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Tachyon Condensation on the Brane Antibrane System

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Abstract

A coincident D-brane - anti-D-brane pair has a tachyonic mode. We present an argument showing that at the classical minimum of the tachyonic potential the negative energy density associated with the potential exactly cancels the sum of the tension of the brane and the anti-brane, thereby giving a configuration of zero energy density and restoring space-time supersymmetry.

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It has been known for sometime that a coincident D-brane – anti-D-brane pair contains a tachyonic excitation[1, 2, 3, 4]. In a previous paper[5] it was conjectured that at the minimum of the tachyon potential, the negative contribution to the energy density from the potential exactly cancels the sum of the tensions of the brane and the anti-brane, thereby giving a configuration of zero energy density (and hence restoring space-time supersymmetry). Although this proposal might sound radical, there are many known examples where tachyon condensation restores space-time supersymmetry[6, 7, 8], and the present example is just an extension of these earlier examples. In this paper we shall offer a 'proof' of this conjecture. Although this will not be a rigorous mathematical proof; at least it should serve the purpose of putting this conjecture on a firmer footing.

The strategy that we shall adopt will follow closely that of [7]; and indeed most of the results we shall require are already contained there. (For earlier references related to this subject, see [9].) For definiteness we shall focus on the Dirichlet 2-brane, but the extension to other branes should be straightforward. We take the coincident 2-brane anti-2-brane system, and wrap it on a two dimensional torus $T^2 = S^1 \times S^1$, of radii R_1 and R_2 respectively. (For simplicity we shall set $\alpha' = 1$ and measure all lengths and masses in string units.) We now introduce one unit of magnetic flux on both the brane and the anti-brane; and the signs of the magnetic fields are chosen such that each corresponds to $+1$ unit of D0-brane charge. The strength of the magnetic field on each brane is given by:

$$F_{12} = \frac{2\pi}{V}, \tag{1}$$

where $V = 4\pi^2 R_1 R_2$ is the area of the torus. In the limit $R_1 \to \infty$, $R_2 \to \infty$, the magnetic field strength goes to zero and we expect to recover the physics of the original brane – anti-brane system.

We shall however begin by analyzing this system in the $R_1, R_2 \to 0$ limit. In this limit the system is best described by going to the T-dual version, in which we make $R_i \to (1/R_i)$ duality transformation on both the circles. This gives a dual torus $\widetilde{T}^2$ with radii

$$\widetilde{R}_i = \frac{1}{R_i}, \qquad i = 1, 2, \tag{2}$$

and coupling constant

$$\widetilde{g} = \frac{g}{R_1 R_2}, \tag{3}$$

where g is the coupling constant of the original theory.[2] Under this duality transformation the 0-brane and the 2-brane charges get interchanged. Thus the wrapped D2-brane with one unit of magnetic flux gets converted to a wrapped D2-brane with 1 unit of magnetic flux, whereas the wrapped anti-D2-brane with one unit of magnetic flux gets converted to a wrapped D2-brane with -1 unit of magnetic flux. When $\widetilde{R}_1$ and $\widetilde{R}_2$ are large, this system may be described by an effective supersymmetric U(2) gauge theory on this dual torus[10] in the presence of a background gauge field of the form:

$$A_1 = 0, \qquad A_2 = \frac{2\pi x^1 \sigma_3}{\widetilde{V}}, \tag{4}$$

where x^1, x^2 denote the directions of the two circles of $\widetilde{T}^2$, A_μ denotes the component of the U(2) gauge field along the μth circle, σ_i are the Pauli matrices, and $\widetilde{V} = 4\pi^2 \widetilde{R}_1 \widetilde{R}_2$ is the volume of the dual torus $\widetilde{T}^2$. This gauge field A satisfies the boundary conditions

$$\begin{aligned} A(x^1 = 2\pi\widetilde{R}_1, x^2) &= \Omega_1 \circ A(x^1 = 0, x^2)\,, \\ A(x^1, x^2 = 2\pi\widetilde{R}_2) &= \Omega_2 \circ A(x^1, x^2 = 0)\,, \end{aligned} \tag{5}$$

where Ω_μ are gauge transformation parameters:

$$\Omega_1 = \exp(i x^2 \sigma_3 / \widetilde{R}_2)\,, \qquad \Omega_2 = 1\,, \tag{6}$$

and $\Omega_\mu \circ A$ denotes the gauge transform of A by Ω_μ. Ω_1 and Ω_2 satisfy the relation:

$$\Omega_2(x^1 = 2\pi\widetilde{R}_1)\Omega_1(x^2 = 0) = \Omega_1(x^2 = 2\pi\widetilde{R}_2)\Omega_2(x^1 = 0)\,. \tag{7}$$

From eq.(4) we see that the gauge field configuration lies fully inside the SU(2) part of the gauge group, and does not have any U(1) component. If we now consider fluctuations of the SU(2) gauge fields around the background gauge field configuration given in (4), satisfying the boundary conditions given in (5), (6), we shall find tachyonic modes[7]. It is easy to verify that the mass spectrum of the tachyonic modes is identical to that calculated directly from the spectrum of the original string theory before duality transformation. In the gauge theory the presence of these tachyonic modes simply reflect the fact that the gauge field configuration given in (4), which gives rise to a non-vanishing field strength and hence a positive definite energy density on the brane, does not correspond to the

[2] During the process of varying the parameters R_1 and R_2 to take various limits, we shall keep both g and $\widetilde{g}$ small, so that the classical description is good in the original theory as well as in the dual theory.

minimum energy configuration subject to the boundary conditions (5), (6). To see this let us note that with the help of a gauge transformation $g(x^1, x^2)$ on $A_\mu(x)$, we can make both Ω_1 and Ω_2 to be identity, since, as can be seen from (7), there is no obstruction to this choice[11]. In particular, we can choose $g(x^1, x^2)$ such that

$$\begin{aligned} & g(x^1 = 0, x^2) = \exp(ix^2\sigma_3/\widetilde{R}_2)\,, \quad g(x^1 = 2\pi\widetilde{R}_1, x^2) = 1\,, \\ & g(x^1, x^2 = 2\pi\widetilde{R}_2) = g(x^1, x^2 = 0)\,, \quad \frac{\partial}{\partial x^1} g(x^1, x^2) = 0 \text{ at } x^1 = 0, 2\pi\widetilde{R}_1\,. \end{aligned} \tag{8}$$

The existence of a $g(x^1, x^2)$ satisfying these conditions can be proved by noting that $g(x^1 = 0, x^2)$ describes a map from S^1 to the SU(2) group manifold. Since SU(2) is simply connected, it is possible to find an interpolating map $g(x^1, x^2)$ such that $g(x^1 = 2\pi\widetilde{R}_1, x^2)$ is the identity element of the group. The transformed field $g \circ A_\mu$ satisfies periodic boundary condition along x^1 and x^2, and thus the lowest energy configuration corresponds to $g \circ A_\mu = 0$ and hence $g \circ F_{\mu\nu} = 0$. We can go back to the original gauge in which Ω_μ are given by (5) with the help of a reverse gauge transformation $g^{-1}(x^1, x^2)$. This will give rise to a non-trivial gauge field configuration, but the field strength $F_{\mu\nu}$ will continue to vanish.

This shows that the classical minimum energy configuration after 'tachyon condensation' corresponds to zero field strength for both the U(1) and the SU(2) gauge fields, and hence the energy per unit area will be given by the sum of the tensions of the two D2-branes. Since in the unit we are using each D-brane has tension $1/(4\pi^2\widetilde{g})$, $\widetilde{g}$ being the string coupling constant, and the total area of each brane is $4\pi^2\widetilde{R}_1\widetilde{R}_2$, we see that the total mass of this system is given by:

$$M = \frac{2\widetilde{R}_1\widetilde{R}_2}{\widetilde{g}}\,. \tag{9}$$

Note that this saturates the BPS bound for a pair of D2-branes wrapped on the dual torus $\widetilde{T}^2$. It is also possible to argue that once quantum fluctuations are taken into account, there is a quantum ground state of the system with exactly the same mass[12, 13].[3]

Since the classical ground state of the system saturates BPS bound, we would expect that it would continue to saturate BPS bound as we change the parameters of the theory continuously, and hence its mass at the classical minimum will continue to be given by

[3] These states are needed for U-duality, as they are related to the Kaluza-Klein modes carrying two units of momentum along the internal directions of the torus.

(9). Let us now express the mass formula (9) in terms of the original variables R_1, R_2 and g. Since the string metric does not change under a T-duality transformation, eq.(9), when expressed in terms of the original variables using eqs.(2), (3), takes the form:

$$M = \frac{2}{g} \, . \tag{10}$$

Now recall that the original system was a D2-brane and an anti-D2-brane wrapped on a torus T^2 of area $4\pi^2 R_1 R_2$, with one unit of magnetic flux on each of the branes. Thus from (10) we see that at the minimum of the tachyon potential, the energy per unit area on the brane $-$ anti-brane system is given by:

$$\frac{M}{4\pi^2 R_1 R_2} = \frac{1}{2\pi^2 R_1 R_2 g} \, . \tag{11}$$

If we now take the limit $R_1, R_2 \to \infty$, the energy per unit area goes to zero. On the other hand, as has already been argued before, in this limit the magnetic field strength on the brane and the anti-brane goes to zero and we expect to recover the physics of the brane $-$ anti-brane system without any magnetic flux. Thus we deduce that at the classical minimum of the potential, the energy per unit area of the brane $-$ anti-brane system vanishes.

This concludes our 'proof'. If we assume that a similar result holds for the D-string anti-D-string pair in type I string theory, then in the spirit of [5] we can identify the stable non-BPS SO(32) spinor states in type I theory[14] as the 'tachyonic soliton' on the D-string - anti-D-string pair in which the tachyon field changes from $-T_0$ to $+T_0$ as we go from far left to the far right on the string. Here $\pm T_0$ denote the locations of the minima of the tachyonic potential at which the tension of the D-string anti-D-string pair is canceled by the tachyon potential. In order to see that this represents an SO(32) spinor state, we can compactify type I theory on a circle of large radius, and consider a D-string anti-D-string pair wound around that circle. If one of them carries a non-trivial Z_2 Wilson line[15] and the other carries trivial Z_2 Wilson line, then the combined system represents an SO(32) spinor, being a combination of a spinor and a singlet state. In this case the tachyon associated with the open string stretched between the D-string and the anti-D-string must be anti-periodic as we go around the circle. The kink solution described above precisely satisfies such a boundary condition. Bound states of wound D-strings carrying spinorial and winding charges have been discussed recently in [16].

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