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Black Hole Entropy Function and the Attractor Mechanism in Higher Derivative Gravity

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Abstract

We study extremal black hole solutions in D dimensions with near horizon geometry $AdS_2 \times S^{D-2}$ in higher derivative gravity coupled to other scalar, vector and anti-symmetric tensor fields. We define an entropy function by integrating the Lagrangian density over S^{D-2} for a general $AdS_2 \times S^{D-2}$ background, taking the Legendre transform of the resulting function with respect to the parameters labelling the electric fields, and multiplying the result by a factor of 2π. We show that the values of the scalar fields at the horizon as well as the sizes of AdS_2 and S^{D-2} are determined by extremizing this entropy function with respect to the corresponding parameters, and the entropy of the black hole is given by the value of the entropy function at this extremum. Our analysis relies on the analysis of the equations of motion and does not directly make use of supersymmetry or specific structure of the higher derivative terms.

Contents

1 Introduction and Summary

Analysis of supersymmetric black holes in string theory have led to many new insights into the classical and quantum aspects of black holes. In particular a rich structure has emerged in the context of half-BPS black holes in $\mathcal{N} = 2$ supersymmetric string theories in four dimensions. One of the important features of these black holes is the attractor mechanism [1, 2, 3] by which the values of the scalar fields at the horizon are determined only by the charges carried by the black hole and are independent of the asymptotic values of the scalar fields. The entropy of these black holes agrees with the microscopic counting of the states of the brane system they describe, not only in the supergravity approximation, but also after the inclusion of higher derivative corrections to the generalized prepotential[4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14]. More recently it has been shown that the Legendre transform of the black hole entropy with respect to the electric charges is directly related to the generalized prepotential, and this has led to a new conjectured relation between the black hole entropy and topological string partition function[15, 16, 17, 18]. Finally, applying the results for these black holes to the special case of black holes in heterotic string theory with purely electric charges, one finds agreement between black hole entropy and the degeneracy of elementary string states[19, 20, 21, 22, 23, 24, 25, 26, 27] even though the black hole entropy vanishes in the supergravity approximation[28, 29, 30].

All of these results have been derived by making heavy use of supersymmetry. In particular while taking into account the effect of higher derivative terms one includes in the string theory effective action only a special class of terms which can be computed using the partition function of topological string theory[31, 32, 33]. These corrections are controlled by a special function known as the generalized prepotential[8, 12, 13]. While these constitute an important set of terms in the string theory effective acion, they are by no means the only terms, and at present there is no understanding of why these terms should play a special role in the study of black holes. In fact there are counterexamples, involving elementary string states in type II string theory, for which the corrections to the generalized prepotential are not enough to produce the desired result for the black hole entropy[24]. Thus it seems important to study the role of the complete set of higher derivative terms on the near horizon geometry of the black hole.

In this paper we study the effect of higher derivative terms on the entropy of extremal black holes in D dimensions following the general formalism developed in [34, 35, 36, 37]. We do not make use of supersymmetry directly, but define extremal black holes to be those objects whose near horizon geometry is given by $AdS_2 \times S^{D-2}$.[1] We also define the entropy of the extremal black hole to be the extremal limit of the entropy of a non-extremal black hole so that we can use the general formula for the entropy given in [34, 35, 36, 37] even though strictly extremal black holes do not have a bifurcate horizon. Our main results may be summarized as follows.

1. Let $S_{BH}(\vec{q}, \vec{p})$ denote the entropy of a D-dimensional extremal black hole, with near horizon geometry $AdS_2 \times S^{D-2}$, as a function of electric charges $\{q_i\}$ associated with one form gauge fields and magnetic charges $\{p_a\}$ associated with $(D-3)$ form gauge fields. We choose a coordinate system in which the AdS_2 part of the metric is proportional to $-r^2 dt^2 + dr^2/r^2$. Then the Legendre transform of $S_{BH}(\vec{q}, \vec{p})/2\pi$ with respect to the variables q_i is equal to the the integral of the Lagrangian density over the $(D-2)$ dimensional sphere S^{D-2} enclosing the black hole. The variable conjugate to q_i represents the radial electric field e_i at the horizon associated with the i-th gauge field.

2. Consider a general $AdS_2 \times S^{D-2}$ background parametrized by the sizes of AdS_2 and S^{D-2}, the electric and magnetic fields and the values of various scalar fields. We define an entropy function by integrating the Lagrangian density evaluated for this background over S^{D-2}, taking the Legendre transform of this integral with respect to the parameters e_i labelling the electric fields and multiplying the result by 2π. The result is a function of the values u_s of the scalar fields, the sizes v_1 and v_2 of AdS_2 and S^{D-2}, the electric charges q_i conjugate to the variables e_i, and the

[1] Eventually supersymmetry may play a role in establishing the existence of a solution that interpolates between the near horizon $AdS_2 \times S^{D-2}$ geometry and the asymptotic Minkowski space-time.

magnetic charges p_a labelling the background magnetic fields. We show that for given $\vec{q}$ and $\vec{p}$, the values u_s of the scalar fields as well as the sizes v_1 and v_2 of AdS_2 and S^{D-2} are determined by extremizing the entropy function with respect to the variables u_i, v_1 and v_2. Furthermore the entropy itself is given by the value of the entropy function at the horizon.

3. For extremal black hole solutions without Ramond-Ramond (RR) charges in tree level string theory the Lagrangian density at the horizon vanishes due to the dilaton field equation. In this case the entropy of the black hole is given simply by 2π times the product of the electric field at the horizon and the electric charge of the black hole.

These results rely on the assumption that the Lagrangian density can be expressed in terms of gauge invariant field strengths and does not involve the gauge fields explicitly. Thus if Chern-Simons terms are present we either need to remove them by going to the dual field variables, or if that is not possible, consider black hole solutions which are not affected by these Chern-Simons terms.

2 Entropy of Extremal Black Holes

We begin by considering a four dimensional theory of gravity coupled to a set of abelian gauge fields $A_\mu^{(i)}$ and neutral scalar fields $\{\phi_s\}$. Suppose $\sqrt{-\det g}\,\mathcal{L}$ is the lagrangian density, expressed as a function of the metric $g_{\mu\nu}$, the scalar fields $\{\phi_s\}$, the gauge field strengths $F_{\mu\nu}^{(i)}$, and covariant derivatives of these fields. We consider a spherically symmetric extremal black hole solution with near horizon geometry $AdS_2 \times S^2$. The most general field configuration, consistent with the $SO(2,1)\times SO(3)$ symmetry of $AdS_2\times S^2$, is of the form:

$$\begin{aligned}
& ds^2 \equiv g_{\mu\nu}dx^\mu dx^\nu = v_1\left(-r^2dt^2+\frac{dr^2}{r^2}\right)+v_2\left(d\theta^2+\sin^2\theta d\phi^2\right)\\
& \phi_s = u_s\\
& F_{rt}^{(i)} = e_i, \qquad F_{\theta\phi}^{(i)} = \frac{p_i}{4\pi}\sin\theta\,,
\end{aligned} \tag{2.1}$$

where v_1, v_2, $\{u_s\}$, $\{e_i\}$ and $\{p_i\}$ are constants. For this background the nonvanishing components of the Riemann tensor are:[2]

$$\begin{aligned} R_{\alpha\beta\gamma\delta} &= -v_1^{-1}(g_{\alpha\gamma}g_{\beta\delta} - g_{\alpha\delta}g_{\beta\gamma})\,, \qquad \alpha,\beta,\gamma,\delta = r,t\,, \\ R_{mnpq} &= v_2^{-1}\,(g_{mp}g_{nq} - g_{mq}g_{np})\,, \qquad m,n,p,q = \theta,\phi\,. \end{aligned} \tag{2.2}$$

It follows from the general form of the background that the covariant derivatives of the scalar fields ϕ_s, the gauge field strengths $F^{(i)}_{\mu\nu}$ and the Riemann tensor $R_{\mu\nu\rho\sigma}$ all vanish for the near horizon geometry. By the general symmetry consideration it follows that the contribution to the equation of motion from any term in the action that involves covariant derivatives of the gauge field strengths, scalars or the Riemann tensor vanish identically for this background and we can restrict our attention to only those terms which do not involve covariant derivatives of these fields.[3]

Let us denote by $f(\vec{u},\vec{v},\vec{e},\vec{p})$ the Lagrangian density $\sqrt{-\det g}\,\mathcal{L}$ evaluated for the near horizon geometry (2.1) and integrated over the angular coordinates[27]:

$$f(\vec{u},\vec{v},\vec{e},\vec{p}) = \int d\theta\, d\phi\, \sqrt{-\det g}\,\mathcal{L}\,. \tag{2.3}$$

The scalar and the metric field equations in the near horizon geometry correspond to extremizing f with respect to the variables $\vec{u}$ and $\vec{v}$:

$$\frac{\partial f}{\partial u_s} = 0, \qquad \frac{\partial f}{\partial v_i} = 0\,. \tag{2.4}$$

On the other hand the non-trivial components of the gauge field equations and the Bianchi identities take the form:

$$\partial_r\left(\frac{\partial\sqrt{-\det g}\,\mathcal{L}}{\partial F^{(i)}_{rt}}\right) = 0, \qquad \partial_r F^{(i)}_{\theta\phi} = 0\ . \tag{2.5}$$

Both sets of equations in (2.5) are automatically satisfied by the background (2.1), with the constants of integration having the interpretation as electric and magnetic charges of

[2]In our convention $R^{\mu}{}_{\nu\rho\sigma} = \partial_\rho\Gamma^\mu_{\nu\sigma} - \partial_\sigma\Gamma^\mu_{\nu\rho} + \Gamma^\mu_{\tau\rho}\Gamma^\tau_{\nu\sigma} - \Gamma^\mu_{\tau\sigma}\Gamma^\tau_{\nu\rho}$ where $\Gamma^\mu_{\nu\rho}$ is the Christoffel symbol.

[3]We are assuming that all terms in the action depend explicitly only on the gauge field strengths and not on gauge fields. This condition is violated for example in string theory by Chern-Simons type coupling of the gauge fields to three form field strengths. However, as is well known, we can get rid of such terms by dualizing the two form field to a scalar axion a. This field couples to the gauge fields only through field strengths. If we encounter a theory where it is impossible to carry this out for all fields, our analysis will still be valid if these additional terms do not affect the equation of motion and the entropy for the specific black hole solution under study.

the black hole. From this it follows that the constants p_i appearing in (2.1) correspond to magnetic charges of the black hole, and

$$\frac{\partial f}{\partial e_i} = q_i \tag{2.6}$$

where q_i denote the electric charges carried by the black hole.

For fixed $\vec{p}$ and $\vec{q}$, (2.4) and (2.6) give a set of equations which are equal in number to the number of unknowns $\vec{u}$, $\vec{v}$ and $\vec{e}$. In a generic case we may be able to solve these equations completely to determine the background in terms of only the electric and the magnetic charges $\vec{q}$ and $\vec{p}$. [4] This is consistent with the attractor mechanism for supersymmetric background which says that the near horizon configuration of a black hole depends only on the electric and magnetic charges carried by the black hole and not on the asymptotic values of these scalar fields. We shall return to a more detailed discussion of this mechanism in section 3.

Let us now turn to the analysis of the entropy associated with this black hole. A general formula for the entropy in the presence of higher derivative terms has been given in [34, 35, 36, 37]. The formula simplifies enormously here since the covariant derivatives of all the tensors vanish, and we get a simple formula:

$$S_{BH} = 8\pi \, \frac{\partial \mathcal{L}}{\partial R_{rtrt}} \, g_{rr} \, g_{tt} \, A_H \, , \tag{2.7}$$

where A_H is the area of the event horizon and $\frac{\partial \mathcal{L}}{\partial R_{\mu\nu\rho\sigma}}$ is defined through the equation

$$\delta \, \mathcal{L} = \frac{\partial \mathcal{L}}{\partial R_{\mu\nu\rho\sigma}} \, \delta \, R_{\mu\nu\rho\sigma} \, . \tag{2.8}$$

In computing $\delta\mathcal{L}$ we can ignore all terms in $\mathcal{L}$ which involve covariant derivatives of the Riemann tensor, and treat the components of the Riemann tensor as independent variables.

In order to simplify this formula let us denote by $f_\lambda(\vec{u}, \vec{v}, \vec{e}, \vec{p})$ an expression similar to the right hand side of (2.3) except that each factor of R_{rtrt} in the expression of $\mathcal{L}$ is

[4] We should note however that the situation in string theory is not completely generic. For example in $\mathcal{N} = 2$ supersymmetric string theories there is no coupling of the hypermultiplet scalars to the vector multiplet fields or the curvature tensor to lowest order in α', and hence in this approximation the function f does not depend on the hypermultiplet scalars. Thus the equations (2.4), (2.6) do not fix the values of the hypermultiplet scalars in this approximation.

multiplied by a factor of λ. Then we have the relation:

$$\frac{\partial f_\lambda(\vec{u},\vec{v},\vec{e},\vec{p})}{\partial\lambda}\bigg|_{\lambda=1} = \int d\theta\, d\phi\, \sqrt{-\det g}\, R_{\alpha\beta\gamma\delta}\, \frac{\partial\mathcal{L}}{\partial R_{\alpha\beta\gamma\delta}}\,, \tag{2.9}$$

where the repeated indices $\alpha,\beta,\gamma,\delta$ are summed over the coordinates r and t. Now since by symmetry consideration $(\partial\mathcal{L}/\partial R_{\alpha\beta\gamma\delta})$ is proportional to $(g^{\alpha\gamma}g^{\beta\delta} - g^{\alpha\delta}g^{\beta\gamma})$, we have

$$\frac{\partial\mathcal{L}}{\partial R_{\alpha\beta\gamma\delta}} = -v_1^2\,(g^{\alpha\gamma}g^{\beta\delta} - g^{\alpha\delta}g^{\beta\gamma})\,\frac{\partial\mathcal{L}}{\partial R_{rtrt}}\,. \tag{2.10}$$

The constant of proportionality has been fixed by taking $(\alpha\beta\gamma\delta) = (rtrt)$. Using (2.2) and (2.10) we can rewrite (2.9) as

$$\frac{\partial\mathcal{L}}{\partial R_{rtrt}}\,A_H = \frac{1}{4}\,v_1^{-2}\;\frac{\partial f_\lambda(\vec{u},\vec{v},\vec{e},\vec{p})}{\partial\lambda}\bigg|_{\lambda=1}\,. \tag{2.11}$$

Substituting this into (2.7) gives[27]

$$S_{BH} = -2\pi\;\frac{\partial f_\lambda(\vec{u},\vec{v},\vec{e},\vec{p})}{\partial\lambda}\bigg|_{\lambda=1}\,. \tag{2.12}$$

We shall now reexpress the right hand side of (2.12) in terms of derivatives of f with respect to the variables $\vec{u}$, $\vec{v}$, $\vec{e}$ and $\vec{p}$. Since the expression for $\mathcal{L}$ is invariant under reparametrization of the r,t coordinates, every factor of R_{rtrt} in the expression for f_λ must appear in the combination $\lambda\, g^{rr}g^{tt}R_{rtrt} = -\lambda v_1^{-1}$, every factor of $F^{(i)}_{rt}$ must appear in the combination $\sqrt{-g^{rr}g^{tt}}F^{(i)}_{rt} = e_i v_1^{-1}$, and every factor of $F^{(i)}_{\theta\phi} = e_i$ and $\phi_s = u_s$ must appear without any accompanying power of v_1. The contribution from all terms which involve covariant derivatives of $F^{(i)}_{\mu\nu}$, $R_{\mu\nu\rho\sigma}$ or ϕ_s vanish; hence there is no further factor of v_1 coming from contraction of the metric with these derivative operators. The only other v_1 dependence of $f_\lambda(\vec{u},\vec{v},\vec{e},\vec{p})$ is through the overall multiplicative factor of $\sqrt{-\det g} \propto v_1$. Thus $f_\lambda(\vec{u},\vec{v},\vec{e},\vec{p})$ must be of the form $v_1 g(\vec{u}, v_2, \vec{p}, \lambda v_1^{-1}, \vec{e} v_1^{-1})$ for some function g, and we have

$$\lambda\frac{\partial f_\lambda(\vec{u},\vec{v},\vec{e},\vec{p})}{\partial\lambda} + v_1\frac{\partial f_\lambda(\vec{u},\vec{v},\vec{e},\vec{p})}{\partial v_1} + e_i\frac{\partial f_\lambda(\vec{u},\vec{v},\vec{e},\vec{p})}{\partial e_i} - f_\lambda(\vec{u},\vec{v},\vec{e},\vec{p}) = 0\,. \tag{2.13}$$

Setting $\lambda = 1$ in (2.13), using the equation of motion of v_1 as given in (2.4), and substituting the result into eq.(2.12) we get

$$S_{BH} = 2\pi\left(e_i\,\frac{\partial f}{\partial e_i} - f\right)\,. \tag{2.14}$$

This together with (2.6) shows that $S_{BH}(\vec{q},\vec{p})/2\pi$ may be regarded as the Legendre transform of the function $f(\vec{u},\vec{v},\vec{e},\vec{p})$ with respect to the variables e_i after eliminating $\vec{u}$ and $\vec{v}$ through their equations of motion (2.4).

The analysis can be easily generalized to higher dimensional theories as follows. In D space-time dimensions we consider an extremal black hole solution with near horizon geometry $AdS_2 \times S^{D-2}$. The relevant fields which can take non-trivial expectation value near the horizon are scalars $\{\phi_s\}$, metric $g_{\mu\nu}$, gauge fields $A^{(i)}_\mu$ and $(D-3)$-form gauge fields $B^{(a)}_{\mu_1\cdots\mu_{D-3}}$. If $H^{(a)}_{\mu_1\cdots\mu_{D-2}}$ denote the field strength associated with the B field, then the general background consistent with the $SO(2,1)\times SO(D-1)$ symmetry of the background geometry is of the form:

$$\begin{aligned}
& ds^2 \equiv g_{\mu\nu}dx^\mu dx^\nu = v_1\left(-r^2dt^2 + \frac{dr^2}{r^2}\right) + v_2\, d\Omega_{D-2}^2 \\
& \phi_s = u_s \\
& F^{(i)}_{rt} = e_i, \qquad H^{(a)}_{l_1\cdots l_{D-2}} = p_a\, \epsilon_{l_1\cdots l_{D-2}} \sqrt{\det h^{(D-2)}}\, /\Omega_{D-2}\,.
\end{aligned} \tag{2.15}$$

where $d\Omega_{D-2} = h^{(D-2)}_{ll'}dx^l dx^{l'}$ denotes the line element on the unit $(D-2)$-sphere, Ω_{D-2} denotes the area of the unit $(D-2)$-sphere, x^{l_i} with $2 \le l_i \le (D-1)$ are coordinates along this sphere and ϵ denotes the totally anti-symmetric symbol with $\epsilon_{2\ldots(D-1)} = 1$. We now define

$$f(\vec{u},\vec{v},\vec{e},\vec{p}) = \int dx^2\cdots dx^{D-1}\sqrt{-\det g}\,\mathcal{L}\,, \tag{2.16}$$

as in (2.3). Analysis identical to that for $D=4$ now tells us that the constants p_a represent magnetic type charges carried by the black hole, and the equations which determine the values of $\vec{u}$, $\vec{v}$ and $\vec{e}$ are

$$\frac{\partial f}{\partial u_s} = 0, \qquad \frac{\partial f}{\partial v_i} = 0\,, \qquad \frac{\partial f}{\partial e_i} = q_i\,, \tag{2.17}$$

where q_i denote the electric charges carried by the black hole. Also using (2.7) which is valid in any dimension, we can show that the entropy of the black hole is given by 2π times the Legendre transform of f:

$$S_{BH} = 2\pi\left(e_i\frac{\partial f}{\partial e_i} - f\right)\,. \tag{2.18}$$

as in (2.14).

At string tree level, and in the absence of Ramond-Ramond background fields (which includes all black holes in heterotic string theory) the Lagrangian density at the horizon and hence the function f vanishes due to the dilaton field equation. Thus eqs.(2.17), (2.18) give:

$$S_{BH} = 2\pi \, q_i \, e_i \, . \tag{2.19}$$

In other words the entropy of these black holes is given by 2π times the product of the electric charge and the electric field at the horizon. It will be interesting to see if this quantity admits a simple interpretation in the world-sheet conformal field theory that describes this background.

3 Attractor Mechanism and the Entropy Function

We can now reformulate the attractor mechanism in a more suggestive manner. Let us define

$$F(\vec{u}, \vec{v}, \vec{q}, \vec{p}) = 2\pi \left(e_i \, \frac{\partial f(\vec{u}, \vec{v}, \vec{e}, \vec{p})}{\partial e_i} - f(\vec{u}, \vec{v}, \vec{e}, \vec{p}) \right) , \tag{3.1}$$

with e_i determined by the equation:

$$\frac{\partial f(\vec{u}, \vec{v}, \vec{e}, \vec{p})}{\partial e_i} = q_i \, . \tag{3.2}$$

In that case it follows from (2.17) that the values of $\vec{u}$ and $\vec{v}$ at the horizon are determined by extremizing the function $F(\vec{u}, \vec{v}, \vec{q}, \vec{p})$ with respect to $\vec{u}$ and $\vec{v}$:

$$\frac{\partial F(\vec{u}, \vec{v}, \vec{q}, \vec{p})}{\partial u_s} = 0 \, , \qquad \frac{\partial F(\vec{u}, \vec{v}, \vec{q}, \vec{p})}{\partial v_i} = 0 \, . \tag{3.3}$$

Furthermore, eq.(2.18) shows that the black hole entropy S_{BH} is given by the value of the function F at this extremum:

$$S_{BH}(\vec{q}, \vec{p}) = F(\vec{u}, \vec{v}, \vec{q}, \vec{p}) \, , \tag{3.4}$$

with $\vec{u}$, $\vec{v}$ given by eq.(3.3). This suggests that we call $F(\vec{u}, \vec{v}, \vec{q}, \vec{p})$ the entropy function. Finally, the near horizon electric field e_i are given by

$$e_i = \frac{1}{2\pi} \frac{\partial F}{\partial q_i} \, . \tag{3.5}$$

4 Relation to Earlier Results

We are now in a position to discuss the relation between our results and the observation of [15] that the Legendre transform of the entropy of a black hole in $\mathcal{N} = 2$ supersymmetric string theory is given by the imaginary part of the generalized prepotential of the theory. In the argument of the prepotential the real parts of the complex vector multiplet scalar fields are replaced, up to a constant of proportionality, by the magnetic charges of the black hole, whereas the imaginary parts of these scalar fields are replaced by the variables conjugate to the electric charges of the black hole. This result follows from our results together with the following observations (see *e.g.* [12]):

1. For the near horizon configuration of the black hole in $\mathcal{N} = 2$ supersymmetric string theory, all terms in the Lagrangian density vanish, except for a single term proportional to the imaginary part of the generalized prepotential .

2. For the near horizon geometry the real parts of the vector multiplet scalar fields are proportional to the magnetic field at the horizon whereas the imaginary parts of these scalar fields are proportional to the electric field at the horizon.

A little algebra shows that all the normalization factors also work out correctly and we can reproduce the abovementioned observation of [15] from our results.

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References

[1] S. Ferrara, R. Kallosh and A. Strominger, Phys. Rev. D **52**, 5412 (1995) [arXiv:hep-th/9508072].

[2] A. Strominger, Phys. Lett. B **383**, 39 (1996) [arXiv:hep-th/9602111].

[3] S. Ferrara and R. Kallosh, Phys. Rev. D **54**, 1514 (1996) [arXiv:hep-th/9602136].

[4] J. M. Maldacena, A. Strominger and E. Witten, JHEP **9712**, 002 (1997) [arXiv:hep-th/9711053].

[5] B. de Wit, Nucl. Phys. Proc. Suppl. **49**, 191 (1996) [arXiv:hep-th/9602060].

[6] B. de Wit, Fortsch. Phys. **44**, 529 (1996) [arXiv:hep-th/9603191].

[7] K. Behrndt, G. Lopes Cardoso, B. de Wit, D. Lust, T. Mohaupt and W. A. Sabra, Phys. Lett. B **429**, 289 (1998) [arXiv:hep-th/9801081].

[8] G. Lopes Cardoso, B. de Wit and T. Mohaupt, Phys. Lett. B **451**, 309 (1999) [arXiv:hep-th/9812082].

[9] G. Lopes Cardoso, B. de Wit and T. Mohaupt, Fortsch. Phys. **48**, 49 (2000) [arXiv:hep-th/9904005].

[10] G. Lopes Cardoso, B. de Wit and T. Mohaupt, Nucl. Phys. B **567**, 87 (2000) [arXiv:hep-th/9906094].

[11] G. Lopes Cardoso, B. de Wit and T. Mohaupt, Class. Quant. Grav. **17**, 1007 (2000) [arXiv:hep-th/9910179].

[12] T. Mohaupt, Fortsch. Phys. **49**, 3 (2001) [arXiv:hep-th/0007195].

[13] G. Lopes Cardoso, B. de Wit, J. Kappeli and T. Mohaupt, JHEP **0012**, 019 (2000) [arXiv:hep-th/0009234].

[14] G. L. Cardoso, B. de Wit, J. Kappeli and T. Mohaupt, Fortsch. Phys. **49**, 557 (2001) [arXiv:hep-th/0012232].

[15] H. Ooguri, A. Strominger and C. Vafa, Phys. Rev. D **70**, 106007 (2004) [arXiv:hep-th/0405146].

[16] E. Verlinde, arXiv:hep-th/0412139.

[17] H. Ooguri, C. Vafa and E. Verlinde, arXiv:hep-th/0502211.

[18] R. Dijkgraaf, R. Gopakumar, H. Ooguri and C. Vafa, arXiv:hep-th/0504221.

[19] A. Dabholkar, arXiv:hep-th/0409148.

[20] A. Dabholkar, R. Kallosh and A. Maloney, arXiv:hep-th/0410076.

[21] A. Sen, arXiv:hep-th/0411255.

[22] V. Hubeny, A. Maloney and M. Rangamani, arXiv:hep-th/0411272.

[23] D. Bak, S. Kim and S. J. Rey, arXiv:hep-th/0501014.

[24] A. Sen, arXiv:hep-th/0502126.

[25] A. Dabholkar, F. Denef, G. W. Moore and B. Pioline, arXiv:hep-th/0502157.

[26] A. Sen, arXiv:hep-th/0504005.

[27] A. Sen, arXiv:hep-th/0505122.

[28] A. Sen, Mod. Phys. Lett. A **10**, 2081 (1995) [arXiv:hep-th/9504147].

[29] A. W. Peet, Nucl. Phys. B **456**, 732 (1995) [arXiv:hep-th/9506200].

[30] A. Sen, JHEP **9802**, 011 (1998) [arXiv:hep-th/9712150].

[31] M. Bershadsky, S. Cecotti, H. Ooguri and C. Vafa, Nucl. Phys. B **405**, 279 (1993) [arXiv:hep-th/9302103].

[32] I. Antoniadis, E. Gava, K. S. Narain and T. R. Taylor, Nucl. Phys. B **413**, 162 (1994) [arXiv:hep-th/9307158].

[33] M. Bershadsky, S. Cecotti, H. Ooguri and C. Vafa, Commun. Math. Phys. **165**, 311 (1994) [arXiv:hep-th/9309140].

[34] R. M. Wald, Phys. Rev. D **48**, 3427 (1993) [arXiv:gr-qc/9307038].

[35] T. Jacobson, G. Kang and R. C. Myers, Phys. Rev. D **49**, 6587 (1994) [arXiv:gr-qc/9312023].

[36] V. Iyer and R. M. Wald, Phys. Rev. D **50**, 846 (1994) [arXiv:gr-qc/9403028].

[37] T. Jacobson, G. Kang and R. C. Myers, arXiv:gr-qc/9502009.