hep-th/0603149

Higher Derivative Corrections to Non-supersymmetric Extremal Black Holes in $\mathcal{N} = 2$ Supergravity

Bindusar Sahoo and Ashoke Sen

Harish-Chandra Research Institute
Chhatnag Road, Jhusi, Allahabad 211019, INDIA

E-mail: bindusar@mri.ernet.in, ashoke.sen@cern.ch, sen@mri.ernet.in

Abstract

Using the entropy function formalism we compute the entropy of extremal supersymmetric and non-supersymmetric black holes in $\mathcal{N} = 2$ supergravity theories in four dimensions with higher derivative corrections. For supersymmetric black holes our results agree with all previous analysis. However in some examples where the four dimensional theory is expected to arise from the dimensional reduction of a five dimensional theory, there is an apparent disagreement between our results for non-supersymmetric black holes and those obtained by using the five dimensional description. This indicates that for these theories supersymmetrization of the curvature squared term in four dimension does not produce all the terms which would come from the dimensional reduction of a five dimensional action with curvature squared terms.

Contents

1 Introduction

During the last several years study of higher derivative corrections to the entropy of extremal supersymmetric black holes have provided fruitful results in string theory[1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13]. In many examples these corrections match the appropriate corrections to the statistical entropy of the corresponding microscopic system. Given this success one might ask: are there similar results for non-supersymmetric black holes? While in general studying higher derivative corrections to the entropy of a generic black hole is a difficult problem, a general method for computing the entropy of extremal, but not necessarily supersymmetric black holes was developed in [14, 15]. This method does not provide an explicit construction of the full black hole solution, but gives a way to compute the near horizon field configuration and entropy of an extremal black hole with a given set of charges assuming the existence of the black hole solution. Various other recent approaches to studying non-supersymmetric black holes in string theory can be found in [16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27].

In this paper we apply the method developed in [14, 15] to compute the entropy of extremal black holes in four dimensional $\mathcal{N} = 2$ supergravity theories with curvature squared type corrections. We use fully supersymmetrized version of the action given in [28, 29] and construct the entropy function for a general extremal black hole solution following the procedure given in [14, 15]. Extremizing the entropy function with respect to the parameters labelling the near horizon background gives a set of algebraic equations for these parameters and the value of the entropy function at the extremum gives the entropy of the corresponding black hole. We show that these extremization equations admit a class of solutions which coincide with the supersymmetric extremal black holes studied in [1, 2, 3, 4, 5, 6, 7, 30, 31, 32, 33, 34]. In particular we recover the supersymmetric attractor equations of [1, 2, 3, 4, 5, 6, 7] in the presence of higher derivative terms. But our method allows us to go beyond the supersymmetric configurations and study higher derivative corrections to the entropy of extremal but non-supersymmetric black holes as well. We illustrate this by several examples.

Although no explicit study of higher derivative corrections to these non-supersymmetric solutions has been carried out before, there is a general argument due to Kraus and Larsen[35, 36] which gives an expression for the entropy of a class of extremal non-supersymmetric black holes when the four dimensional theory comes from the dimensional reduction of a five dimensional theory and the near horizon geometry of the black hole solution, expressed in the five dimensional language, has the structure of $AdS_3 \times S^2$. Unfortunately we find that our results do not agree with the prediction of Kraus and Larsen. The only possible explanation for this discrepancy seems to be that adding the minimal set of terms in the four dimensional action that is required for supersymmetrization of the curvature squared term does not reproduce all the terms which arise from dimensional reduction of the curvature squared term in five dimensions.

The rest of the paper is organized as follows. In section 2 we review the bosonic part of the $\mathcal{N} = 2$ supergravity action with curvature squared corrections. In section 3 we propose our ansatz for the near horizon geometry of extremal black holes in these theories, and construct the entropy function for these black holes. The parameters labelling the near horizon geometry of the black hole are obtained by extremizing the entropy function with respect to these parameters. In section 4 we verify that the entropy function constructed in section 3 is invariant under electric-magnetic duality transformation. In section 5 we show that the equations obtained by extremizing the entropy function admit a class of solutions which obey the well known supersymmetric attractor equations derived in [1, 2, 3, 4, 5, 6, 7]. However this does not exhaust the set of solutions of the extremization equations and there are in general other solutions which describe the near horizon geometry of non-supersymmetric extremal black holes. In section 6 we use our formalism to study supersymmetric extremal black holes in tree level heterotic string theory compactified on $T^4 \times T^2$ or $K3 \times T^2$ and reproduce the known results for the entropy and near horizon geometry of these black holes. Section 7 is devoted to the study of non-supersymmetric extremal black holes in the same theory. We find an expression for the entropy of a class of extremal non-supersymmetric black holes in a power series expansion in inverse power of the magnetic charge. In section 8 we extend our analysis of non-supersymmetric black holes to a more general class of models describing M-theory compactification on Calabi-Yau manifolds, and explicitly compute the first correction to the entropy of these black holes due to higher derivative terms. Finally in section 9 we compare our results of sections 6-8 to the predictions of [35, 36], assuming that the four dimensional theory under consideration comes from dimensional reduction of a five dimensional theory, and that the near horizon $AdS_2 \times S^2$ geometry gets lifted to a near horizon $AdS_3 \times S^2$ geometry in five dimensions. We find that the results do not agree. Although we do not have a complete understanding of the origin of this discrepancy, we suggest a possible explanation for this apparent disagreement between the two results.

2 $\mathcal{N} = 2$ Supergravity Action with Higher Derivative Corrections

The off-shell formulation of $\mathcal{N} = 2$ supergravity action in four dimensions was developed in [37, 38, 39, 40, 41, 42, 43, 28, 29]. Here we shall review this formulation following the notation of [6]. The basic bosonic fields in the theory are a set of $(N+1)$ complex scalar fields X^I with $0 \leq I \leq N$ (of which one can be gauged away using a scaling symmetry), $(N+1)$ gauge fields A^I_μ and the metric $g_{\mu\nu}$. Besides this the theory contains several non-dynamical fields. These include a complex anti-self-dual antisymmetric tensor field $T^-_{\mu\nu}$, a real scalar field D, a $U(1)$ gauge field $\mathcal{A}_\mu$, an $SU(2)$ gauge field $\mathcal{V}^i{}_{j\mu}$, a vector field V_μ, a set of $SU(2)$ triplet scalar fields Y^I_{ij} with $0 \leq I \leq N$, an SU(2) triplet scalar field M_{ij} and an SU(2) matrix valued scalar field Φ^α_i which transforms as a fundamental of the gauged $SU(2)$ and also a fundamental of a global $SU(2)$ symmetry (see eq.(3.111) of [6]).[1] Here i, j $(1 \leq i, j \leq 2)$ are indices labelling the fundamental representation of the gauged $SU(2)$ and fields in the triplet representation are obtained by taking the symmetric combination of a pair of indices i, j. The $SU(2)$ indices are raised and lowered by the antisymmetric tensors ε^{ij} and ε_{ij} with $\varepsilon^{12} = \varepsilon_{12} = 1$. α $(1 \leq \alpha \leq 2)$ labels the fundamental representation of the global $SU(2)$. We define

$$\begin{aligned}
& F^I_{\mu\nu} = \partial_\mu A^I_\nu - \partial_\nu A^I_\mu, \quad {}^*F^{I\mu\nu} = \frac{1}{2}(\sqrt{-\det g})^{-1}\epsilon^{\mu\nu\rho\sigma}F^I_{\rho\sigma}, \quad F^{I\pm}_{\mu\nu} = \frac{1}{2}(F^I_{\mu\nu} \pm i\ {}^*F^I_{\mu\nu}) \\
& f_\mu^\nu = -\frac{1}{2}R_\mu^\nu - \frac{1}{4}(D - \frac{1}{3}R)\delta_\mu^\nu + \frac{1}{2}\epsilon_\mu{}^{\nu\rho\sigma}(\sqrt{-\det g})^{-1}\partial_\rho\mathcal{A}_\sigma + \frac{1}{32}T^-_{\mu\rho}T^{+\nu\rho} \\
& \mathcal{R}_{\mu\nu}{}^{\rho\sigma} = R_{\mu\nu}{}^{\rho\sigma} + \left(f_\mu^\rho\delta_\nu^\sigma - f_\nu^\rho\delta_\mu^\sigma - f_\mu^\sigma\delta_\nu^\rho + f_\nu^\sigma\delta_\mu^\rho\right) - \frac{1}{32}(T^{-\rho\sigma}T^+_{\mu\nu} + T^-_{\mu\nu}T^{+\rho\sigma}) \\
& \mathcal{R}^\pm{}_{\mu\nu}{}^{\rho\sigma} = \frac{1}{2}\left(\mathcal{R}_{\mu\nu}{}^{\rho\sigma} \pm \frac{i}{2}(\sqrt{-\det g})^{-1}\epsilon^{\rho\sigma\tau\delta}\mathcal{R}_{\mu\nu\tau\delta}\right) \\
& \mathcal{F}^i{}_{j\mu\nu} = \partial_\mu\mathcal{V}^i{}_{j\nu} - \partial_\nu\mathcal{V}^i{}_{j\mu} + \frac{1}{2}\mathcal{V}^i{}_{k\mu}\mathcal{V}^k{}_{j\nu} - \frac{1}{2}\mathcal{V}^i{}_{k\nu}\mathcal{V}^k{}_{j\mu} \\
& \widehat{A} = T^{-\mu\nu}T^-_{\mu\nu}, \quad \widehat{B}_{ij} = -8\varepsilon_{ki}\mathcal{F}^k{}_{j\mu\nu}T^{-\mu\nu} - 8\varepsilon_{kj}\mathcal{F}^k{}_{i\mu\nu}T^{-\mu\nu}\,, \\
& \widehat{F}^{-\mu\nu} = -16\mathcal{R}_{\rho\sigma}{}^{\mu\nu}T^{-\rho\sigma} \\
& \widehat{C} = 64\mathcal{R}^-_{\mu\nu\rho\sigma}\mathcal{R}^{-\mu\nu\rho\sigma} + 32\mathcal{F}^{-i}{}_{j\mu\nu}\mathcal{F}^{-j\mu\nu}{}_i \\
& \qquad -8T^{-\mu\nu}\{(\nabla_\mu - i\mathcal{A}_\mu), (\nabla^\rho - i\mathcal{A}^\rho)\}T^+_{\rho\nu} + 16T^{-\mu\nu}f_\mu^\rho T^+_{\rho\nu}
\end{aligned} \tag{2.1}$$

Here $\epsilon^{\mu\nu\rho\sigma}$ denotes the totally antisymmetric tensor density with $\epsilon^{tr\theta\phi} = 1$ and $T^{+\mu\nu} = (T^{-\mu\nu})^*$. Anti-selfduality of $T^-_{\mu\nu}$ imposes the condition:

$$T^{-\mu\nu} = -\frac{i}{2}\,(\sqrt{-\det g})^{-1}\,\epsilon^{\mu\nu\rho\sigma}\,T^-_{\rho\sigma}\,. \tag{2.2}$$

Note that our notation for the Riemann tensor $R_{\mu\nu\rho\sigma}$ differs from that of [6] by a $-$ sign. We take

$$\begin{aligned}
& \Gamma^\mu_{\nu\rho} = \frac{1}{2}g^{\mu\sigma}\left(\partial_\nu g_{\sigma\rho} + \partial_\rho g_{\sigma\nu} - \partial_\sigma g_{\nu\rho}\right) \\
& R^\mu{}_{\nu\rho\sigma} = \partial_\rho\Gamma^\mu_{\nu\sigma} - \partial_\sigma\Gamma^\mu_{\nu\rho} + \Gamma^\mu_{\tau\rho}\Gamma^\tau_{\nu\sigma} - \Gamma^\mu_{\tau\sigma}\Gamma^\tau_{\nu\rho} \\
& R_{\nu\sigma} = R^\mu{}_{\nu\mu\sigma}, \qquad R = g^{\nu\sigma}R_{\nu\sigma}\,.
\end{aligned} \tag{2.3}$$

[1] Following [6] we shall be using a non-linear multiplet as the second compensator field in our description of the theory. We could also work with a description of the theory where we use *e.g.* a hypermultiplet as the second compensator field[7]. The expression for the entropy function given in eq.(3.9) in independent of which description we use. Also we shall be using K-gauge condition from the beginning where we set the gauge field associated with the dilatation symmetry of conformal supergravity to zero[6].

For later use we also define

$$G^-_{I\mu\nu} = -16\pi i \frac{\partial \mathcal{L}}{\partial F^{-I\mu\nu}}\,, \tag{2.4}$$

where $\sqrt{-\det g}\,\mathcal{L}$ is the Lagrangian density. In carrying out the differentiation on the right hand side of (2.4) we must treat the $F^{-I}_{\mu\nu}$ for different $\{I,\mu,\nu\}$ as independent variables so that under a variation of $F^I_{\mu\nu}$

$$\delta\mathcal{L} = \frac{i}{16\pi}\left(G^-_{I\mu\nu}\delta F^{-I\mu\nu} - G^+_{I\mu\nu}\delta F^{+I\mu\nu}\right)\,. \tag{2.5}$$

The action involving these fields is written in terms of the prepotential $F(\vec{X},\widehat{A})$, – a meromorphic function of the complex fields X^I and the composite auxiliary field $\widehat{A}$. F satisfies the condition:

$$F(\lambda\vec{X},\lambda^2\widehat{A}) = \lambda^2 F(\vec{X},\widehat{A})\,. \tag{2.6}$$

We define[2]

$$F_I = \frac{\partial F}{\partial X^I}, \quad F_{\widehat{A}} = \frac{\partial F}{\partial \widehat{A}}, \quad F_{IJ} = \frac{\partial^2 F}{\partial X^I \partial X^J}, \quad F_{\widehat{A}I} = \frac{\partial^2 F}{\partial X^I \partial \widehat{A}}, \quad F_{\widehat{A}\widehat{A}} = \frac{\partial^2 F}{\partial \widehat{A}^2}\,. \tag{2.7}$$

In terms of the prepotential the bosonic part of the action is given by (see eq.(3.111) of [6])

$$S = \int d^4x\,\sqrt{-\det g}\,\mathcal{L}\,, \tag{2.8}$$

where

$$\begin{aligned} 8\,\pi\,\mathcal{L} \;=\; & -\frac{i}{2}(X^I\bar{F}_I - \bar{X}^I F_I)R + \Big[i(\partial_\mu F_I + i\mathcal{A}_\mu F_I)(\partial^\mu \bar{X}^I - i\mathcal{A}^\mu \bar{X}^I) \\ & + \frac{i}{4}F_{IJ}(F^{-I}_{\mu\nu} - \frac{1}{4}\bar{X}^I T^-_{\mu\nu})(F^{-J\mu\nu} - \frac{1}{4}\bar{X}^J T^{-\mu\nu}) \\ & + \frac{i}{8}\bar{F}_I(F^{-I}_{\mu\nu} - \frac{1}{4}\bar{X}^I T^-_{\mu\nu})T^{-\mu\nu} - \frac{i}{8}F_{IJ}Y^I_{ij}Y^{Jij} + \frac{i}{32}\bar{F}\,\widehat{A} \\ & + \frac{i}{2}F_{\widehat{A}}\widehat{C} - \frac{i}{8}F_{\widehat{A}\widehat{A}}(\widehat{B}_{ij}\widehat{B}^{ij} - 2\widehat{F}^-_{\mu\nu}\widehat{F}^{-\mu\nu}) \\ & + \frac{i}{2}\widehat{F}^-_{\mu\nu}F_{\widehat{A}I}(F^{-I}_{\mu\nu} - \frac{1}{4}\bar{X}^I T^-_{\mu\nu}) - \frac{i}{4}\widehat{B}_{ij}F_{\widehat{A}I}Y^{Iij} \\ & + h.c.\Big] \\ & - i(X^I\bar{F}_I - \bar{X}^I F_I)\Big(\nabla^\mu V_\mu - \frac{1}{2}V^\mu V_\mu - \frac{1}{4}|M_{ij}|^2 \end{aligned}$$

[2] Note that we are using the same symbol F for the prepotential and the gauge field strengths. This should not cause any confusion since the index structures of these two sets of quantities are quite different.

$$+(\partial^\mu \Phi^i_\alpha + \frac{1}{2}\mathcal{V}^{i\mu}{}_j \Phi^j_\alpha)(\partial_\mu \Phi^\alpha_i + \frac{1}{2}\mathcal{V}_{i\mu}{}^k \Phi^\alpha_k)\Big) \,. \tag{2.9}$$

Here $\Phi^j_\alpha = (\Phi^\alpha_j)^*$ and ∇_μ denotes ordinary covariant derivative. Furthermore, the fields are subject to the constraint

$$\nabla^\mu V_\mu - \frac{1}{2} V^\mu V_\mu - \frac{1}{4}|M_{ij}|^2 + (\partial^\mu \Phi^i_\alpha + \frac{1}{2}\mathcal{V}^{i\mu}_{\ j}\Phi^j_\alpha)(\partial_\mu \Phi^\alpha_i + \frac{1}{2}\mathcal{V}_{i\mu}^{\ k}\Phi^\alpha_k) - D + \frac{1}{3}R = 0 \,. \tag{2.10}$$

3 Entropy Function for Extremal Black Holes

In the theory described in section 2 we consider extremal black holes with near horizon geometry of the form:[3]

$$\begin{aligned}
& ds^2 = v_1(-r^2 dt^2 + dr^2/r^2) + v_2(d\theta^2 + \sin^2\theta d\phi^2) \\
& F^I_{rt} = e_I, \quad F^I_{\theta\phi} = p^I \sin\theta, \quad X^I = x^I, \quad T^-_{rt} = v_1\, w \\
& D - \frac{1}{3}R = 0, \quad \mathcal{A}_\mu = 0, \quad \mathcal{V}^i{}_{j\mu} = 0, \quad V_\mu = 0, \quad M_{ij} = 0, \quad \Phi^\alpha_i = \delta^\alpha_i \,, \quad Y^I_{ij} = 0 \,.
\end{aligned} \tag{3.1}$$

As can be seen from the action (2.9) and the constraint (2.10), this is a consistent truncation, respecting the symmetries of $AdS_2 \times S^2$. In particular the equations of motion for the fields D, $\mathcal{A}_\mu$, $\mathcal{V}^i{}_{j\mu}$, V_μ, M_{ij} and Φ^α_i subject to the constraint (2.10), as well as the constraint itself, are automatically satisfied for the background (3.1). We now define the entropy function $\mathcal{E}(v_1, v_2, w, \vec{x}, \vec{e}, \vec{q}, \vec{p})$ as follows[14]

$$\mathcal{E}(v_1, v_2, w, \vec{x}, \vec{e}, \vec{q}, \vec{p}) = 2\pi \left(-\frac{1}{2}\, \vec{q}.\vec{e} - \int d\theta d\phi \, \sqrt{-\det g}\, \mathcal{L} \right) \,, \tag{3.2}$$

where the $\sqrt{-\det g}\,\mathcal{L}$ appearing on the right hand side of eq.(3.2) is to be evaluated for the background (3.1) and the integral over θ, ϕ is to be evaluated at fixed r, t. For an extremal black hole carrying electric charge vector $\vec{q}$ and magnetic charge vector $\vec{p}$ the parameters $\vec{x}$, $\vec{e}$, v_1, v_2, w labelling the near horizon geometry are obtained by extremizing the function $\mathcal{E}$ with respect to e^I, x^I, v_1, v_2 and w:

$$\frac{\partial \mathcal{E}}{\partial v_i} = 0, \qquad \frac{\partial \mathcal{E}}{\partial x^I} = 0, \qquad \frac{\partial \mathcal{E}}{\partial w} = 0, \qquad \frac{\partial \mathcal{E}}{\partial e^I} = 0 \,, \tag{3.3}$$

[3] Note that the normalization of the magnetic charge vector $\vec{p}$ used here differs from that of [14] by a factor of 4π. Similarly the normalization of the electric charge vector $\vec{q}$ introduced in (3.2) differs from that of [14] by a factor of $-\frac{1}{2}$. These normalizations have been chosen so as to be consistent with the ones used in [6].

and the entropy associated with the black hole is given by the value of the function $\mathcal{E}$ at the extremum[14, 15]:

$$S_{BH} = \mathcal{E}\,. \tag{3.4}$$

Comparing (2.4) with the equation obtained by extremizing (3.2) with respect to e^I:

$$q_I = -2\,\frac{\partial}{\partial e^I}\int d\theta d\phi\,\sqrt{-\det g}\,\mathcal{L}\,, \tag{3.5}$$

and using the definition of e^I given in (3.1), we can show that near the horizon

$$G_{I\theta\phi} = q_I\,\sin\theta\,. \tag{3.6}$$

We can now calculate the various quantities defined in eq.(2.1) for the background described in (3.1). In particular we get

$$\begin{aligned}
& f^r_r = f^t_t = \frac{1}{2}v_1^{-1} - \frac{1}{32}w\bar{w}\,, \qquad f^\theta_\theta = f^\phi_\phi = -\frac{1}{2}v_2^{-1} + \frac{1}{32}w\bar{w} \\
& f^\mu_\nu = 0 \quad \text{otherwise}, \\
& \mathcal{R}_{m\alpha}{}^{n\beta} = -\mathcal{R}_{\alpha m}{}^{n\beta} = -\mathcal{R}_{m\alpha}{}^{\beta n} = \mathcal{R}_{\alpha m}{}^{\beta n} = \frac{1}{2}(v_1^{-1} - v_2^{-1})\delta^n_m\delta^\beta_\alpha \\
& \qquad \text{for} \quad \alpha,\beta = r,t, \quad m,n = \theta,\phi \\
& \mathcal{R}_{\mu\nu}{}^{\rho\sigma} = 0 \quad \text{otherwise},
\end{aligned} \tag{3.7}$$

$$\begin{aligned}
& \widehat{A} = -4w^2, \qquad \widehat{B}_{ij} = 0, \qquad \widehat{F}^-_{\mu\nu} = 0, \\
& \widehat{C} = 16w\bar{w}\left(-v_1^{-1} - v_2^{-1} + \frac{1}{8}w\bar{w}\right) + 128(v_1^{-1} - v_2^{-1})^2\,.
\end{aligned} \tag{3.8}$$

For the action given in (2.9), a straightforward calculation now gives:

$$\begin{aligned}
\mathcal{E} \;&=\; -\pi q_I e^I - \pi v_1 v_2\Big[i(v_1^{-1} - v_2^{-1})(x^I\bar{F}_I - \bar{x}^I F_I) \\
&\quad -\Big\{\frac{i}{4}v_1^{-2}F_{IJ}(e^I - iv_1v_2^{-1}p^I - \frac{1}{2}\bar{x}^I v_1 w)(e^J - iv_1v_2^{-1}p^J - \frac{1}{2}\bar{x}^J v_1 w) + h.c.\Big\} \\
&\quad -\Big\{\frac{i}{4}v_1^{-1}w\bar{F}_I(e^I - iv_1v_2^{-1}p^I - \frac{1}{2}\bar{x}^I v_1 w) + h.c.\Big\} \\
&\quad +\Big\{\frac{i}{8}\bar{w}^2 F + h.c.\Big\} + 8\,i\,\bar{w}w\Big(-v_1^{-1} - v_2^{-1} + \frac{1}{8}\bar{w}w\Big)\Big(F_{\widehat{A}} - \bar{F}_{\widehat{A}}\Big) \\
&\quad +64\,i\,(v_1^{-1} - v_2^{-1})^2\Big(F_{\widehat{A}} - \bar{F}_{\widehat{A}}\Big)\Big] \\
&\equiv\; -\pi q_I e^I - \pi\, g(v_1, v_2, w, \vec{x}, \vec{e}, \vec{p})\,.
\end{aligned} \tag{3.9}$$

The entropy function defined here has a scale invariance

$$x^I \to \lambda x^I, \quad v_i \to \lambda^{-1}\bar\lambda^{-1} v_i, \quad e^I \to e^I. \quad w \to \lambda w, \quad q_I \to q_I, \quad p^I \to p^I\,. \tag{3.10}$$

This descends from the invariance of the lagrangian density (2.9) under local scale transformation, and is usually eliminated by using some gauge fixing condition. We shall however find it convenient to work with the gauge invariant equations of motion obtained by extremizing (3.9) with respect to v_1, v_2, w, $\vec{x}$ and $\vec{e}$.

Since (3.9) is quadratic in the electric field variables e^I we can explicitly eliminate them by solving their equations of motion to express the entropy function as a function of the other variables. A tedious but straightforward algebra shows that after eliminating the variables e^I the entropy function reduces to:

$$\begin{aligned}
\mathcal{E}(v_1, v_2, w, \vec{x}, \vec{q}, \vec{p}) \;=\; & \pi\Bigg[i(v_1 - v_2)(x^I \bar F_I - \bar x^I F_I) \\
& + v_1 v_2^{-1} \begin{pmatrix} p^I & q_I \end{pmatrix} \begin{pmatrix} (\bar{\mathbf{F}}\mathbf{N}^{-1}\mathbf{F})_{IJ} & -(\bar{\mathbf{F}}\mathbf{N}^{-1})_I{}^J \\ -(\mathbf{N}^{-1}\mathbf{F})^I{}_J & (\mathbf{N}^{-1})^{IJ} \end{pmatrix} \begin{pmatrix} p^J \\ q_J \end{pmatrix} \\
& - \frac{i}{2} v_1 \left\{ w \begin{pmatrix} p^I & q_I \end{pmatrix} \begin{pmatrix} (\bar{\mathbf{F}}\mathbf{N}^{-1}\mathbf{F})_{IJ} & -(\bar{\mathbf{F}}\mathbf{N}^{-1})_I{}^J \\ -(\mathbf{N}^{-1}\mathbf{F})^I{}_J & (\mathbf{N}^{-1})^{IJ} \end{pmatrix} \begin{pmatrix} \bar x^J \\ \bar F_J \end{pmatrix} - h.c. \right\} \\
& + 8 v_1 v_2 \bar w^3 w^3 (\mathbf{N}^{-1})^{IJ} \bar F_{\widehat{A}I} F_{\widehat{A}J} \\
& + \left\{ 4 i v_1 v_2 \bar w^2 w^4 \left(F_{\widehat{A}\widehat{A}} + i\,(\mathbf{N}^{-1})^{IJ} F_{\widehat{A}I} F_{\widehat{A}J} \right) + h.c. \right\} \\
& + \frac{i}{8} v_1 v_2 w \bar w (x^I \bar F_I - \bar x^I F_I) + 8 i w \bar w (v_1 + v_2 - \frac{1}{16} v_1 v_2 w \bar w)(F_{\widehat{A}} - \bar F_{\widehat{A}}) \\
& - 64\, i\, v_1 v_2 \left(v_1^{-1} - v_2^{-1} \right)^2 \left(F_{\widehat{A}} - \bar F_{\widehat{A}} \right) \Bigg]\,,
\end{aligned} \tag{3.11}$$

where

$$N_{IJ} = i\left(\bar F_{IJ} - F_{IJ}\right)\,, \tag{3.12}$$

and $\mathbf{N}$, $\mathbf{F}$ denote matrices with matrix elements N_{IJ} and F_{IJ} respectively. Note that by an abuse of notation we have continued to use the symbol $\mathcal{E}$ to denote the entropy function even after elimination of the variables e^I. In arriving at (3.11) we have used the relations

$$x^I F_I + 2\widehat{A} F_{\widehat{A}} = 2\,F, \qquad x^I F_{IJ} + 2\widehat{A} F_{J\widehat{A}} = F_J, \qquad x^I F_{I\widehat{A}} + 2\widehat{A} F_{\widehat{A}\widehat{A}} = 0\,, \tag{3.13}$$

which follow from (2.6).

4 Symplectic Invariance of the Entropy Function

As has been discussed in [1, 2, 3, 4, 5, 6, 7], the equations of motion derived from the Lagrangian density (2.9) retain their form under a symplectic transformation:

$$\begin{pmatrix} \check{X}^I \\ \check{F}_J \end{pmatrix} = \begin{pmatrix} U^I{}_K & Z^{IL} \\ W_{JK} & V_J{}^L \end{pmatrix} \begin{pmatrix} X^K \\ F_L \end{pmatrix}, \qquad \begin{pmatrix} \check{F}^{\pm I}_{\mu\nu} \\ \check{G}^{\pm}_{J\mu\nu} \end{pmatrix} = \begin{pmatrix} U^I{}_K & Z^{IL} \\ W_{JK} & V_J{}^L \end{pmatrix} \begin{pmatrix} F^{\pm K}_{\mu\nu} \\ G^{\pm}_{L\mu\nu} \end{pmatrix}, \tag{4.1}$$

with all other fields, including the metric $g_{\mu\nu}$ and the auxiliary field $T^-_{\mu\nu}$, remaining invariant. Here U, Z, W and V are each $(N+1)\times(N+1)$ matrix, satisfying the conditions

$$U^T W - W^T U = 0, \qquad Z^T V - V^T Z = 0, \qquad U^T V - W^T Z = \mathbf{1}, \tag{4.2}$$

so that $\begin{pmatrix} U & Z \\ W & V \end{pmatrix}$ is a symplectic matrix. Eq.(4.1) not only tells us how the fundamental fields X^I and $F^I_{\mu\nu}$ transform under this transformation, but also implicitly tells us how the prepotential F transforms to a new prepotential $\check{F}$ (so that $\check{F}_I = \partial\check{F}/\partial\check{X}^I$). Since in general $\check{F}$ and F have different functional forms, the transformation (4.1) is not a symmetry. In special cases where $\check{F}$ and F have the same form, the symplectic transformations generate (continuous) duality symmetries of the classical theory.

From (3.1), (3.6) it follows that under a symplectic transformation the parameters labelling the near horizon geometry of a black hole transform as

$$\begin{pmatrix} \check{x}^I \\ \check{F}_J \end{pmatrix} = \begin{pmatrix} U^I{}_K & Z^{IL} \\ W_{JK} & V_J{}^L \end{pmatrix} \begin{pmatrix} x^K \\ F_L \end{pmatrix}, \qquad \begin{pmatrix} \check{p}^I \\ \check{q}_J \end{pmatrix} = \begin{pmatrix} U^I{}_K & Z^{IL} \\ W_{JK} & V_J{}^L \end{pmatrix} \begin{pmatrix} p^K \\ q_L \end{pmatrix},$$

$$\check{v}_1 = v_1, \qquad \check{v}_2 = v_2, \qquad \check{w} = w. \tag{4.3}$$

We shall now verify that the entropy function (3.11) is invariant under the symplectic transformation. Using the relations (see *e.g.* eqs.(3.88), (3.97)-(3.99) of [6])

$$\begin{aligned} &\check{F}_{\widehat{A}} = F_{\widehat{A}}, \qquad \check{\mathbf{F}} = (V\mathbf{F} + W)(U + Z\mathbf{F})^{-1}, \qquad \check{\mathbf{N}}^{-1} = \bar{\mathcal{S}}\mathbf{N}^{-1}\mathcal{S}^T = \mathcal{S}\mathbf{N}^{-1}\bar{\mathcal{S}}^T, \\ &\check{F}_{\widehat{A}I} = F_{\widehat{A}J}(\mathcal{S}^{-1})^J{}_I, \qquad \check{F}_{\widehat{A}\widehat{A}} = F_{\widehat{A}\widehat{A}} - F_{\widehat{A}I}F_{\widehat{A}J}\mathcal{Z}^{IJ} \end{aligned} \tag{4.4}$$

where

$$\mathcal{S}^I{}_J = U^I{}_J + Z^{IK}F_{KJ}, \qquad \mathcal{Z}^{IK} = (\mathcal{S}^{-1})^I{}_J Z^{JK}, \tag{4.5}$$

it is easy to check that

$$\check{F}_{\widehat{A}\widehat{A}} + i(\check{\mathbf{N}}^{-1})^{IJ}\check{F}_{\widehat{A}I}\check{F}_{\widehat{A}J} = F_{\widehat{A}\widehat{A}} + i(\mathbf{N}^{-1})^{IJ}F_{\widehat{A}I}F_{\widehat{A}J}, \tag{4.6}$$

and

$$\begin{pmatrix} \check{\bar{\mathbf{F}}}\check{\mathbf{N}}^{-1}\check{\mathbf{F}} & -\check{\bar{\mathbf{F}}}\check{\mathbf{N}}^{-1} \\ -\check{\mathbf{N}}^{-1}\check{\mathbf{F}} & \check{\mathbf{N}}^{-1} \end{pmatrix} = \begin{pmatrix} V & -W \\ -Z & U \end{pmatrix} \begin{pmatrix} \bar{\mathbf{F}}\mathbf{N}^{-1}\mathbf{F} & -\bar{\mathbf{F}}\mathbf{N}^{-1} \\ -\mathbf{N}^{-1}\mathbf{F} & \mathbf{N}^{-1} \end{pmatrix} \begin{pmatrix} V^T & -Z^T \\ -W^T & U^T \end{pmatrix} . \tag{4.7}$$

Using (4.1)-(4.7) it is straightforward to verify that the entropy function given in (3.11) is invariant under a sympletic transformation. This is in accordance with the general result on duality invariance of the entropy function discussed in [15].

5 Supersymmetric Attractors

It can be easily seen that the extremization equations (3.3) can be satisfied by setting

$$v_1 = v_2 = \frac{16}{\bar{w}w} , \tag{5.1}$$

$$e^I - i v_1 v_2^{-1} p^I - \frac{1}{2}\bar{x}^I v_1 w = 0 \tag{5.2}$$

$$(\bar{w}^{-1}\bar{F}_I - w^{-1}F_I) = -\frac{i}{4}\, q_I . \tag{5.3}$$

Taking the real and imaginary parts of eq.(5.2) gives

$$e^I = 4(\bar{w}^{-1}\bar{x}^I + w^{-1}x^I) , \tag{5.4}$$

and

$$(\bar{w}^{-1}\bar{x}^I - w^{-1}x^I) = -\frac{1}{4}\, i\, p^I . \tag{5.5}$$

The black hole entropy computed using Wald's formalism[44, 45, 46, 47] is equal to the entropy function evaluated for this background[14] and is given by

$$S_{BH} = 2\pi \left[-\frac{1}{2}\, \vec{q}\cdot\vec{e} - 16\, i\, (w^{-2}F - \bar{w}^{-2}\bar{F}) \right] . \tag{5.6}$$

If we choose w=constant gauge (which corresponds to $\widehat{A}$=$-4w^2$=constant), then eqs.(5.1)-(5.5) describe the usual attractor equations for the near horizon geometry of extremal black holes, and (5.6) gives the expression for the entropy of these black holes as written down in [48]. For example (5.6) shows that in the gauge w =real constant, the Legendre transform of the black hole entropy with respect to the electric charges q_I is proportional to the imaginary part of the prepotential F. Furthermore eqs.(5.1), (5.2) shows that the argument x^I of the prepotential is proportional to $e^I + ip^I$, i.e. its real part is the variable conjugate to the electric charge q_I and its imaginary part is the magnetic charge p^I. These are some of the observations made in [48].

Note that the attractor equations (5.1)-(5.5) provide sufficient but not necessary conditions for extremizing the entropy function. In section 7 we shall find near horizon configurations which extremize the entropy function but do not satisfy eqs.(5.1)-(5.5).

6 Supersymmetric Black Holes in the STU Model

Let us now restrict our attention to a specific theory with three vector multiplets and a prepotential

$$F(X^0, X^1, X^2, X^3, \widehat{A}) = -\frac{X^1 X^2 X^3}{X^0} - C\widehat{A}\,\frac{X^1}{X^0}\,. \tag{6.1}$$

For $C = 1/64$ this describes a subsector of the low energy effective action for tree level heterotic string theory on $T^4 \times T^2$ or $K3 \times T^2$, with the identification

$$\frac{X^1}{X^0} = iS\,, \quad \frac{X^2}{X^0} = iT\,, \quad \frac{X^3}{X^0} = iU\,, \tag{6.2}$$

where S, T and U denote the usual axion-dilaton field, the Kahler modulus of T^2 and the complex structure modulus of T^2 respectively. The corresponding gauge fields A^0_μ, ... A^3_μ may be identified as the components of the metric and the rank two anti-symmetric tensor field with one index along one of the directions of T^2 and the other index along a non-compact direction.[4]

For the choice of the prepotential given in (6.1) the equations of motion derived from the lagrangian density (2.9) are invariant under the $SO(2,2) = SL(2,R) \times SL(2,R)$ T-duality transformation:

$$\begin{aligned}
X^0 \to cX^3 + dX^0\,, \quad & X^1 \to -cF_2 + dX^1\,, \\
X^2 \to -cF_1 + dX^2\,, \quad & X^3 \to aX^3 + bX^0\,, \\
F_0 \to aF_0 - bF_3\,, \quad & F_1 \to aF_1 - bX^2\,, \\
F_2 \to aF_2 - bX^1\,, \quad & F_3 \to -cF_0 + dF_3\,, \\
F^{-3}_{\mu\nu} \to a\,F^{-3}_{\mu\nu} + b\,F^{-0}_{\mu\nu}\,, \quad & F^{-0}_{\mu\nu} \to c\,F^{-3}_{\mu\nu} + d\,F^{-0}_{\mu\nu}\,,
\end{aligned}$$

[4] In order to make this identification we need to dualize the gauge field $\mathcal{A}^1_\mu$. This is reflected in the relation (6.6) between the charges $(\vec{q}, \vec{p})$ in this theory and the charges $(\vec{Q}, \vec{P})$ in heterotic string compactification.

$$
\begin{aligned}
G^-_{1\mu\nu} &\to a\,G^-_{1\mu\nu} - b\,F^{-2}_{\mu\nu}\,, \quad F^{-2}_{\mu\nu} \to -c\,G^-_{1\mu\nu} + d\,F^{-2}_{\mu\nu} \\
G^-_{3\mu\nu} &\to d\,G^-_{3\mu\nu} - c\,G^-_{0\mu\nu}, \quad G^-_{0\mu\nu} \to -b\,G^-_{3\mu\nu} + a\,G^-_{0\mu\nu}, \\
F^{-1}_{\mu\nu} &\to d\,F^{-1}_{\mu\nu} - c\,G^-_{2\mu\nu}\,, \quad G^-_{2\mu\nu} \to -b\,F^{-1}_{\mu\nu} + a\,G^-_{2\mu\nu}
\end{aligned} \tag{6.3}
$$

and

$$
\begin{aligned}
X^0 &\to rX^2 + sX^0\,, \quad X^1 \to -rF_3 + sX^1\,, \\
X^3 &\to -rF_1 + sX^3\,, \quad X^2 \to kX^2 + lX^0\,, \\
F_0 &\to kF_0 - lF_2\,, \quad F_1 \to kF_1 - lX^3\,, \\
F_3 &\to kF_3 - lX^1\,, \quad F_2 \to -rF_0 + sF_2\,, \\
F^{-2}_{\mu\nu} &\to k\,F^{-2}_{\mu\nu} + l\,F^{-0}_{\mu\nu}, \quad F^{-0}_{\mu\nu} \to r\,F^{-2}_{\mu\nu} + s\,F^{-0}_{\mu\nu}, \\
G^-_{1\mu\nu} &\to k\,G^-_{1\mu\nu} - l\,F^{-3}_{\mu\nu}\,, \quad F^{-3}_{\mu\nu} \to -r\,G^-_{1\mu\nu} + s\,F^{-3}_{\mu\nu} \\
G^-_{2\mu\nu} &\to s\,G^-_{2\mu\nu} - r\,G^-_{0\mu\nu}, \quad G^-_{0\mu\nu} \to -l\,G^-_{2\mu\nu} + k\,G^-_{0\mu\nu}, \\
F^{-1}_{\mu\nu} &\to s\,F^{-1}_{\mu\nu} - r\,G^-_{3\mu\nu}\,, \quad G^-_{3\mu\nu} \to -l\,F^{-1}_{\mu\nu} + k\,G^-_{3\mu\nu}\,,
\end{aligned} \tag{6.4}
$$

where

$$
ad - bc = 1, \qquad ks - lr = 1, \qquad a, b, c, d, k, l, r, s \in \mathbb{R}\,. \tag{6.5}
$$

These are special cases of the symplectic transformations discussed in section 4 for which $\check{F}$ has the same functional form as F.

We now define:

$$
\begin{aligned}
&Q_1 = q_2, \quad Q_2 = -p^1, \quad Q_3 = q_3, \quad Q_4 = q_0\,, \\
&P_1 = p^3, \quad P_2 = p^0, \quad P_3 = p^2, \quad P_4 = q_1\,,
\end{aligned} \tag{6.6}
$$

$$
Q^2 = 2(Q_1Q_3 + Q_2Q_4), \quad P^2 = 2(P_1P_3 + P_2P_4), \quad Q \cdot P = (Q_1P_3 + Q_3P_1 + Q_2P_4 + Q_4P_2)\,, \tag{6.7}
$$

where p^I and q_I have been defined in eqs.(3.1) and (3.6). From (6.3)-(6.6) it follows that the duality transformations act on $\vec{P}$, $\vec{Q}$ as $SO(2,2)$ transformations:

$$
\begin{pmatrix} Q_1 \\ Q_2 \\ Q_3 \\ Q_4 \end{pmatrix} \to \Omega \begin{pmatrix} Q_1 \\ Q_2 \\ Q_3 \\ Q_4 \end{pmatrix}, \qquad \begin{pmatrix} P_1 \\ P_2 \\ P_3 \\ P_4 \end{pmatrix} \to \Omega \begin{pmatrix} P_1 \\ P_2 \\ P_3 \\ P_4 \end{pmatrix}\,, \tag{6.8}
$$

where Ω is the $SO(2,2)$ matrix

$$\Omega = \begin{pmatrix} s & 0 & 0 & -r \\ 0 & s & r & 0 \\ 0 & l & k & 0 \\ -l & 0 & 0 & k \end{pmatrix} \begin{pmatrix} a & b & 0 & 0 \\ c & d & 0 & 0 \\ 0 & 0 & d & -c \\ 0 & 0 & -b & a \end{pmatrix} . \tag{6.9}$$

Q^2, P^2 and $Q\cdot P$ defined in (6.7) are the three independent duality invariant combinations which can be formed out of $\vec{Q}$ and $\vec{P}$. Thus the black hole entropy depends only on these combinations.

Supersymmetric black holes in this theory have been analyzed in detail in [1, 2, 3, 4, 5, 6, 7]. These black holes exist for $P^2 > 0$, $(Q\cdot P)^2 < Q^2P^2$ and the entropy associated with these black holes can be obtained by extremizing the entropy function with respect to various near horizon parameters, and plugging them into (5.6). The solution satisfies the supersymmetric attractor equations given in (5.1)-(5.5). Due to the duality symmetry (6.8) we can choose to work in a special frame in which $P_2 = 0$, i.e. $p^0 = 0$. By solving the attractor equations (5.1)-(5.5) and using the definitions (6.6), (6.7) we get, in the $w = 1$ gauge,

$$\begin{aligned}
x^0 &= -\frac{1}{8}Q_2\sqrt{\frac{P^2(P^2+512C)}{P^2Q^2-(P\cdot Q)^2}} \\
\frac{x^1}{x^0} &= -\frac{P\cdot Q}{P^2} + i\sqrt{\frac{P^2Q^2-(P\cdot Q)^2}{P^2(P^2+512C)}} \\
\frac{x^2}{x^0} &= -\frac{1}{2Q_2P_1}(Q_2P_4+Q_1P_3-P_1Q_3) - i\frac{P_3}{Q_2}\sqrt{\frac{P^2Q^2-(P\cdot Q)^2}{P^2(P^2+512C)}} \\
\frac{x^3}{x^0} &= -\frac{1}{2Q_2P_3}(Q_2P_4-Q_1P_3+P_1Q_3) - i\frac{P_1}{Q_2}\sqrt{\frac{P^2Q^2-(P\cdot Q)^2}{P^2(P^2+512C)}} \\
v_1 &= v_2 = 16, \qquad e^I = 8\,Re(x^I) \quad \text{for} \quad 0 \le I \le 3, \qquad w = 1\,.
\end{aligned} \tag{6.10}$$

The entropy associated with these black holes is given by

$$S_{BH} = \pi\sqrt{P^2Q^2-(P\cdot Q)^2}\sqrt{1+\frac{512C}{P^2}}\,. \tag{6.11}$$

This result was derived in [4] and reviewed in eq.(6.64) of [6].

The solution simplifies for a specific class of black holes for which $P\cdot Q = 0$. In this case we can get supersymmetric black holes if $Q^2 > 0$, $P^2 > 0$. A representative element satisfying this condition is

$$P_1 = P_3 = P_0, \qquad Q_2 = Q_4 = -Q_0, \qquad P_2 = P_4 = Q_1 = Q_3 = 0, \qquad Q_0, P_0 > 0\,, \tag{6.12}$$

with $P_0 > 0$, $Q_0 > 0$. In this case

$$Q^2 = 2Q_0^2, \qquad P^2 = 2P_0^2\,, \tag{6.13}$$

and, according to (6.6),

$$p^1 = Q_0, \quad p^2 = P_0, \quad p^3 = P_0, \quad q_0 = -Q_0\,, \tag{6.14}$$

with all other charges zero. Eqs.(6.10), (6.11) now reduce to:

$$\begin{aligned} & x^0 = \frac{1}{8}\sqrt{P_0^2 + 256C}, \quad x^1 = \frac{i}{8}Q_0, \quad x^2 = \frac{i}{8}P_0, \quad x^3 = \frac{i}{8}P_0, \\ & e^0 = \sqrt{P_0^2 + 256C}, \quad e^1 = e^2 = e^3 = 0\,, \quad w = 1\,, \\ & v_1 = 16, \quad v_2 = 16\,, \end{aligned} \tag{6.15}$$

and

$$S_{BH} = 2\pi Q_0 \sqrt{P_0^2 + 256C} = \pi\,\sqrt{Q^2\,P^2}\,\sqrt{1 + 512\frac{C}{P^2}}\,. \tag{6.16}$$

7 Non-supersymmetric Extremal Black Holes in the STU Model

For $C = 0$ the theory described in section 6 also contains extremal non-supersymmetric black holes for[33, 16, 17, 18, 19, 20, 21, 22]

$$Q^2 P^2 < (Q \cdot P)^2\,. \tag{7.1}$$

As before, the entropy of these black holes can be obtained by extremizing the entropy function. Due to the duality symmetries given in (6.8), (6.9), we can simplify the calculation by choosing a representative $\vec{Q}$, $\vec{P}$ satisfying

$$P_2 = P_4 = Q_2 = Q_4 = 0\,, \tag{7.2}$$

and then rewriting the final result in a duality invariant form. It turns out that in this case the resulting entropy function, after elimination of the auxiliary variable w, and the electric field variables e^I, has a Z_2 symmetry which allows us to set[5]

$$Im(T) = Im(U) = 0\,. \tag{7.3}$$

The final result for the entropy function after extremization is

$$S_{BH} = \pi\sqrt{(Q\cdot P)^2 - Q^2P^2}\,. \tag{7.4}$$

For simplicity we shall focus our attention on a special class of these black holes for which

$$Q\cdot P = 0, \qquad P^2 > 0, \qquad Q^2 < 0\,. \tag{7.5}$$

In this case instead of using the configuration (7.2) we shall use a representative element

$$-Q_2 = Q_4 = Q_0, \quad P_1 = P_3 = P_0, \quad Q_1 = Q_3 = P_2 = P_4 = 0, \qquad Q_0, P_0 > 0\,, \tag{7.6}$$

so that we have

$$P^2 = 2P_0^2, \qquad Q^2 = -2Q_0^2\,, \tag{7.7}$$

and, according to (6.6),

$$p^1 = Q_0, \quad p^2 = P_0, \quad p^3 = P_0, \quad q_0 = Q_0\,. \tag{7.8}$$

Note that the charge assignment (7.6) differs from that of (6.12) by simple reversal of the sign of Q_4, i.e. of q_0. For $C = 0$ the bosonic part of the action, after elimination of the auxiliary field w, has a Z_2 symmetry that allows us to relate the black hole solutions for the charge configurations (6.12) and (7.6) by simple reversl of the sign of e^0. Thus the near horizon geometry for the black hole solution corresponding to the charges given in (7.6) is obtained from (6.15) by setting $C = 0$, changing the sign of e^0, and finally determining w by solving its equation of motion. This gives

$$\begin{aligned}
&x^0 = \frac{1}{8}P_0, \quad x^1 = \frac{i}{8}Q_0, \quad x^2 = \frac{i}{8}P_0, \quad x^3 = \frac{i}{8}P_0, \\
&e^0 = -P_0, \quad e^1 = e^2 = e^3 = 0, \quad w = \frac{1}{2}\,, \\
&v_1 = 16, \quad v_2 = 16\,.
\end{aligned} \tag{7.9}$$

[5]Physically this corresponds to a solution in heterotic string theory on $T^4\times T^2$ or $K3\times T^2$ where the electric and magnetic charges associated with only one of the two circles of T^2 are present. Thus $T^4\times S^1$ or $K3\times S^1$ part factorizes from the black hole geometry.

One can verify explicitly that this configuration extremizes the entropy function (3.9) for $F(\vec{X},\widehat{A}) = -X^1X^2X^3/X^0$. The corresponding entropy is

$$S_{BH} = 2\pi\, Q_0\, P_0\,, \tag{7.10}$$

in accordance with the general result (7.4). Our goal will be to analyze higher derivative corrections to the near horizon geometry of these non-supersymmetric black holes by keeping $C \neq 0$. In order to do so, it will be convenient to choose the gauge

$$w = \frac{1}{2} \tag{7.11}$$

so that the leading order solution given in (7.9) already satisfies the gauge condition. In this gauge the entropy function evaluated for $\vec{P}$, $\vec{Q}$ of the form given in (7.6) can be shown to be invariant under the transformation

$$x^0 \to (x^0)^*, \qquad x^i \to -(x^i)^* \quad \text{for} \quad 1 \le i \le 3, \qquad e^i \to -e^i \quad \text{for} \quad 1 \le i \le 3\,. \tag{7.12}$$

Thus we can look for a solution to the extremization equation within the subspace which is invariant under the transformation (7.12), i.e. we take

$$x^0 = (x^0)^*, \qquad x^i = -(x^i)^* \quad \text{for} \quad 1 \le i \le 3, \qquad e^i = 0 \quad \text{for} \quad 1 \le i \le 3\,. \tag{7.13}$$

It will be convenient to introduce rescaled real varibles y^0, y^1, y^2, y^3, $\tilde{e}^0$ through

$$x^0 = P_0 y^0, \quad x^1 = iQ_0 y^1, \quad x^2 = iP_0 y^2, \quad x^3 = iP_0 y^3, \quad e^0 = P_0 \tilde{e}^0\,. \tag{7.14}$$

Substituting (7.8), (7.11)-(7.14) into (3.9) we get

$$\begin{aligned}
\mathcal{E} \;=\; & \pi Q_0 P_0 \Bigg[-\tilde{e}^0 - \frac{v_1}{v_2}\frac{y^1+y^2+y^3}{y^0} + \left\{\frac{v_1}{y^0} - \frac{\tilde{e}^0}{(y^0)^2}\right\}(y^1y^2+y^2y^3+y^1y^3) \\
& + \left\{ -\frac{(\tilde{e}^0)^2 v_2}{v_1 (y^0)^3} + \frac{\tilde{e}^0 v_2}{(y^0)^2} + \frac{8(v_2-v_1)}{y^0} - \frac{v_1v_2}{2y^0} \right\} y^1y^2y^3 \\
& + \frac{C}{P_0^2}\Bigg\{ -\frac{\tilde{e}^0}{(y^0)^2} + \frac{v_1}{2y^0} - \frac{(\tilde{e}^0)^2 v_2 y^1}{v_1 (y^0)^3} + \frac{\tilde{e}^0 v_2 y^1}{2(y^0)^2} + \frac{8v_2y^1}{y^0} - \frac{3v_1v_2y^1}{16y^0} \\
& - \frac{128 y^1}{y^0}\left(\frac{v_1}{v_2} - \frac{v_2}{v_1}\right)^2 \Bigg\}\Bigg]
\end{aligned} \tag{7.15}$$

Note that $Q_0P_0 = \frac{1}{2}\sqrt{-Q^2P^2}$ appears as an overall factor in the above expression, and the rest of the expression is a function of the combination $C/P^2 = C/(2P_0^2)$. Thus the black hole entropy, obtained by extremizing (7.15) with respect to v_1, v_2, $\tilde{e}^0$, y^0, y^1, y^2 and y^3, must be of the form:

$$S_{BH} = \sqrt{-Q^2P^2}\, f\left(\frac{C}{P^2}\right), \tag{7.16}$$

for some function $f(u)$. We shall try to analyze $f(u)$ as a power series expansion in u. The leading contribution, which corresponds to setting the term involving C/P^2 in (7.15) to zero, is given by

$$f(0) = \pi\,. \tag{7.17}$$

The corresponding values of v_1, v_2, $\tilde{e}^0$ and y^I are given by

$$v_1 = 16, \quad v_2 = 16, \quad \tilde{e}^0 = -1, \quad y^0 = 1/8, \quad y^1 = 1/8, \quad y^2 = 1/8, \quad y^3 = 1/8\,. \tag{7.18}$$

These results are in agreement with (7.9), (7.10).

The order u term in $f(u)$ can be obtained by evaluating the order C/P^2 term in (7.15) in the background (7.18). This gives

$$f(u) = \pi(1 + 80\,u + \mathcal{O}(u^2))\,. \tag{7.19}$$

In order to determine the higher order corrections to $f(u)$ we need to solve the extremization equations iteratively as a power series in u. The result for the first few terms is

$$\begin{aligned} f(u) \;=\; & \pi(1 + 80u - 3712u^2 - 243712u^3 - 18325504u^4 - 9538502656u^5 \\ & +7416509890560u^6 + 1770853956059136u^7 + 326801388942131 20u^8 \\ & -194861291843407052800u^9 - 115321933038468181524480u^{10} + \ldots) \end{aligned} \tag{7.20}$$

8 Black Holes in M-theory on Calabi-Yau Manifolds

In this section we shall repeat the analysis of the previous sections for a slightly general class of theories, described by a prepotential of the form:

$$F = -d_{ABC}\frac{X^A X^B X^C}{X^0} - d_A \frac{X^A}{X^0}\widehat{A}\,, \tag{8.1}$$

where the indices A, B, C run from 1 to N, and d_{ABC} and d_A are real constants. The corresponding action describes the low energy effective action of M-theory compactified on $S^1\times$ a large volume Calabi-Yau space $\mathcal{M}$ with N four cycles labeled by the index A $(1 \le A \le N)$. The gauge field $\mathcal{A}^0_\mu$ comes from the components of the metric with one index along S^1 and the other index along a non-compact direction. On the other hand the gauge field $\mathcal{A}^A_\mu$ arises from the three form field C_{MNP} with two of the indices along the two cycle of $\mathcal{M}$ that is dual to the A-th four cycle, and the third index along a non-compact direction. d_{ABC} are the intersection numbers of the four cycles, and d_A are the second Chern class of the four cycles up to a normalization factor[34]. This class of theories clearly includes the prepotential (6.1) as a special case.

First consider a black hole solution for which[6]

$$p^0 = 0, \qquad q_A = 0, \qquad q_0 < 0, \qquad d_{ABC}p^Ap^Bp^C + 256 d_A p^A > 0\,. \tag{8.2}$$

In this case it is easy to show that the following is a solution to the supersymmetric attractor equations (5.1)-(5.5):

$$\begin{aligned}
&v_1 = v_2 = 16, \qquad w = 1\,,\\
&x^A = \frac{1}{8} i p^A, \qquad x^0 = \frac{1}{8}\sqrt{\frac{d_{ABC}p^Ap^Bp^C + 256 d_A p^A}{-q_0}}\,,\\
&e^0 = \sqrt{\frac{d_{ABC}p^Ap^Bp^C + 256 d_A p^A}{-q_0}}, \qquad e^A = 0 \quad \text{for} \quad A = 1, 2, \ldots N\,.
\end{aligned} \tag{8.3}$$

The entropy associated with this solution is given by

$$S_{BH} = 2\pi\sqrt{-q_0(d_{ABC}p^Ap^Bp^C + 256 d_A p^A)}\,. \tag{8.4}$$

These results were first obtained in [2].

For $d_A = 0$ the theory, after elimination of the auxiliary fields, has a Z_2 symmetry that allows us to construct a non-supersymmetric black hole solution from the one described above by reversing the signs of q_0 and e^0[18]. In the M-theory description this corresponds to reversing the sign of the S^1 coordinate. We can construct the near horizon field configuration associated with this solution from the one given in (8.3) by setting $d_A = 0$, reversing the sign of q_0 and e^0 leaving v_1, v_2 and the x^I's unchanged, and then finding w

[6] These black holes have been analyzed in detail in [34]. Some recent discussion of these solutions can be found in [49].

by extremizing the entropy function with respect to this variable. This gives:

$$
\begin{aligned}
& v_1 = v_2 = 16, \\
& x^A = \frac{1}{8} i p^A, \qquad x^0 = \frac{1}{8}\sqrt{\frac{d_{ABC} p^A p^B p^C}{q_0}}, \\
& e^0 = -\sqrt{\frac{d_{ABC} p^A p^B p^C}{q_0}}, \qquad e^A = 0 \quad \text{for} \quad A = 1, 2, \ldots N, \\
& w = \frac{1}{2},
\end{aligned} \tag{8.5}
$$

for

$$
p^0 = 0, \qquad q_A = 0, \qquad q_0 > 0, \qquad d_{ABC} p^A p^B p^C > 0 \,. \tag{8.6}
$$

It is easy to verify that this configuration extremizes the entropy function. The entropy associated with this solution is given by

$$
S_{BH} = 2\pi \sqrt{q_0 (d_{ABC} p^A p^B p^C)} \,. \tag{8.7}
$$

We shall now calculate corrections to this formula due to the higher derivative terms proportional to d_A. First we note that for the prepotential given in (8.1), the function g defined through eq.(3.9) is invariant under a Z_2 symmetry:

$$
p^A \to p^A, \quad p^0 \to -p^0, \quad e^A \to -e^A, \quad e^0 \to e^0, \quad X^A \to -\bar{X}^A, \quad X^0 \to \bar{X}^0, \quad w \to \bar{w} \,. \tag{8.8}
$$

From the M-theory perspective this corresponds to a change of sign of the non-compact directions accompanied by a reversal of the sign of the 3-form field. Thus for studying the near horizon background associated with the $p^0 = 0$, $q_A = 0$ black hole we can consider a Z_2 invariant configuration:

$$
p^0 = 0, \quad e^A = 0, \quad x^A = i y^A, \quad x^0 = y^0, \quad w,\, y^I = \text{real} \,. \tag{8.9}
$$

In this case the function g defined through eq.(3.9) takes the form

$$
\begin{aligned}
g \;=\; & \frac{8}{y^0}(v_1 - v_2)(d_{ABC} y^A y^B y^C + 2\, w^2 d_A y^A) \\
& + \frac{v_2}{v_1}\left(\frac{1}{y^0}\right)^3 (d_{ABC} y^A y^B y^C + 4 w^2 d_A y^A)\left(e^0 - \frac{1}{2} y^0 v_1 w\right)^2 \\
& + \left(\frac{1}{y^0}\right)^2 (3 d_{ABC} y^B y^C + 4 w^2 d_A)\left(e^0 - \frac{1}{2} y^0 v_1 w\right)\left(p^A - \frac{1}{2} y^A v_2 w\right)
\end{aligned}
$$

$$
\begin{aligned}
&+3\,\frac{v_1}{v_2}\frac{1}{y^0}d_{ABC}\left(p^A-\frac{1}{2}y^Av_2w\right)\left(p^B-\frac{1}{2}y^Bv_2w\right)\,y^C\\
&+\frac{1}{2}\left(\frac{1}{y^0}\right)^2 v_2w(d_{ABC}y^Ay^By^C+4w^2d_Ay^A)\left(e^0-\frac{1}{2}y^0v_1w\right)\\
&-\frac{1}{2}\frac{1}{y^0}v_1w(3d_{ABC}y^By^C+4w^2d_A)\left(p^A-\frac{1}{2}y^Av_2w\right)\\
&-\frac{1}{4}\frac{1}{y^0}w^2v_1v_2(d_{ABC}y^Ay^By^C+4w^2d_Ay^A)+16\frac{1}{y^0}w^2\left(-v_1-v_2+\frac{1}{8}w^2v_1v_2\right)\,d_Ay^A\\
&+128\frac{1}{y^0}v_1v_2(v_1^{-1}-v_2^{-1})^2\,d_Ay^A\,. \qquad (8.10)
\end{aligned}
$$

This function has a scaling symmetry:

$$
g(v_1,v_2,w,\lambda^{-1}y^0,\{y^A\},\lambda^{-1}e^0,\{p^A\})=\lambda\,g(v_1,v_2,w,y^0,\{y^A\},e^0,\{p^A\})\,, \qquad (8.11)
$$

which corresponds to scaling of the S^1 coordinate in the M-theory description. Now recall that the entropy function

$$
\mathcal{E}=-\pi q_0e^0-\pi\,g(v_1,v_2,w,x^0,\{x^A\},e^0,\{p^A\})\,, \qquad (8.12)
$$

has to be extremized with respect to the variables v_1, v_2, e^0, y^0, y^A and w. This can be done by first extremizing g with respect to v_1, v_2, w, y^0 and y^A and then extremizing the resulting expression for $\mathcal{E}$ with respect to e^0. Due to the scaling behaviour given in (8.11), extremization of g with respect to v_1, v_2, y^0 and y^A gives a term of the form

$$
g=-\frac{K(\{p^A\})}{|e^0|}+\frac{L(\{p^A\})}{e^0}\,, \qquad (8.13)
$$

for some functions $K(\{p^A\})$, $L(\{p^A\})$. The first term on the right hand side of this equation is invariant under $e^0\to -e^0$ whereas the second term changes sign under this transformation. Thus the second term reflects the effect of parity non-invariant terms in M-theory on the Calabi-Yau manifold $\mathcal{M}$.[7] Substituting (8.13) into (8.12) gives

$$
\mathcal{E}=-\pi q_0e^0+\pi\,\frac{K(\{p^A\})}{|e^0|}-\pi\,\frac{L(\{p^A\})}{e^0}\,. \qquad (8.14)
$$

Extremizing this with respect to e^0 now gives

$$
\begin{aligned}
S_{BH}=\mathcal{E} &= 2\pi\sqrt{(K(\{p^A\})-L(\{p^A\}))\,|q_0|}\,, \qquad \text{for } q_0<0\,,\\
&= 2\pi\sqrt{(K(\{p^A\})+L(\{p^A\}))\,q_0}\,, \qquad \text{for } q_0>0\,, \qquad (8.15)
\end{aligned}
$$

[7]Here parity transformation refers to the change of sign of the S^1 coordinate without any change in sign of the 3-form field.

assuming that $K(\{p^A\}) \geq |L(\{p^A\})|$.

Eq.(8.15) gives the general form of the entropy in this theory. Comparing (8.15) with (8.4) for $d_A = 0$ and (8.7) we see that to leading order $K = d_{ABC}p^A p^B p^C$, $L = 0$. We shall now calculate the first non-leading correction to K and L, i.e. corrections of order d_A. Eq.(8.4) shows that

$$K(\{p^A\}) - L(\{p^A\}) = d_{ABC}p^A p^B p^C + 256 d_A p^A \,, \tag{8.16}$$

exactly. To calculate $K + L$ we need to calculate the entropy of the black hole for $q_0 > 0$. For this we note that since the entropy is the value of the entropy function $\mathcal{E}$ at its extremum, an error of order d_A in determining the near horizon background will affect the value of the entropy function only at quadratic order in d_A. Thus to first order in d_A the computation of the entropy for $q_0 > 0$ involves evaluating the full entropy function in the near horizon background given in (8.5). This is a straightforward task and yields:

$$S_{BH} = 2\pi\sqrt{q_0(d_{ABC}p^A p^B p^C)}\left(1 + \frac{40 d_A p^A}{d_{ABC}p^A p^B p^C}\right) + \mathcal{O}(d_A d_B) \,. \tag{8.17}$$

This corresponds to

$$K(\{p^A\}) + L(\{p^A\}) = d_{ABC}p^A p^B p^C + 80 d_A p^A + \mathcal{O}(d_A d_B) \,. \tag{8.18}$$

For the choice $d_{ABC}p^A p^B p^C = p^1 p^2 p^3$ and $d_A p^A = C p^1$, (8.17) agrees with (7.16), (7.19) to first order in C.

9 A Puzzle

For theories obtained by dimensional reduction of five dimensional supersymmetric theories of gravity on a circle, the entropy of a class of black holes can be analyzed using a five dimensional picture[35, 36]. The black holes discussed in sections 6-8 fall into this class. For these black holes the three dimensional geometry that includes the compact direction S^1, the AdS_2 component of the near horizon geometry, and the effect of the electric field e^0 (regarded as a component of the metric with one index along S^1 and the other index along the time direction) describes a locally AdS_3 space[50], or more precisely the near horizon geometry of an extremal BTZ black hole[51]. Together with the S^2 factor this gives a locally $AdS_3 \times S^2$ near horizon geometry. The entropy of such a black hole can then be analyzed using either an Euclidean action formalism[35, 36, 52] or using Wald's formalism[53, 54]. The answer takes the form

$$S_{BH} = 2\,\pi\,\left(\sqrt{\frac{c_R h_R}{6}} + \sqrt{\frac{c_L h_L}{6}}\right)\,, \tag{9.1}$$

where c_R, c_L, h_R and h_L are expressed as functions of various charges. For the black hole solutions described in section 8 one finds[35, 36],

$$\begin{aligned} &h_L = -q_0, \quad h_R = 0, \quad \text{for} \quad q_0 < 0\,, \\ &h_L = 0, \quad h_R = q_0, \quad \text{for} \quad q_0 > 0\,, \end{aligned} \tag{9.2}$$

and

$$c_L = 6(d_{ABC}p^A p^B p^C + 256 d_A p^A), \qquad c_R = 6(d_{ABC}p^A p^B p^C + 128 d_A p^A)\,. \tag{9.3}$$

On the other hand, (8.15)-(8.18) can be put in the form given in (9.1), (9.2) with

$$\begin{aligned} c_L &= 6\left(K(\{p^A\}) - L(\{p^A\})\right) = 6(d_{ABC}p^A p^B p^C + 256 d_A p^A), \\ c_R &= 6\left(K(\{p^A\}) + L(\{p^A\})\right) = 6(d_{ABC}p^A p^B p^C + 80 d_A p^A + \mathcal{O}(d_A d_B))\,. \end{aligned} \tag{9.4}$$

Comparing (9.3) and (9.4) we see that our value of c_L agrees with that of [35, 36], but our value of c_R differs from that of [35, 36].

It is worthwhile reviewing the argument leading to the computation of $c_R - c_L = 12L$ from the five dimensional perspective. Action (2.8), (2.9) with F given in (8.1) has a term proportional to

$$d_A \int_4 Re(X^A/X^0)\, Tr(R\wedge R) \tag{9.5}$$

from the term in the action proportional to $F_{\widehat{A}}\widehat{C}$. Here $\int_n$ denotes an n-dimensional integral. Since $Re(X^A/X^0)$ can be identified as the component of the gauge field $\mathcal{A}^A$ along S^1 in the five dimensional description, the term (9.5) arises from a term proportional to

$$d_A \int_5 \mathcal{A}^A \wedge Tr(R\wedge R)\, = d_A \int_5 d\mathcal{A}^A \wedge \Omega_3\,, \tag{9.6}$$

in five dimensions. Here Ω_3 is the gravitational Chern-Simons term. We can now regard the near horizon geometry of the black hole solution as a solution in three dimensional theory, obtained by dimensional reduction of the five dimensional theory on the S^2 factor.

Since p^A denotes the flux of the gauge field strength $\mathcal{F}^A = d\mathcal{A}^A$ through S^2, the three dimensional theory has a term in the action proportional to

$$d_A\, p^A \int_3 \Omega_3\,. \tag{9.7}$$

Furthermore this is the only parity non-invariant term in the action that affects the black hole solution under study. Other possible parity non-invarint terms involving gauge Chern-Simons terms and covariant derivatives of field strengths and curvature tensor do not contribute in the background we are considering. The quantity L can now be computed in terms of the coefficient of this parity non-invariant term using the method of [54] and gives the answer

$$L = -64\, d_A p^A\,. \tag{9.8}$$

This disagrees with the four dimensional result computed from (8.16), (8.18)

$$L = -88\, d_A p^A + \mathcal{O}(d_A d_B)\,. \tag{9.9}$$

The origin of this discrepancy is not completely clear to us. Here we discuss various possibilities. However as indicated in the discussion, we have been able to rule out most of these possibilities except the first one.

1. The analysis of [35, 36] applies to the problem at hand only if the action and the black hole solution that we have used arises, up to a field redefinition, from dimensional reduction of a gauge and general cordinate invariant five dimensional theory. This can be shown to be true in the absence of higher derivative terms, but has not so far been demonstrated for the theory including the higher derivative corrections. If the dimensional reduction of the five dimensional theory produces the four dimensional theory analyzed here together with an extra set of terms which are supersymmetric by themselves, the discrepancy may be attributed to these missing terms in our four dimensional action.

2. The analysis of [35, 36] uses the Euclidean action formalism as well as the formalism based on calculation of anomalies in the boundary theory to compute the entropy of a black hole with near horizon $AdS_3 \times S^2$ geometry. In the absence of the Chern-Simons term the result for the black hole entropy agrees with the one computed using Wald's formalism[53]. However Wald's formalism cannot be applied directly in the

presence of Chern-Simons terms in the action since the Lagrangian density is not manifestly general coordinate invariant. In contrast our analysis in four dimension is based on Wald's formalism since the four dimensional Lagrangian density may be written in a manifestly general coordinate invariant form. One might wonder if the discrepancy between our result and that of [35, 36] can be attributed to a difference between these two formalisms. This possibility however has been ruled out in [54] where the entropy of an extremal BTZ black hole in the presence of gravitational Chern-Simons term (and other higher derivative terms) was computed using Wald's formalism by regarding this as a two dimensional configuration and shown to agree with the results of the Euclidean computation.

3. In the analysis of [35, 36] the quantities h_R, h_L are defined as appropriate conserved charges in the five dimensional theory, while the quantity q_0 is defined as a conserved charge in the four dimensional theory. The relation (9.2) between h_R, h_L and the charge q_0 could in principle be renormalized in the presence of higher derivative terms. However we have been able to rule out this possibility as well by regarding the BTZ black hole as a two dimensional configuration and expressing the entropy of an extremal BTZ black hole directly in terms of the gauge charge in the two dimensional theory[54]. The formula for the entropy takes the same form as in (9.1) with h_R, h_L replaced by $\pm q_0$ as indicated in (9.2). After inclusion of the S^2 factor this shows that there is no renormalization factor between the conserved charges in five and four dimensions due to the higher derivative terms.

4. The analysis of [35, 36] relied on an indirect computation of $c_R + c_L$ based on supersymmetry relations. It is conceivable that there are subtle effects which affect the various relations used in [35, 36] in arriving at the final formula for $c_R + c_L$. This however does not affect the calculation of $c_R - c_L = 12L(\{p^A\})$ which can be related directly to the gravitational Chern-Simons term in the five dimensional action[35, 36, 54]. Since our result for c_L agrees with the five dimensional result while the result for c_R does not agree, we have a mismatch between the values of $L(\{p^A\})$ calculated using the two descriptions. This cannot be attributed to a failure of the arguments based on supersymmetry relations.

In view of the discussion above the only possible explanation seems to be that the four dimensional action given in (2.8), (2.9) fails to capture some of the terms which come from dimensional reduction of a five dimensional supersymmetric theory. A probable reason for this is the following.[8] The five dimensional supergravity multiplet, when dimensionally reduced to four dimensions, contains a gravity multiplet and an additional vector multiplet. Thus if we add to the five dimensional action supersymmetrized curvature squared terms then upon dimensional reduction to four dimensions, it will contain supersymmetrized curvature squared terms and also another set of terms which involve supersymmetrization of the four derivative term involving the additional vector multiplet fields. In contrast the action used in [2, 3, 4, 5, 6, 7] contains the minimal set of terms which are required for supersymmetrizing the curvature squared terms. Thus this action could miss the additional terms involving vector multiplet fields which would arise from the dimensional reduction of the five dimensional action.

In view of this it is all the more surprising that for BPS black holes the result of [2, 3, 4, 5, 6, 7] agrees with the one obtined using the five dimensional picture[35, 36]. Clearly some additional non-renormalization theorems which hold only for supersymmetric black holes are at work here. Presumably when the missing terms are included it will not change the result for supersymmetric black hole, but the entropy of the special class of non-supersymmetric black holes analyzed in sections 6-8 will agree with the corresponding results derived from the five dimensional analysis. Once these terms are found, we can calculate their effect on the entropy function and apply it to calculate the entropy of black holes whose near horizon geometry do not necessarily have the $AdS_3 \times S^2$ form.

Acknowledgement: We would like to thank G. Lopes Cardoso, B. de Wit, J. Kappeli and T. Mohaupt for useful correspondence and their comments on an earlier version of the manuscript. We would also like to thank Atish Dabholkar, Justin David, Rajesh Gopakumar, N Iizuka, A. Iqubal, Dileep Jatkar, M. Shigemori and Sandip Trivedi for useful discussions at various stages of this work.

References

[1] K. Behrndt, G. Lopes Cardoso, B. de Wit, D. Lust, T. Mohaupt and W. A. Sabra, Phys. Lett. B **429**, 289 (1998) [arXiv:hep-th/9801081].

[8] This explanation was offered to us by G. Lopes Cardoso, B. de Wit, J. Kappeli and T. Mohaupt.

[2] G. Lopes Cardoso, B. de Wit and T. Mohaupt, Phys. Lett. B **451**, 309 (1999) [arXiv:hep-th/9812082].

[3] G. Lopes Cardoso, B. de Wit and T. Mohaupt, Fortsch. Phys. **48**, 49 (2000) [arXiv:hep-th/9904005].

[4] G. Lopes Cardoso, B. de Wit and T. Mohaupt, Nucl. Phys. B **567**, 87 (2000) [arXiv:hep-th/9906094].

[5] G. Lopes Cardoso, B. de Wit and T. Mohaupt, Class. Quant. Grav. **17**, 1007 (2000) [arXiv:hep-th/9910179].

[6] T. Mohaupt, Fortsch. Phys. **49**, 3 (2001) [arXiv:hep-th/0007195].

[7] G. Lopes Cardoso, B. de Wit, J. Kappeli and T. Mohaupt, JHEP **0012**, 019 (2000) [arXiv:hep-th/0009234].

[8] G. L. Cardoso, B. de Wit, J. Kappeli and T. Mohaupt, Fortsch. Phys. **49**, 557 (2001) [arXiv:hep-th/0012232].

[9] A. Dabholkar, arXiv:hep-th/0409148.

[10] A. Dabholkar, R. Kallosh and A. Maloney, arXiv:hep-th/0410076.

[11] A. Sen, arXiv:hep-th/0411255.

[12] V. Hubeny, A. Maloney and M. Rangamani, arXiv:hep-th/0411272.

[13] D. Bak, S. Kim and S. J. Rey, arXiv:hep-th/0501014.

[14] A. Sen, arXiv:hep-th/0506177.

[15] A. Sen, arXiv:hep-th/0508042.

[16] K. Goldstein, N. Iizuka, R. P. Jena and S. P. Trivedi, arXiv:hep-th/0507096.

[17] R. Kallosh, arXiv:hep-th/0510024.

[18] P. K. Tripathy and S. P. Trivedi, arXiv:hep-th/0511117.

[19] A. Giryavets, arXiv:hep-th/0511215.

[20] K. Goldstein, R. P. Jena, G. Mandal and S. P. Trivedi, arXiv:hep-th/0512138.

[21] R. Kallosh, N. Sivanandam and M. Soroush, arXiv:hep-th/0602005.

[22] R. Kallosh, arXiv:hep-th/0603003.

[23] P. Prester, JHEP **0602**, 039 (2006) [arXiv:hep-th/0511306].

[24] M. Alishahiha and H. Ebrahim, arXiv:hep-th/0601016.

[25] A. Sinha and N. V. Suryanarayana, arXiv:hep-th/0601183.

[26] B. Chandrasekhar, S. Parvizi, A. Tavanfar and H. Yavartanoo, arXiv:hep-th/0602022.

[27] S. Parvizi and A. Tavanfar, arXiv:hep-th/0602292.

[28] B. de Wit, Nucl. Phys. Proc. Suppl. **49**, 191 (1996) [arXiv:hep-th/9602060].

[29] B. de Wit, Fortsch. Phys. **44**, 529 (1996) [arXiv:hep-th/9603191].

[30] S. Ferrara, R. Kallosh and A. Strominger, Phys. Rev. D **52**, 5412 (1995) [arXiv:hep-th/9508072].

[31] A. Strominger, Phys. Lett. B **383**, 39 (1996) [arXiv:hep-th/9602111].

[32] S. Ferrara and R. Kallosh, Phys. Rev. D **54**, 1514 (1996) [arXiv:hep-th/9602136].

[33] S. Ferrara, G. W. Gibbons and R. Kallosh, Nucl. Phys. B **500**, 75 (1997) [arXiv:hep-th/9702103].

[34] J. M. Maldacena, A. Strominger and E. Witten, JHEP **9712**, 002 (1997) [arXiv:hep-th/9711053].

[35] P. Kraus and F. Larsen, arXiv:hep-th/0506176.

[36] P. Kraus and F. Larsen, arXiv:hep-th/0508218.

[37] B. de Wit, J. W. van Holten and A. Van Proeyen, Nucl. Phys. B **167**, 186 (1980).

[38] E. Bergshoeff, M. de Roo and B. de Wit, Nucl. Phys. B **182**, 173 (1981).

[39] B. de Wit, J. W. van Holten and A. Van Proeyen, Nucl. Phys. B **184**, 77 (1981) [Erratum-ibid. B **222**, 516 (1983)].

[40] B. de Wit and A. Van Proeyen, Nucl. Phys. B **245**, 89 (1984).

[41] B. de Wit, P. G. Lauwers and A. Van Proeyen, Nucl. Phys. B **255**, 569 (1985).

[42] B. de Wit, R. Philippe and A. Van Proeyen, Nucl. Phys. B **219**, 143 (1983).

[43] B. de Wit, P. G. Lauwers, R. Philippe, S. Q. Su and A. Van Proeyen, Phys. Lett. B **134**, 37 (1984).

[44] R. M. Wald, Phys. Rev. D **48**, 3427 (1993) [arXiv:gr-qc/9307038].

[45] T. Jacobson, G. Kang and R. C. Myers, Phys. Rev. D **49**, 6587 (1994) [arXiv:gr-qc/9312023].

[46] V. Iyer and R. M. Wald, Phys. Rev. D **50**, 846 (1994) [arXiv:gr-qc/9403028].

[47] T. Jacobson, G. Kang and R. C. Myers, arXiv:gr-qc/9502009.

[48] H. Ooguri, A. Strominger and C. Vafa, arXiv:hep-th/0405146.

[49] A. Fayyazuddin, [arXiv:hep-th/0603141].

[50] A. Strominger, JHEP **9802**, 009 (1998) [arXiv:hep-th/9712251].

[51] M. Banados, C. Teitelboim and J. Zanelli, Phys. Rev. Lett. **69**, 1849 (1992) [arXiv:hep-th/9204099].

[52] S. N. Solodukhin, arXiv:hep-th/0509148.

[53] H. Saida and J. Soda, Phys. Lett. B **471**, 358 (2000) [arXiv:gr-qc/9909061].

[54] B. Sahoo and A. Sen, arXiv:hep-th/0601228.