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# String Network

Ashoke Sen [1]

*Mehta Research Institute of Mathematics*
*and Mathematical Physics*
*Chhatnag Road, Jhoosi, Allahabad 221506, INDIA*

**Abstract**

Type IIB string theory admits a BPS configuration in which three strings (of different type) meet at a point. Using this three string configuration we construct a string network and study its properties. In particular we prove supersymmetry of this configuration. We also consider string lattices, which can be used to construct BPS states in toroidally compactified string theory.

[1] E-mail: sen@mri.ernet.in

Type IIB string theory in ten dimensions is known to have a stable configuration in which three strings of different type meet[1, 2, 3, 4]. If the three strings are of type $(p_i, q_i)$ $(1 \leq i \leq 3)$[5, 6, 7] then charge conservation requires

$$\sum_{i=1}^{3} p_i = \sum_{i=1}^{3} q_i = 0. \tag{1}$$

The configuration is shown in Fig.1. The angles between different strings are adjusted

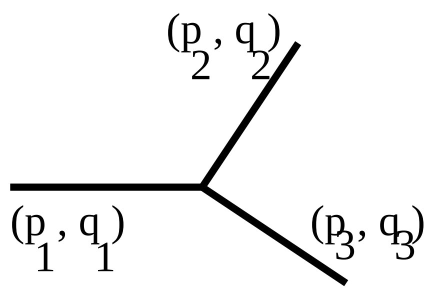


Figure 1: Three string junction.

such that the net force on the vertex due to the tensions between different strings cancel[1]. If $T_{p,q}$ denotes the tension of a $(p,q)$ string and $\widehat{n}_i$ denotes the direction of the $i$th string meeting at the vertex, then we must have

$$\sum_{i=1}^{3} T_{p_i,q_i} \widehat{n}_i = 0\,. \tag{2}$$

The BPS nature of this configuration was proved recently in [4] (See refs.[8, 9, 10, 11, 12] for related work). Given such a configuration, one can also construct string network by joining many of these vertices together[1], as shown in Fig.2, with eqs.(1), (2) satisfied at each vertex. In this paper we shall discuss some of the properties of such a network, including their supersymmetry properties. For simplicity, we shall restrict our analysis to planar network where all the strings lie in a single plane.

**String Network**: Eq.(1) guarantees charge conservation at each vertex. Let us now examine the consequence of applying eq.(2) at each vertex, with $\widehat{n}_i$ now representing a two dimensional unit vector. First, we recall from [6] that

$$T_{p,q} = \frac{1}{\sqrt{\tau_2}} |p + q\tau|\,, \tag{3}$$

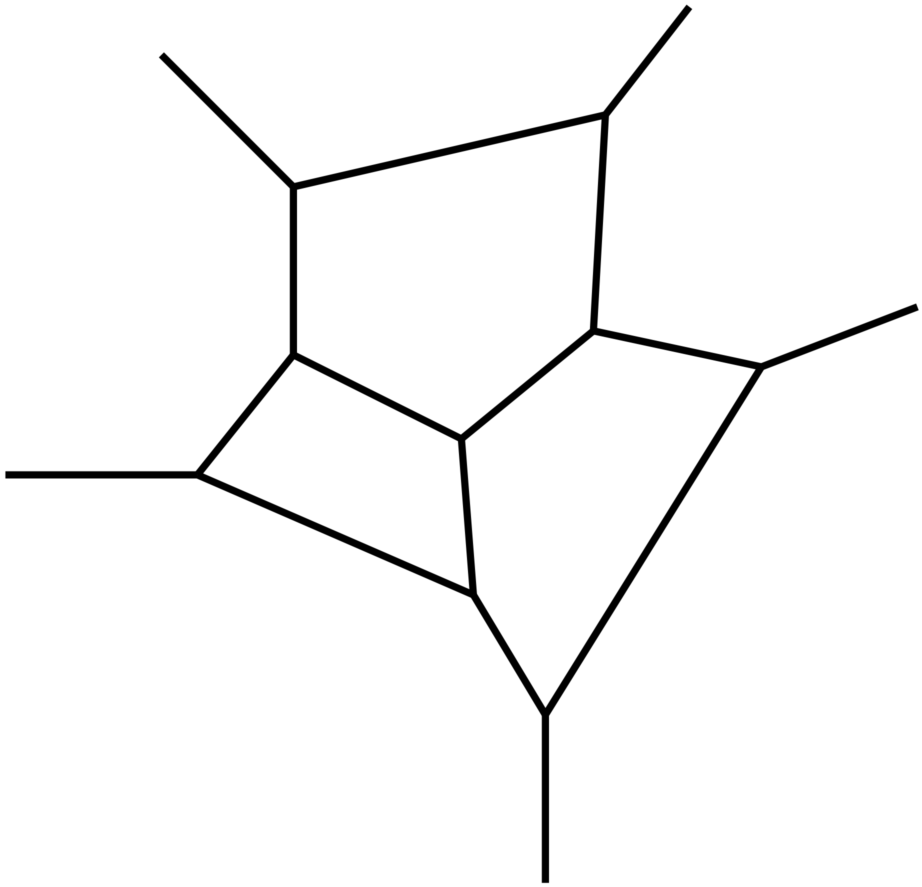

Figure 2: String network

where $\tau = \tau_1 + i\tau_2$ denotes the axion-dilaton modulus of type IIB. Let us denote by $\phi(p,q,\tau)$ the argument of $(p+q\tau)$:

$$(p+q\tau) = |p+q\tau| e^{i\phi(p,q,\tau)} \,. \tag{4}$$

From (1), (3) and (4) we see that at each vertex,

$$\sum_{i=1}^{3} T_{p_i,q_i} e^{i\phi(p_i,q_i,\tau)} = 0 \,. \tag{5}$$

This shows that eq.(2) can be automatically satisfied if we choose:

$$\widehat{n}_i = (\, \cos\phi(p_i,q_i,\tau), \sin\phi(p_i,q_i,\tau)) \,. \tag{6}$$

(This corresponds to orienting a $(p,q)$ string along the vector $(p+q\tau)$ in the complex plane.) (6) must be satisfied by all links in the network. From this we see that the planar network has the beautiful property that the orientation of a given link is determined solely by the charges $(p,q)$ carried by the link, and not its location in the network. Thus for example, two links in the network, each carrying a (2,3) string, will always have the same orientation, irrespective of however far apart they are in the network. Conversely, we see that as long as the network has the property that any $(p,q)$ type string is oriented along the vector $(p+q\tau)$ in the complex plane, the force balance condition at any junction is automatically satisfied.

**Supersymmetry**: We shall now argue that this network is invariant under one fourth of the space-time supersymmetry of type IIB string theory. We proceed as follows. Let us consider a $(p,q)$ string stretched along the 9th direction. Let $\epsilon_L$ and $\epsilon_R$ be the two real supersymmetry transformation parameters of type IIB string theory, associated with the left and the right moving sector of the world-sheet of the fundamental string. The $(p,q)$ string configuration described above is invariant under half of the supersymmetry of type IIB string theory, generated by $(\epsilon_L, \epsilon_R)$ satisfying[2]

$$\epsilon_L + i\epsilon_R = e^{i\phi(p,q,\tau)}\Gamma_1 \cdots \Gamma_8(\epsilon_L - i\epsilon_R)\,, \tag{7}$$

where $\Gamma_\mu$ are the ten dimensional gamma matrices. This equation can be proved by first noting that for the (1,0) string this gives:

$$\epsilon_L = \Gamma_1 \cdots \Gamma_8 \epsilon_L, \qquad \epsilon_R = -\Gamma_1 \cdots \Gamma_8 \epsilon_R\,, \tag{8}$$

which is the correct formula.[3] Furthermore, eq.(7) is invariant under the SL(2,Z) transformation:

$$\tau \to \frac{a\tau+b}{c\tau+d}, \qquad \begin{pmatrix} p \\ q \end{pmatrix} \to \begin{pmatrix} a & -b \\ -c & d \end{pmatrix} \begin{pmatrix} p \\ q \end{pmatrix}, \tag{9}$$

$$(\epsilon_L - i\epsilon_R) \to \exp\Big(\frac{i}{2}\arg(c\tau+d)\Big)(\epsilon_L - i\epsilon_R)\,. \tag{10}$$

The transformation (10) of $(\epsilon_L - i\epsilon_R)$ can be derived by repeating the arguments of [13] for type IIB theory. This shows that eq.(7) can be derived by making an SL(2,Z) transformation of the corresponding equations (8) for the fundamental string.

[2]Here we are only considering asymptotic $\epsilon_L$ and $\epsilon_R$, and not taking into account the space dependence that will be induced due the classical background fields produced by the string.

[3]Note that type IIB string theory has left-right exchange symmetry on the world-sheet under which $\epsilon_L \leftrightarrow \epsilon_R$. However, the presence of a fundamental string of a given orientation breaks this symmetry, thereby giving rise to different conditions on $\epsilon_L$ and $\epsilon_R$.

Let us now use this formula to test the supersymmetry of the string network. Let us consider the $i$th link carrying charge $(p_i, q_i)$ and oriented at an angle $\phi(p_i, q_i, \tau)$ relative to the 9-axis in the 8-9 plane. Eq.(7) is then modified to:

$$\epsilon_L + i\epsilon_R = e^{i\phi(p_i,q_i,\tau)}\Gamma_1 \cdots \Gamma_7(\Gamma_8 \cos\phi(p_i, q_i, \tau) + \Gamma_9 \sin\phi(p_i, q_i, \tau))(\epsilon_L - i\epsilon_R) \,. \tag{11}$$

This equation can be satisfied *for all $i$* by requiring that

$$\begin{aligned} &\epsilon_L = \Gamma_1 \cdots \Gamma_8 \epsilon_L, \qquad \epsilon_R = -\Gamma_1 \cdots \Gamma_8 \epsilon_R \,, \\ &\epsilon_L = \Gamma_1 \cdots \Gamma_7 \Gamma_9 \epsilon_R \,. \end{aligned} \tag{12}$$

This reduces the supersymmetry to one fourth of the original amount. Since eqs.(12) are independent of the composition of the network, but dependent only on its orientation, we see that a configuration of two or more parallel networks is invariant under the same supersymmetries even if the different networks have different compositions. Therefore we expect such configurations to form a stable system.

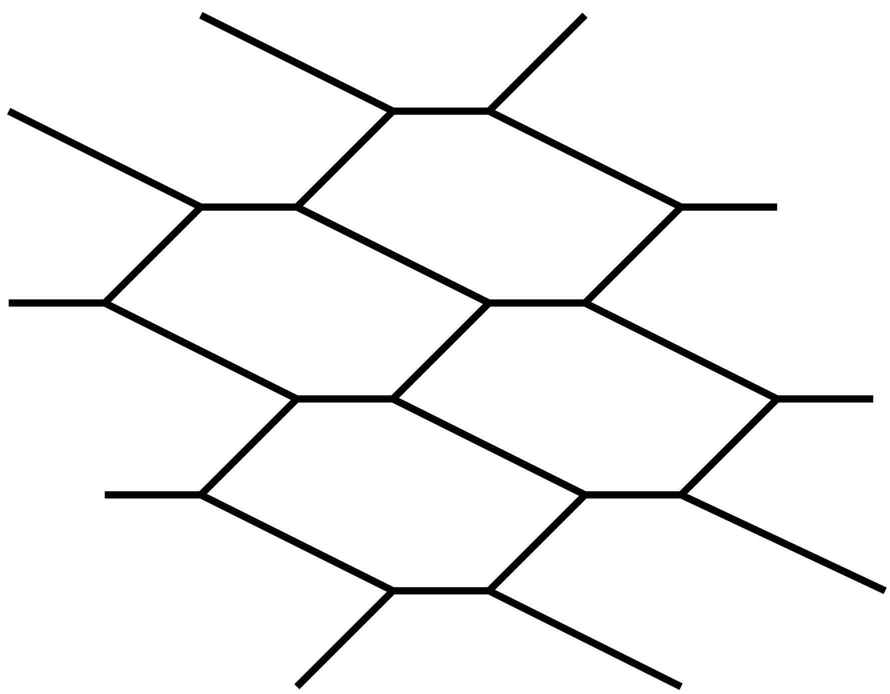

Figure 3: String lattice

**String Lattice**: By taking a periodic network we can construct 'string lattices'. An example has been shown in Fig.3. Here the links of three different orientations correspond to strings carrying three different kinds of charges. Note that although the orientations of the links are fixed by the charges they carry, their lengths are arbitrary. Thus the lattice displayed in Fig.3 is characterized by the three length parameters $l_1$, $l_2$ and $l_3$, – the lengths of the three different kinds of links in the lattice. These three parameters determine the length of the two independent basis vectors of the lattice and the angle between these basis vectors.

This configuration can also be interpreted as a BPS state of type IIB string compactified on a two dimensional torus, with periodicity matching the periodicity of the lattice. Thus $l_1$, $l_2$ and $l_3$ determine the shape and size of the torus. Conversely, for a given torus, one needs to fix $l_1$, $l_2$ and $l_3$ appropriately so that the network fits on the torus.

The mass of the BPS state is given by the total mass of all the strings that lie inside a unit cell. For the lattice shown in Fig.3, the unit cell can be identified as the parallelogram obtained by joining the centers of four adjacent hexagons. If $(p_i, q_i)$ denote the charges carried by the $i$th type of link in the lattice, then the mass of the BPS state represented by this lattice is given by

$$\sum_{i=1}^{3} l_i T(p_i, q_i)\,. \tag{13}$$

One can bring this formula in a more conventional form as follows. If $\widehat{n}_1$, $\widehat{n}_2$ and $\widehat{n}_3$ denote the unit vectors along the three links meeting at a vertex, then the two independent basis vectors of the lattice can be taken as:

$$\vec{a} = l_1\widehat{n}_1 - l_3\widehat{n}_3, \qquad \vec{b} = l_2\widehat{n}_2 - l_3\widehat{n}_3\,. \tag{14}$$

The area $A$ and the modular parameter $\lambda = \lambda_1 + i\lambda_2$ of the torus are then given by

$$A = |\vec{a}\times\vec{b}|\,, \tag{15}$$

$$\lambda_1 = \vec{a}\cdot\vec{b}/\vec{a}^2, \qquad \lambda_2 = |\vec{a}\times\vec{b}|/\vec{a}^2\,. \tag{16}$$

It is a straighforward (although somewhat tedious) exercise to see that in terms of $A$ and $\lambda$, the mass formula (13) can be rewritten as

$$m^2 = A\,(\,p_1 \quad q_1 \quad p_2 \quad q_2\,)\,(M \pm L)\begin{pmatrix} p_1 \\ q_1 \\ p_2 \\ q_2 \end{pmatrix}\,, \tag{17}$$

where,

$$M = \frac{1}{\lambda_2}\begin{pmatrix} \mathcal{M} & \lambda_1\mathcal{M} \\ \lambda_1\mathcal{M} & |\lambda|^2\mathcal{M} \end{pmatrix}, \qquad L = \begin{pmatrix} 0 & \mathcal{L} \\ -\mathcal{L} & 0 \end{pmatrix}, \tag{18}$$

$$\mathcal{M} = \frac{1}{\tau_2}\begin{pmatrix} 1 & \tau_1 \\ \tau_1 & |\tau|^2 \end{pmatrix}, \qquad \mathcal{L} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}. \tag{19}$$

For given $(p_i, q_i)$, the sign in front of $L$ in eq.(17) is chosen such that the contribution from this term to $m^2$ is positive.

This is the $SL(2,Z)_S \times SL(2,Z)_U$ invariant BPS formula for type IIB string theory on $T^2$. Note that in this case the full U-duality group is $SL(3,Z) \times SL(2,Z)$, but the mass formula (17) does not display this symmetry manifestly, since we have considered states carrying no momenta along the internal directions of the torus, and also have set the anti-symmetric tensor field background to zero. Also note that using an appropriate SL(3,Z) transformation we can convert the charges $(q_1, q_2)$ associated with Ramond-Ramond tensor field to internal momenta on the torus. Thus under this duality transformation the BPS states represented by the string lattice get mapped to elementary string states. It will be interesting to compare the degeneracy of states considered here with that of the elementary string states.

This finishes our short note on string network. At present the utility of the string network, besides describing BPS states in toroidally compactified type IIB string theory, is not clear. However, in future a manifestly SL(2,Z) invariant non-perturbative formulation of string theory may be made possible by regarding the string network, instead of string loops, as fundamental objects. This would be similar in spirit to recent developments in canonical quantum gravity, in which loops have been replaced by spin networks[14].

Note added: After writing this paper I became aware of refs.[15, 16] where supersymmetric configurations of webs of strings and five-branes have been discussed. Some related work has also been reported in refs.[17, 18, 19].

# References


[1] J. Schwarz, hep-th/9607201.

[2] O. Aharony, J. Sonnenschein and S. Yankielowicz, Nucl. Phys. **B474** (1996) 309 [hep-th/9603009].

[3] M. Gaberdiel and B. Zwiebach, hep-th/9709013.

[4] K. Dasgupta and S. Mukhi, hep-th/9711094.

[5] J. Harvey and A. Strominger, Nucl. Phys. **B449** (1995) 535 [hep-th/9504047].

[6] J. Schwarz, Phys. Lett. **B360** (1995) 13.

[7] E. Witten, Nucl. Phys. **B460** (1996) 335 [hep-th/9510135].

[8] C. Callan and J. Maldacena, hep-th/9708147.

[9] G. Gibbons, hep-th/9709027.

[10] S. Lee, A. Peet and L. Thorlacius, hep-th/9710097.

[11] L. Thorlacius, hep-th/9710181.

[12] A. Hashimoto, hep-th/9711097.

[13] T. Ortin, Phys. Rev. **D51** (1995) 790 [hep-th/9404035].

[14] C. Rovelli, gr-qc/9710008.

[15] O. Aharony and A. Hanany, hep-th/9704170.

[16] O. Aharony, A. Hanany and B. Kol, hep-th/9710116.

[17] B. Kol, hep-th/9705031.

[18] A. Brandhuber, N. Itzhaki, J. Sonnenschein, S. Theisen and S. Yankielowicz, hep-th/9709010.

[19] N. Leung and C. Vafa, hep-th/9711013.