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Uniqueness of Tachyonic Solitons

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Abstract

It has been conjectured that condensation of tachyons on a bosonic D-brane gives rise to vacuum / soliton solutions which are independent of the initial magnetic field on the D-brane. We present evidence for this conjecture using results from two dimensional conformal field theory. In particular we identify a continuous path in the configuration space of open string fields which interpolates between D-brane configurations with two different quantized magnetic flux.

The dynamics of tachyon, massless scalars and gauge fields on a D-p-brane of 26 dimensional bosonic string theory in flat Minkowski signature space-time in static gauge is described by the action

$$S_{BI} = -\frac{1}{g_c}\frac{1}{(2\pi)^p}\int d^{p+1}x\,\mathcal{V}(T)\sqrt{-\det\Big(g_{\mu\nu}+\partial_\mu Y^i\partial_\nu Y^i+2\pi F_{\mu\nu}\Big)}+\ldots\,, \qquad (1)$$

where x^μ denote the world-volume coordinates of the brane ($0\le\mu\le p$), y^i denote the space-time coordinates transverse to the brane ($p+1\le i\le 25$), Y^i denote the massless scalar fields on the D-brane world-volume associated with the coordinates y^i, g_c is the closed string coupling constant, g_{MN} is the closed string metric (which we take to be constant with $g_{i\mu}=0$, $g_{ij}=\delta_{ij}$), $F_{\mu\nu}=\partial_\mu A_\nu-\partial_\nu A_\mu$ denote the gauge field strength, T denotes the tachyon field, and $\mathcal{V}(T)$ denotes the tachyon potential with the D-brane tension term included. $\mathcal{V}(T)$ has a maximum at $T=T_{max}$ describing the original D-brane configuration and has been conjectured to have a local minimum at $T=T_{min}$ where $\mathcal{V}(T_{min})=0$[1, 2]. We have chosen $\alpha'=1$ and have normalized $\mathcal{V}$ so that $\mathcal{V}(T=T_{max})=1$. $\ldots$ denotes terms containing derivatives of T, $\partial_\mu Y$ and $F_{\mu\nu}$.

It has also been conjectured that the minimum of the potential $T=T_{min}$ describes the vacuum without a D-brane[1], and that at this minimum all different values of $F_{\mu\nu}$ and Y^i actually describe the same physical configuration in open string field theory[3]. It follows from this, using the techniques of non-commutative field theory[4, 5, 6], that many apparently different soliton solutions, for which T approaches T_{min} asymptotically, but the asymptotic values of $F_{\mu\nu}$ are different, also describe the same configuration[3]. This in turn resolves some of the puzzles raised in refs.[7, 8] in interpreting non-commutative tachyonic solitons as D-branes[9, 10, 11].

Some evidence for this conjecture comes from the string field theory results of refs.[12, 13]. In this paper we provide an evidence for this conjecture using the techniques of two dimensional conformal field theory (CFT). For simplicity of notation, we shall take our initial system to be a D-2 brane, and assume that the directions tangential to the 2-brane have been compactified on a torus T^2. In this case, the above conjecture has a somewhat dramatic consequence. Since the total magnetic flux through T^2 is quantized, in terms of the field variables appearing in the Born-Infeld action it is not possible to continuously deform a configuration with a given magnetic flux to a configuration with a different magnetic flux *even via off-shell field configurations.* On the other hand if the conjecture is correct, then starting from a configuration with a given magnetic flux at $T=T_{max}$, we can deform T to T_{min}, and since at $T=T_{min}$ all magnetic field background describes the same configuration, we can change the value of the magnetic field to any other value allowed by the quantization rules. We can then deform the tachyon field back to $T = T_{max}$. This gives a continuous path in the configuration space of open string field theory interpolating between D-branes with different amounts of magnetic flux.

A T-dual version of this phenomenon is as follows. Let us take a D1-brane, and take the direction x^1 tangential to the D-brane, and a direction (say y^2) transverse to the D-brane to be compact. As usual, we denote by Y^2 the massless scalar field on the D-brane world-volume associated with the coordinate y^2. Now consider a classical field configuration on the D1-brane world-volume theory of the form:

$$Y^2 = ax^1 \, , \tag{2}$$

where a is a constant. In this case compactness of the 1 and 2 directions imply that only discrete values of a are allowed. In particular, if both 1 and 2 directions have the same periodicity (say 2π), then a must be an integer,[1] since as x^1 changes by 2π, Y^2 must change by an integer multiple of 2π. Again, the Born-Infeld action describing the D-brane world-volume theory does not allow a continuous deformation of fields which interpolates between configurations with different values of a. However, if the conjecture stated above is correct, then we should be able to interpolate between these two field configurations by starting with a given value of a, taking T to T_{min} where all values of a correspond to the same configuration, and then changing T back to T_{max} with a taking a different value.

In this paper we explicitly demonstrate this phenomenon using the techniques of two dimensional CFT. For definiteness we shall focus on the problem of interpolating between two magnetic field backgrounds in D2-brane wrapped on a torus. Since it is difficult to describe tachyon condensation into vacuum using the CFT techniques, one would have thought that even if the interpolation outlined above had been possible, it will be difficult to demonstrate this using CFT techniques since the path passes through the vacuum configuration. But as we shall see, we do not need to go all the way down to the vacuum configuration for this interpolation; it is possible to find a path via a codimension one soliton, *i.e.* a D1-brane in this case. Since formation of a codimension one soliton via tachyon condensation is a well understood process in conformal field theory[15, 1, 17, 18], we can use CFT techniques for studying this process.

[1]This describes a D-string pointing along the vector $(1, a)$ in the (x^1, y^2) plane for integer a. There are also other allowed D-string configurations pointing along the vector (p, q) for any relatively prime pair of integers p, q. They can be represented as classical configurations in the world-volume theory of p D-strings lying along x^1.

More specifically, we shall show that under certain conditions the initial system of D2-brane with magnetic flux can be taken to a D1-brane via marginal deformation, and furthermore, that this D1-brane can be taken to a D2-brane without magnetic flux via a relevant deformation. In the language of string field theory this implies that in the configuration space of string fields there is a continuous path which interpolates between a D2-brane configuration with magnetic flux and a D2-brane configuration without magnetic flux. This is precisely what is expected according to the conjecture stated earlier.

We label the D2-brane world volume by coordinates (x^0, x^1, x^2) with $x^1 \equiv x^1 + 2\pi$, $x^2 \equiv x^2 + 2\pi$, take the background anti-symmetric tensor field to be 0, and the background closed string metric to be

$$g_{\mu\nu} = \begin{pmatrix} R_1^2 & \\ & R_2^2 \end{pmatrix} \qquad \text{for} \quad \mu = 1, 2\,, \tag{3}$$

with the rest of the components of g_{MN} being equal to those of the Minkowski metric η_{MN}. Thus R_1 and R_2 are the radii of the circles, measured in the closed string metric, along x^1 and x^2 directions respectively. With the normalization of $F_{\mu\nu}$ used in writing eq.(1), the quantization law of the magnetic flux is given by:

$$F_{12} = \frac{n}{2\pi}\,. \tag{4}$$

From eq.(1) we see that the mass of the wrapped D2-brane is given by

$$M_{D2} = \frac{1}{g_c}\sqrt{R_1^2 R_2^2 + (2\pi F_{12})^2} = \frac{1}{g_c}\sqrt{R_1^2 R_2^2 + n^2}\,. \tag{5}$$

Now, according to the results of ref.[6, 14] we can describe the string field theory living on this system around this background magnetic field by starting with a string field theory written in a background metric $G_{\mu\nu}$, effective coupling g_o, and zero background magnetic field, and then replacing all products appearing in the action and the equations of motion of this theory by non-commutative $*$-products defined with non-commutativity parameter $\Theta^{\mu\nu}$. $G_{\mu\nu}$, $\Theta^{\mu\nu}$ and g_o are given in terms of $g_{\mu\nu}$, $F_{\mu\nu}$ and g_c as:

$$G^{-1} + \frac{\Theta}{2\pi} = (g + 2\pi F)^{-1}\,, \qquad g_o = g_c\sqrt{\frac{\det G}{\det(g + 2\pi F)}}\,. \tag{6}$$

This gives, using eqs.(3), (4)

$$G = (R_1^2 R_2^2 + n^2)\begin{pmatrix} R_2^{-2} & \\ & R_1^{-2} \end{pmatrix}, \qquad \Theta = \frac{2\pi n}{R_1^2 R_2^2 + n^2}\begin{pmatrix} & -1 \\ 1 & \end{pmatrix}, \qquad g_o = g_c\frac{\sqrt{R_1^2 R_2^2 + n^2}}{R_1 R_2}\,. \tag{7}$$

From this we see that the radius of the x^1 direction measured in the metric $G_{\mu\nu}$ is given by $\sqrt{(R_1^2 R_2^2 + n^2)/R_2^2}$. Let us adjust R_1, R_2 and n such that this radius is unity, *i.e.*

$$R_1^2 R_2^2 + n^2 = R_2^2 \,. \tag{8}$$

We shall now show that when eq.(8) is satisfied, there is a marginal deformation which takes the CFT describing the D2-brane system under study to a D1-brane along x^2. For this let us consider an auxiliary system where the non-commutativity parameter is set to zero, keeping $G_{\mu\nu}$ and g_o fixed at values given in eqs.(7). This will correspond to a D2-brane wrapped on T^2 with zero background magnetic field, and closed string metric and coupling constant given by $G_{\mu\nu}$ and g_o defined in eq.(7). When eq.(8) is satisfied, the radius in the x^1 direction is unity for this auxiliary system, and the results of refs.[15, 1] show that there is an exact marginal deformation in the boundary CFT describing this auxiliary system, generated by the operator $\cos(X^1)$. Furthermore, for a specific value of the deformation parameter, the deformed CFT associated with this auxiliary system represents a D1-brane lying along the x^2 direction. In the language of string field theory, the existence of this marginal deformation implies the existence of a one parameter family of solutions, with fields depending on the x^1 direction[12].

Let us now go back to the original system, describing the D2-brane on T^2 with n units of magnetic flux on it. Equations of motion in the string field theory describing this system differs from those in the auxiliary system by the replacement of all the ordinary products by $*$-products[6, 14]. But the $*$-product reduces to the ordinary product for field configurations which depend on only one direction. Thus the one parameter family of solutions in the string field theory describing the auxiliary system are also classical solutions in the string field theory describing the original system.

In the language of two dimensional conformal field theory, this means that for the D2-brane on T^2 in the background given in eqs.(3), (4), the boundary CFT admits an exactly marginal perturbation when eq.(8) is satisfied, (Special cases of such deformations for $R_1 = R_2$ were discussed in ref.[16].) One would naturally suspect that just as in the case of the auxiliary system, this CFT also flows to a D1-brane along x^2 under this marginal deformation. That this is indeed so can be argued by noting that the correlation functions of open string vertex operators in the boundary CFT, which carry momentum only along the X^1 direction, are identical to that in the auxiliary CFT. Thus we can borrow the results of [15, 1] and conclude that there is a special point along the direction of marginal deformation where the Neumann boundary condition along x^1 gets converted to a Dirichlet boundary condition. This gives a D1-brane along the x^2 direction.

There are various consistency checks that one can perform to confirm this result:

1. The mass of a D1-brane wrapped along x^2 is given by

$$M_{D1} = \frac{R_2}{g_c} \,. \tag{9}$$

   When eq.(8) is satisfied, the mass of the D1-brane given in (9) matches with that of the D2-brane given in (5).

2. If there is a marginal deformation interpolating between the initial D2-brane configuration and the final D1-brane configuration, then the CFT describing the final D1-brane system must also admit a marginal deformation. In the analysis of refs.[15, 1] this came from massless open string modes winding around the 1 direction. The ground state of such an open string has mass^2 equal to

$$(R_1^2 - 1) = -n^2/R_2^2 \,, \tag{10}$$

   using (8). This is strictly negative for $n \neq 0$, and describes a tachyonic mode rather than a massless mode. This poses a puzzle. However note that if we consider an open string state winding once along the x^1 direction and carrying n units of momentum along the x^2 direction, then it has mass^2

$$R_1^2 + \frac{n^2}{R_2^2} - 1 = 0 \,. \tag{11}$$

   Thus in this case these open string modes give rise to the marginal deformation which takes us back to the original D2-brane system with a magnetic field on it. This calculation also illustrates the importance of quantization law of F; if n in eq.(8) had not been an integer, the spectrum of open strings on the D-string wrapped along x^2 would not contain a massless state of this kind.

3. The analysis can be given a more intuitive interpretation in a T-dual language. For this let us make an $R \to 1/R$ duality transformation along the x^2 direction so that the dual $\tilde{x}^2$ direction now has radius R_2^{-1}. In that case the initial configuration describes a D-string pointing along the vector $(1, n)$ in the $(x^1, \tilde{x}^2)$ plane. The total length of such a D-string inside a unit cell is $2\pi\sqrt{R_1^2 + n^2/R_2^2}$. It can be easily verified that when condition (11) is satisfied, the open string state on this D-string, carrying unit

momentum along the D-string, is exactly marginal. Using the results of refs.[1, 15] we can show that this marginal deformation takes the D-string to a D0-brane. We can now go back to the original description by a reverse $R \to 1/R$ duality transformation along the $\tilde{x}^2$ direction. This gives us a D-string stretched along x^2.

Thus we have established that there is a marginal deformation which takes us from an initial configuration of D2-brane with n units of magnetic flux to a D1-brane lying along x^2. We shall now show that there is a relevant deformation of the CFT which takes this D1-brane to a D2-brane wrapped on T^2, but with no magnetic flux. As seen from eq.(10), the ground state of an open string with unit winding along the x^1 direction, and no momentum along the x^2 direction, represents a tachyonic mode, and hence a relevant deformation of the boundary CFT. We shall now investigate the effect of switching on this relevant perturbation on the conformal field theory describing the D-string. This is indeed a well studied problem, and can be recognised as such by going to the T-dual description in which we make an $R \to (1/R)$ duality transformation along the x^1 direction. This takes the D-string to a D2-brane wrapped on the dual torus, and the open string states with unit winding along x^1 to open string states with unit momentum along the dual $\tilde{x}^1$ direction. As has been shown in refs.[17, 18], the effect of perturbation by open string vertex operators carrying unit momentum along the $\tilde{x}^1$ direction is to take the D2-brane to a D1-brane lying along x^2. Going back to the original description by another $R \to 1/R$ duality transformation along $\tilde{x}^1$, we see that the final configuration is a D2-brane wrapped on the original torus T^2, *without any background magnetic field*.

Thus by a combination of marginal and relevant deformations we can take a D2-brane wrapped on T^2 with *n units of magnetic flux* to a D2-brane wrapped on T^2 with *no magnetic flux*. In open string field theory the effect of a marginal deformation can be represented by a continuous deformation of string field configuration via *on-shell field configurations*, whereas a relevant deformation of the kind discussed here can be represented by a continuous deformation of string field configuration via *off-shell field configurations*. (To see how relevant deformation can be regarded as a continuous deformation via off-shell field configuration, we can take the solution given in ref.[19], and continuously deform each component of the string field from 0 to the final value appropriate for the solution.) Thus the result of this paper implies that in string field theory describing dynamics of a D2-brane wrapped on T^2, there is a continuous deformation via off-shell string field configurations which can interpolate between a configuration with n units of magnetic flux through T^2 and a configuration with 0 unit of magnetic flux through T^2. This establishes the desired result.

One can give a slight twist to this tale by taking R_1 to be 1 instead of the value given in eq.(8). In this case the x^1 radius of the auxiliary system, given by $\sqrt{1+n^2R_2^{-2}}$, is larger than 1, and as a result the open string vertex operator with unit momentum along the x^1 direction is relevant rather than marginal. Nevertheless, the analysis of refs.[17, 18] tells us that under this relevant deformation the auxiliary CFT flows to that describing a D-string along x^2. The arguments given earlier then shows that the same must be true also for the original system consisting of a D2-brane with n units of magnetic flux.

Now start with a different configuration, – a D2-brane wrapped on the same torus but without any magnetic flux. For $R_1 = 1$ the ground state of the open string with unit momentum along x^1 is massless and describes an exact marginal deformation which takes the D2-brane to a D1-brane lying along x^2. Thus we see that starting with two different D2-brane configurations, one with magnetic flux and one without magnetic flux, we reach the same D1-brane configuration. This in turn shows that the codimension one soliton formed by tachyon condensation on the original D2-brane is independent of the magnetic flux on the brane. This is precisely what is predicted according to the conjecture that we are trying to verify.

We conclude with the following observations:

- The analysis of this paper can also be applied to the D-brane anti-D-brane system or non-BPS D-brane in type II string theories. Magnetic field on a non-BPS D-brane does not give rise to any Ramond-Ramond (RR) charge, and so the possibility of changing this magnetic field does not violate any conservation law. For a D-brane anti-D-brane system only a special combination of the magnetic field on the two branes can be switched on this way, – the one which does not give rise to any RR charge.[2] Thus there is again no conflict with conservation of RR charges.

- The main lesson learnt from the analysis of this paper is that whereas the description of the world-volume theory of the D-brane in terms of the effective action involving tachyon and the massless fields captures many of the important features, it fails to capture all the important properties of the system. This point has already been advocated forcefully in ref.[13] in a different context. Here we see another illustration of the same phenomenon. Field configurations which are disconnected in the low energy effective field theory get connected to each other in the full string theory. On

[2] I would like to thank S. Minwalla for discussion on this point.

the other hand, description of the D-brane system in terms of conformal field theory is suitable for studying condensation of tachyons into a lower dimensional soliton, but not into the vacuum. Thus full-fledged string field theory seems to be the only framework for studying all aspects of the problem.

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References

[1] A. Sen, "Descent relations among bosonic D-branes," Int. J. Mod. Phys. **A14**, 4061 (1999) [hep-th/9902105].

[2] V.A. Kostelecky and S. Samuel, "The Static Tachyon Potential in the Open Bosonic String Theory," Phys. Lett. **B207** (1988) 169;
A. Sen and B. Zwiebach, "Tachyon Condensation in String Field Theory," hep-th/9912249;
N. Moeller and W. Taylor, "Level truncation and the tachyon in open bosonic string field theory", hep-th/0002237.

[3] A. Sen, "Some issues in non-commutative tachyon condensation," hep-th/0009038.

[4] A. Connes, M. R. Douglas and A. Schwarz, "Noncommutative geometry and matrix theory: Compactification on tori," JHEP **9802**, 003 (1998) [hep-th/9711162];
M. R. Douglas and C. Hull, "D-branes and the noncommutative torus," JHEP **9802**, 008 (1998) [hep-th/9711165].

[5] V. Schomerus, "D-branes and deformation quantization," JHEP **9906**, 030 (1999) [hep-th/9903205].

[6] N. Seiberg and E. Witten, "String theory and noncommutative geometry," JHEP **9909**, 032 (1999) [hep-th/9908142].

[7] R. Gopakumar, S. Minwalla and A. Strominger, "Symmetry restoration and tachyon condensation in open string theory," hep-th/0007226.

[8] N. Seiberg, "A note on background independence in noncommutative gauge theories, matrix model and tachyon condensation," hep-th/0008013.

[9] R. Gopakumar, S. Minwalla and A. Strominger, "Noncommutative solitons," JHEP **0005**, 020 (2000) [hep-th/0003160].

[10] K. Dasgupta, S. Mukhi and G. Rajesh, "Noncommutative tachyons," JHEP **0006**, 022 (2000) [hep-th/0005006].

[11] J. A. Harvey, P. Kraus, F. Larsen and E. J. Martinec, "D-branes and strings as non-commutative solitons," JHEP **0007**, 042 (2000) [hep-th/0005031].

[12] A. Sen and B. Zwiebach, "Large marginal deformations in string field theory," hep-th/0007153.

[13] W. Taylor, "Mass generation from tachyon condensation for vector fields on D-branes," hep-th/0008033.

[14] T. Kawano and T. Takahashi, hep-th/9912274.

[15] A. Recknagel and V. Schomerus, "Boundary deformation theory and moduli spaces of D-branes," Nucl. Phys. **B545**, 233 (1999) hep-th/9811237;
C.G. Callan, I.R. Klebanov, A.W. Ludwig and J.M. Maldacena, "Exact solution of a boundary conformal field theory," Nucl. Phys. **B422**, 417 (1994) hep-th/9402113;
J. Polchinski and L. Thorlacius, "Free fermion representation of a boundary conformal field theory," Phys. Rev. **D50**, 622 (1994) hep-th/9404008.

[16] C. G. Callan, I. R. Klebanov, J. M. Maldacena and A. Yegulalp, "Magnetic fields and fractional statistics in boundary conformal field theory," Nucl. Phys. **B443**, 444 (1995) [hep-th/9503014].

[17] P. Fendley, H. Saleur and N. P. Warner, "Exact solution of a massless scalar field with a relevant boundary interaction," Nucl. Phys. **B430**, 577 (1994) [hep-th/9406125].

[18] J. A. Harvey, D. Kutasov and E. J. Martinec, "On the relevance of tachyons," hep-th/0003101.

[19] N. Moeller, A. Sen and B. Zwiebach, "D-branes as tachyon lumps in string field theory," hep-th/0005036.