# Test of Siegel Gauge for the Lump Solution 

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#### Abstract

We test the validity of the Siegel gauge condition for the lump solution of cubic open bosonic string field theory by checking the equations of motion of the string field components outside the Siegel gauge. At level $(3,6)$ approximation, the linear and quadratic terms of the equations of motion of these fields are found to cancel within about $20 \%$.


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## 1 Introduction

String field theory [1] , 2] has turned out to be a very powerful tool for directly verifying the various conjectures 3, 因, 5] about tachyon condensation on unstable D-branes of bosonic and superstring theories. There are two main conjectures: 1) the state corresponding to the tachyon condensed to the minimum of its potential is the closed string vacuum state without any D-brane, and 2) suitable classical solutions involving the tachyon field represent the various lower dimensional D-branes. Both the conjectures make statements about nonperturbative field configurations on a D-brane. It is natural that in a second quantized string theory which defines string theory off-shell, one should be able to verify these conjectures directly. Various works in second quantized string theories have been done towards this direction both in the context of bosonic string [7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25] and superstring theories [26, 27, 28, 29, 30].

Witten's cubic string field theory [1] is one of the candidates for the second quantized string theory in the bosonic case. Although the action of this theory is at most cubic in fields, there are infinite number of terms in the action. Therefore performing computations is in general difficult. But fortunately the argument of universality 31, 32 and application of symmetry properties, - all of which help in truncating the string field consistently to a subspace of the full configuration space, - combined with the level truncation method introduced by Kostelecky and Samuel [6] have made computations possible. The work of refs. [7], [9] has verified the first conjecture in bosonic string theory to a very high degree of accuracy. In the work of ref. [12] the verification of the second conjecture has been performed by explicitly constructing the codimension one lump solution on a D-brane wrapped on a circle, and verifying that the energy of the solution corresponds to that of the unwrapped D-brane of one lower dimension. Computations have been done for
various values of the radius of the circle and with a modified level truncation scheme. This analysis has also been extended to the higher codimension solutions [13, 14].

The cubic string field theory is a gauge theory. Therefore in this case one has to deal with the gauge-fixing conditions. Because of the infinite number of fields present in the theory, it is difficult to show the reasonability of a given gauge choice in general. Both, the construction of the nonperturbative vacuum solution [7, (9] and the lump solution [12, [13, (14] have been carried out using the Siegel gauge condition. But acceptance of these solutions is subject to the validity of this gauge chosen for the solutions. In ref. 18] the stringy BRST invariance of the nonperturbative vacuum solution has been explicitly checked in the level truncation scheme. This is equivalent to checking the validity of the Siegel gauge for the solution by ensuring that the equations of motion of the fields outside the Siegel gauge are automatically satisfied by the solution obtained in the Siegel gauge. In this paper we attempt to check the validity of the Siegel gauge for the lump solution given in ref. (12] using similar method. We will perform computations only for the value of the radius $\sqrt{3}$ and use the modified level truncation method introduced in ref. 12].

The rest of the paper is organized as follows. In sec 2 we give the description of the setup in which we perform our computation. This is basically a review of the setup considered in ref. [12], the only exception being that the set of string fields is extended to include the fields outside the Siegel gauge. In sec. 3 we explain, in the context of the lump solution, the approach taken to test the validity of a gauge choice for a specific solution. Finally in sec.母 we give our results up to the level $(3,6)$ in the modified level truncation scheme. Although this is done only for radius $R=\sqrt{3}$, the analysis can be easily extended to the other values of the radius analysed in ref. [12].

## 2 Review of the Setup

Here we review the general setup considered in ref. [12] in computing the lump solution in a modified level truncation scheme. We will follow the same notations and conventions adopted in this paper.

- Action: We consider a D-brane in the 26 dimensional bosonic string theory. The cubic open string field theory action on the D-brane is given by:

$$
\begin{equation*}
S=-\frac{1}{g_{o}^{2}}\left(\frac{1}{2}\left\langle\Phi, Q_{B} \Phi\right\rangle+\frac{1}{3}\langle\Phi, \Phi * \Phi\rangle\right), \tag{2.1}
\end{equation*}
$$

where the string field $\Phi$ is a ghost number 1 state in the Hilbert space of the combined matter-ghost conformal field theory. The BRST charge $Q_{B}$, the BPZ inner product $\langle A, B\rangle$ and the star product $A * B$ have their usual meaning. This action possesses gauge invariance with the following gauge transformation law:

$$
\begin{equation*}
\delta|\Phi\rangle=Q_{B}|\Lambda\rangle+|\Phi * \Lambda\rangle-|\Lambda * \Phi\rangle \tag{2.2}
\end{equation*}
$$

where $|\Lambda\rangle$ is a ghost number zero state.

- Background: The background considered in ref. [12] is of quite general type. The total matter conformal field theory CFT on the open string world-sheet is,

$$
\begin{equation*}
\mathrm{CFT}=\operatorname{CFT}(X)+\mathrm{CFT}^{\prime}, \tag{2.3}
\end{equation*}
$$

with

$$
\begin{equation*}
\mathrm{CFT}^{\prime}=\operatorname{CFT}(Y) \oplus \operatorname{CFT}\left(X^{0}\right) \oplus \operatorname{CFT}(\mathcal{M}) \tag{2.4}
\end{equation*}
$$

Here $X$ is the world-sheet scalar field corresponding to a direction $x$ along the Dbrane which is compactified on a circle of radius $R$ and the scalar fields $Y$ and $X^{0}$ correspond to respectively a space-like non-compact direction $y$ transverse to the D-brane, and the time direction $x^{0} . \mathcal{M}$ is an arbtrary manifold describing the rest of the compactification of space-time with the only restriction that any noncompact direction of $\mathcal{M}$ is transverse to the D -brane ${ }^{2}$. This effectively makes the D -brane a D-string aligned along $x$ in the non-compact part of the space-time.

- Consistent truncation: For a lump solution which varies only along $x$, the background fields can carry momentum only along the $x$ direction. As was shown in ref. (12], in constructing this solution we can use a truncated version of the string field theory where we take the string field to be a linear combination of states created by the ghost oscillators, and the Virasoro generators of $\mathrm{CFT}(X)$ and $\mathrm{CFT}^{\prime}$, on parity even primary states of $\operatorname{CFT}(X)$. Furthermore, the Siegel gauge condition excludes the excitations involving the ghost oscillator $c_{0}$. Since in the gauge invariant action one has to include these states, the truncated spectrum is generated by acting the oscillators

$$
\begin{equation*}
\left\{L_{-1}^{X}, L_{-2}^{X}, \cdots ; L_{-2}^{\prime}, L_{-3}^{\prime}, \cdots ; c_{1}, c_{0}, c_{-1}, c_{-2}, \cdots ; b_{-2}, b_{-3}, \cdots\right\} \tag{2.5}
\end{equation*}
$$

on either

[^1]- the zero momentum parity-even primary states of $\operatorname{CFT}(X)$ (with the null states removed), or
- the Fock vacuum states of the form $\cos \left(\frac{n}{R} X(0)\right)|0\rangle$.

Among these states we must keep only those which have ghost number 1 and are twist even. The latter property requires that the total contribution to $L_{0}$ eigenvalue of the state from the matter and ghost oscillators is odd.

- Modified level truncation scheme: Ref. 12 has introduced a modified level truncation scheme which is applicable in a more general situation. The level of a field is defined as the difference between the $L_{0}$ eigenvalue of the corresponding state and the zero momentum tachyon state. Then the level $(M, N)$ approximation to the action is obtained by keeping all fields with level $M$ and below, and keeping all terms in the action with total level $N$ and below.


## 3 Testing Validity of the Siegel Gauge for the Lump Solution

While making a gauge choice it has to be ensured that any field configuration can be brought to lie on the gauge slice under a gauge tranformation. Whether a gauge choice is good in this sense or not can depend on the class of field configurations considered in a given problem. For example, for the twist even cofigurations, the Siegel gauge, namely $b_{0}|\Phi\rangle=0$, is a good gauge choice near $\Phi=0$. But the vacuum solution found in refs. [6, 7, 9] or the lump solution found in ref. 12] is far away from $\Phi=0$. Thus the arguments showing the validity of the Siegel gauge near $\Phi=0$ are not applicable here. In order to show that the Siegel gauge is a valid gauge choice for a solution of this kind, one needs to verify that the solution obtained in the Siegel gauge satisfies the full set of gauge invariant equations of motion. Since the solutions in the Siegel gauge are constructed by setting to zero the variation of the gauge fixed action with respect to the fields satisfying the Siegel gauge condition, what needs to be checked is that the equations of motion (derived from the gauge invariant action) of the fields which do not satisfy the Siegel gauge condition are also satisfied. For the vacuum solution this was verified in ref. [18]. To check the validity of Siegel gauge for the lump solution in this approach one follows the following steps in the modified level truncation scheme:

- Compute the action $S$ (eqn. (2.1)) by expanding the string field $|\Phi\rangle$ in the truncated Hilbert space without implementing any gauge condition. ${ }^{\text {P }}$
- Find the equations of motion of the fields outside the siegel gauge.
- Check if these equations of motion are satisfied by the lump solution obtained in the gauge fixed theory.

A given term in the action $S$ can be linear, quadratic or cubic in a given field. Since in the lump solution all the fields outside the Siegel gauge are set to zero, in verifying the equations of motion we need to take the first derivative of the action with respect to various fields and then set the fields outside the Siegel gauge to zero. Thus only those terms in the action which are linear in fields outside the Siegel gauge contribute to the equation of motion of these fields. These are the terms we need to compute.

Let us take the following expansion for the truncated string field:

$$
\begin{equation*}
|\Phi\rangle=\sum_{a} \phi_{a}\left|\Phi_{a}\right\rangle+\sum_{n} \phi_{n}\left|\Phi_{n}\right\rangle, \tag{3.1}
\end{equation*}
$$

where the indices $a$ and $n$ run over respectively the states inside the Siegel gauge and outside the Siegel gauge. Then the part of the action which contributes to the equation of motion of a specific field $\phi_{n}$ is:

$$
\begin{equation*}
-\frac{1}{g_{o}^{2}}\left(\sum_{a} C_{a n} \phi_{a}+\sum_{a, b} C_{a b n} \phi_{a} \phi_{b}\right) \phi_{n} \tag{3.2}
\end{equation*}
$$

where $C_{\alpha \beta}=C_{\beta \alpha}=\left\langle\Phi_{\alpha}, Q_{B} \Phi_{\beta}\right\rangle$ and $C_{\alpha \beta \gamma}=C_{\alpha \gamma \beta}=\left\langle\Phi_{\alpha}, \Phi_{\beta} * \Phi_{\gamma}\right\rangle$.'国 Therefore the correponding equation of motion is:

$$
\begin{equation*}
\sum_{a} C_{a n} \phi_{a}+\sum_{a, b} C_{a b n} \phi_{a} \phi_{b}=0 . \tag{3.3}
\end{equation*}
$$

One has to check the above equation in the modified level truncation scheme.

## 4 Checking Equations of Motion for Fields Outside the Siegel Gauge

Here we will check the equations of motion for fields outside the Siegel gauge in the modified level truncation scheme. We will present results upto the level $(3,6)$.

[^2]We start by making a list of the relevant fields. In table 1 some classes of states have been listed with their vertex operators and levels. The relevant fields which get involved in the computations are the coefficients of these states in the expansion of the string field. The states which are inside the Siegel gauge, namely $\left|T_{n}\right\rangle,\left|U_{n}\right\rangle,\left|V_{n}\right\rangle,\left|W_{n}\right\rangle$ and $\left|Z_{n}\right\rangle$ are precisely the ones which have been considered in ref. [12] for constructing the lump solution. The states $\left|R_{n}\right\rangle$ and $\left|S_{n}\right\rangle$ are outside the Siegel gauge as they involve the $c_{0}$ oscillator.

Table 2 shows the fields ${ }^{5}$ and their levels which get involved in our computation upto level $(3,6)$ for $R=\sqrt{3}$. The fields in the square brackets i.e. $r_{0}, r_{1}$ and $s_{1}$ are the ones outside the Siegel gauge. We will check the equations of motion for these fields.

| State | Vertex Operator | Level <br> $R^{2}=3$ |
| :--- | :--- | :---: |
| $\left\|T_{n}\right\rangle=c_{1}\left\|\frac{n}{R}\right\rangle$ | $c \cos \left(\frac{n}{R} X\right)$ | $n^{2} / R^{2}$ |
| $\left\|U_{n}\right\rangle=c_{-1}\left\|\frac{n}{R}\right\rangle$ | $\frac{1}{2} \partial^{2} c \cos \left(\frac{n}{R} X\right)$ | $2+n^{2} / R^{2}$ |
| $\left\|V_{n}\right\rangle=c_{1} L_{-2}^{X}\left\|\frac{n}{R}\right\rangle$ | $T^{X} c \cos \left(\frac{n}{R} X\right)$ | $2+n^{2} / R^{2}$ |
| $\left\|W_{n}\right\rangle=c_{1} L_{-2}^{\prime}\left\|\frac{n}{R}\right\rangle$ | $T^{\prime} c \cos \left(\frac{n}{R} X\right)$ | $2+n^{2} / R^{2}$ |
| $\left\|Z_{n}\right\rangle=c_{1} L_{-1}^{X} L_{-1}^{X}\left\|\frac{n}{R}\right\rangle$ | $c \partial^{2}\left(\cos \left(\frac{n}{R} X\right)\right)$ | $2+n^{2} / R^{2}$ |
| $\left\|R_{n}\right\rangle=b_{-2} c_{0} c_{1}\left\|\frac{n}{R}\right\rangle$ | $b \partial c c \cos \left(\frac{n}{R} X\right)$ | $2+n^{2} / R^{2}$ |
| $\left\|S_{n}\right\rangle=c_{0} L_{-1}^{X}\left\|\frac{n}{R}\right\rangle$ | $\partial c \partial\left(\cos \left(\frac{n}{R} X\right)\right)$ | $2+n^{2} / R^{2}$ |

Table 1: The Hilbert space states relevant for constructing the lump solution.
The notations that we will use is as follows. Linear part of eqn. (3.3), i.e. $\sum_{a} C_{a n} \phi_{a}$ computed using the level $(M, 2 M)$ approximation to the action, will be denoted by $L(M, 2 M)$. Similarly $Q(M, 2 M)$ will denote the quadratic part of eqn. (3.3), namely, $\sum_{a, b} C_{a b n} \phi_{a} \phi_{b}$.

## Results

We define $K=\frac{3 \sqrt{3}}{4}$. For $R=\sqrt{3}$ we have the following results:

## The Field $r_{0}$ :

$$
\begin{equation*}
L(2,4)=\frac{1}{2} v_{0}+\frac{25}{2} w_{0}-3 u_{0} \tag{4.1}
\end{equation*}
$$

[^3]| Level | Fields |
| :---: | :---: |
| 0 | $t_{0}$ |
| $1 / 3$ | $t_{1}$ |
| $4 / 3$ | $t_{2}$ |
| 2 | $u_{0}, v_{0}, w_{0},\left[r_{0}\right]$ |
| $7 / 3$ | $u_{1}, v_{1}, w_{1}, z_{1},\left[r_{1}\right],\left[s_{1}\right]$ |
| 3 | $t_{3}$ |

Table 2: Fields in the Hilbert space up to level 3.

$$
\begin{equation*}
Q(2,4)=-K t_{0}^{2}-\frac{1}{2} K^{1-2 / R^{2}} t_{1}^{2}+\frac{\sqrt{3}}{2} t_{0} u_{0}+\frac{5}{12 \sqrt{3}} t_{0} v_{0}+\frac{125}{12 \sqrt{3}} t_{0} w_{0} \tag{4.2}
\end{equation*}
$$

Since contribution to the linear term $L$ should come only from a certain level which is $(2,4)$ in this case, $L$ will have the same expression for any higher level. $Q$ in general varies as one changes the level.

$$
\begin{align*}
Q(7 / 3,14 / 3)= & Q(2,4)-\frac{1}{2} K^{1-8 / R^{2}} t_{2}^{2}+\frac{\sqrt{3}}{4} K^{-2 / R^{2}} t_{1} u_{1} \\
& +\frac{125}{24 \sqrt{3}} K^{-2 / R^{2}} t_{1} w_{1}+\left(\frac{8}{27 R^{2}}+\frac{5}{54}\right) K^{1-2 / R^{2}} t_{1} v_{1} \\
& +\left(\frac{11}{8 R^{2}}-\frac{1}{R^{4}}\right) K^{-1-2 / R^{2}} t_{1} z_{1}  \tag{4.3}\\
Q(3,6)= & Q(7 / 3,14 / 3)+\frac{703}{324 \sqrt{3}} u_{0}^{2}-\frac{179}{972} K v_{0}^{2}-\frac{9475}{972} K w_{0}^{2} \\
- & \frac{5 \sqrt{3}}{108} u_{0} v_{0}-\frac{125 \sqrt{3}}{108} u_{0} w_{0}-\frac{625}{1458} K v_{0} w_{0} \tag{4.4}
\end{align*}
$$

## The Field $r_{1}$ :

Here $L$ gets contribution at the level $(7 / 3,14 / 3)$.

$$
\begin{gather*}
L(7 / 3,14 / 3)=-\frac{3}{2} u_{1}+\frac{1}{2}\left(\frac{4}{R^{2}}+\frac{1}{2}\right) v_{1}+\frac{25}{4} w_{1}+\frac{3}{R^{2}} z_{1}  \tag{4.5}\\
Q(7 / 3,14 / 3)=-K^{1-2 / R^{2}} t_{0} t_{1}-\frac{1}{2} K^{1-6 / R^{2}} t_{1} t_{2}+\frac{\sqrt{3}}{4} K^{-2 / R^{2}} t_{0} u_{1}
\end{gather*}
$$

$$
\begin{align*}
&+\frac{1}{2}\left(\frac{1}{R^{2}}+\frac{5}{16}\right) K^{-1-2 / R^{2}} t_{0} v_{1}+\frac{125}{24 \sqrt{3}} K^{-2 / R^{2}} t_{0} w_{1} \\
&+\left(\frac{11}{8 R^{2}}-\frac{1}{R^{4}}\right) K^{-1-2 / R^{2}} t_{0} z_{1}+\frac{\sqrt{3}}{4} K^{-2 / R^{2}} t_{1} u_{0} \\
&+\left(\frac{5}{54}-\frac{16}{27 R^{2}}\right) K^{1-2 / R^{2}} t_{1} v_{0}+\frac{125}{24 \sqrt{3}} K^{-2 / R^{2}} t_{1} w_{0}  \tag{4.6}\\
& Q(3,6)=Q(7 / 3,14 / 3)+\frac{\sqrt{3}}{8} K^{-6 / R^{2}} t_{2} u_{1}+\left(\frac{5}{64}-\frac{3}{4 R^{2}}\right) K^{-1-6 / R^{2}} t_{2} v_{1} \\
&+\frac{125}{108} K^{1-6 / R^{2}} t_{2} w_{1}+\frac{1}{2}\left(\frac{11}{8 R^{2}}-\frac{9}{R^{4}}\right) K^{-1-6 / R^{2}} t_{2} z_{1} \tag{4.7}
\end{align*}
$$

## The Field $s_{1}$ :

$$
\begin{gather*}
L(7 / 3,14 / 3)=\frac{1}{R^{2}} u_{1}-\frac{3}{R^{2}} v_{1}-\frac{2}{R^{2}}\left(1+\frac{2}{R^{2}}\right) z_{1}  \tag{4.8}\\
Q(7 / 3,14 / 3)=-\frac{1}{2 R^{2}} K^{-1-2 / R^{2}} t_{0} u_{1}+\frac{1}{2 R^{2}} K^{-1-2 / R^{2}} t_{1} u_{0}  \tag{4.9}\\
Q(3,6)=Q(7 / 3,14 / 3)+\frac{3}{4 R^{2}} K^{-1-6 / R^{2}} t_{2} u_{1} \tag{4.10}
\end{gather*}
$$

Now to get the numerical values of $L$ and $Q$ for different fields at a given level one has to substitute the lump solution obtained in ref. 12] at that level. Table 3 displays these solutions at different levels and table 4 shows the numerical values that we obtain after substituting these solutions in the equations for $L$ and $Q$ given above.

From table we see that for the equation of motion of each field, there is a high degree of cancellation between the linear and the quadratic terms in the equation of motion. The last column explains clearly the degree of this cancellation, - we see that the sum of the quadratic and the linear term is typically about $10-20 \%$ of the linear term alone. This may not seem to be a very good result, but we note that in the corresponding calculation for the vacuum solution at level $(2,6)\left(t_{0}=.544, u_{0}=.190, v_{0}=w_{0}=.0560\right)$ [6, [32], the linear and the quadratic terms in the equation of motion of $r_{0}$ are given by respectively 0.158 and -0.124 . These add up to about $22 \%$ of the linear term. Thus at this level our

| Field | $(2,4)$ | $(7 / 3,14 / 3)$ | $(3,6)$ |
| :---: | :---: | :---: | :---: |
| $t_{0}$ | 0.25703 | 0.265131 | 0.269224 |
| $t_{1}$ | -0.384575 | -0.394396 | -0.394969 |
| $t_{2}$ | -0.107424 | -0.12046 | -0.125011 |
| $u_{0}$ | 0.0888087 | 0.0900609 | 0.0969175 |
| $v_{0}$ | -0.00675676 | -0.0175367 | -0.0172906 |
| $w_{0}$ | 0.0317837 | 0.0299617 | 0.0320394 |
| $u_{1}$ | $\ldots$ | -0.0643958 | -0.0648543 |
| $v_{1}$ | $\ldots$ | 0.0540447 | 0.0505836 |
| $w_{1}$ | $\ldots$ | -0.0187778 | -0.0189058 |
| $z_{1}$ | $\ldots$ | -0.0698363 | -0.0665402 |
| $t_{3}$ | $\ldots$ | $\ldots$ | -0.0142169 |

Table 3: The values of various modes of the string field at the stationary point of the potential for $R=\sqrt{3}$ calculated at various levels of approximation.

| Field | Level | L | Q | $\mathrm{L}+\mathrm{Q}$ | $(\mathrm{L}+\mathrm{Q}) / \mathrm{L}$ |
| :---: | :--- | :---: | :---: | :---: | :---: |
| $r_{0}$ | $(2,4)$ | 0.127492 | -0.098026 | 0.0294657 | 0.231119 |
|  | $(7 / 3,14 / 3)$ | 0.0955702 | -0.0838568 | 0.0117134 | 0.122563 |
|  | $(3,6)$ | 0.101095 | -0.0881436 | 0.0129511 | 0.128108 |
| $r_{1}$ | $(7 / 3,14 / 3)$ | -0.0410629 | 0.0323025 | -0.00876042 | 0.213342 |
|  | $(3,6)$ | -0.0410517 | 0.0325827 | -0.00846901 | 0.206301 |
| $s_{1}$ | $(7 / 3,14 / 3)$ | 0.00208592 | -0.00198787 | 0.0000980474 | 0.0470043 |
|  | $(3,6)$ | 0.00173186 | -0.00131896 | 0.000412898 | 0.238413 |

Table 4: The linear $(L)$, quadratic $(Q)$ and total $(Q+L)$ contribution to the equations of motion of the fields outside the Siegel gauge. The last column shows the degree to which the linear and the quadratic terms cancel.
results for the lump solution are as good as those for the vacuum solution. As has been verified in ref. [18], the cancellation betwen the linear and quadratic terms in the equations of motion of the vacuum solution improves to about $1 \%$ when we use the solution at level $(10,20)$ approximation. 7 We could therfore expect a similar improvement of the results for the lump solution when we go to higher level.

This provides evidence that the equations of motion of the fields outside the Siegel gauge are automatically satisfied by the solutions obtained in ref.[12] in the Siegel gauge.

From the last column of table one may notice that for the field $s_{1}$ the cancellation at level $(7 / 3,14 / 3)$ is much better than that at level $(3,6)$. This at a first glance, may seem to show that the level truncation scheme fails to work in this case. This however is not the case, as can be seen from that fact that the cancellation for the field $r_{1}$ at level $(7 / 3$, $14 / 3)$ is much worse than that for $s_{1}$, and so the high degree of cancellation for the field $s_{1}$ should be treated as accidental. Indeed, at any given level of approximation it is always possible to take the independent fields at level $7 / 3$ to be appropriate linear combinations

[^4]of $r_{1}$ and $s_{1}$ so that the derivative of the action with respect to one of these fields is very small (or even zero) when we substitute the Siegel gauge solution obtained at that level. It so happens that in the level $(7 / 3,14 / 3)$ approximation $s_{1}$ is the linear combination for which the equation of motion is satisfied very closely. But clearly this does not establish that the level $(7 / 3,14 / 3)$ approximation is better than the level $(3,6)$ approximation, what is required for the approximation to be good is that the contribution to the equations of motion of both the fields should be small. As can be seen from table 4 , the cancellation for $r_{1}$ is actually better at level $(3,6)$ than at level $(7 / 3,14 / 3)$. Thus we cannot conclude that the cancellation becomes worse when we go from level $(7 / 3,14 / 3)$ to level $(3,6)$. It is true however that on the whole, the degree of cancellation does not improve either in going from level $(7 / 3,14 / 3)$ to level $(3,6)$. This could be due to the fact that we do not introduce too many new fields in going from the level $(7 / 3,14 / 3)$ to the level $(3,6)$ approximation.

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[^1]:    ${ }^{2}$ This restriction was made in ref. 12] just to make the D-brane have a finite mass.

[^2]:    ${ }^{3}$ i.e. using the oscillators in (2.5)
    ${ }^{4}$ That the coefficients $C_{\alpha \beta \gamma}$ are same even for non-cyclic permutations of the indices $\alpha, \beta, \gamma$, is a property of the truncated spectrum. Here the indices $\alpha, \beta, \gamma$ run over all fields in the truncated spectrum.

[^3]:    ${ }^{5}$ Following ref. 12], we denote a field by the lowercase symbol corresponding to the uppercase symbol used for the corresponding state.

[^4]:    ${ }^{6}$ Note that although the authors of ref. 18 used the solutions obtained at the level $(10,20)$ approximation, in computing the contribution to the equations of motion of the field $r_{0}$ they only used the level $(2,6)$ approximation. Presumably contribution from fields at higher level do not alter the results.

