strange at first that this string actually moves in $(25+1)$ dimensions. The reason it can happen is that at the tachyonic vacuum the D-brane has vanishing tension, and hence it does not cost the D-brane any additional energy to adjust its world-volume to contain any given fundamental string world-sheet embedded in $(25+1)$ dimensional space-time. Thus the string world-sheet always lies inside the D- p -brane world-volume, as should be the case. The non-trivial fact here is that the dynamics of the string tangential to the D-brane world-volume, which is described by the gauge fields, and the dynamics transverse to the D-brane world-volume, which is described by the massless scalar fields associated with the transverse motion of the D-brane, are together described by the Nambu-Goto action in the full $(25+1)$ dimensional target space-time.

Although the dynamics of the string soliton constructed this way agrees with that of the fundamental string, there are some caveats. First of all the tension of the string is governed by the total amount of electric flux it carries, and only after properly taking into account the quantization rule for the electric flux one can show that the tension matches that of the fundamental string. Within the classical field theory which we shall be studying, there is no rationale for this quantization law. A related problem is as follows. Although the string solution constructed here has the correct degrees of freedom describing the dynamics of a fundamental string, it also has additional degrees of freedom corresponding to the energy density spreading out in the direction transverse to the original string solution instead of being confined in a thin tube along the string. We show that these problems can be avoided by making the solitonic string driven by an external open string. For this we consider the case where one of the directions transverse to the D- p -brane is compact, and we begin with a configuration of open strings starting on the D-brane, and ending on its image under translation along the compact direction. We then ask what happens when the tachyon on the D-brane rolls down to its ground state.

³This question was partially addressed in [15] where it was shown that in the approximation where the contribution to the Hamiltonian is dominated by the electric flux on the D-brane world-volume, there is a symmetry that exchanges the velocity tangential to the D-brane with the velocity transverse to the D-brane.

⁴Earlier string field theory analysis has provided evidence for the restoration of the translational invariance along directions transverse to the D-brane world-volume[18, 19].

We argue that at the tachyonic vacuum, the two ends of the original open string are connected by a flux line on the D-brane, with the total amount of flux fixed by the source (and the sink) of flux, namely the end points of the original open string on the brane. Furthermore, the condition for minimum energy prevents the flux from spreading, since the source and the sink of flux are point-like objects on the D- p -brane world-volume. The net result is a single fundamental string winding along the compact direction. Using a T-duality transformation along the compact direction we can then argue that the T-dual D- $(p + 1)$ -brane at the tachyonic vacuum must contain closed string excitations carrying momentum along the compact direction.

Related earlier work in refs.[20] analysed the dynamics of tensionless D-branes in a different formalism and found that the D-brane world-volume is foliated by string world-sheet. It will be interesting to explore the precise relation between these results and the static gauge results of refs.[14, 8, 15, 16] and the present paper.

The paper is organised as follows. In section 2 we review the result for the effective action on the D-brane world-volume at the tachyonic vacuum[17] and its Hamiltonian formulation[15]. In section 3 we show that given any solution of the equations of motion of a Nambu-Goto string moving in $(25+1)$ dimensional space-time, we can construct a solution of the equations of motion of the D- p -brane world-volume theory with energy density localised along the world-sheet of the corresponding Nambu-Goto string solution. This establishes that the D- p -brane world-volume theory admits string-like soliton solutions whose dynamics is governed by the Nambu-Goto action in $(25+1)$ dimensions. In section 4 we use this result to argue that the open string field theory, describing the D-brane world-volume theory at the tachyonic vacuum, must contain closed string excitations.

2 Low Energy Effective Field Theory on the D-brane at the Tachyonic Vacuum

We shall analyse the dynamics of massless fields living on a D- p brane at the tachyonic vacuum in the static gauge. Let us denote by x^μ ($0 \leq \mu \leq p$) the world-volume coordinates on the D-brane, by A_μ the U(1) gauge field living on the D-brane, and by Y^I ($p + 1 \leq I \leq 25$) the massless scalars representing the transverse coordinates of the brane. The action is given by[17]:

$$S = -V(T) \int d^{p+1}x \sqrt{-\det(\eta_{\mu\nu} + F_{\mu\nu} + \partial_\mu Y^I \partial_\nu Y^I)}, \quad (2.1)$$

where $V(T)$ is the tachyon potential which vanishes at the tachyonic vacuum $T = T_0$. We shall work in the gauge $A_0 = 0$, and denote by $\pi^i(x)$ and $p_I(x)$ the momenta conjugate to A_i and Y^I respectively for $1 \leq i \leq p$. As was shown in [15], the dynamics of the brane at the tachyonic vacuum is best described in the Hamiltonian formalism, with the Hamiltonian

$$H = \int d^p x \mathcal{H}, \quad (2.2)$$

with

$$\mathcal{H} = \sqrt{\pi^i \pi^i + p_I p_I + (\pi^i \partial_i Y^I)^2 + b_i b_i}, \quad (2.3)$$

where

$$b_i \equiv F_{ij} \pi^j + \partial_i Y^I p_I. \quad (2.4)$$

$F_{ij} = \partial_i A_j - \partial_j A_i$ is the magnetic field strength. The π^i 's satisfy a constraint:

$$\partial_i \pi^i = 0. \quad (2.5)$$

In writing down the Hamiltonian (2.2), (2.3) we have taken the tachyon field T to be frozen at its minimum $T = T_0$.⁵

Let us denote by $E_i = \partial_0 A_i$ the electric field strength. Then the Bianchi identities and the equations of motion derived from the Hamiltonian given in (2.2), (2.3) are given by

$$\partial_{[i} F_{jk]} = 0, \quad \partial_0 F_{ij} = \partial_i E_j - \partial_j E_i, \quad (2.6)$$

$$E_i = \frac{1}{\mathcal{H}} (\pi^i + \partial_i Y^I \pi^j \partial_j Y^I - F_{ij} b_j), \quad (2.7)$$

$$\partial_0 \pi^i + \partial_j \left(\frac{1}{\mathcal{H}} (\pi_j b_i - \pi_i b_j) \right) = 0, \quad (2.8)$$

$$\partial_0 Y^I = \frac{1}{\mathcal{H}} (p_I + \partial_i Y^I b_i), \quad (2.9)$$

$$\partial_0 p^I = \partial_i \left(\frac{1}{\mathcal{H}} (\pi^i \pi^j \partial_j Y^I + b_i p^I) \right). \quad (2.10)$$

For this system, there are conserved Noether currents $T_{\mu\nu}$ and $T_{\mu I}$ ($0 \leq \mu, \nu \leq p$, $(p+1) \leq I \leq 25$) associated with the translation along the spatial coordinates x^μ labelling the D- p -brane world-volume, as well as translation along the coordinates Y^I transverse to the world-volume. These are given by:

$$\begin{aligned} T_{00} &= \mathcal{H}, & T_{k0} &= -b_k, & T_{0i} &= -b_i, & T_{ki} &= \frac{1}{\mathcal{H}} (\pi^k \pi^i - b_k b_i), \\ T_{0I} &= p_I, & T_{kI} &= \frac{1}{\mathcal{H}} (\pi^k \pi^j \partial_j Y^I + b_k p^I), \end{aligned} \quad (2.11)$$

⁵Proposals for the effective action including tachyon kinetic term have been put forward in [21].

and satisfy

$$\eta^{\mu\nu}\partial_\mu T_{\nu\rho} = 0, \quad \eta^{\mu\nu}\partial_\mu T_{\nu I} = 0. \quad (2.12)$$

3 Fundamental String Solution

In this section we shall demonstrate that the equations of motion discussed in section 2 admit fundamental string solutions whose dynamics is identical to that of a Nambu-Goto string moving in (25+1) dimensional space-time. Using the results of [22], ref.[15] showed that if we set the Y^I 's to 0, then the dynamics of the solitonic string is described by the Nambu-Goto action in ($p+1$) dimensional space-time. The new result is the incorporation of the Y^I 's. Since the dynamics of a Nambu-Goto string in (25+1) dimensional space-time has full (25+1) dimensional Poincare invariance, our result gives strong support to the conjecture that the tachyonic vacuum of the D- p -brane represents a configuration where the full (25+1) dimensional Poincare invariance is restored.

Our strategy will be as follows. We shall show that for every configuration of a Nambu-Goto string satisfying the string equations of motion we can construct a solution of the equations of motion (2.5) - (2.10), with energy density localised along the string. For this we start by writing down the action of the Nambu-Goto string in (25+1) dimensional space-time:

$$S_{NG} = - \int d\tau d\sigma \sqrt{-\det(\eta_{MN}\partial_\alpha Z^M \partial_\beta Z^N)} \quad (3.1)$$

where ξ^α for $\alpha = 0, 1$ denote the world-volume coordinates of the string: $(\xi^0, \xi^1) \equiv (\tau, \sigma)$, Z^M ($0 \leq M \leq 25$) denote the space-times coordinates of the string, and η_{MN} is the Minkowski metric $diag(-1, 1, 1, \dots, 1)$. We shall choose the static gauge: ($Z^0 = \tau, Z^1 = \sigma$) and go to the Hamiltonian formalism. If we denote by P_s the momenta conjugate to Z^s for $2 \leq s \leq 25$, the Hamiltonian is given by:

$$H_{NG} \equiv \int d\sigma \mathcal{H}_{NG} = \int d\sigma \sqrt{1 + P_s P_s + \partial_\sigma Z^s \partial_\sigma Z^s + (P_s \partial_\sigma Z^s)^2}. \quad (3.2)$$

The equations of motion following from this Hamiltonian is given by:

$$\partial_\tau Z^s = \frac{1}{\mathcal{H}_{NG}} (P_s + \partial_\sigma Z^s P_t \partial_\sigma Z^t), \quad (3.3)$$

$$\partial_\tau P_s = \partial_\sigma \left(\frac{1}{\mathcal{H}_{NG}} (\partial_\sigma Z^s + P_s P_t \partial_\sigma Z^t) \right). \quad (3.4)$$

In these equations s and t indices take values $2, 3, \dots, 25$. For future use, we shall define

$$P_1 = - \sum_{s=2}^{25} P_s \partial_\sigma Z^s, \quad Z^1(\tau, \sigma) = \sigma. \quad (3.5)$$

With these definitions, it is straightforward to verify that eqs.(3.3), (3.4) are satisfied also for $s = 1$. (The sum over t in these equations still runs from 2 to 25).

Let $(Z^s(\tau, \sigma), P_s(\tau, \sigma))$ for $2 \leq s \leq 25$ be a solution of eqs.(3.3), (3.4). Now consider the following field configuration on the D- p -brane:

$$\begin{aligned}\pi^i(x^0, \dots x^p) &= \partial_\sigma Z^i(\tau, \sigma) f(x^0, \dots x^p) \Big|_{(\tau, \sigma) = (x^0, x^1)}, \\ p_I(x^0, \dots x^p) &= P_I(\tau, \sigma) f(x^0, \dots x^p) \Big|_{(\tau, \sigma) = (x^0, x^1)}\end{aligned}\tag{3.6}$$

where we have used the convention that the indices i, j, k run from 1 to p , the indices I, J, K run from $(p + 1)$ to 25, and the indices s, t run from 2 to 25. $f(x^0, \dots x^p)$ is an arbitrary function of the variables $(x^m - Z^m(x^0, x^1))$ for $2 \leq m \leq p$, and hence satisfies:

$$\partial_\sigma Z^i \partial_i f \Big|_{(\tau, \sigma) = (x^0, x^1)} = 0, \quad (\partial_\tau Z^i \partial_i f + \partial_0 f) \Big|_{(\tau, \sigma) = (x^0, x^1)} = 0.\tag{3.7}$$

The fields $Y^I(x^0, \dots x^p)$ and $F_{ij}(x^0, \dots x^p)$ are subject to the following set of conditions:

$$(\partial_\sigma Z^j \partial_j Y^I - \partial_\sigma Z^I) \Big|_{(\tau, \sigma) = (x^0, x^1)} = 0,\tag{3.8}$$

and

$$(F_{ij} \partial_\sigma Z^j + \partial_i Y^I P_I + P_i) \Big|_{(\tau, \sigma) = (x^0, x^1)} = 0,\tag{3.9}$$

but are otherwise unspecified. Using eqs.(3.6), (3.8) and (3.9) we can easily verify that for this background,

$$\begin{aligned}\mathcal{H}(x^0, \dots x^p) &= H_{NG}(\tau = x^0, \sigma = x^1) f(x^0, \dots x^p), \\ b_i(x^0, \dots x^p) &= -P_i(\tau = x^0, \sigma = x^1) f(x^0, \dots x^p), \\ \pi^j \partial_j Y^I(x^0, \dots x^p) &= \partial_\sigma Z^I(\tau = x^0, \sigma = x^1) f(x^0, \dots x^p).\end{aligned}\tag{3.10}$$

Using eqs.(3.3)-(3.7) and (3.10) we can now verify that eqs.(2.5), (2.8) and (2.10) are satisfied by this background. Thus in order to construct a solution of the full set of equations of motion (2.5)-(2.10) we need to show that it is possible to find $F_{\mu\nu}$ and Y^I satisfying the constraints (2.6), (2.7), (2.9), (3.8) and (3.9).

First we shall establish the existence of Y^I 's satisfying eqs.(2.9) and (3.8). (Note that the eq.(3.9) imposes a constraint on Y^I of the form $\partial_\sigma Z^i (\partial_i Y^I P_I + P_i) \Big|_{(\tau, \sigma) = (x^0, x^1)} = 0$, but due to eq.(3.5), this is automatically satisfied once eq.(3.8) is satisfied.) Using eqs.(3.6), (3.10), we shall now write eqs.(2.9) and (3.8) as follows:

$$\begin{aligned}\partial_0 Y^I &= \frac{1}{\mathcal{H}_{NG}} (P_I - \partial_i Y^I P_i), \\ \partial_1 Y^I &= (-\partial_\sigma Z^m \partial_m Y^I + \partial_\sigma Z^I),\end{aligned}\tag{3.11}$$

where the indices m, n, q run from 2 to p , and it will be understood from now on that τ and σ are to be identified with x^0 and x^1 respectively. We can now treat eqs.(3.11) as the equations determining the x^0 and x^1 evolution of the functions Y^I . (We replace the $\partial_1 Y^I$ appearing on the right hand side of the first equation by the right hand side of the second equation.) Existence of a solution to these equations requires the integrability condition:

$$\partial_1 \left(\frac{1}{\mathcal{H}_{NG}} (P_I - \partial_m Y^I P_m + P_1 (\partial_\sigma Z^m \partial_m Y^I - \partial_\sigma Z^I)) \right) - \partial_0 \left((-\partial_\sigma Z^m \partial_m Y^I + \partial_\sigma Z^I) \right) = 0. \quad (3.12)$$

It is a straightforward although tedious exercise to show that once eqs.(3.3), (3.4) are satisfied, eq.(3.12) is satisfied.

Thus it remains to show the existence of a set of $F_{\mu\nu}$ satisfying eqs.(2.6), (2.7) and (3.9). We begin with the F_{mn} 's ($2 \leq m, n, q \leq p$). We take them to satisfy the following identities:

$$\partial_{[m} F_{nq]} = 0, \quad (3.13)$$

and

$$\begin{aligned} \partial_1 F_{mn} + \partial_1 Z^q [x^0, x^1] \partial_q F_{mn} &= 0, \\ \partial_0 F_{mn} + \partial_0 Z^q [x^0, x^1] \partial_q F_{mn} &= 0. \end{aligned} \quad (3.14)$$

To see that it is possible to choose F_{mn} 's satisfying these conditions, we regard eqs.(3.14) as the evolution equation for F_{mn} in x^0 and x^1 from an initial configuration satisfying the Bianchi identities (3.13). It is easy to verify that the evolution equations (3.14) preserve the Bianchi identities at all values of x^0 and x^1 . It is also easy to verify the integrability of the equations (3.14):

$$\partial_0 (\partial_1 Z^q [x^0, x^1] \partial_q F_{mn}) - \partial_1 (\partial_0 Z^q [x^0, x^1] \partial_q F_{mn}) = 0. \quad (3.15)$$

Given F_{mn} satisfying (3.13), (3.14), we can use (3.10) to write eqs.(2.7) and (3.9) as follows:

$$F_{0i} = \frac{1}{\mathcal{H}_{NG}} (\partial_\sigma Z^i + \partial_i Y^I \partial_\sigma Z^I + F_{ij} P_j), \quad (3.16)$$

and

$$F_{i1} = -F_{in} \partial_\sigma Z^n - \partial_i Y^I P_I - P_i. \quad (3.17)$$

If eq.(3.17) is satisfied for $i = m$, then it is also automatically satisfied for $i = 1$ with the help of eqs.(3.5) and (3.11). Thus the independent equations in (3.17) are:

$$F_{m1} = -F_{mn} \partial_\sigma Z^n - \partial_m Y^I P_I - P_m. \quad (3.18)$$

This gives F_{m1} in terms of F_{mn} and other known quantities. Replacing the F_{m1} 's appearing on the right hand side of eq.(3.16) by the right hand side of eq.(3.18), we can now regard eqs.(3.16) as expressions for F_{01} and F_{0m} in terms of F_{mn} and other known quantities.

We now need to check that F_{0i} and F_{m1} defined through eqs.(3.16), (3.18) satisfy the remaining Bianchi identities:

$$\begin{aligned}\partial_0 F_{mn} + \partial_m F_{n0} + \partial_n F_{0m} &= 0, \\ \partial_1 F_{mn} + \partial_m F_{n1} + \partial_n F_{1m} &= 0, \\ \partial_0 F_{m1} + \partial_m F_{10} + \partial_1 F_{0m} &= 0.\end{aligned}\tag{3.19}$$

It is straightforward to verify that all of these identities are consequences of eqs.(3.3), (3.4), (3.11), (3.13) and (3.14). This completes the construction of a solution of the complete set of equations of motion (2.5)-(2.10) of the D- p -brane world-volume field theory.

We shall now make a special choice of the function f :

$$f(x^0, \dots, x^p) = \prod_{m=2}^p \delta(x^m - Z^m(x^0, x^1)),\tag{3.20}$$

which satisfies eq.(3.7). Furthermore we take

$$Y^I(x^0, x^1, x^m = Z^m(x^0, x^1)) = Z^I(x^0, x^1),\tag{3.21}$$

which can be seen to be compatible with eqs.(3.11).⁶ As can be seen from eqs.(3.10), for the choice of f given in eq.(3.20), the energy density is localised along the surface $x^m = Z^m(x^0, x^1)$ for $2 \leq m \leq p$. Using eq.(3.21) we see that in the full (25+1) dimensional space-time, this describes the surface $x^s = Z^s(x^0, x^1)$ for $2 \leq s \leq 25$. This is precisely the world-sheet of the string described by the Nambu-Goto action (3.1). Thus our analysis shows that whenever the Nambu-Goto equation has a solution described by $Z^s(\sigma, \tau)$, there is a corresponding solution in the D- p -brane world-volume field theory with energy density localised along the world-sheet of the string. In other words, the D- p -brane world-volume theory contains a solution whose dynamics is exactly that of a Nambu-Goto string in (25+1)-dimensions.

Note, however, that the freedom of replacing the δ -function by an arbitrary function of $x^m - Z^m(x^0, x^1)$ means that besides the usual degrees of freedom of the fundamental

⁶Indeed, eqs.(3.11) and (3.14) can be interpreted as the requirement of vanishing of the derivatives of $(Y^I - Z^I)$ and F_{mn} along directions tangential to the string world-sheet. Thus we can solve these equations by taking $Y^I - Z^I$ and F_{mn} to be arbitrary functions g^I and g_{mn} respectively of $x^2 - Z^2(x^0, x^1), \dots, x^p - Z^p(x^0, x^1)$. Eq.(3.21) can then be satisfied by requiring that $g^I(0, \dots, 0) = 0$. The Bianchi identities (3.13) are satisfied by requiring that the functions g_{mn} satisfy $\partial_{[q} g_{mn]} = 0$.

string, our solution has additional degrees of freedom which corresponds to the freedom of spreading out the electric flux in directions transverse to the string. Also the overall normalization on the right hand side of eq.(3.20), which fixes the tension / charge of the string, is arbitrary. We shall return to these questions in the next section. There are also additional degrees of freedom stemming from the fact that eqs.(3.11), (3.13) and (3.14) do not determine Y^I and F_{mn} completely for a given configuration of the Nambu-Goto string. This is analogous to the spurious degeneracy found in [23]. It has been argued in [24] that this apparent degeneracy is due to the wrong choice of variables in describing the theory, and will disappear once we use the correct set of variables.

We shall end this section by writing down the expressions for the conserved Noether currents for the background described above. This is a straightforward exercise, and the results are as follows:

$$\begin{aligned}
T_{00}(x^0, \dots x^p) &= \mathcal{H}_{NG}(\tau, \sigma) \prod_{m=2}^p \delta(x^m - Z^m(\tau, \sigma)) \Big|_{(\tau, \sigma)=(x^0, x^1)}, \\
T_{0k}(x^0, \dots x^p) &= T_{k0}(x^0, \dots x^p) = P_k(\tau, \sigma) \prod_{m=2}^p \delta(x^m - Z^m(\tau, \sigma)) \Big|_{(\tau, \sigma)=(x^0, x^1)}, \\
T_{ki}(x^0, \dots x^p) &= \frac{1}{\mathcal{H}_{NG}} (\partial_\sigma Z^k \partial_\sigma Z^i - P_k P_i) \prod_{m=2}^p \delta(x^m - Z^m(\tau, \sigma)) \Big|_{(\tau, \sigma)=(x^0, x^1)}, \\
T_{0I}(x^0, \dots x^p) &= P_I(\tau, \sigma) \prod_{m=2}^p \delta(x^m - Z^m(\tau, \sigma)) \Big|_{(\tau, \sigma)=(x^0, x^1)}, \\
T_{kI}(x^0, \dots x^p) &= \frac{1}{\mathcal{H}_{NG}} (\partial_\sigma Z^k \partial_\sigma Z^I - P_k P_I) \prod_{m=2}^p \delta(x^m - Z^m(\tau, \sigma)) \Big|_{(\tau, \sigma)=(x^0, x^1)}.
\end{aligned} \tag{3.22}$$

Verification of the conservation laws (2.12) for $T_{\mu\nu}$ and $T_{\mu I}$ is a straightforward application of eqs.(3.3), (3.4). It is also a simple exercise to verify that the corresponding conserved charges $\int d^p x T_{00}$, $\int d^p x T_{0i}$ and $\int d^p x T_{0I}$ agree with the Noether charges of the Nambu-Goto string associated with translation invariance along x^0 , x^i and x^I directions respectively.

4 Closed Strings in the D-brane World-Volume Theory

In this section we shall use the results of the previous section to argue that at the tachyonic vacuum the D-brane world-volume theory must contain closed string excita-

tions. Identification of closed strings as closed flux lines has been discussed earlier in refs.[13, 14, 8, 15, 16]. The present construction is closely related, but differs in that here part of the closed string is formed by an external open string.

We begin with a thought experiment. Consider three well separated D-branes A , B and C , and a state on the world-volume of this system consisting of a fundamental string stretched from A to B , and another fundamental string stretched from B to C . Let us now ask: what happens to this state when the tachyon field on the brane B rolls down to its (local) minimum, but the branes A and C remain unchanged. The D-brane world-volume field theory analysis tells us that the ends of the AB and BC strings are source and sink of one unit of electric flux (measured in natural units) on the world-volume of the brane B . Thus the fate of the system is clear: the final configuration will consist of the AB string, the BC string, and an electric flux line (described by the solution given in section 3) on the B -brane world-volume connecting the end point of the AB string to the starting point of the BC string. Note that the condition for minimum energy will prevent the flux from spreading out as its source and sink are point objects.⁷ Furthermore there is precisely one unit of electric flux and hence its tension is equal to that of a fundamental string[14]. Thus it is natural to interpret the flux line as a fundamental string stretched from the end point of the AB string to the starting point of the BC string. (This string, as well as the external AB and BC strings, can adjust their positions and orientations so as to minimise the energy of the configuration). Thus the net result of this process is a single open string stretched between A and C . It is as if the tachyon condensation on the world-volume of the brane B joins the ends of the AB and BC strings by a fundamental string. Even before tachyon condensation on the brane B , the ends of AB and BC strings could join to produce an AC string. But there it was a perturbative quantum process, whereas the process described here is a non-perturbative classical process.

Now we consider a different system: take a single D-brane and an open string with both ends on this D-brane. One can use the same argument to conclude that when the tachyon condenses to its ground state, the two ends of the open string will be connected by a flux line, and once we identify the flux line as the fundamental string, we get a closed string state! This argument can be made more concrete as follows. Take a D-brane with one of its transverse directions compact, and consider an open string stretched from the D-brane to its image under translation along the compact direction. Now let us ask what happens to this open string state when the tachyon on the D-brane condenses to

⁷Of course, one would still need to understand why local fluctuations on the string involving the spreading of the flux is absent. Some discussion on this can be found in refs.[14, 8].

its ground state. Since the original state carries fundamental string winding charge this state cannot disappear. To see what happens it is simplest to go to the infinite cover; in this case we have initially an infinite number of parallel D-branes at regular spacing, and between any two neighbouring D-branes we have an open string suspended between the two. Thus on any of the D-branes we have an open string ending and another open string starting giving a source and a sink of electric flux. When the tachyon condenses to its ground state, each D-brane develops a flux line joining the source to the sink. If we identify this flux line as a fundamental string as before, the result is a single infinitely long string. After modding out by the discrete translation symmetry to compactify the direction, this is nothing but a closed string wrapped around the compact direction!

Thus we conclude that if we start with a D-brane with a transverse direction compact, then after tachyon condensation the open string field theory on the D-brane world-volume must contain excitations corresponding to closed string winding states along the compact direction. Let us now make a T-duality transformation along the compact direction. This converts the D- p brane to a D- $(p + 1)$ brane, but is otherwise a symmetry of the open string field theory order by order in open string perturbation theory. On the other hand this transforms the closed string winding modes to closed string momentum modes. Thus if we start with a D-brane with one of its tangential directions compact, then after tachyon condensation the corresponding open string field theory will contain excitations corresponding to closed string states carrying momentum along the compact direction.

It will be interesting to see if we can study this phenomenon directly in open string field theory.

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References

- [1] A. Sen, “Descent relations among bosonic D-branes,” *Int. J. Mod. Phys.* **A14**, 4061 (1999) [hep-th/9902105].
- [2] A. Sen, “Stable non-BPS bound states of BPS D-branes,” *JHEP* **9808**, 010 (1998) [hep-th/9805019];
 A. Sen, “Tachyon condensation on the brane antibrane system,” *JHEP* **9808**, 012 (1998) [hep-th/9805170];
 A. Sen, “BPS D-branes on non-supersymmetric cycles,” *JHEP* **12**, 021 (1998) hep-th/9812031.

- [3] E. Witten, “D-branes and K-theory,” JHEP **12**, 019 (1998) hep-th/9810188; “Overview of K-theory applied to strings,” hep-th/0007175.
- [4] P. Horava, “Type IIA D-branes, K-theory, and matrix theory,” Adv. Theor. Math. Phys. **2**, 1373 (1999) hep-th/9812135.
- [5] A. Recknagel and V. Schomerus, “Boundary deformation theory and moduli spaces of D-branes,” Nucl. Phys. **B545**, 233 (1999) hep-th/9811237; C.G. Callan, I.R. Klebanov, A.W. Ludwig and J.M. Maldacena, “Exact solution of a boundary conformal field theory,” Nucl. Phys. **B422**, 417 (1994) hep-th/9402113; J. Polchinski and L. Thorlacius, “Free fermion representation of a boundary conformal field theory,” Phys. Rev. **D50**, 622 (1994) hep-th/9404008.
- [6] A. Sen, “SO(32) spinors of type I and other solitons on brane-antibrane pair,” JHEP **9809**, 023 (1998) [hep-th/9808141]; J. A. Harvey, D. Kutasov and E. J. Martinec, “On the relevance of tachyons,” hep-th/0003101; P. Fendley, H. Saleur and N. P. Warner, “Exact solution of a massless scalar field with a relevant boundary interaction,” Nucl. Phys. **B430**, 577 (1994) [hep-th/9406125]; J. Majumder and A. Sen, “Vortex pair creation on brane-antibrane pair via marginal deformation,” JHEP **0006**, 010 (2000) [hep-th/0003124].
- [7] V.A. Kostelecky and S. Samuel, “The Static Tachyon Potential in the Open Bosonic String Theory,” Phys. Lett. **B207** (1988) 169; V. A. Kostelecky and R. Potting, “Expectation Values, Lorentz Invariance, and CPT in the Open Bosonic String,” Phys. Lett. **B381**, 89 (1996) [hep-th/9605088]; A. Sen, “Universality of the Tachyon Potential,” JHEP **9912**, 027 (1999) [hep-th/9911116]; A. Sen and B. Zwiebach, “Tachyon Condensation in String Field Theory,” JHEP **0003**, 002 (2000) [hep-th/9912249]; N. Berkovits, “The Tachyon Potential in Open Neveu-Schwarz String Field Theory,” JHEP **0004**, 022 (2000) [hep-th/0001084]; W. Taylor, “D-brane effective field theory from string field theory”, Nucl. Phys. **B585**, 171 (2000) [hep-th/0001201]; N. Moeller and W. Taylor, “Level truncation and the tachyon in open bosonic string field theory”, Nucl. Phys. **B583**, 105 (2000) [hep-th/0002237]; J.A. Harvey and P. Kraus, “D-Branes as unstable lumps in bosonic open string field

- theory”, JHEP **0004**, 012 (2000) [hep-th/0002117];
- N. Berkovits, A. Sen and B. Zwiebach, “Tachyon condensation in superstring field theory”, hep-th/0002211;
- R. de Mello Koch, A. Jevicki, M. Mihailescu and R. Tatar, “Lumps and p-branes in open string field theory”, Phys. Lett. **B482**, 249 (2000) [hep-th/0003031];
- P. De Smet and J. Raeymaekers, “Level four approximation to the tachyon potential in superstring field theory”, JHEP **0005**, 051 (2000) [hep-th/0003220];
- A. Iqbal and A. Naqvi, “Tachyon condensation on a non-BPS D-brane,” hep-th/0004015;
- N. Moeller, A. Sen and B. Zwiebach, “D-branes as tachyon lumps in string field theory,” JHEP **0008**, 039 (2000) [hep-th/0005036];
- L. Rastelli and B. Zwiebach, “Tachyon potentials, star products and universality,” hep-th/0006240;
- R. de Mello Koch and J. P. Rodrigues, “Lumps in level truncated open string field theory,” hep-th/0008053;
- N. Moeller, “Codimension two lump solutions in string field theory and tachyonic theories,” hep-th/0008101;
- A. Kosteleyky and R. Potting, “Analytical construction of a nonperturbative vacuum for the open bosonic string,” hep-th/0008252;
- H. Hata and S. Shinohara, “BRST invariance of the non-perturbative vacuum in bosonic open string field theory,” JHEP **0009**, 035 (2000) [hep-th/0009105];
- B. Zwiebach, “Trimming the Tachyon String Field with $SU(1,1)$ ”, hep-th/0010190.
- [8] J. A. Harvey, P. Kraus, F. Larsen and E. J. Martinec, “D-branes and strings as non-commutative solitons,” JHEP **0007**, 042 (2000) [hep-th/0005031].
- [9] R. Gopakumar, S. Minwalla and A. Strominger, “Noncommutative solitons,” JHEP **0005**, 020 (2000) [hep-th/0003160];
- K. Dasgupta, S. Mukhi and G. Rajesh, “Noncommutative tachyons,” JHEP **0006**, 022 (2000) [hep-th/0005006];
- E. Witten, “Noncommutative tachyons and string field theory,” hep-th/0006071;
- J. A. Harvey, P. Kraus and F. Larsen, “Exact noncommutative solitons,” hep-th/0010060.
- [10] A.A. Gerasimov and S.L. Shatashvili, *On exact tachyon potential in open string field theory*, hep-th/0009103;
- D. Kutasov, M. Marino and G. Moore, *Some exact results on tachyon condensation*

- in string field theory*, hep-th/0009148;
D. Kutasov, M. Marino and G. Moore, “Remarks on tachyon condensation in superstring field theory,” hep-th/0010108;
D. Ghoshal and A. Sen, “Normalisation of the background independent open string field theory action,” hep-th/0009191;
J. A. Minahan and B. Zwiebach, “Field theory models for tachyon and gauge field string dynamics,” JHEP **0009**, 029 (2000) [hep-th/0008231]; “Effective tachyon dynamics in superstring theory,” hep-th/0009246.
- [11] E. Witten, “Noncommutative Geometry And String Field Theory,” Nucl. Phys. **B268**, 253 (1986);
N. Berkovits, “Super-Poincare Invariant Superstring Field Theory,” Nucl. Phys. **B450** (1995) 90, [hep-th/9503099];
N. Berkovits, “A New Approach to Superstring Field Theory,” proceedings to the 32nd International Symposium Ahrenshoop on the Theory of Elementary Particles, Fortschritte der Physik (Progress of Physics) **48** (2000) 31, [hep-th/9912121].
- [12] A. Sen, “Non-BPS states and branes in string theory,” hep-th/9904207.
- [13] P. Yi, “Membranes from five-branes and fundamental strings from Dp branes,” Nucl. Phys. **B550**, 214 (1999) [hep-th/9901159].
- [14] O. Bergman, K. Hori and P. Yi, “Confinement on the brane,” Nucl. Phys. **B580**, 289 (2000) [hep-th/0002223].
- [15] G. Gibbons, K. Hori and P. Yi, “String fluid from unstable D-branes,” hep-th/0009061.
- [16] F. Larsen, “Fundamental Strings as Noncommutative Solitons,” hep-th/0010181.
- [17] A. Sen, “Supersymmetric world-volume action for non-BPS D-branes,” JHEP **9910**, 008 (1999) [hep-th/9909062].
- [18] A. Sen and B. Zwiebach, “Large marginal deformations in string field theory,” hep-th/0007153.
- [19] W. Taylor, “Mass generation from tachyon condensation for vector fields on D-branes,” hep-th/0008033.

- [20] U. Lindstrom and R. von Unge, “A picture of D-branes at strong coupling,” Phys. Lett. **B403**, 233 (1997) [hep-th/9704051];
H. Gustafsson and U. Lindstrom, “A picture of D-branes at strong coupling. II: Spinning partons,” Phys. Lett. **B440**, 43 (1998) [hep-th/9807064];
U. Lindstrom, M. Zabzine and A. Zheltukhin, “Limits of the D-brane action,” JHEP **9912**, 016 (1999) [hep-th/9910159].
- [21] M. R. Garousi, “Tachyon couplings on non-BPS D-branes and Dirac-Born-Infeld action,” Nucl. Phys. **B584**, 284 (2000) [hep-th/0003122];
E. A. Bergshoeff, M. de Roo, T. C. de Wit, E. Eyras and S. Panda, “T-duality and actions for non-BPS D-branes,” JHEP **0005**, 009 (2000) [hep-th/0003221];
J. Kluson, “Proposal for non-BPS D-brane action,” hep-th/0004106.
- [22] H. B. Nielsen and P. Olesen, “Local field theory of the dual string,” Nucl. Phys. **B57**, 367 (1973).
- [23] R. Gopakumar, S. Minwalla and A. Strominger, “Symmetry restoration and tachyon condensation in open string theory,” hep-th/0007226;
C. Sochichiu, “Noncommutative tachyonic solitons: Interaction with gauge field,” hep-th/0007217;
N. Seiberg, “A note on background independence in noncommutative gauge theories, matrix model and tachyon condensation,” hep-th/0008013.
- [24] A. Sen, “Some issues in non-commutative tachyon condensation,” hep-th/0009038;
“Uniqueness of tachyonic solitons,” hep-th/0009090;
O. Andreev, “Some Computations of Partition Functions and Tachyon Potentials in Background Independent Off-Shell String Theory”, hep-th/0010218.