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'Blowing up' D-branes on Non-supersymmetric Cycles

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Abstract

In the orbifold limit of K3, one can give exact conformal field theory description of D-branes wrapped on certain non-supersymmetric cycles of K3. We study the effect of switching on the 'non-geometric blow up modes' corresponding to anti-symmetric tensor gauge field flux through the 2-cycles on these D-branes. Working to first order in the blow up parameter, we determine the region of the moduli space in which these D-branes are stable. Across the boundary of this region, the D-brane wrapped on the non-supersymmetric cycle decays to a pair of D-branes, each wrapped on a supersymmetric cycle, via a second order phase transition.

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1 Introduction and Summary

BPS D-branes[1, 2] have proved to be extremely useful in studying various aspects of string dualities, stringy black holes and other properties of string theory. The original formulation of D-branes was given for flat D-branes in flat space-time. Significant progress was made in [3] in the study of curved D-branes. In particular [3] showed how to find an exact boundary conformal field theory description of D-branes wrapped on certain 2-cycles of K3 in the orbifold limit. The 2-cycles studied there correspond to the cycles associated with the blow up of the orbifold fixed points.[2] Although in the orbifold limit these 2-cycles have zero size, D-branes wrapped on these 2-cycles have finite tension due to the presence of the anti-symmetric two form flux through these cycles[4]. These cycles are supersymmetric, so that D-branes wrapped on these cycles correspond to BPS D-branes.

If we take two such branes, associated with two different 2-cycles (*i.e.* different fixed points P and Q of the orbifold) then the combined system could be non-supersymmetric although the individual branes are BPS. It was shown in [5, 6] that in certain region of the moduli space, there is a single non-BPS D-brane configuration carrying the same charge quantum numbers as this combined system, but with tension less than the sum of the tension of the two individual D-branes. Thus we can regard this single brane as a (classical) bound state of the two BPS branes[7, 8, 9]. It can also be interpreted as a D-brane wrapped around a single 2-cycle of K3 which is homologically equivalent to the sum of the two individual cycles associated with the fixed points P and Q. Since the wrapped D-brane is non-BPS, the associated cycle is non-supersymmetric. Refs.[5, 6] gave an exact boundary conformal field theory description of this non-BPS brane.

In the orbifold limit the relevant modulus which controls the stability of the non-BPS brane is the radius $\widetilde{R}$ of the circle of the original torus passing through the orbifold fixed

[2]Throughout this paper we shall use the words fixed points and orbifold planes interchangeably, both refering to five dimensional fixed planes spanning the non-compact directions.

points P and Q.[3] When this radius is less than a critical radius $\widetilde{R}_c$, the brane wrapped on the non-supersymmetric cycle has lower tension than the sum of the tensions of the two supersymmetric branes. This is reflected in the fact that in this region of moduli space the non-BPS brane has no tachyonic mode and hence is stable, whereas the system containing the pair of BPS branes has a tachyonic mode and hence is unstable. When the radius is larger than the critical radius the situation is reversed. Now the tension of the non-BPS brane is larger than the sum of the tension of the two BPS branes. Furthermore the tachyonic mode disappears from the system containing pair of BPS branes, and a tachyonic mode appears on the non-BPS brane. Thus in this region of the moduli space the stable system is clearly the configuration of two BPS branes.[4]

At the critical radius $\widetilde{R}_c$ both systems have a massless open string mode representing the limit of the tachyonic mode from their respective region of instability. We shall refer to this as the tachyonic mode although at the critical radius it is not tachyonic. One can show that this represents an exactly marginal deformation which interpolates between the system containing a pair of BPS branes and the non-BPS brane[8, 5, 10, 11, 12]. Let us denote by α the parameter labelling the marginal deformation, normalized so that $\alpha = 0$ (mod 2) represents the pair of BPS branes and $\alpha = 1$ (mod 2) represents the non-BPS brane. α can be interpreted as the vacuum expectation value (vev) of the tachyonic mode on the brane. Away from the critical radius the tachyonic mode develops a potential. This potential energy (density) $V(\alpha)$ is periodic under $\alpha \to \alpha + 2$ due to a periodicity in the underlying conformal field theory[8], and has the following qualitative behaviour:

1. At the critical radius $V(\alpha)$ vanishes, since the tachyonic deformation is exactly marginal.

2. For $\widetilde{R} > \widetilde{R}_c$, $V(\alpha)$ has a minimum at $\alpha = 0$ and a maximum at $\alpha = 1$. This shows that the $\alpha = 0$ configuration, representing a pair of BPS branes, is the stable configuration, whereas the $\alpha = 1$ configuration representing the non-BPS brane is unstable.

[3] For simplicity we are assuming that the torus used in the construction of the orbifold is a product of four circles and the points P and Q lie along one of the circles. We also take the radii of the other circles to be sufficiently large in order to avoid other kinds of instability[5, 6] than the ones which will be discussed here.

[4] Throughout this paper we shall restrict our analysis to open string tree level. Thus the process of formation of the bound state via tachyon condensation discussed here is distinct from the bound state formation via possible attractive force due to closed string exchange interaction. The former is an open string tree level effect whereas the latter is open string one loop effect.

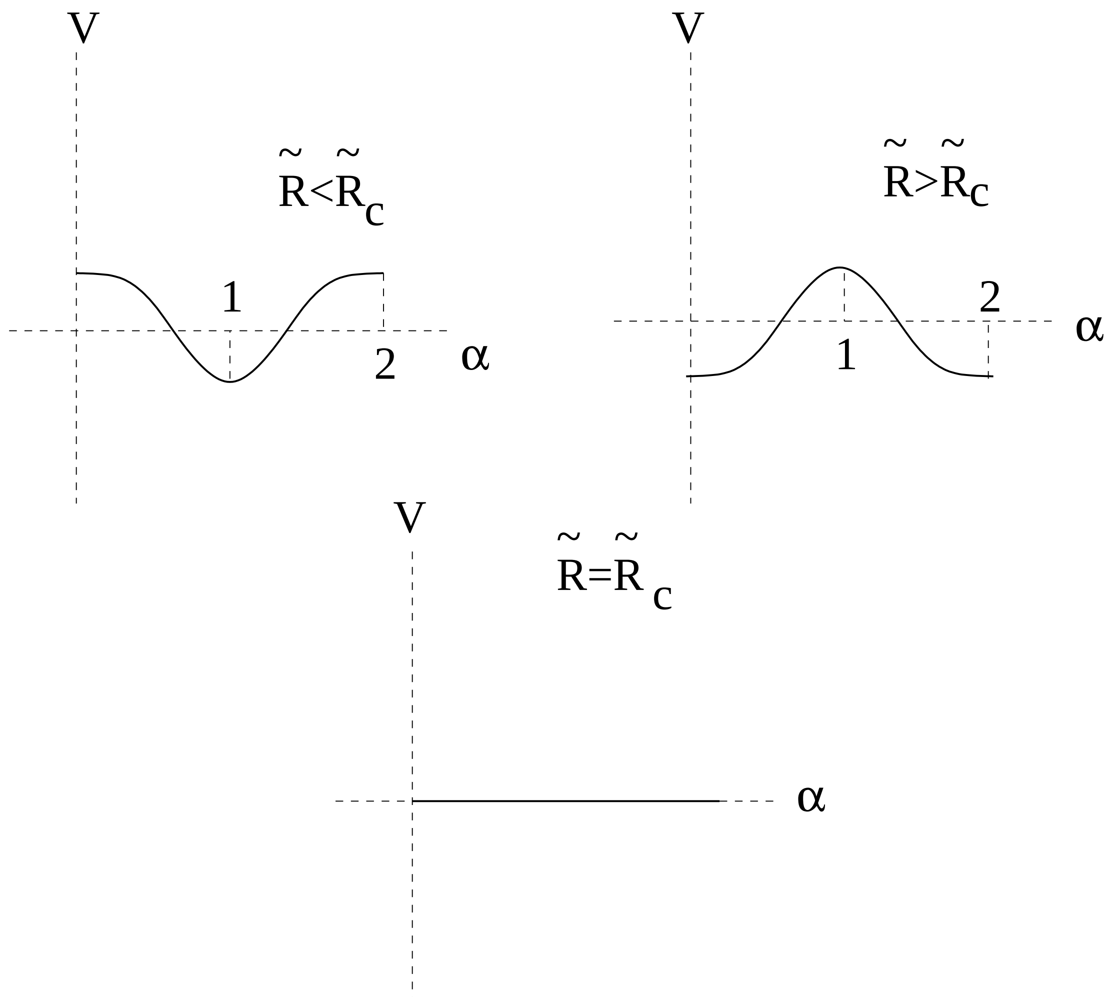

Figure 1: The potential $V(\alpha)$ for different values of $\widetilde{R}$.

3. For $\widetilde{R} < \widetilde{R}_c$, $V(\alpha)$ has a minimum at $\alpha = 1$ and a maximum at $\alpha = 0$, showing that the $\alpha = 1$ configuration, representing a non-BPS brane, is the stable configuration. The $\alpha = 0$ configuration, being the maximum of $V(\alpha)$, is unstable.

This has been sketched in Fig.1. Note that the location of the ground state in the α space jumps discontinuously as we pass through the critical radius, as shown in Fig.2.

This is the result in the orbifold limit of the theory. In this paper we shall study how this picture gets modified when we blow up the orbifold fixed points. In fact the particular

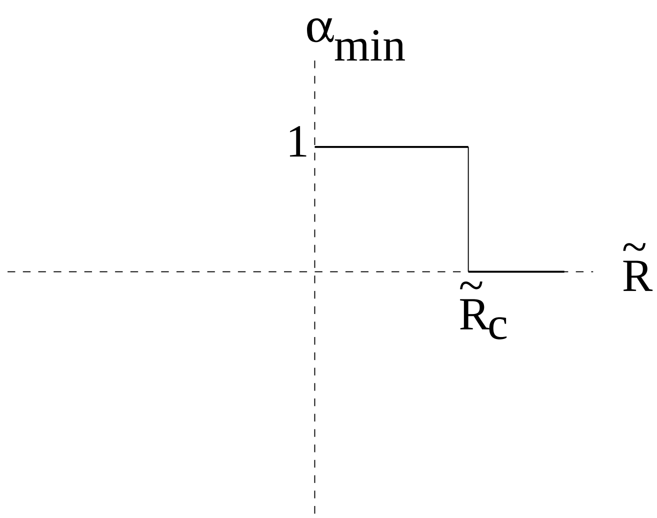

Figure 2: Location of the minimum of $V(\alpha)$ for different values of $\widetilde{R}$.

moduli which we shall turn on are not the geometric blow up modes, but deformations corresponding to changing the flux of the anti-symmetric tensor field through the 2-cycle. Although these particular moduli do not correspond to the geometric blow up parameters, but deformations of the Kahler class associated with the cycles by an imaginary part, we shall refer to these as the blow up modes. We find that to first order in the blow up parameters, the potential $V(\alpha)$ depends on only one of these parameters, which is the difference in the antisymmetric tensor field flux through the two 2-cycles. We compute the complete potential $V(\alpha)$ to first order in the blow up parameter ζ and first order in the difference $(\widetilde{R} - \widetilde{R}_c)$. The result is:

$$V(\alpha) \propto (\frac{1}{4}(\widetilde{R}_c - \widetilde{R}) \cos(\alpha\pi) + \zeta \cos(\frac{1}{2}\alpha\pi)) \,. \tag{1.1}$$

From this we can study the locations of the extrema of the potential for various ranges of $\widetilde{R}$. We can also identify the nature of the absolute minimum of $V(\alpha)$ in different ranges of $\widetilde{R}$ by continuously connecting it to a minimum of $V(\alpha)$ for $\zeta = 0$, where the identification is known. As is clear from eq.(1.1), $V(\alpha)$ is invariant under $\alpha \to \alpha + 4$, and $\alpha \to -\alpha$. Using this we can restrict α to the range $0 \leq \alpha \leq 2$. In this range the minimum of $V(\alpha)$ has the following structure:

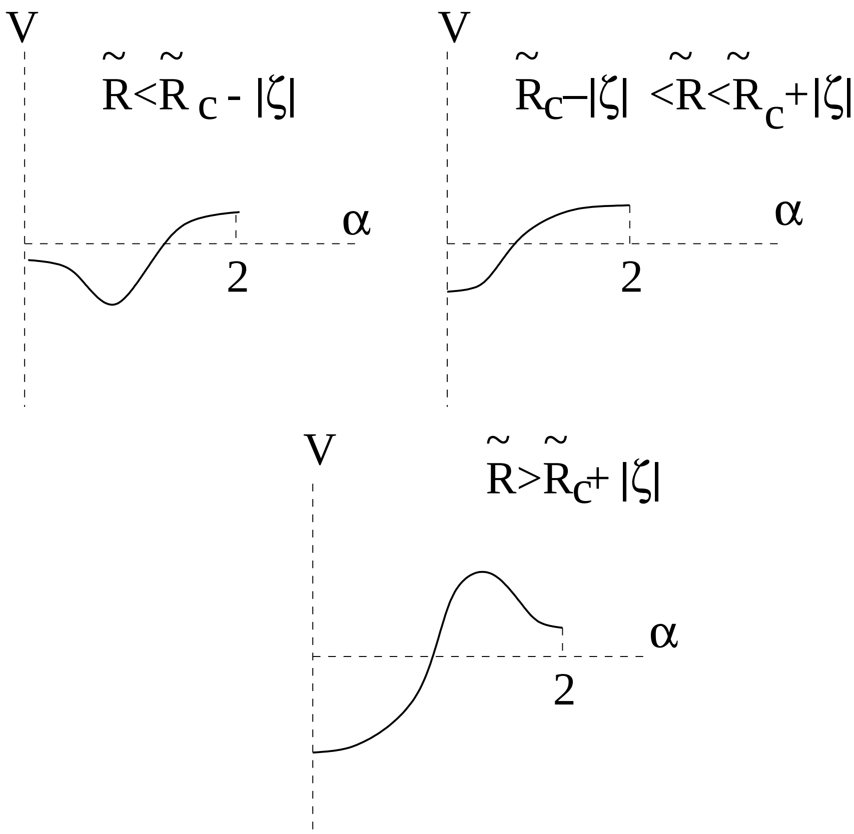

Figure 3: The potential $V(\alpha)$ for different values of $\widetilde{R}$ for $\zeta < 0$.

1. For $\widetilde{R} > (\widetilde{R}_c - |\zeta|)$ the absolute minimum of the potential corresponds to a pair of BPS D-branes. This minimum is at $\alpha = 0$ ($\alpha = 2$) for $\zeta < 0$ ($\zeta > 0$). The $\alpha = 2$ configuration differs from the $\alpha = 0$ configuration in that the D0-brane charge carried by the pair of wrapped membranes get exchanged.

2. For $\widetilde{R} < (\widetilde{R}_c - |\zeta|)$ the absolute minimum of the potential corresponds to a single non-BPS D-brane. This minimum is at $\alpha = \frac{2}{\pi}\cos^{-1}(\zeta/(\widetilde{R} - \widetilde{R}_c))$. For $|\zeta| << |\widetilde{R} - \widetilde{R}_c|$ the minimum approaches $\alpha = 1$ in agreement with the answer for $\zeta = 0$.

Fig.3 gives a sketch of the potential for various ranges of values of $\widetilde{R}$ for $\zeta < 0$. Fig.4 shows the phase diagram in the $(\widetilde{R}, \zeta)$ plane.

From this we see that for $\zeta \neq 0$ the critical radius is shifted to $(\widetilde{R}_c - |\zeta|)$. Also as $\widetilde{R}$

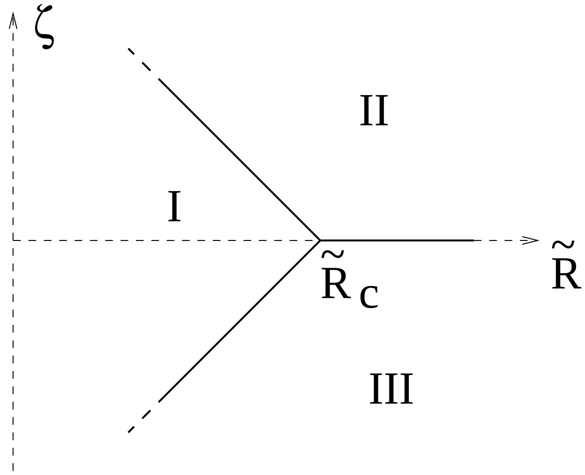

Figure 4: Phase diagram in the $\widetilde{R} - \zeta$ plane. Phase I ($\alpha_{min} = (2/\pi)\cos^{-1}(\zeta/(\widetilde{R} - \widetilde{R}_c))$ corresponds to a D2-brane wrapped on a non-supersymmetric cycle, phase II ($\alpha_{min} = 2$) corresponds to a pair of D2-branes, each wrapped on a supersymmetric cycle, and phase III ($\alpha_{min} = 0$) corresponds to a pair of D2-branes wrapped on the same supersymmetric cycles as in phase II, but carrying opposite D0-brane charges compared to those in phase II. The shape of the curves displayed here is valid only to first order in ζ and $(\widetilde{R} - \widetilde{R}_c)$.

approaches $(\widetilde{R}_c - |\zeta|)$ from below (region I in Fig.4), the location of the absolute minimum approaches the point $\alpha = 0$ ($\alpha = 2$) for $\zeta < 0$ ($\zeta > 0$). This is the same as the location of the minimum for $\widetilde{R} > (\widetilde{R}_c - |\zeta|)$. Fig.5 shows a sketch of the evolution of the minimum of $V(\alpha)$ as a function of $\widetilde{R}$ for a fixed $\zeta < 0$. Thus *there is no discontinuous jump in the location of the minimum as we pass through the phase boundary between regions I and III and between regions I and II.* A detailed analysis of the potential shows that in this case the phase transition from the non-BPS D-brane to the pair of BPS D-branes is second order.[5] On the other hand the minimum of $V(\alpha)$ jumps discontinuously from $\alpha = 0$ to $\alpha = 2$ as ζ changes sign keeping $\widetilde{R} > \widetilde{R}_c$. Thus the phase transition between regions II and III across the $\zeta = 0, \widetilde{R} > \widetilde{R}_c$ line is first order.

The rest of the paper is organised as follows. Although our analysis is valid for any even (odd) dimensional D-brane in type IIA (IIB) string theory wrapped on (non)-supersymmetric cycles of K3, for convenience we shall focus on a particular case – D2-

[5]Possibility of the existence of such critical points in non-BPS D-branes was speculated by C. Vafa[13].

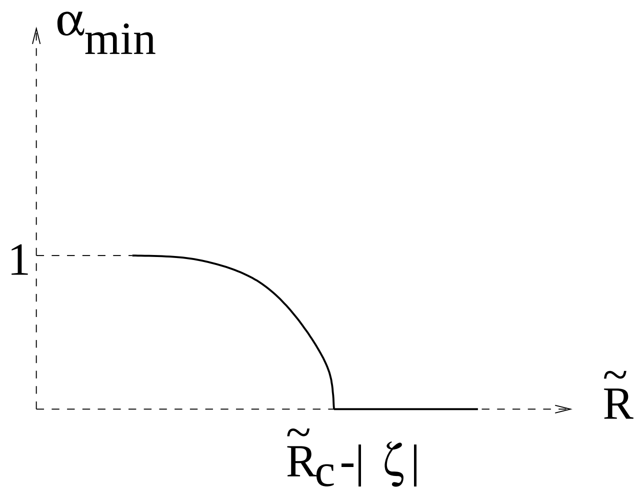

Figure 5: Location of the minimum α_{min} of $V(\alpha)$ for different values of $\widetilde{R}$ for $\zeta < 0$. As $|\zeta| \to 0$, this approaches a step function.

brane of type IIA string theory wrapped on the cycles of K3. In section 2 we give a description of these states using the (non-BPS) D-branes of type II string theory, and give a precise statement of the problem that we want to solve. In section 3 we solve the problem by finding the tachyon potential, and finding its extrema.

2 Statement of the Problem

In this section we review some of the earlier results which will be required for our analysis, carry out some preliminary analysis of the problem, and give a precise statement of the problem that we shall solve in the next section. The system that we shall analyse is the same one as in [5], namely a non-BPS D-string of IIA wrapped on a circle $\widetilde{S}^1$ of radius $\widetilde{R}$, modded out by $\mathcal{I}_4$ where $\mathcal{I}_4$ reverses the direction tangential to $\widetilde{S}^1$ and three other directions.[6] This can be regarded as a two brane wrapped on a non-supersymmetric cycle of $K3$ in the orbifold limit if the other three directions are compact[5, 6]. We shall take the radii of these three directions to be sufficiently large so that there are no tachyonic modes from open strings wound in these directions. In order to use some of the already known

[6]The D-string is taken to be at the origin of these other three coordinates, so that the original configuration is invariant under $\mathcal{I}_4$.

results, we shall first make a T-duality transformation on $\widetilde{S}^1$, so that the background now represents type IIB string theory on the dual S^1 modded out by $(-1)^{F_L} \cdot \mathcal{I}_4$, and the non-BPS D-string of the original type IIA theory becomes a non-BPS D-particle of IIB[14, 15] stuck to one of the orbifold planes. This system is identical to the one analysed in [7, 15], and can also be identified to the system analysed in [8] before the orbifold projection. We shall denote by $R = \alpha' \widetilde{R}^{-1}$ the radius of the dual S^1 and by x the coordinate of the dual S^1 and take the non-BPS D-particle to be located at $x = 0$. We shall refer to this new description of the system as the IIB description, and the original description as the IIA description. From now on we shall continue to use the type IIB description unless mentioned otherwise.

Although the non-BPS D-particle in type IIB string theory has a tachyonic mode, it is projected out by the orbifolding operation[15, 8]. However, as we reduce the radius R of S^1, the open string stretched between the D-particle and its image across S^1 develops a tachyonic mode. Let R_c denote the critical radius below which this tachyonic mode appears. In the $\alpha' = 1$ unit that we shall be using, $R_c = \frac{1}{\sqrt{2}}$. The physical interpretation of the appearance of the tachyonic mode can be understood by studying the mass of the non-BPS D-particle, as well as the total mass of a D-string $\bar{\text{D}}$-string pair of type IIB string theory, wrapped on S^1. If g denotes the type IIB string coupling constant, then the mass of the non-BPS D-particle stuck to the orbifold plane is given by:

$$m_{D0} = \frac{1}{2} \cdot \sqrt{2} \cdot \frac{1}{g} = \frac{1}{\sqrt{2} g} \, . \tag{2.1}$$

The various factors in this expression can be understood as follows. Since taking orbifold by $\mathcal{I}_4 \cdot (-1)^{F_L}$ cuts space into half its original size, it keeps only half of the D-particle at $x = 0$. Thus its mass is half of that of the non-BPS D-particle in type IIB string theory, which in turn is equal to $(\sqrt{2}/g)$. By the same argument, after the orbifold projection the mass of a D-string wrapped on S^1 is computed by multiplying its tension by πR instead of $2\pi R$, since one fundamental region of the orbifold contains a piece of the D-string stretched from the fixed point at $x = 0$ to the fixed point at $x = \pi R$. The total mass of the D-string $\bar{\text{D}}$-string pair stretched from $x = 0$ to $x = \pi R$ is given by:

$$m_{D1\bar{D}1} = 2 . \frac{1}{2\pi g} . \pi R = \frac{R}{g} \, . \tag{2.2}$$

We see that at $R = R_c = \frac{1}{\sqrt{2}}$ the two systems are degenerate. Below the critical radius the D-string $\bar{\text{D}}$-string pair has lower energy. Thus it would be natural to associate the tachyonic instability of the D0-brane to the possibility of its decay into a D-string $\bar{\text{D}}$-string pair, provided the two systems carry the same charge quantum numbers.

In order to investigate whether the D-particle carries the same charge quantum numbers as a D-string $\bar{\text{D}}$-string pair, we use the boundary state[16, 17, 18, 19, 20, 21] description of the two systems. The boundary state describing the D-particle was constructed in [15]. It is a linear combination of untwisted sector Neveu-Schwarz-Neveu-Schwarz (NSNS) states and twisted sector Ramond-Ramond (RR) states, from which it follows that it is charged under the gauge field at $x = 0$ originating in the twisted RR sector. The situation with a D-string ($\bar{\text{D}}$-string) is somewhat more complex. The boundary state describing a BPS D-string ($\bar{\text{D}}$-string) is characterized by a Z_2 Wilson line $e^{i\theta} = \pm 1$ along S^1, and two more parameters ϵ_1, ϵ_2 which can take values ± 1[7]:

$$\begin{aligned} |\theta, \epsilon_1, \epsilon_2\rangle &= \frac{1}{2}(|\theta, U\rangle_{NSNS} + \epsilon_1|\theta, U\rangle_{RR}) + \frac{1}{2\sqrt{2}}\epsilon_2(|T_1\rangle_{NSNS} + \epsilon_1|T_1\rangle_{RR}) \\ &\quad + \frac{1}{2\sqrt{2}} e^{i\theta}\epsilon_2(|T_2\rangle_{NSNS} + \epsilon_1|T_2\rangle_{RR})\,. \end{aligned} \tag{2.3}$$

Here U stands for untwisted sector, T_1 stands for twisted sector at $x = 0$ and T_2 stands for twisted sector at $x = \pi R$. ϵ_1 takes value $+1$ (-1) for a D-string ($\bar{\text{D}}$-string), and $\epsilon_1\epsilon_2$ denotes the sign of the twisted sector RR charge carried by the $x = 0$ end of the D-string. $e^{i\theta}\epsilon_1\epsilon_2$ denotes the sign of the twisted sector RR charge carried by the $x = \pi R$ end of the string.

Thus we see that after modding out by $(-1)^{F_L} \cdot \mathcal{I}_4$ a D-string ($\bar{\text{D}}$-string) carries twisted sector Ramond-Ramond (RR) charge at its two ends. If we choose a D-string state that carries + charge at the $x = 0$ end and $-$ charge at the $x = \pi R_c$ end corresponding to $(\theta, \epsilon_1, \epsilon_2) = (\pi, +, +)$ (configuration (a) in Fig.6 of [7]), and a $\bar{\text{D}}$-string that carries + charge at both ends corresponding to $(\theta, \epsilon_1, \epsilon_2) = (0, -, -)$ (configuration (g) of Fig.6 of [7]), then the D-string $\bar{\text{D}}$-string pair is neutral under the twisted sector gauge field at $x = \pi R_c$ and carries +ve charge under the twisted sector gauge field at $x = 0$. This matches with the charge quantum number of the non-BPS D-particle at $x = 0$, as can be seen from the fact that the massless RR component of the boundary state describing a non-BPS D-particle at $x = 0$[15](published version) is given precisely by twice that appearing in eq.(2.3).[7] Hence the D-particle can decay into this pair of D-string states. The appearance

[7]This relative factor of (1/2) between the twisted sector RR charge carried by the end of a BPS D-string and by a non-BPS D-particle can be seen as follows. The open string partition function for

of the tachyonic mode on the D-particle below $R = R_c$ signals the possibility of this decay. From this analysis we see that this configuration requires the D-string to carry a Z_2 Wilson line, whereas the D̄-string does not carry any Wilson line. In the dual type IIA description the D-string D̄-string pair, with the D-string carrying a Z_2 Wilson line, corresponds to a D0-D̄0 pair situated at diametrically opposite points on the dual circle $\widetilde{S}^1$. After modding out by $\mathcal{I}_4$ this can be reinterpreted as a pair of BPS D2-branes of IIA, wrapped on the two cycles associated with the two orbifold fixed points on $\widetilde{S}^1$[3].

Note that we could also consider a D-string state that carries + charge at both ends corresponding to $(\theta, \epsilon_1, \epsilon_2) = (0, +, +)$ (configuration (c) in Fig.6 of [7]), and a D̄-string that carries + charge at the $x = 0$ end and − charge at the $x = \pi R_c$ end corresponding to $(\theta, \epsilon_1, \epsilon_2) = (\pi, -, -)$ (configuration (e) of Fig.6 of [7]). This has the same RR charge as the previous configuration, and so the D-particle can also decay into this state. This differs from the previous configuration by having the Wilson line on the D̄-string rather than on the D-string. In the dual type IIA description this again corresponds to a D0-D̄0 pair on $\widetilde{S}^1$, but with their positions reversed.

A more systematic analysis of the transition from the D-particle state to a D-string D̄-string pair was carried out in refs.[8, 5]. It was shown that the lowest mode of the tachyon, which is massless at $R = R_c$, represents an exactly marginal deformation. By switching on this marginal deformation one can continuously interpolate between the boundary conformal field theories (BCFT) describing the non-BPS D-particle and the D-string D̄-string pair. We could see this marginal deformation either by starting from the D-particle side, or by starting from the D-string − D̄-string side. We shall find it more convenient to do the analysis from the D-string D̄-string side, so that we can use the results of [8].

On an infinite D-string D̄-string system there is a tachyonic mode with mass$^2 = -\frac{1}{2}$. Upon wrapping the D-string D̄-string pair on a circle of radius$R$, with a Z_2 Wilson line on the D-string, the tachyon field T is anti-periodic under $x \to x + 2\pi R$, and has a Fourier

the non-BPS D-string has a projection operator $\frac{1+(-1)^F}{2}\frac{1+g_1}{2}\frac{1+g_2}{2}$, where F denotes world-sheet fermion number, g_1 denotes $(-1)^{F_L}$ accompanied by the transformation $(x^6 \ldots x^9 \equiv x) \to (-x^6, \ldots - x^9)$, and g_2 denotes $(-1)^{F_L}$ accompanied by the transformation $(x^6 \ldots x^9) \to (-x^6, \ldots - x^8, 2\pi R - x^9)$[7]. On the other hand, partition function of open strings living on a non-BPS D-particle at $x \equiv x^9 = 0$ has a projection operator $\frac{1+(-1)^F \cdot g_1}{2}$[15]. Thus the coefficient of $(-1)^F \cdot g_1$ in the two cases differ by a factor of 4. Since this coefficient is related to the norm of the twisted sector RR component of the boundary state describing the system, we conclude that the twisted sector RR component of the boundary state describing a D-particle has an extra factor of 2 compared to that describing a D-string.

expansion of the form:

$$T(x) = \sum_{n \in Z} T_{n+\frac{1}{2}} e^{i(n+\frac{1}{2})\frac{x}{R}} \,. \tag{2.4}$$

The effective mass^2 of the mode $T_{n+\frac{1}{2}}$ is given by

$$m^2_{n+\frac{1}{2}} = \frac{(n+\frac{1}{2})^2}{R^2} - \frac{1}{2} \,. \tag{2.5}$$

Thus for $R \le R_c = \frac{1}{\sqrt{2}}$ there are no tachyonic modes on this system. For $R > R_c$, $T_{\pm\frac{1}{2}}$ becomes tachyonic, indicating that the system becomes unstable in this range of R. At $R = R_c$, $(T_{\frac{1}{2}} \pm T_{-\frac{1}{2}})$ can be shown to be exactly marginal, and one can deform the BCFT by switching on vev of this field. But from eq.(2.59) of [7] one finds that only the mode $(T_{\frac{1}{2}} - T_{-\frac{1}{2}})$ is invariant under $(-1)^{F_L} \cdot \mathcal{I}_4$ and can be switched on. It was shown in [8] that by switching on the vev of $(T_{\frac{1}{2}} - T_{-\frac{1}{2}})$ we can reach the BCFT describing the non-BPS D-particle of type IIB string theory.[8] We denote by α the suitably normalized vev of this mode of the tachyon at $R = R_c$, with $\alpha = 0$ representing the D-string $\bar{\mathrm{D}}$-string system, and $\alpha = 1$ representing the D-particle, as in [8].

The marginality of $(T_{\frac{1}{2}} - T_{-\frac{1}{2}})$ can be seen by rewriting the BCFT at $R = R_c$ in terms of a different set of variables. As discussed in [8], the effect of switching on the tachyon vev is to insert the following operator at the boundary of the world-sheet:

$$Tr \exp\Big(i \frac{\alpha}{2\sqrt{2}} \sigma_1 \int dt \partial_t \phi_B\Big) \,. \tag{2.6}$$

Here $\int dt$ denotes integration along the boundary of the world-sheet, $\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ is a Chan Paton factor, Tr denotes trace over the Chan Paton factors, and ϕ_B denotes the boundary value of the world-sheet field $\phi = \phi_L + \phi_R$, where the field ϕ is related to the bosonic coordinate field $X = (X_L + X_R)$ along S^1 and its right- and left-moving world-sheet partners ψ, $\widetilde{\psi}$, through the fermionization $-$ bosonization relations:

$$e^{i\sqrt{2}X_R} = \frac{1}{\sqrt{2}}(\xi + i\eta), \qquad e^{i\sqrt{2}X_L} = \frac{1}{\sqrt{2}}(\widetilde{\xi} + i\widetilde{\eta}), \tag{2.7}$$

[8]Ref.[8] introduced the vertex operators $V_\pm$ for $T_{\pm\frac{1}{2}}$, which, in the -1 picture[22], were taken to be proportional to $\pm e^{(\pm i/\sqrt{2})X}$. But if T_r is to label the rth mode of the tachyon field $T(x)$, then its vertex operator in the -1 picture should be proportional to $e^{irX/R}$ without any extra r dependent sign. For this reason the vertex operator for $(T_{\frac{1}{2}} - T_{-\frac{1}{2}})$ of this paper corresponds to what was called $(V_+ + V_-)$ in [8].

$$e^{i\sqrt{2}\phi_R} = \frac{1}{\sqrt{2}}(\xi + i\psi), \qquad e^{i\sqrt{2}\phi_L} = \frac{1}{\sqrt{2}}(\widetilde{\xi} + i\widetilde{\psi}). \tag{2.8}$$

ξ, η, $\widetilde{\xi}$, $\widetilde{\eta}$ are world-sheet fermion fields. (2.6) can be interpreted as a Wilson line along the bosonic coordinate ϕ, and clearly represents a marginal deformation.

Since at $R = R_c$ the tachyon becomes marginal, the tachyon potential $V(\alpha)$ vanishes at $R = R_c$. For $R < R_c$, $V(\alpha)$ has a minimum at $\alpha = 0$, representing the fact that the D-string $\bar{\text{D}}$-string configuration corresponds to the minimum energy configuration, whereas for $R > R_c$, $V(\alpha)$ has a minimum at $\alpha = 1$ indicating that the non-BPS D-particle represents the minimum energy configuration. Thus the point $R = R_c$ marks the phase boundary between the stable D-particle configuration, and the stable D-string $\bar{\text{D}}$-string configuration. The question that we shall be interested in is: how does this picture get modified when we go to a different point in the moduli space of K3 (in the original type IIA description)? We shall only analyse the effect of small deformations around the original configuration. These may be divided into two classes: moduli of type IIA string theory on the torus (constant metric and antisymmetric tensor field background), and the blow up modes of the fixed points corresponding to twisted sector closed string states from the NSNS sector. There are four such blow up modes from each orbifold fixed point. Three of these modes correspond to geometric blow up parameters, and the fourth one corresponds to antisymmetric tensor field flux through the 2-cycle associated with the fixed point. In the dual type IIB description that we have been using, we have the moduli corresponding to constant metric and anti-symmetric tensor field background in the dual torus, and the twisted sector modes. In this case however the twisted sector modes from the NSNS sector have a different interpretation. Each orbifold plane obtained by modding out by $\mathcal{I}_4 \cdot (-1)^{F_L}$ has a hidden NS five brane, since it is S-dual to the coincident orientifold 5-plane D-5-brane system[23]. Switching on the twisted sector modes associated with a given orbifold plane corresponds to moving the NS five-brane away from the orbifold plane.

It can be easily verified that switching on the constant metric or anti-symmetric tensor field background does not modify the physics at the phase boundary between a stable configuration of D-particle and the D-string $\bar{\text{D}}$-string system. To see this note that if we denote by R the radius of S^1 measured in the new metric, then in terms of R, the tachyon mass formula (2.5) as well as the D-brane mass formulae (2.1), (2.2) remain unchanged. Thus at the critical radius $R = \frac{1}{\sqrt{2}}$ the D-particle becomes degenerate with the D-string

$\bar{\text{D}}$-string system, and the tachyonic modes $T_{\pm\frac{1}{2}}$ become massless. We can use the same bosonization and fermionization formulae to show that at $R = R_c$, $(T_{\frac{1}{2}} - T_{-\frac{1}{2}})$ represents an exactly marginal deformation and interpolates between the BCFT describing these two systems.

Thus it remains to study what happens when we switch on the twisted sector massless NSNS fields. We shall now argue that only one of the eight blow up modes associated with the two fixed points actually affect the masses of the D-brane system to first order and hence could modify the physics at the phase boundary to this order. To do this we use the boundary state description of the D-string $\bar{\text{D}}$-string system given in [7] and that of the non-BPS D-particle given in [15]. First of all, the boundary state describing the non-BPS D-particle is a linear combination of the boundary state from untwisted sector NSNS sector, and twisted sector RR sector[15]. Since it has no component from the twisted sector NSNS sector, we see that the D-particle does not have a direct coupling to these states and hence its mass does not depend on these twisted sector moduli to first order. On the other hand, from eq.(2.3) we see that the D-string ($\bar{\text{D}}$-string) boundary state has components along the twisted sector NSNS states, and so the mass of the D-string ($\bar{\text{D}}$-string) has linear dependence on the particular moduli fields which appear in the boundary state. There are two such moduli, one from $|T_1\rangle_{NSNS}$ and the other from $|T_2\rangle_{NSNS}$. The physical interpretation is quite clear. These moduli represent the motion of the NS five branes along the circle S^1, and since the ends of the D-string ($\bar{\text{D}}$-string) lie on the NS five-branes, their lengths and hence their masses depend on the location of the five-brane along S^1.[9] Also note that for a given change in the location of the five brane the change in the mass can have either sign (as reflected in the coefficient of $|T_1\rangle_{NSNS}$ and $|T_2\rangle_{NSNS}$ in (2.3), which can be of either sign). This can be traced to the fact that the D-string ($\bar{\text{D}}$-string) can either end on the NS 5-brane or its image under $\mathcal{I}_4 \cdot (-1)^{F_L}$.[10] As we move the five brane in one direction, its image moves in the opposite direction. Thus whether the mass increases or decreases for a given movement of the five-brane is determined by whether the D-string ($\bar{\text{D}}$-string) ends on the five-brane or its image.

As has been discussed earlier, the D-string $\bar{\text{D}}$-string system under consideration corresponds to the configurations (a) and (g) of ref.[7], characterized by $(\theta, \epsilon_1, \epsilon_2)$ values $(\pi, +, +)$ and $(0, -, -)$ respectively. Using eq.(2.3) we see that the twisted sector compo-

[9]Moving the five-branes in directions transverse to S^1 does not affect the length and hence the mass of the D-string ($\bar{\text{D}}$-string) to first order.

[10]The S-dual version of this was explained in Fig.2 of [7].

nent of the boundary state of the combined system is given by:

$$\frac{1}{\sqrt{2}}(|T_1\rangle_{RR} - |T_2\rangle_{NSNS}) \,. \tag{2.9}$$

The component $|T_1\rangle_{RR}$ indicates that it carries twisted sector RR charge associated with the $x = 0$ plane, as must be the case since it has the same charge as the D-particle situated at $x = 0$. On the other hand, the component $|T_2\rangle_{NSNS}$ indicates that it couples to NSNS sector twisted sector modes associated with the orbifold plane at $x = \pi R$. Thus the mass of the combined system depends only on the location of the NS five-brane associated with the orbifold plane at $x = \pi R$. This can be explained by taking both the D-string and the $\bar{\text{D}}$-string to end on the NS 5-brane at $x = \pi R$, but having one of them end on the 5-brane and the other end on its image near $x = 0$.[11] In that case the motion of the NS 5-brane near $x = 0$ will not change the total mass of the system, but the motion of the NS 5-brane near $x = \pi R$ will change the total mass.

Let us denote by ζ the specific massless mode that appears in $|T_2\rangle_{NSNS}$. From eq.(2.9) we see that this is the only NSNS twisted sector mode on which the mass of the D-string $\bar{\text{D}}$-string pair depends to first order, and we shall study how the transition between the D-particle state and the D-string $\bar{\text{D}}$-string state is affected upon switching on this mode. In the original type IIA description, ζ denotes the difference in the flux of the antisymmetric tensor field through the two 2-cycles of K3, associated with the two orbifold fixed points.

Our strategy will be to determine the tachyon potential completely for $R \simeq R_c$, $\zeta \simeq 0$ to linear order in $(R - R_c)$ and ζ, ignoring terms quadratic in $(R - R_c)$ and ζ, as well as terms of order $(R - R_c)\zeta$, and then study its minimum as a function of α for various values of R and ζ. Thus the general form of the potential will be

$$V(\alpha) \simeq \frac{1}{g}[(R - R_c)f(\alpha) + \zeta g(\alpha)] \,, \tag{2.10}$$

where $f(\alpha)$ and $g(\alpha)$ are two functions to be determined. Note that since we are working close to the point $R = R_c$, $\zeta = 0$, we can continue to use the parameter α to label the nearly massless tachyonic mode. Also note that we have extracted an overall power

[11] This must be the case if both of them have to carry the same charge at the $x = 0$ end and opposite charge at the $x = \pi R$ end, since the D-string and the $\bar{\text{D}}$-string, ending on the same five-brane, carry opposite charge. On the other hand if the D-string and the $\bar{\text{D}}$-string end on the five brane and its image respectively, then their ends carry the same charge, since the gauge field on the five-brane world-volume that survives the orbifold projection is the difference between the U(1) gauge field on the five brane and its image. This is exactly analogous to the situation describing a D5-brane near an orientifold 5-plane.

of the inverse string coupling (g^{-1}) outside the potential since the g dependence of the world-volume action of a D-brane always comes through such an overall multiplicative factor.

In the next section we shall determine the functions $f(\alpha)$ and $g(\alpha)$, and study the extremum of the potential as a function of α. By studying how the minimum of the potential varies as we change the parameters R and ζ, we shall be able to determine the phase diagram of the D-brane system under study in the $R-\zeta$ plane.

3 Determination of the Tachyon Potential and the Phase Diagram

First we shall determine $f(\alpha)$. For this we set $\zeta = 0$, so that the tachyon potential has the form:

$$V(\alpha) = \frac{1}{g}[(R-R_c)f(\alpha) + O((R-R_c)^2)]\,, \tag{3.1}$$

for $R \simeq R_c$. We can determine $f(\alpha)$ by noting that $(\partial V/\partial\alpha) \simeq g^{-1}(R-R_c)f'(\alpha)$ corresponds to the one point function of the tachyon to order $(R-R_c)$. This was computed in [8] and the answer was found to be proportional to $\sin\pi\alpha$. Integrating this we see that $f(\alpha)$ must be proportional to $\cos(\pi\alpha)$. The constant of proportionality can also be easily found by noting that the difference between $V(\alpha)$ at $\alpha = 0$ and at $\alpha = 1$ must be equal to the difference between the total mass of the D-string $\bar{\text{D}}$-string pair wrapped on S^1, and the mass of the non-BPS D-particle. Using eqs.(2.1), (2.2) we get

$$V(\alpha=0) - V(\alpha=1) = \frac{1}{g}(R-R_c)\,. \tag{3.2}$$

This gives[12]

$$f(\alpha) = \frac{1}{2}\cos(\alpha\pi)\,. \tag{3.3}$$

From eqs.(3.1) and (3.3) we see that the minimum of V is at $\alpha = 0$ for $R < R_c$, and is at $\alpha = 1$ for $R > R_c$. This is consistent with the fact that the D-string $\bar{\text{D}}$-string pair is the stable configuration for $R < R_c$, and the D-particle is the stable configuration for $R > R_c$.

We shall now use a shortcut for determining $f(\alpha)$, which we shall generalise later for determining $g(\alpha)$. In the derivation given above, we have used the tachyon one point

[12] This analysis determines $V(\alpha)$ up to an additive α-independent constant which has no relevance for finding the extrema of $V(\alpha)$ in the α-space.

function to first order in $(R-R_c)$ to compute $f'(\alpha)$. This represents a two point function on the disk in the theory at $R=R_c$, with one insertion of the closed string vertex operator V_B corresponding to radius deformation at the center, and one insertion of the tachyon vertex operator at the boundary[8]. We need to choose the picture numbers[22] of these vertex operators so that the total picture number is -2. Let us take V_B to be in $(-1,-1)$ picture, and the tachyon vertex operator in the 0-picture. Besides this there is an insertion of the exponential of the integrated zero picture tachyon vertex operator at the boundary as given in eq.(2.6). Since the α dependence of the one point function comes from only the matter part of the correlation function, let us restrict ourselves to this sector. In this case it is clear that if we start from an amplitude where we only have the insertion of V_B at the center, and the exponential of the integrated tachyon vertex operator at the boundary, then by differentiating it with respect to α we bring down an extra factor of tachyon vertex operator at the boundary. Since this two point function has been argued to be proportional to $f'(\alpha)$, we see that the original amplitude is proportional to $f(\alpha)$ itself.

The main lesson from here is that $f(\alpha)$ may be computed directly by computing the matter part of the disk amplitude with a single insertion of V_B at the center (reflecting that we are working to order $(R-R_c)$) and insertion of the exponential of integrated tachyon vertex operator at the boundary given in eq.(2.6). The computation of $g(\alpha)$ will be done in the same way, with V_B replaced by the appropriate twisted sector vertex operator V_{TW}.[13]

Let us now turn to the determination of $g(\alpha)$. Since we have already argued that the energy of the D-string $\bar{\text{D}}$-string system depends linearly on ζ, we see that $g(0)$ must be a non-zero constant. We shall absorb this constant into the definition of ζ and set $g(0)=1$.[14] On the other hand by analysing the boundary state describing the non-BPS D-particle we have argued before that the mass of this D-particle does not depend on ζ. Hence $g(1)$ must vanish.

The complete determination of $g(\alpha)$ is done by computing the disk amplitude with an insertion of the ζ vertex operator at the center, and the exponential of the integrated

[13]If one could construct the boundary state describing the system at $R=R_c$ for all α analogously to ref.[10], then one could read out $g(\alpha)$ by computing the component of the boundary state along V_{TW}.

[14]With this normalization the ζ dependent contribution to the mass of the D-string $\bar{\text{D}}$-string pair is given by (ζ/g). This should be equated to the total tension $(1/\pi g)$ of the D-string $\bar{\text{D}}$-string pair multiplied by the shift in the position of the NS 5-brane. Thus $\pi\zeta$ measures the shift in the position of the NS 5-brane.

tachyon vertex operator at the boundary. We use the notation and the normalization conventions of [8]. The vertex operator V_{TW} for a twisted sector state associated with the orbifold plane at $x = \pi R_c$ has the property that as we go around such a vertex operator on the fundamental string world-sheet, the various world-sheet fields undergo the following changes:

$$X \to (2\pi R_c - X), \qquad \psi \to -\psi, \qquad \widetilde{\psi} \to -\widetilde{\psi} \,. \tag{3.4}$$

Using eqs.(2.7), (2.8) we see that this transformation is equivalent to,

$$\xi \to -\xi, \qquad \widetilde{\xi} \to -\widetilde{\xi}, \qquad \psi \to -\psi, \qquad \widetilde{\psi} \to -\widetilde{\psi}, \qquad \eta \to \eta, \qquad \widetilde{\eta} \to \widetilde{\eta}\,, \tag{3.5}$$

or to,

$$\phi_L \to \phi_L + \frac{\pi}{\sqrt{2}}, \qquad \phi_R \to \phi_R + \frac{\pi}{\sqrt{2}} \,. \tag{3.6}$$

Thus as we go around V_{TW} on the string world-sheet, $\phi = \phi_L + \phi_R$ changes by $\sqrt{2}\pi$. Since $\int dt \partial_t \phi_B$ measures the total change of ϕ as we go around the boundary of the disk, we see that if there is an insertion of V_{TW} at the center of the disk, then

$$Tr \exp(i\frac{\alpha}{2\sqrt{2}}\sigma_1 \int dt \partial_t \phi_B) = Tr \exp(i\frac{\alpha}{2\sqrt{2}} \cdot \sigma_1 \cdot \sqrt{2}\pi) = 2\cos(\frac{1}{2}\alpha\pi) \,. \tag{3.7}$$

This shows that $g(\alpha)$ is proportional to $\cos(\frac{1}{2}\alpha\pi)$. Using the normalization $g(0) = 1$, we get

$$g(\alpha) = \cos(\frac{1}{2}\alpha\pi) \,. \tag{3.8}$$

This satisfies the condition $g(1) = 0$ derived earlier. Thus the full tachyon potential to this order is given by:

$$V(\alpha) \simeq \frac{1}{g}(\frac{1}{2}(R - R_c)\cos(\alpha\pi) + \zeta\cos(\frac{1}{2}\alpha\pi)) \,. \tag{3.9}$$

Note that the potential is periodic in α with periodicity 4. This may appear as a surprise, as in ref.[8] it was found that the BCFT describing the D-string $\bar{\text{D}}$-string system is periodic in α with periodicity 2. We can understand the origin of this apparent discrepancy as follows. Let us set $\zeta = 0$, $R = R_c$, and start from the D-particle state represented by the point $\alpha = 1$. We can perturb this system by the marginal tachyonic deformation and study the fate of the BCFT as a function of the new deformation parameter $(\alpha - 1)$. The T-dual version of this analysis in the type IIA description was carried out in [5]. In this analysis the starting configuration was a non-BPS D-string of IIA wrapped on a circle. Switching on the marginal deformation corresponding to $(\alpha - 1) = 1$ corresponds to the creation of a kink-antikink pair on the circle, which is to be interpreted as a D0-brane $\bar{\text{D}}$0-brane pair of type IIA string theory situated at diametrically opposite points on a circle[5, 24].[15] On the other hand if we take $(\alpha - 1) = -1$, the effect is to create an antikink-kink pair. Thus the result is again a D0-$\bar{\text{D}}$0 pair, but with their positions reversed. This has the following interpretation in the dual type IIB description. If we take the $\alpha = 0$ configuration to represent a D-string $\bar{\text{D}}$-string pair with a Wilson line on the D-string, then the $\alpha = 2$ configuration denotes a D-string $\bar{\text{D}}$-string pair with a Wilson line along the $\bar{\text{D}}$-string. These correspond to the same BCFT before the orbifold projection, but differ in the orbifold theory when twisted sector modes are switched on. As discussed in section 2, since the Wilson line is on the $\bar{\text{D}}$ string, the $\alpha = 2$ configuration corresponds to the pair of states carrying $(\theta, \epsilon_1, \epsilon_2)$ quantum numbers $(0, +, +)$ and $(\pi, -, -)$ respectively (pair of states (c) and (e) in the language of [7]). Using eq.(2.3) we see that the twisted sector component of the boundary state describing the combined system is given by:

$$\frac{1}{\sqrt{2}}(|T_1\rangle_{RR} + |T_2\rangle_{NSNS}) . \qquad (3.10)$$

Comparing with eq.(2.9) we see that it carries the same twisted sector RR charge as the system at $\alpha = 0$, but its coupling to the twisted sector NSNS state is opposite to that of the system at $\alpha = 0$. Thus the ζ dependent component of the tachyon potential should have opposite signs at $\alpha = 0$ and at $\alpha = 2$, as is the case for the potential given in eq.(3.9).

Using the periodicity $\alpha \to \alpha + 4$ we can restrict α to the range $-2 < \alpha \leq 2$. Also, by making a gauge transformation on the D-string $\bar{\text{D}}$-string system at $\alpha = 0$, we can change the sign of the tachyon, which corresponds to the transformation $\alpha \to -\alpha$. Thus α and $-\alpha$ denote equivalent configurations, and the physical range of α can be taken to be $0 \leq \alpha \leq 2$. Finally, without any loss of generality we can take ζ to be negative in our analysis, since the sign of ζ can be changed by a redefinition $\alpha \to 2 - \alpha$.

Let us define:

$$u = 2(R - R_c) . \qquad (3.11)$$

By analysing the potential (3.9) with $\zeta < 0$, we get the following results:

1. For $u < (-|\zeta|)$, $V(\alpha)$ has a pair of minima at $\alpha = 0$ and at $\alpha = 2$, and a maximum at $\alpha = (2/\pi)\cos^{-1}(-\zeta/u)$. The absolute minimum is at $\alpha = 0$. As $\zeta \to 0$, this

[15] Note that $(\alpha - 1)$ was denoted by α in [5], since there the starting configuration was the non-BPS D-string.

state goes over smoothly to the D-string $\bar{\rm D}$-string system with the Wilson line on the D-string. Thus even for non-zero ζ we can identify the system at $\alpha = 0$ to a D-string $\bar{\rm D}$-string pair.

2. For $(-|\zeta|) < u < (|\zeta|)$, $V(\alpha)$ has a minimum at $\alpha = 0$ and a maximum at $\alpha = 2$. As u passes through the point $u = -|\zeta|$, the minimum at $\alpha = 0$ evolves smoothly. Thus even in this range of u, we can interprete the stable minimum at $\alpha = 0$ as a D-string $\bar{\rm D}$-string pair.

3. For $u > (|\zeta|)$, $V(\alpha)$ has a pair of maxima at $\alpha = 0$ and at $\alpha = 2$, and a minimum at $\alpha = (2/\pi)\cos^{-1}(-\zeta/u)$. As $\zeta \to 0$, this minimum evolves smoothly to the stable non-BPS D-particle corresponding to the point $\alpha = 1$. Thus by continuity we can conclude that for $u > |\zeta|$ the stable minimum at $\alpha = (2/\pi)\cos^{-1}(-\zeta/u)$ denotes the stable non-BPS D-particle.

The results for $\zeta > 0$ can be obtained by using the symmetry $\zeta \to -\zeta$, $\alpha \to 2-\alpha$. In this case the $\alpha = 2$ configuration, representing a D-string $\bar{\rm D}$-string pair with Wilson line on the $\bar{\rm D}$ string, corresponds to the stable minimum for $u < |\zeta|$, and the $\alpha = \frac{2}{\pi}\cos^{-1}(-\frac{\zeta}{u})$ configuration, representing a non-BPS D-particle at $x = 0$, corresponds to the stable minimum for $u > |\zeta|$.

If we want to translate these results to the dual IIA description, we only need to note that the radius $\widetilde{R}$ of the circle in this description is given by $(1/R)$. Thus $(\widetilde{R} - \widetilde{R}_c) \simeq \frac{1}{R_c^2}(R_c - R) = 2(R_c - R)$. Thus the parameter u can be identified as $(\widetilde{R}_c - \widetilde{R})$. This reproduces the results quoted in the introduction.

The phase diagram in the $u-\zeta$ plane is quite simple. For $\zeta < 0$, $u < (|\zeta|)$ the D-string $\bar{\rm D}$-string with Wilson line on the D-string is the stable configuration. In the dual theory describing type IIA on K3, this corresponds to a pair of D2-branes, each wrapped on a supersymmetric cycle of K3. For $\zeta > 0$, $u < |\zeta|$, the D-string $\bar{\rm D}$-string system with Wilson line on the $\bar{\rm D}$-string is the stable configuration. In the dual IIA theory this again corresponds to a pair of wrapped D2-branes, but carrying opposite D0-brane charges compared to the previous configuration. For $u > (|\zeta|)$ the non-BPS D-particle is the stable configuration for all ζ. In the dual type IIA theory this represents a D2-brane wrapped on the non-supersymmetric cycle. The phase diagram in the $(\widetilde{R} = \widetilde{R}_c - u, \zeta)$ plane has been shown in Fig.4.

The location of the minimum α_{min} of $V(\alpha)$ evolves continuously as u crosses the phase boundary $|\zeta|$. It is instructive to study the nature of the transition across the line $u = |\zeta|$. For this note that for $\zeta < 0$, $u > |\zeta|$ the potential has two maxima and a minimum in the range $0 \le \alpha \le 2$. As u approaches $|\zeta|$ from above, the maximum at $\alpha = 0$, the minimum at $\alpha = (2/\pi)\cos^{-1}(-\zeta/u)$, and its image under $\alpha \to -\alpha$ merge together to become a single minimum at $\alpha = 0$. Thus we can conclude that the phase transition across the $\zeta < 0$, $u = |\zeta|$ line is second order. As is the characteristic of such a phase transition, at $u = |\zeta|$ the first three α derivatives of $V(\alpha)$ vanish at $\alpha = 0$. The same result holds for the line $\zeta > 0$, $u = |\zeta|$. On the other hand, phase transition across the line $\zeta = 0$, $u < 0$ is first order, as the location of the minimum of $V(\alpha)$ jumps discontinuously from $\alpha = 0$ to $\alpha = 2$ across this line.

Note that for $u > |\zeta|$ the value of the potential at the minimum α_{min} is given by:

$$V(\alpha_{min}) = -\frac{1}{g}[\frac{1}{2}(R - R_c) + \frac{\zeta^2}{4(R - R_c)}]\,. \tag{3.12}$$

From this one can calculate the mass of the non-BPS D-particle as follows. First of all we note that the total mass of the system for a given value of α must be related to $V(\alpha)$ by an additive constant:

$$M(\alpha) = C + V(\alpha)\,. \tag{3.13}$$

C is determined by demanding the $M(\alpha = 0)$ reproduces correctly the mass of the D-string $\bar{\text{D}}$-string pair. Since using footnote 14 we see that $\pi\zeta$ corresponds to the shift of the NS 5-brane near $x = \pi R$, the net distance of this NS 5-brane from the $x = 0$ plane is $\pi(R + \zeta)$. Thus the mass of the D-string $\bar{\text{D}}$-string system, obtained by multiplying their length by the tension, is given by $(R + \zeta)/g$. This gives:

$$C = \frac{1}{g}(R + \zeta) - V(\alpha = 0) = \frac{1}{2g}(R + R_c)\,. \tag{3.14}$$

Thus the mass of the stable non-BPS D-particle for $R > R_c + \frac{1}{2}|\zeta|$ is given by:

$$M(\alpha_{min}) = C + V(\alpha_{min}) = \frac{1}{g}(R_c - \frac{\zeta^2}{4(R - R_c)})\,. \tag{3.15}$$

References

[1] J. Polchinski, Phys. Rev. Lett. **75**, 4724 (1995) hep-th/9510017.

[2] J. Dai, R. Leigh, and J. Polchinski, Mod. Phys. Lett. **A4** (1989) 2073;
R. Leigh, Mod. Phys. Lett. **A4** (1989) 2767;
P.Horava, Phys. Lett. **B231**, 251 (1989);
M.B. Green, Phys. Lett. **B266**, 325 (1991); Phys. Lett. **B329**, 435 (1994); Phyl. Lett. **B354**, 271 (1995); hep-th/9510016;
J. Polchinski, Phys. Rev. **D50** (1994) 6041 [hep-th/9407031];
J. Polchinski, S. Chaudhury and C. Johnson, [hep-th/9602052];
J. Polchinski, [hep-th/9611050].

[3] M.R. Douglas and G. Moore, hep-th/9603167.

[4] P. Aspinwall, Phys. Lett. **B357** 329 (1995) hep-th/9507012.

[5] A. Sen, JHEP **12**, 021 (1998) hep-th/9812031.

[6] O. Bergman and M.R. Gaberdiel, JHEP **03**, 013 (1999) hep-th/9901014.

[7] A. Sen, JHEP **08**, 010 (1998) hep-th/9805019.

[8] A. Sen, JHEP **09**, 023 (1998) hep-th/9808141.

[9] E. Witten, JHEP **12** (1998) 019 hep-th/9810188.

[10] M. Frau, L. Gallot, A. Lerda and P. Strigazzi, hep-th/9903123.

[11] A. Sen, hep-th/9904207.

[12] A. Lerda and R. Russo, hep-th/9905006.

[13] C. Vafa, Private communication.

[14] A. Sen, JHEP **10** (1998) 021 hep-th/9809111.

[15] O. Bergman and M.R. Gaberdiel, Phys. Lett. **B441**, 133 (1998) hep-th/9806155.

[16] J. Polchinski and Y. Cai, Nucl.Phys. **B296** 91 (1988).

[17] C. Callan, C. Lovelace, C. Nappi and S. Yost, Nucl.Phys. **B308** 221 (1988).

[18] T. Onogi and N. Ishibashi, Mod. Phys. Lett. **A4**, 161 (1989).

[19] N. Ishibashi, Mod. Phys. Lett. **A4**, 251 (1989).

[20] C. Lovelace, Phys. Lett. **B34** (1971) 500;
L. Clavelli and J. Shapiro, Nucl. Phys. **B57** (1973) 490;
M. Ademollo, R.D'Auria, F. Gliozzi, E. Napolitano, S. Sciuto and P. di Vecchia, Nucl. Phys. **B94** (1975) 221;
C. Callan, C. Lovelace, C. Nappi and S. Yost, Nucl.Phys. **B293** (1987) 83;
M. Bianchi and A. Sagnotti, Phys. Lett. **247B** (1990) 517; Nucl. Phys. B361 (1991) 519;
P. Horava, Nucl. Phys. **B327** 461 (1989).

[21] O. Bergman and M. Gaberdiel, Nucl.Phys. **B499** 183 (1997) hep-th/9701137;
M. Li, Nucl.Phys. **B460** 351 (1996) hep-th/9510161;
H. Ooguri, Y. Oz and Z. Yin, Nucl.Phys. **B477** 407 (1996) hep-th/9606112;
K. Becker, M.Becker, D. Morrison, H. Ooguri, Y. Oz and Z. Yin, Nucl. Phys. **B480** 225 (1996) hep-th/9608116;
M. Kato and T. Okada, Nucl. Phys. **B499** 583 (1997) 583 hep-th/9612148;
S. Stanciu, hep-th/9708166;
A. Recknagel and V. Schomerus, hep-th/9712186;
J. Fuchs and C. Schweigert, hep-th/9712257;
S. Stanciu and A. Tseytlin, hep-th/9805006;
M. Gutperle and Y. Satoh, hep-th/9808080;
F. Hussain, R. Iengo, C. Nunez and C. Scrucca, Phys. Lett. **B409** 101 (1997) hep-th/9706186;
M. Bertolini, R. Iengo and C. Scrucca, hep-th/9801110;
M. Bertolini, P. Fre, R. Iengo and C. Scrucca, hep-th/9803096;
P. Di Vecchia, M. Frau, A. Lerda, I. Pesando, R. Russo and S. Sciuto, Nucl. Phys. **B507** 259 (1997) hep-th/9707068;
M. Billo, P. Di Vecchia, M. Frau, A. Lerda, I. Pesando, R. Russo and S. Sciuto, Nucl. Phys. **B526** 199 (1998) hep-th/9802088; hep-th/9805091;
A. Recknagel and V. Schomerus, hep-th/9811237.

[22] D. Friedan, E. Martinec and S. Shenker, Nucl. Phys. **B271**, 93 (1986).

[23] A. Sen, Nucl. Phys. **B474**, 361 (1996) hep-th/9604070.

[24] P. Horava, hep-th/9812135.