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# Geometric Tachyon to Universal Open String Tachyon

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**Abstract**

A system of $k$ Neveu-Schwarz (NS) 5-branes of type II string theory with one transverse direction compactified on a circle admits various unstable D-brane systems, – some with geometric instability arising out of being placed at a point of unstable equilibrium in space and some with the usual open string tachyonic instability but no geometric instability. We discuss the effect of NS 5-branes on the descent relations among these branes and their physical interpretation in the T-dual ALF spaces. We argue that if the tachyon potential controlling these descent relations obeys certain conditions, then in certain region in the parameter space labelling the background the two types of unstable branes become identical via a second order phase transition, with the geometric tachyon in one system getting mapped to the usual open string tachyon of the other system. This would provide a geometric description of the tachyonic instability of the usual non-BPS Dp-brane in ten dimensional flat space-time.

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# 1 Introduction and Summary

Type IIA and IIB string theories are known to admit unstable non-BPS D-branes. These D-branes have tachyonic modes obtained by quantizing open strings living on these branes. The physics of the tachyonic mode is by now well understood [1–3]. However there is no clear geometric interpretation of these modes.

Some time ago Kutasov identified a D-brane system with a different kind of instability [4,5]. This involves $k$ Neveu-Schwarz (NS) 5-branes with a transverse circle, and BPS Dp-branes with world-volume parallel to the NS 5-branes, placed as a point on the transverse circle diametrically opposite to the NS 5-branes.[1] At this point the potential energy density of the D-brane has a saddle point. As a result this is a point of unstable equilibrium and the Dp-brane has a tachyonic mode associated with the displacement of the brane along the circle. Although this is a geometric mode, it was found in the analysis of [4, 5] that the behaviour of this geometric tachyon is in many ways very similar to the behaviour of the open string tachyon on a conventional non-BPS D-brane in flat space-time background. Various other aspects of the dynamics of this system have been investigated in [6–11].

In this paper we shall introduce several other unstable D-brane systems in the same background geometry and study and compare their properties. These additional D-branes are non-BPS D(p+1)-

[1] Even though it is a non-supersymmetric configuration, we shall continue to refer to this D-brane as BPS D-brane in order to distinguish it from the usual non-BPS D-branes carrying open string tachyonic modes even in flat space-time.

branes extending along the transverse circle – either wrapping the circle or ending on the NS 5-branes – and other world-volume directions parallel to those of the Dp-brane described in the last paragraph.[2] These D-branes are unstable due to the usual open string tachyon living on their world-volume; however they do not have any additional geometrical instability.

A T-duality transformation on the transverse circle takes the original closed string field configuration to type IIB/IIA string theory on ALF spaces [12,13]. When the $k$ NS 5-branes are all coincident in the original description, the T-dual geometry involves ALF spaces with $A_{k-1}$ singularities but when the NS 5-branes are separated the singularity in the T-dual geometry is resolved by blowing up the collapsed 2-cycles. The duality map relates the geometrically unstable Dp-branes in the original background to BPS D(p+1)-branes wrapped on the equators of these blown up 2-cycles, and the non-BPS D(p+1)-branes of the original background to non-BPS Dp-branes and non-BPS D(p+2)-branes wrapped on these 2-cycles. By following the duality map we can derive various properties of these branes. For example, one such property is that the tension of a BPS D(p+1)-brane wrapped on the equator of a 2-cycle remains finite even when the 2-cycle collapses to zero size if the D-brane carries a non-zero Wilson line along the collapsed cycle.

It was already noted in [4, 5] that the geometric tachyons have many properties in common with conventional open string tachyons. Since we now have both types of D-branes in the same background geometry, we can try to compare their properties in detail and explore if this analogy can be made into an equivalence. Indeed one finds that the two types of D-branes exihibit very similar behaviour. For example in the NS 5-brane background the condensation of the geometric tachyon on the geometrcally unstable BPS Dp-brane and the condensation of the open string tachyon on the non-BPS D(p+1)-brane, – either into the vacuum or into a kink solution that depends on any of the $p$ coordinates common to both D-branes – produces identical configurations. Furthermore in the absence of the NS 5-branes the non-BPS D(p+1)-branes and the BPS Dp-branes are related via usual open string tachyon condensation [14]. This continues to hold even in the presence of NS 5-branes, although we find that the precise form of these relations are modified. Thus these two types of D-branes may be considered as two different classical solutions in the same theory, – the world-volume theory of the non-BPS D(p+1)-brane. This leads to the following question: Can these two solutions merge as we vary the external parameters *e.g.* the radius of the circle or the number of NS 5-branes? If so, then at that point the two systems will become identical, with the geometric tachyon on one getting identified with the open string tachyon on the other. Such merger of solutions

[2]This system is different from the one analyzed in [6] where non-BPS Dp-branes with world-volume transverse to the circle $S^1$ was considered.

has indeed been observed in a closely related system earlier [15].

This analysis however is plagued by the difficulty that the various properties of the systems with geometric tachyons like the tension or the tachyon mass$^2$ were calculated using the Dirac-Born-Infeld (DBI) action and can in principle be affected by $\alpha'$ corrections.[3] When the number of NS 5-branes is large, the $\alpha'$ corrections remain under control even up to the zero radius limit. In this case we find that the two solutions do not merge. The BPS Bp-brane in unstable equilibrium remains lighter than the non-BPS D(p+1)-brane wrapped on the circle all the way from infinite radius to zero radius, and a tachyonic kink configuration on the latter takes us to the former configuration. A naive comparison without taking into account $\alpha'$-corrections shows that in the particular case involving two coincident NS 5-branes and zero radius of the circle transverse to the NS 5-brane, the D-brane with geometric instability and the usual non-BPS D-brane have identical tension and tachyon mass$^2$, – a coincidence already noted in [5]. In the absence of a non-renormalization theorem we cannot reach a definite conclusion.[4] However if it turns out that there is an underlying non-renormalization theorem protecting the tension and tachyon mass$^2$ of the system with geometric tachyon, then it would be a strong indication that the solutions describing the two types of D-branes merge at this point, with the geometric tachyon of one system getting mapped to the usual open string tachyon of the other system.

In the T-dual geometry the non-BPS D-brane under consideration is a non-BPS Dp-brane placed at a point in the ALE space with $A_1$ singularity. This might lead one to conclude that this correspondence, even if true, is not so exciting. However we should recall that the interesting part of the usual open string tachyon dynamics is universal and does not depend on the geometry of the transverse space in which the D-brane is placed. In this particular example, the tachyon vacuum solution, the kink solutions along any of the $p$ directions tangential to the brane, or the rolling tachyon solution on this Dp-brane are identical to those on a Dp-brane in flat space-time. Thus this correspondence, if true, would provide us with a geometric understanding of the tachyon dynamics on a non-BPS Dp-brane in flat space-time as well.

An interesting case is that of a single NS 5-brane background with a transverse circle of radius $R$. Its T-dual geometry is Taub-NUT space of size $\widetilde{R} = 1/R$, which reduces to flat space-time in the infinite $\widetilde{R}$ limit. In the original NS 5-brane background we can construct a geometrically unstable D-brane configuration by placing a BPS Dp-brane at a point on the transverse circle diametrically opposite to the NS 5-brane. This configuration should exist for large radius of the transverse circle,

[3]We show that the non-BPS D(p+1)-branes extending along the transverse circle do not suffer from this problem.

[4]In a closely related situation where the NS 5-brane has non-compact transverse directions, it is known that the DBI action produces certain results exactly [16].

and the interesting question is: what happens to this brane in the small radius limit when the dual geometry is flat space-time? For multiple NS 5-branes the dual geometry has (collapsed) 2-cycles and the D-brane described above has a natural description as BPS D(p+1)-branes wrapped on the equator of such a 2-cycle. However the geometry dual to a single NS 5-brane does not possess a 2-cycle, and hence there is no analogous interpretation for this D-brane. One can consider several possibilities: 1) it could describe a new unstable Dp-brane, 2) it could disappear from the spectrum by having either zero or infinite tension, 3) instead of remaining localized, it could blow up and fill the whole space in this limit or 4) it could describe the usual unstable Dp-brane. Of these the fourth possibility is most exciting, since it will provide us with a direct geometric interpretation of the usual open string tachyon on an unstable Dp-brane in flat space-time as a geometric instability in the dual description.

The issue involved is of course the same issue raised earlier in the more general context of multiple NS 5-brane background, but it will be useful to describe it again in this special context. The system of one NS 5-brane with a transverse circle contains a non-BPS D(p+1)-brane wrapped on the transverse circle. Its dual description is the usual non-BPS Dp-brane sitting at the center of the Taub-NUT space, – precisely the configuration with which we would like to identify the system described in the last paragraph. Furthermore for large radius of the circle transverse to the five brane, a kink solution of the open string tachyon on this non-BPS D(p+1)-brane can produce a BPS Dp-brane sitting diametrically across the NS 5-brane, – the earlier system. Thus two systems can be viewed as different classical solutions in the same theory, and we would like to ask if the two solutions can merge at some critical radius as we reduce the radius of the transverse circle. If they do then it would mean that below that critical radius the two solutions become identical. In the dual description it would imply that the 'new' non-BPS Dp-brane constructed via the procedure described in the previous paragraph and the usual non-BPS Dp-brane, sitting at the center of Taub-NUT, will be identical when the size of the Taub-NUT space exceeds a critical value.

We can reformulate this problem in terms of an effective potential for the tachyonic mode whose condensation takes us from the non-BPS D(p+1)-brane to the geometrically unstable BPS Dp-brane.[5] In this description the non-BPS D(p+1)-brane and the BPS Dp-brane represent two different extrema of the tachyon potential. We show that the quadratic term in the effective potential changes sign at some critical radius $R_c$ of the circle transverse to the NS 5-brane. If the coefficient of the quartic term is positive at this critical radius then the two extrema merge at this critical radius and remain

[5]This tachyonic mode should be distinguished from the tachyon zero mode whose condensation takes us to the tachyon vacuum.

identical below this radius. This represents a second order phase transition for the BPS Dp-brane at which the spontaneously broken $(-1)^{F_L}$ symmetry is restored. On the other hand if the coefficient of the quartic term in the potential is negative at the critical radius, then the two extrema do not merge and the D-branes do not become identical. With our present level of knowledge we cannot determine what really happens; however we show that the first possibility is more economical since it does not require the existence of any other extrema of the potential. In contrast if the sign of the quartic term is negative at the critical radius then new extrema of the potential appear below the critical radius, signalling new D-brane configurations. If these extrema survive down to zero radius then we have the problem of explaining what they are in the dual flat space-time geometry.

This effective potential approach can also be applied for other values of $k$, but the potential will have different features for different $k$. If it turns out that there is a non-renormalization theorem for the tension and the tachyon mass$^2$ of the geometrically unstable D-branes for $k \geq 2$, then the picture is somewhat trivial for $k \geq 2$ coincident NS 5-branes. For $k = 2$ the critical radius where the two types of D-branes become identical is at $R = 0$, whereas for $k \geq 3$ the two types of D-branes remain distinct all the way down to $R = 0$. However the picture becomes much richer once we consider a more general configuration where the NS 5-branes are separated from each other. After all, if we consider the configuration of $k$ NS 5-branes equally spaced on a circle of radius $R$, then it is a $k$-fold cover of the configuration describing a single NS 5-brane with a transverse circle of radius $R/k$. Thus the merger of the two D-brane configurations for $R \leq R_c$ in the $k = 1$ case will imply that for $k$ equally spaced NS 5-branes, a BPS Dp-brane sitting midway between two neighbouring NS 5-branes and a non-BPS D(p+1)-brane stretched between the two neighbouring NS 5-branes must become identical below the critical value $kR_c$ of $R$. In the full moduli space spanned by $R$ and the separation between the NS 5-branes we would expect a codimension one critical surface that separates the region in which the two types of D-branes are distinct from the region in which they are identical. In the special case of $k = 2$ if we denote by $2c$ the angular separation between the two NS 5-branes on the circle then the critical curve in the $(R, c)$ plane should pass through the points $(0, 0)$ and $(2R_c, \pi/2)$.

If the non-renormalization theorems do not hold then the detailed picture described above will not be correct, *e.g.* for $k = 2$ the critical curve will not pass through the $(0, 0)$ point. However the general picture, i.e. the existence of a critical surface that separates a region where the two D-branes are identical from the region where they are not identical, is based on the sign of certain coefficient in the tachyon potential, and will still hold if this coefficient has the correct sign.

Finally we should remark that even though we have dealt with unstable systems with tachyons, these may also be useful in getting stable non-supersymmetric configurations after certain orbifolding that projects out the tachyon mode.

The rest of the paper is organized as follows. In §2 we describe various unstable D-brane systems in a background of multiple NS 5-branes with a transverse circle and discuss descent relation between these different D-branes for large radius of the transverse circle. In §3 we discuss the description of these unstable D-branes in the dual ALF geometry. In §4 we describe the zero radius limit of the original configuration that converts the dual ALF geometry to ALE geometry and study the fate of the descent relations in this limit. §5 describes comparison between different D-brane systems and possible identification of a BPS D-brane with geometric tachyon with a non-BPS D-brane with the usual open string tachyon. In §6 we discuss the case of a single NS 5-brane with a transverse circle, and determine under what condition a geometrically unstable D-brane in this background in the zero radius limit would describe the usual unstable D-brane in a dual flat space-time geometry. We conclude in §7 with some comments.

# 2 Unstable D-brane Configurations and Their Descent Relations in NS 5-brane Background

We begin by considering a system of $k$ NS 5-branes in type IIA/IIB string theory stretched along the $(x^0, \dots x^5)$ plane and placed at $(x^6, \dots x^9) = (0, \dots 0)$. Let $x^6$ be a compact coordinate with period $2\pi R$. The string metric, the dilaton $\Phi$, and the NS sector 3-form field strength $\mathcal{H}$ produced by this background are given by [17, 18][6]

$$
\begin{aligned}
ds^2 &= \eta_{\mu\nu}\, dx^\mu dx^\nu + H(\vec{r}, y)(dy^2 + d\vec{r}^2)\,, \\
e^{2\Phi} &= g^2\, H(\vec{r}, y)\,, \\
\mathcal{H}_{mnp} &= -\epsilon_{mnpq}\partial^q \Phi\,,
\end{aligned} \tag{2.1}
$$

where $\mu, \nu$ run from 0 to 5, $m, n, p, q$ run from 6 to 9,

$$
\vec{r} \equiv (x^7, x^8, x^9), \qquad y \equiv x^6\,, \tag{2.2}
$$

and

$$
H(\vec{r}, y) = 1 + \frac{k}{2Rr}\, \frac{\sinh(r/R)}{\cosh(r/R) - \cos(y/R)}\,, \qquad r \equiv |\vec{r}|\,. \tag{2.3}
$$

This background is invariant under the transformation

$$
\sigma: \quad y \to -y, \qquad \vec{r} \to -\vec{r}\,. \tag{2.4}
$$

[6]We shall use $\alpha' = 1$ convention throughout this paper.

This symmetry will play an important role in our analysis.

If the $k$ 5-branes are not coincident but are placed at different points $(\vec{r}_i, y_i)$ $(1 \le i \le k)$ then the solution is still described by (2.1), but with $H$ given by

$$H(\vec{r}, y) = 1 + \sum_{i=1}^{k} \frac{1}{2R|\vec{r} - \vec{r}_i|} \frac{\sinh(|\vec{r} - \vec{r}_i|/R)}{\cosh(|\vec{r} - \vec{r}_i|/R) - \cos\left((y - y_i)/R\right)} . \tag{2.5}$$

We shall consider various types of non-supersymmetric D-brane configurations in the background geometry described in eqs.(2.1)-(2.3). The first type of such configurations, which we shall call G-type D-branes because they will turn out to have **g**eometric instability, is obtained by placing a BPS Dp-brane ($p \le 5$) along $(x^0, \ldots x^p)$ at $\vec{r} = \vec{0}$, $y = \pi R$ and arbitrary values of $x^{p+1}, \ldots x^5$ [5]. We shall summarize the main results of [5]. The DBI action on the Dp-brane in this background will be given by

$$-\, g^{-1}\, \mathcal{T}_p \int d^{p+1}\xi \, \left(H(\vec{Z}, Y)\right)^{-1/2} \sqrt{-\det G} \tag{2.6}$$

where $\{\xi^\alpha\}$ $(0 \le \alpha \le p)$ are the Dp-brane world-volume coordinates – taken to coincide with $(x^0, \ldots x^p)$, $\vec{Z}$ and $Y$ denote respectively the $\vec{r}$ and $y$ coordinates of the D-brane world-volume, $g^{-1}\,\mathcal{T}_p$ denotes the tension of a BPS Dp-brane at $\infty$, and

$$G_{\alpha\beta} = \eta_{\alpha\beta} + H(\vec{Z}, Y)(\partial_\alpha \vec{Z} \cdot \partial_\beta \vec{Z} + \partial_\alpha Y \partial_\beta Y) , \qquad 0 \le \alpha, \beta \le p , \tag{2.7}$$

is the induced metric on the Dp-brane world-volume. Note that we have ignored the motion of the brane along the plane of the 5-brane as well as the dynamics of gauge fields on the brane; these will not play any role in our analysis.

The overall multiplicative factor of $(H(\vec{Z}, Y))^{-1/2}$ provides a potential for the motion of the brane. For $H$ given in (2.3) this potential has an absolute minimum at $Y = 0$, $\vec{Z} = 0$ where it vanishes, and a saddle point at $Y = \pi R$, $\vec{Z} = 0$ where it has a minimum as a function of $\vec{Z}$ but a maximum as a function of $Y$. Thus it represents a point of unstable equilibrium [5] and $Y$ becomes a tachyonic field on the Dp-brane world-volume. One can easily calculate the tension $\tau_p$ of the Dp-brane situated at $(\vec{Z} = \vec{0}, Y = \pi R)$ and the mass squared $m_T^2$ of the tachyon on this D-brane coming from the unstable geometric mode. The answers are

$$\begin{aligned} \tau_p &= g^{-1}\mathcal{T}_p H(\vec{0}, \pi R)^{-1/2} = g^{-1}\,\mathcal{T}_p \left(1 + \frac{k}{4R^2}\right)^{-1/2} , \\ m_T^2 &= \left[H^{-1/2} \frac{\partial^2 H^{-1/2}}{\partial y^2}\right]_{\vec{r}=0, y=\pi R} = -\frac{k}{(k + 4R^2)^2} . \end{aligned} \tag{2.8}$$

We should keep in mind however that the DBI action (2.6) receives higher derivative corrections. Thus the results (2.8) can also receive higher derivative corrections. In particular for low values of $k$ and $R \sim 1$, the spatial curvature as well as the derivatives of the dilaton are of order one near $(\vec{r} = 0, y = \pi R)$ and hence the corrections can be of order unity. We shall return to a discussion about these corrections later.

It was noted in [5] that the tachyonic mode described by $Y$ has many properties in common with the usual open string tachyon on an unstable D-brane. Clearly the minimum of the tachyon potential is at $Y = 0$ where the tension of the Dp-brane, being proportional to $H(\vec{0}, 0)^{-1/2} = 0$, vanishes. Furthermore one can consider tachyonic kink configurations in this theory located at $x^p = 0$, described by the solution $Y = 0$ for $x^p < 0$ and $Y = 2\pi R$ for $x^p > 0$. This describes a BPS Dp-brane located at $x^p = 0$ and stretched along $x^0, \ldots x^{p-1}$, $y$ [5]. Neither of these general properties is expected to be modified by $\alpha'$ corrections.

For future reference it will be useful to consider the situation where the $k$ NS 5-branes are displaced away from the $(\vec{r}, y) = (\vec{0}, 0)$ point symmetrically to $\{(\vec{r}_i, y_i),\ 1 \le i \le k\}$ so that $(\vec{r}, y) = (\vec{0}, \pi R)$ is still a point of unstable equilibrium for the Dp-brane. In this case (2.5) shows that the tension of the Dp-brane situated at $(\vec{0}, \pi R)$ is given by

$$\tau_p = g^{-1}\, \mathcal{T}_p\, H(\vec{0}, \pi R)^{-1/2} = g^{-1}\, \mathcal{T}_p \left( 1 + \sum_{i=1}^{k} \frac{1}{2R|\vec{r}_i|}\, \frac{\sinh(|\vec{r}_i|/R)}{\cosh(|\vec{r}_i|/R) + \cos{(y_i/R)}} \right)^{-1/2} . \tag{2.9}$$

A second type of non-supersymmetric D-brane configuration in the same background is obtained by placing a non-BPS D(p+1) brane [1–3, 19] spanning the coordinates $x^0, \ldots x^p$ and $y$. We shall call these U-type D-branes to indicate that they carry the **u**sual open string tachyon. The world-volume action describing the dynamics of the massless modes on this brane is given by

$$-\frac{1}{\sqrt{2}\pi}\, g^{-1}\, \mathcal{T}_p \int d^{p+1}\xi\, dy\, H(\vec{Z}, y)^{-1/2} \sqrt{-\det \mathcal{G}}\,, \tag{2.10}$$

where $(\{\xi^\alpha\}, y)$ denote the world-volume coordinates and

$$\mathcal{G}_{\alpha\beta} = \eta_{\alpha\beta} + H(\vec{Z}, y)\partial_\alpha \vec{Z} \cdot \partial_\beta \vec{Z}, \quad \mathcal{G}_{yy} = H(\vec{Z}, y) \left( 1 + \partial_y \vec{Z} \cdot \partial_y \vec{Z} \right), \quad \mathcal{G}_{\alpha y} = \mathcal{G}_{y\alpha} = H(\vec{Z}, y)\partial_\alpha \vec{Z} \cdot \partial_y \vec{Z}\,. \tag{2.11}$$

As before we have ignored the dynamics of the gauge fields on the brane and the motion of the brane along the NS 5-brane world-volume. Another field that has not been included in the action (2.10) but will play an important role in our analysis is the open string tachyon field.

This theory has a classical solution corresponding to

$$\vec{Z} = \vec{c}\,, \tag{2.12}$$

for any constant vector $\vec{c}$, describing the non-BPS D(p+1)-brane situated as $\vec{r} = \vec{c}$ and spanning the $x^0, \ldots x^p, y$ directions. From the point of view of an asymptotic observer this looks like a p-brane since one of its world-volume directions is compact. The tension, defined as the mass per unit $p$-volume, and the mass$^2$ of the open string tachyon living on this D-brane is given by

$$\tau_p' = \frac{1}{\sqrt{2}\pi}\, g^{-1}\, \mathcal{T}_p \int dy\, H(\vec{Z}, y)^{-1/2} \sqrt{-\det \mathcal{G}}\Big|_{\vec{Z}=\vec{c}} = \sqrt{2}\, R\, g^{-1}\, \mathcal{T}_p \qquad m_T'^2 = -\frac{1}{2}\,. \tag{2.13}$$

This D-brane can decay into the closed string vacuum of zero energy density via tachyon condensation [20–23]. Furthermore an $x^p$ dependent tachyonic kink solution on this D-brane localized along the $x^p = 0$ surface produces a BPS Dp-brane stretched along the $x^0, \ldots x^{p-1}$, $y$ direction [14]. Due to the universality of the open string tachyon dynamics [24, 25] neither of these properties is modified by $\alpha'$ corrections. Thus the condensation of the open string tachyon living on this D-brane, either into a kink or into its vacuum, produces identical results as the condensation of the geometric tachyon living on the BPS Dp-brane placed at $y = \pi R$. This provides us with the first hint that there may be some deeper relation between these branes.

We shall now argue that unlike the formulæ (2.8), eqs.(2.13) are not modified by $\alpha'$ correction. We begin with the tension. For this let us switch theories and consider type IIB/IIA theory with the same NS5 background and a BPS D(p+1) brane along $x^0, \ldots x^p, y$. This is a supersymmetric system with no force between the D-brane and the NS 5-brane and hence the tension of the D(p+1)-brane (defined as the mass per unit $p$-volume after integration over the $y$ coordinate) is given by the BPS formula which is independent of $\vec{c}$ and is not modified by $\alpha'$ corrections. The same argument holds for a BPS $\bar{\text{D}}$(p+1)-brane in the same position. From this we can conclude that at string tree level when the interaction between different D-branes can be ignored, the tension of a coincident D(p+1)-brane-$\bar{\text{D}}$(p+1)-brane at an arbitrary position $\vec{c}$ will be equal to twice that of a single D(p+1)-brane. We can now take an orbifold of this system by $(-1)^{F_L}$ to construct a non-BPS D(p+1)-brane of IIA/IIB placed at $\vec{c}$ [14], with its tension given by $\sqrt{2}$ times that of the BPS D(p+1)-brane in IIB/IIA theory. This is precisely the tension given in (2.13). Thus we conclude that there is no $\alpha'$ correction to the expression of the tension given in (2.13) for any $\vec{c}$.

The argument regarding tachyon mass$^2$ is even simpler. Since the tachyon vertex operator on the non-BPS D-brane is proportional to the identity operator in the matter sector, and since the identity operator has dimension zero in all conformal field theories, the mass$^2$ of the tachyon on the non-BPS D(p+1)-brane must be given by (2.13).

A major part of our analysis will focus on studying the relationship between the two types of D-branes introduced above. We shall begin by comparing the action of the transformation $\sigma$ given in

(2.4) on the tachyon on both types of branes. Since $\sigma$ changes $y \to -y$, the geometric tachyon $Y$ on the G-type D-brane, being the $y$ coordinate of the D-brane world-volume, clearly changes sign under $\sigma$. On the other hand it is known from the analysis of [14] that the open string tachyon field $T$ on the unstable D(p+1) brane wrapped along $y$ also changes sign under $\sigma$. Thus if we restrict ourselves to $\sigma$ invariant field configurations, we project out the geometric tachyon on the G-type D-brane as well as the zero mode of the open string tachyon on the U-type D-brane. However, for sufficiently large radius of the transverse circle the lowest lying $\sigma$ invariant mode of the open string tachyon on the U-type D-brane, satisfying $T(y) = -T(-y)$, is also tachyonic, and its condensation produces a Dp-brane at $y = \pi R$ and a $\bar{\text{D}}$p-brane at $y = 0$ [14]. Since the $\bar{\text{D}}$p-brane at $y = 0$, being on top of the NS 5-branes, has no tension and charge, it is indistinguishible from the tachyon vacuum. Thus the resulting configuration is just a Dp-brane at $y = \pi R$, i.e. the G-type Dp-brane. This shows that both G and U-type D-branes can be regarded as different classical solutions in a single theory – the world-volume theory of the U-type brane. If we reduce the radius, then in the absence of 5-branes the situation is reversed at a critical radius, and the U-type D-brane is obtained as a result of winding tachyon condensation on the G-type brane [14]. We shall see in §4-§6 that the situation changes when NS 5-branes are present.

There is a third type of non-supersymmetric brane configurations with properties very similar to those of the U-type branes described above. Here we consider again a non-BPS D(p+1) brane spanning the coordinates $x^0, \ldots x^p$ and $y$, but this time instead of wrapping the $y$ circle it begins at one of the $k$ NS5-branes, goes around the circle, and ends on another NS5-brane. We shall call these S-type D-branes to indicate that they are non-BPS D-branes **s**tretched between the NS 5-branes. When all the 5-branes are coincident this configuration has the same tension and the tachyon mass$^2$ as the original configuration; however this is no longer the case if we separate the five-branes. For example if we take a pair of 5-branes at $y = y_1$ and $y = 2\pi R - y_2$ respectively, then a non-BPS D(p+1)-brane stretching between these five branes will have tension and tachyon mass$^2$ given by

$$\tau_p'' = \frac{1}{\sqrt{2}\pi}\, g^{-1}\, \mathcal{T}_p\, (2\pi R - y_1 - y_2), \qquad m_T''^2 = -\frac{1}{2}\,. \tag{2.14}$$

The tension coincides with $\tau_p'$ given in (2.13) for $y_1 = y_2 = 0$ but not otherwise. In particular if we take $y_1 = y_2 = \pi R$ then the D-brane will have vanishing tension. Another difference between this D-brane and the U-type D-brane considered earlier is that the latter can be moved away from $\vec{r} = 0$ at no cost in energy, but that is not the case for the new system since the 5-branes are located at $\vec{r} = 0$. An argument similar to that for the U-type D-branes shows us that the tension and the tachyon mass formulæ for the S-type D-brane also do not receive any $\alpha'$ corrections. An $x^p$-dependent tachyonic kink on this D-brane localized at $x^p = 0$ will produce a BPS Dp-brane along $x^0, \ldots x^{p-1}, y$, ending on the two NS 5-branes at the two ends. On the other hand a $y$-dependent tachyonic kink localized at $y = \pi R$ will produce a BPS Dp-brane at $y = \pi R$ lying along $x^0, \ldots x^p$, i.e. a G-type unstable Dp-brane configuration.

# 3 Dual Description in ALF Spaces

We now consider a different description of the system related to the one given above by a T-duality transformation along the circle along $y$. This maps the closed string backgound involving the NS 5-branes to a configuration in type IIB/IIA theory of $k$ coincident Kaluza-Klein monopoles, or equivalently ALF space [26] with $A_{k-1}$ singularity at the origin [12, 13]. The dilaton and the metric associated with this background is given by

$$\begin{aligned} ds^2 &= \left(1 + \frac{k\widetilde{R}}{2r}\right)\left(dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2)\right) + \widetilde{R}^2 \left(1 + \frac{k\widetilde{R}}{2r}\right)^{-1} \left(d\psi + \frac{k}{2}\cos\theta d\phi\right)^2 \\ e^{2\Phi} &= \widetilde{g}^2 , \end{aligned} \tag{3.1}$$

with the identifications:

$$(\theta, \phi, \psi) \equiv (2\pi - \theta, \phi + \pi, \psi + \frac{k\pi}{2}) \equiv (\theta + 2\pi, \phi + 2\pi, \psi) \equiv (\theta, \phi + 2\pi, \psi + k\pi) \equiv (\theta, \phi, \psi + 2\pi) . \tag{3.2}$$

Here

$$\widetilde{R} = 1/R, \qquad \widetilde{g}^2 = g^2/R^2 . \tag{3.3}$$

From the point of view of an asymptotic observer $(r, \theta, \phi)$ are the polar coordinates with origin at the location of the monopoles and $\psi$ is the coordinate along the compact direction. For $k = 1$ the geometry is smooth but for $k > 1$ there is an $A_{k-1}$ singularity at the origin $r = 0$. This is best seen by introducing the cartesian coordinate system for the metric near the origin:

$$\begin{aligned} w^1 &= 2\sqrt{r}\cos\frac{\theta}{2}\cos\left(\frac{\psi}{k} + \frac{\phi}{2}\right), \qquad w^2 = 2\sqrt{r}\cos\frac{\theta}{2}\sin\left(\frac{\psi}{k} + \frac{\phi}{2}\right), \\ w^3 &= 2\sqrt{r}\sin\frac{\theta}{2}\cos\left(\frac{\psi}{k} - \frac{\phi}{2}\right), \qquad w^4 = 2\sqrt{r}\sin\frac{\theta}{2}\sin\left(\frac{\psi}{k} - \frac{\phi}{2}\right) . \end{aligned} \tag{3.4}$$

For $k = 1$ the coordinates $\{w^a\}$ are invariant under the identification (3.2), but for $k > 1$ there is an identification under a $\mathbb{Z}_k$ rotation in the $w^1$-$w^2$ and $w^3$-$w^4$ plane.

It will be useful to examine the action of the transformation (2.4) on the dual geometry. This can be figured out by examining its action at large $|\vec{r}|$ where the $(\vec{r}, y)$ space looks like $\mathbb{R}^3 \times S^1$.

Under a T-duality that takes the circle labelled by $y$ to its dual circle labelled by $\psi$, the image of (2.4) is known to be given by $(-1)^{F_L}$ accompanied by $\vec{r} \to -\vec{r}$, $\psi \to -\psi$ transformation. In terms of $(r, \theta, \phi)$ coordinates this translates to

$$(-1)^{F_L} \times \{(r, \theta, \phi, \psi) \to (r, \pi - \theta, \phi + \pi, -\psi)\} \ . \tag{3.5}$$

The solution described in (3.1) is actually a special limit of a general class of non-singular solutions given by

$$\begin{aligned} ds^2 &= U(\vec{r}) \left(dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2)\right) + \widetilde{R}^2 U(\vec{r})^{-1} \left(d\psi + \frac{1}{\widetilde{R}}\vec{\omega}.d\vec{r}\right)^2 \\ e^{2\Phi} &= \widetilde{g}^2 \,, \end{aligned} \tag{3.6}$$

where

$$U(\vec{r}) = 1 + \frac{\widetilde{R}}{2} \sum_{i=1}^{k} \frac{1}{|\vec{r} - \vec{r}_i|}, \qquad \vec{\nabla} \times \vec{\omega} = \vec{\nabla} U \,. \tag{3.7}$$

This space is completely non-singular; the apparent singulaities at $\vec{r} = \vec{r}_i$ being coordinate singularities. There are various non-contractible two cycles described by taking the direct product of the circle labelled by $\psi$ and the straight line in $\vec{r}$ space joining $\vec{r}_i$ and $\vec{r}_j$. Away from $\vec{r}_i$ and $\vec{r}_j$ the resulting two dimensional surface looks like a cylinder but the circle labelled by $\psi$ collapses at the end-points $\vec{r}_i$ and $\vec{r}_j$ giving this surface the topology of a sphere. In the limit $\vec{r}_i \to \vec{r}_j$ the 2-cycle passing through $\vec{r}_i$ and $\vec{r}_j$ collapses and the space becomes singular. Explicit discussion on D-branes wrapped on various 2-cycles of this geometry can be found in [27].

This geometry described in (3.6) is dual to the configuration of NS5-branes described in eq.(2.5) with all the $y_i$'s set to zero. Non-vanishing $y_i$'s correspond to switching on flux of NS sector 2-form field B through various 2-cycles of this geometry [12, 13].

We shall now describe the various unstable D-brane configurations introduced in the previous section in this T-dual background. We begin with the G-type unstable D-branes obtained in the original description by placing a BPS Dp-brane along $x^0, \ldots x^p$ at $(\vec{r} = 0, y = 0)$ or $(\vec{r} = 0, y = \pi R)$. Since T-duality acting on a D-brane localized at a point on a circle maps it to a D-brane wrapped along the dual circle, we expect that the T-dual description of the G-type brane is a BPS D(p+1) brane along $x^0, \ldots x^p$ and $\psi$, placed at $\vec{r} = 0$.[7] The coordinate $y$ of the original Dp-brane

[7]We must caution the reader that this heuristic picture should be used with caution; $\alpha'$ corrections necessarily spread out the D-brane wave-function and hence the D-brane boundary states are not strictly localized either in the original description or in the new description. However when all the $|\vec{r}_i|$'s are large and hence space-time near $\vec{r} = 0$ is nearly flat, this picture becomes accurate.

corresponds to the Wilson line along $\psi$ on the dual D(p+1)-brane. In order to test this we begin with a configuration where in the original description we move the NS 5-branes far away from the origin in a symmetric fashion so that $(\vec{0},0)$ and $(\vec{0},\pi R)$ remain extrema of the potential. In this case from (2.9) we see that the tension of the Dp-brane is given by

$$g^{-1}\,\mathcal{T}_p\left(1+\sum_{i=1}^{k}\frac{1}{2R|\vec{r}_i|}\right)^{-1/2}+\mathcal{O}\left(e^{-|\vec{r}_i|/R}\right)\,. \tag{3.8}$$

On the other hand in the dual description, the mass per unit $p$-volume of a D(p+1)-brane wrapped along the $\psi$ circle at $\vec{r}=0$ is obtained by integrating the tension of the D(p+1)-brane along the $\psi$ circle:

$$\widetilde{g}^{-1}\frac{\mathcal{T}_p}{2\pi}\int d\psi\,\widetilde{R}\,U(\vec{r})^{-1/2}=g^{-1}\,\mathcal{T}_p\left(1+\sum_{i=1}^{k}\frac{1}{2R|\vec{r}_i|}\right)^{-1/2}\,. \tag{3.9}$$

This agrees with (3.8) up to exponentially suppressed terms. At the level of the supergravity approximation we do not see a potential for the Wilson line or the exponentially suppressed terms in (3.8); however these are expected to be induced by the world-sheet instanton corrections [28–30]. Physically the $\psi$-circle at $\vec{r}=\vec{0}$ represents the equator of a blown up 2-cycle. Thus we see that the T-dual description of the G-type Dp-brane is a BPS D(p+1)-brane wrapped along the equator of a 2-cycle.

Once we have made the identification for large $|\vec{r}_i|$ we can now take the $\vec{r}_i\to 0$ limit on both sides. Let us focus on the $y=\pi R$ case. In the original description this gives a BPS Dp-brane of IIA/IIB placed diametrically opposite to a set of $k$ coincident NS 5-branes on a transverse circle. In the T-dual description the circle at $\vec{r}=0$ labelled by $\psi$ collapses to a point in this limit and we get a BPS D(p+1)-brane of IIB/IIA wrapped along this collapsed circle, but carrying half a unit of Wilson line along this circle. According to (3.9) the tension of this brane vanishes in this limit. This result however is likely to be $\alpha'$ corrected for all $R$ since the curvature at $\vec{r}=0$ is strong. For small $\widetilde{R}$ or equivalently large $R$ we can trust the computation of the brane tension in the original geometry, and by exploiting the duality invariance we come to the conclusion that the tension of this brane remains finite and is given by (2.8) in this limit if we switch on half a unit of Wilson line on the brane along the $\psi$ direction. On the other hand if the Wilson line is zero then it corresponds to placing the Dp-brane at $(\vec{r},y)=(\vec{0},0)$, i.e. at the location of the NS 5-branes, in the original description. Thus the tension of the brane vanishes in this case.

The $k=1$ case deserves special attention. In this case there is a single NS 5-brane in the original description, and we cannot move the NS brane away from $(\vec{0},0)$ keeping $(\vec{0},\pi R)$ a point of unstable equilibrium. Nevertheless by using the symmetry of the problem we could interpret the dual of the BPS Dp-brane placed at $(\vec{r}, y) = (\vec{0}, \pi R)$ as a BPS D(p+1)-brane wrapped along the $\psi$ circle at $\vec{r} = 0$ with Wilson line along the $\psi$ direction. Again for sufficiently small $\widetilde{R}$, i.e. large $R$, this D-brane has finite tension given by (2.8) with $k = 1$. This is somewhat surprising considering that the $\psi$ circle collapses at $\vec{r} = 0$ and Taub-NUT space has no singularity at $\vec{r} = 0$. We shall return to this case in §6.

The non-BPS D(p+1)-brane of the original theory along $x^0, \ldots x^p$, $y$ placed at $\vec{r}$ are easy to describe in the dual theory. This goes over to a non-BPS Dp-brane along $x^0, \ldots x^p$ in the dual system placed at fixed values of $\vec{r}$ and $\psi$ in the ALF space, with the location along $\psi$ determined by the Wilson line along $y$ of the original system. In particular the original D(p+1) brane placed at $\vec{r} = 0$ corresponds to a non-BPS Dp-brane placed at $\vec{r} = 0$ in the dual system.

Finally we turn to the S-type D-brane, extending from the $i$th to the $j$th NS 5-brane in the original description. Again the simplest way to find their dual description is to first move the NS 5-branes away from $(\vec{r}, y) = (\vec{0}, 0)$ to positions $(\vec{r}_i, 0)$. In this process the non-BPS D(p+1)-brane extending from the $i$th to the $j$th NS 5-brane gets stretched between the points $(\vec{r}_i, 0)$ and $(\vec{r}_j, 2\pi R)$. Had the $y$ coordinate been zero at both the end points the D(p+1)-brane would have been fully localized in the $y$-direction and a T-duality transformation would have delocalized it along the dual $\psi$ direction, producing a non-BPS D(p+2)-brane wrapped around the 2-cycle passing through $\vec{r}_i$ and $\vec{r}_j$. By following the standard rules of T-duality transformation one can show that the effect of one unit of winding of the original D(p+1)-brane along the $y$ direction is to produce one unit of gauge field strength flux on the dual D(p+2)-brane through this 2-cycle. Thus the S-type brane corresponds in this dual description to non-BPS D(p+2)-brane wrapped on the two cycle passing through $\vec{r}_i$ and $\vec{r}_j$, with one unit of gauge field strength flux turned on through this 2-cycle. When the 5-branes in the original description are coincident, the 2-cycle collapses to zero size with vanishing flux of the $B$ field through them. In this case the D(p+2)-brane would have vanishing tension if it did not have a gauge field flux turned on. The gauge field flux however makes this into a non-BPS Dp-brane. Separating the NS 5-branes in the original description along the $y$ direction corresponds to switching on $B$-flux through appropriate 2-cycles in the dual description. If we consider the special situation where the $i$th and the $j$th NS 5-branes are brought at $y = \pi R$, it corresponds in the dual description to one unit of $B$-flux through the corresponding 2-cycle. In this bulk this is equivalent to having no flux, and hence vanishing cycles, but its effect on the D(p+2)-brane is to switch off the flux of the gauge field strength. As a result the D-brane would now really have vanishing tension, in agreement with what happens in the original description in terms of NS 5-branes.

# 4 Small Radius Limit and the Fate of the Descent Relations

Let us now take the limit [5]

$$R \to 0, \qquad g \to 0, \quad \widetilde{g} \equiv \frac{g}{R} \quad \text{fixed}\,, \tag{4.1}$$

and define new coordinate

$$\widetilde{y} = \frac{y}{R}, \qquad \vec{\widetilde{r}} = \frac{\vec{r}}{R}\,, \tag{4.2}$$

in the original theory. In this limit eqs.(2.1), (2.3) take the form

$$ds^2 = \eta_{\mu\nu}\, dx^\mu dx^\nu + h(\vec{\widetilde{r}}, \widetilde{y})(d\widetilde{y}^2 + d\vec{\widetilde{r}}^{\,2})\,, \qquad e^{2\Phi} = \widetilde{g}^2\, h(\vec{\widetilde{r}}, \widetilde{y})\,, \tag{4.3}$$

$$h(\vec{\widetilde{r}}, \widetilde{y}) = \frac{k}{2\widetilde{r}} \frac{\sinh \widetilde{r}}{\cosh \widetilde{r} - \cos \widetilde{y}}\,, \qquad \widetilde{r} \equiv |\vec{\widetilde{r}}|\,. \tag{4.4}$$

In this coordinate system the BPS Dp-brane in unstable equilibrium is situated at $\vec{\widetilde{r}} = 0$, $\widetilde{y} = \pi$. The formulæ (2.8) for the tension and the tachyon mass$^2$ of this G-type brane take the form [5]:

$$\tau_p = \frac{2}{\sqrt{k}}\, \widetilde{g}^{-1}\, \mathcal{T}_p\,, \qquad m_T^2 = -\frac{1}{k}\,. \tag{4.5}$$

On the other hand in this limit eq.(2.13), describing the tension and tachyon mass$^2$ of the U-type brane, – a D(p+1)-brane wrapped along $\widetilde{y}$, – reduces to

$$\tau_p' = \sqrt{2}\, \widetilde{g}^{-1}\, \mathcal{T}_p\,, \qquad m_T'^2 = -\frac{1}{2}\,. \tag{4.6}$$

Finally the dual ALF space itself reduces to $\mathbb{C}^2/\,\mathbb{Z}_k$ in this limit since the size $\widetilde{R} = 1/R$ of the space goes to $\infty$. The string coupling in this dual theory is given by $g/R = \widetilde{g}$.

Again (4.5) can receive $\alpha'$ corrections. For finite $k$ these corrections can be of order unity. However for large $k$ eq.(4.5) still remains a valid approximation. To see this let us define new coordinates

$$\widehat{y} = \sqrt{k}\, \widetilde{y}, \qquad \vec{\widehat{r}} = \sqrt{k}\, \vec{\widetilde{r}}\,, \tag{4.7}$$

and express (4.3), (4.4) as

$$ds^2 = \eta_{\mu\nu}\, dx^\mu dx^\nu + \widehat{h}(\vec{\widehat{r}}, \widehat{y})(d\widehat{y}^2 + d\vec{\widehat{r}}^{\,2})\,, \qquad e^{2\Phi} = \widetilde{g}^2\, k\, \widehat{h}(\vec{\widehat{r}}, \widehat{y})\,, \tag{4.8}$$

$$\widehat{h}(\vec{\widehat{r}}, \widehat{y}) \equiv \frac{\sqrt{k}}{2\,\widehat{r}} \frac{\sinh(\widehat{r}/\sqrt{k})}{\cosh(\widehat{r}/\sqrt{k}) - \cos(\widehat{y}/\sqrt{k})}\,, \qquad \widehat{r} \equiv |\vec{\widehat{r}}|\,. \tag{4.9}$$

From this we see that for large $k$ the function $\widehat{h}$ is a slowly varying function of $\vec{\widehat{r}}$ and $\widehat{y}$ near $\widehat{r}=0$, $\widehat{y}=\pi\sqrt{k}$. Hence we expect the higher derivative corrections to the solution to be small in this region and the results given in (4.5) should be reliable.

Given that eq.(4.5) can be trusted for large $k$, we can now compare it with (4.6). We see that the tension of the G-type brane is less than that of the U-type brane, and hence ($\sigma$ invariant) tachyon condensation on the G-type brane cannot take us to the U-type brane. This is qualitatively different from what happens in the absence of NS 5-branes; in that case below a critical radius the G-type brane becomes heavier than the U-type brane, and winding tachyon condensation on the former takes it to the U-type brane. In fact in this case we expect the reverse to be true, namely the large radius result that $\sigma$ invariant tachyon condensation on the $y$ wrapped non-BPS D(p+1)-brane takes us to a BPS Dp-brane placed at $y=\pi R$ (plus an anti Dp-brane with vanishing tension at $y=0$) should continue to hold all the way to the small $R$ region. For this we require a $\sigma$ invariant tachyonic mode on the $y$-wrapped D(p+1)-brane. This will happen if the total length of the $y$-circle, measured in the string metric, is large compared to the string length so that the usual tachyonic kink solution satisfying $T(y)=-T(-y)$ still exists on this circle. We see from the metric (4.3) or (4.8) that this is the case for large $k$.[8]

One can give a physical picture of this situation as follows. For small $R$ and large $|\vec{r}|$ where the effect of the NS 5-branes is small, a Dp-brane $\bar{\text{D}}$p-brane pair placed at $y=\pi R$ and $y=0$ respectively has higher tension than a D(p+1)-brane wrapped along $y$, and the former can decay into the latter via tachyon condensation. This is the usual situation in the absence of NS 5-branes. As we approach the NS 5-branes by reducing $|\vec{r}|$,, the tension of the latter remains constant, while the total tension of the former configuration decreases due to the decrease in the value of $e^{\Phi}$. At a certain critical distance away from the NS 5-branes the former configuration will have less tension than the latter configuration, and the D(p+1)-brane wrapped along $y$ becomes unstable against possible decay into a Dp-brane $\bar{\text{D}}$p-brane pair placed at $y=\pi R$ and $y=0$.

An interesting question is: what happens for low values of $k$? We shall explore this in §5 and §6.

# 5 Comparison of Different D-branes for Low $k$

So far we have introduced different types of unstable Dp-brane system in the background of $k$ NS 5-branes of type IIA/IIB string theory with a transverse circle, or equivalently in the dual type IIB/IIA

[8]Naively for any $k$ the length is infinite due to the infinitely long throat near $y=0$. However we cannot trust the calculation for finite $k$ due to large $\alpha'$ corrections.

string theory on ALF spaces, and explored how in certain cases tachyon condensation on one type of brane can take us to another type of brane. We now look for possible relationship between these unstable D-branes that does not involve tachyon condensation. Of these the relationship between the U and S type D-branes is easy to comprehend. If we begin with a configuration where the NS 5-branes are separated along the $y$ direction, then the boundary state of the U-type D-brane at $\vec{r}=0$, describing a non-BPS D(p+1)-brane wrapping the $y$ circle, should coincide with the sum of the boundary states of $k$ different S-type branes, each connecting a given NS 5-brane to its immediate neighbour. When the NS 5-branes coincide, $(k-1)$ of the S-type D-branes collapse to zero size in the $y$ direction whereas the remaining D-brane describes a S-type brane stretching all the way around the $y$ circle. The collapsed D-branes correspond to tensionless non-BPS Dp-branes carrying no charge or tension, and up to these tensionless branes, the U and S type branes can be identified in this case. In the dual ALF geometry this would mean that the boundary state of a non-BPS D(p+2)-brane wrapped on a vanishing cycle carrying one unit of gauge field strength flux through the cycle should coincide with that of a non-BPS Dp-brane upto addition of boundary states describing non-BPS tensionless branes.[9]

Unfortunately, comparison between these and the G-type unstable Dp-brane is plagued by the lack of understanding of $\alpha'$ corrections to (2.8), (4.5). First of all the formulæ (2.8) are most likely going to be modified by $\alpha'$ corrections; there does not seem to be any symmetry at finite $R$ that protects these results from $\alpha'$ corrections. This still leaves open the possibility that the zero radius formulæ (4.5), which can be argued to be valid for large $k$, may be exact. In fact this also cannot be strictly true for all $k$; for $k=1$ the eq.(4.5) gives a tachyon mass$^2$ less than $-1/2$, requiring a negative dimension matter sector operator for the construction of the corresponding vertex operator. Since this is not possible in a unitary theory, we expect (4.5) to be modified for $k=1$. In §6 we shall use the description in terms of tachyon effective potential to suggest a mechanism that could modify the result for $k=1$ without modifying the results for $k\geq 2$. This will essentially involve the G-type D-brane for $k=1$ undergoing a second order phase transition at a finite radius, below which it gets identified with the U-type D-brane.

If we do assume that eqs.(4.5) do not get modified under $\alpha'$ corrections we find that for $k=2$ the tachyon mass$^2$ given in (4.5) and (4.6) are identical [5] and furthermore the tensions given in (4.5) and (4.6) also agree. This is an unexpected result since both in the original geometry and in the dual

[9]Since a CFT whose target space has $A_{k-1}$ singularity is singular one may worry about the meaning of boundary state in this context. We can however make sense of these statements by beginning at the orbifold point where half unit of B-flux is switched on through the various cycles [31, 32] and then considering the limit where these fluxes are turned down to zero.

geometry these D-branes are represented by different kinds of objects. In the original geometry one is a BPS Dp-brane placed at a given point on the circle while the other is a non-BPS D(p+1)-brane spread over the circle. In the dual geometry one is a BPS D(p+1) brane wrapped on the equator of a vanishing 2-cycle, whereas the other can be represented either as a non-BPS Dp-brane or as a non-BPS D(p+2)-brane wrapped on the vanishing 2-cycle with magnetic flux. However since the string world-sheet theory is strongly coupled in this case, neither of these geometric intuitions can be trusted, and if it turns out that there is an underlying non-renormalization theorem for eq(4.5), then it would be a strong indication that the G- and the U- type branes are identical in this case, with the geometric tachyon on one playing the role of the usual open string tachyon on the other. Since for large radius $R$ of the transverse circle the G-type D-brane can be considered as the tachyon kink solution on the U-type brane, the above result can be interpreted as the merger of the tachyon kink solution and the trivial solution describing the original unstable D(p+1)-brane into a single solution at $R = 0$. In §6 we shall discuss what this means for the tachyon effective potential that governs the formation of $y$-dependent tachyon kink on the U-type brane.

The above result, if correct, would identify the usual non-BPS Dp-brane placed in an $A_1$ singular geometry to a BPS Dp-brane placed diametrically across 2 NS 5-branes on a transverse circle in the dual geometry. This may not sound very exciting since we may not care about non-BPS Dp-branes in singular spaces; however recall that the interesting part of dynamics of the open string tachyon describing the tachyon vacuum, the rolling tachyon, or the formation of the tachyon kink along any of the $p$ spatial directions tangential to the Dp-brane is universal and independent of the background geometry of the transverse space in which the Dp-brane is placed [24, 25, 33, 34]. Thus identifying the open string tachyon with the geometric tachyon in this system will give a geometric description of most of the interesting phenomena involving tachyon condensation on a non-BPS Dp-brane in flat space-time background.

Note that even when the tensions associated with two configurations become identical, they could be related by marginal deformation instead of being identical. This is what happens for D-brane systems in the absence of NS 5-branes [2, 14]. However since marginal deformation typically changes the spectrum of open strings on the brane, it would not explain why the tachyon mass$^2$ on the two types of branes agree. In any case existence of a marginal deformation connecting the two configurations is a highly non-generic situation since it requires the tachyon potential to develop a flat direction. It is much more likely for two solutions to merge at a given point in the space of external parameters, as was demonstrated in a closely related example in [15].

It is also instructive to compare the properties of different types of branes when the NS 5-branes are separated from each other. For simplicity we shall consider the case of $k = 2$, with one NS 5-brane at $(\vec{\tilde{r}} = \vec{0}, \tilde{y} = c)$ and the other NS 5-brane at $(\vec{\tilde{r}} = \vec{0}, \tilde{y} = 2\pi - c)$. Since the U-type branes wrap the whole $\tilde{y}$-circle, their properties are not affected by this move and their tensions and tachyon mass$^2$ continue to be given by (4.6). For the S-type brane, i.e. non-BPS D(p+1)-brane stretched between the two five branes, we get from (2.14):

$$\tau''_p = \sqrt{2}\,\tilde{g}^{-1}\,\mathcal{T}_p\left(1 - \frac{c}{\pi}\right), \qquad m''^2_T = -\frac{1}{2}\,. \tag{5.1}$$

On the other hand we can calculate the tension and the tachyon mass$^2$ on a BPS Dp-brane at $\tilde{y} = \pi$ with the help of eqs.(2.5)-(2.7), and get, in the scaling limit (4.1):

$$\tau_p = \sqrt{2}\,\tilde{g}^{-1}\,\mathcal{T}_p\,\cos\frac{c}{2}, \qquad m^2_T = -\frac{1}{2}(2 - \cos c)\,. \tag{5.2}$$

We see from (5.2) that for $c \neq 0$ we have $m^2_T < -\frac{1}{2}$. Since such a tachyon will require a negative dimension matter sector vertex operator this formula certainly needs to be modified. In §6 we shall suggest a mechanism for this modification that is similar to the one that modifies the result for the $k = 1$ case.

One might have hoped that study of D-brane boundary states in a closely related background describing the throat geometry of coincident 5-branes [35, 36] could shed some light on the possible relationship between the G and U/S type branes described here. However as can be seen from (4.3), (4.4), the background describing this system for any given $k$ (say $k = 2$) has no free parameters except the string coupling constant $\tilde{g}$ on which the world-sheet conformal field theory has trivial dependence. Thus we cannot try to analyze this problem with the help of any 'nearby' conformal field theory; we really need to solve the problem exactly in the conformal field theory of interest.

# 6 A Second Order Phase Transition for $k = 1$?

For $k = 1$ we have a single NS 5-brane and there is no distinction between the U and S type branes. Both of them correspond to an unstable Dp-brane sitting at the center of Taub-NUT in the dual description. The G-type branes are however quite mysterious since unlike in the case of multiple NS 5-branes, in this case the dual geometry does not have a 2-cycle and hence we cannot wrap a BPS D(p+1)-brane on the equator of the 2-cycle. Thus these must be some new kind of non-BPS Dp-brane configuration sitting at the center of the Taub-NUT space. A natural question is: what happens to this Dp-brane in the flat space limit?

Since this is an important issue it will be useful to review the steps leading to this question:

1. We consider a single NS 5-brane of type IIA/IIB with a transverse circle, and place a BPS Dp-brane, with its world-volume parallel to that of the NS 5-branes, at a diametrically opposite point on this transverse circle. All other transverse coordinates of the brane are taken to coincide with that of the 5-brane. For large radius of the transverse circle we expect this state to exist since $\alpha'$ corrections are small. Hence it must also have an appropriate description in the T-dual Kaluza-Klein monopole background in type IIB/IIA theory even though this background is highly curved in this regime.

2. We now tune the string coupling and the radius of the circle transverse to the 5-brane to zero keeping their ratio fixed. In the dual description this corresponds to taking the size of the Kaluza-Klein monopole to infinity, keeping the string coupling constant fixed. This gives rise to flat space-time. The BPS Dp-brane parallel to the NS 5-brane in the original description should get mapped to some Dp-brane in type IIB/IIA in flat space-time in the second description. This is the D-brane we want to study.

We can think of several possible scenarios:

1. It describes a genuinely new unstable Dp-brane in flat space-time with finite tension and tachyon mass$^2$.

2. $\alpha'$ correction to (2.8) drives the tension to zero or infinity. In this case the 'new' Dp-brane does not really exist as an independent object in the spectrum.

3. In the flat space limit the brane spreads out over the whole space and does not correspond to a localized Dp-brane with finite tension.

4. The new non-BPS Dp-brane is in fact identical to the usual non-BPS Dp-brane, and the formulæ given in (4.5) are modified for $k = 1$ to those given in (4.6). This amounts to saying that the G and the U-type branes become identical in the $R \to 0$ limit.

It is tempting to speculate that the fourth possibility holds. In that case we shall have a direct geometric interpretation of the open string tachyon on a non-BPS D-brane in flat space-time in terms of the geometric tachyon on a BPS D-brane situated at a point of unstable equilibrium in a dual geometry. We shall now present some observations which indicate that this is a likely possibility.

- Let us express the formulæ for the tension and the tachyon mass$^2$ on this brane given in (2.8) in terms of the variables natural to the dual geometry, i.e. $\widetilde{g}$ and $\widetilde{R}$ given in (3.3). This gives:

$$\tau_p = 2\,\widetilde{g}^{-1}\,\mathcal{T}_p\,(4\widetilde{R}^{-2}+1)^{-1/2}, \qquad m_T^2 = -\frac{1}{(4\widetilde{R}^{-2}+1)^2}\,. \tag{6.1}$$

Thus at small $\widetilde{R}$ where the formula can be trusted, $\tau_p$ and $|m_T^2|$ start at small values but begin increasing as we increase $\widetilde{R}$. Thus it is not inconceivable that as we increase $\widetilde{R}$ to $\infty$, the values of $\tau_p$ and $m_T^2$ approach the tension $\sqrt{2}\,\widetilde{g}^{-1}\,\mathcal{T}_p$ and tachyon mass$^2$ $-1/2$ of a usual non-BPS Dp-brane in flat space-time.

- Since the geometric tachyon on the G-type brane changes sign under the transformation $\sigma$ given in (2.4), the corresponding tachyon in the T-dual description must also change sign under the image of $\sigma$ given in (3.5). The tachyon on the usual non-BPS Dp-brane in Taub-NUT space is also also odd under this transformation. This can be seen either by working in the NS 5-brane background as discussed in §2, or directly in the Taub-NUT geometry due to the presence of the $(-1)^{F_L}$ factor in (3.5). Furthermore, we have also seen that for large $R$, i.e. small $\widetilde{R}$, the G-type brane can be regarded as a $\sigma$ invariant tachyon field configuration on the U-type brane. Thus it is natural to expect that by following this field configuration from small $\widetilde{R}$ to large $\widetilde{R}$ the two types of branes can be related by a $\sigma$ invariant field configuration even for large $\widetilde{R}$.

  Now if we take the usual non-BPS Dp-brane, i.e. the U-type brane in the $\widetilde{R} \to \infty$ limit, then the requirement of $\sigma$ invariance removes the tachyonic mode. Thus all $\sigma$ invariant field configurations on this brane will have higher tension than the tension of the original brane. This would imply that the 'new' Dp-brane must be represented by a classical field configuration on the usual Dp-brane of positive energy density. Furthermore this solution must be invariant under the $p+1$ dimensional Poincare group acting on the Dp-brane world-volume since the 'new' Dp-brane is manifestly invariant under these symmetries. Such a field configuration would essentially require a configuration of constant scalar fields. While we cannot rule out the existence of such solutions, it will certainly be more natural if the 'new' Dp-brane turned out to be the usual non-BPS Dp-brane. Note that this argument is special to the $k=1$ case; for $k \geq 2$ the usual non-BPS Dp-brane is placed in a singular geometry, and there may be additional $\sigma$ invariant tachyonic mode on this brane which could condense and take us to a lower energy density configuration. Indeed we have argued in §4 by working in the NS 5-brane description that at least for large $k$ such $\sigma$ invariant tachyonic modes are present on this brane.

We shall now present a concrete analysis using the language of tachyon effective potential to determine under what condition we can identify the 'new' and the 'usual' non-BPS Dp-branes. Let us begin with a single NS 5-brane with a transverse circle of radius $R$. At large $R$, the G-type D-brane – a Dp-brane placed at $y = \pi R$ – is definitely lighter than the U-type D-brane – a non-BPS D(p+1) brane wrapped along the $y$ direction, and we know that there is a $\sigma$ invariant tachyonic mode on the latter configuration whose condensation produces the former configuration. In fact since the circle size is large we expect many $\sigma$-invariant tachyonic modes. We shall assume that the spectrum is discrete. This may sound unreasonable from the point of view of the NS 5-brane description since the D(p+1)-brane has an infinite length along the $y$-direction due to the infinite throat near the NS 5-brane. However from the point of view of the dual Taub-NUT geometry we have a Dp-brane sitting at the centre of a non-singular geometry, and there is no reason why we should have a continuous spectrum.[10] As we reduce the value of $R$, the modes of the tachyon begin acquiring positive contribution to their mass$^2$, and eventually all $\sigma$-invariant tachyonic modes become massive in the $R \to 0$ limit since in the dual description we have a non-BPS Dp-brane sitting at the origin in flat space time with no $\sigma$-invariant tachyonic mode. We shall begin our analysis in a range of values of $R$ where all but one $\sigma$-invariant tachyonic mode have become massive, and furthermore the magnitude of the mass$^2$ of this remaining tachyon is small compared to the string scale. In this region it should be possible to integrate out all other modes of the tachyon and define a tachyon effective potential $V(\phi)$ for this single mode $\phi$. The U-type brane will correspond to the local maximum of the potential at $\phi = 0$. On the other hand the G-type D-brane ($\bar{\text{D}}$-brane) should be described by some other local extrema at $\pm\phi_0$ of $V(\phi)$ unless they have already merged with the U-type brane by this time. If we reduce the value of $R$ further, then below some critical radius $R_c$ the mode $\phi$ also becomes massive. Our goal will be to explore the fate of the G-type brane during this transition.

$\phi$, being a mode of the open string tachyon on a non-BPS D-brane, is odd under $(-1)^{F_L}$. Thus the effective potential $V(\phi)$ must have $\phi \to -\phi$ symmetry. First let us examine what would happen if $V(\phi)$ were a quartic potential of the form

$$V(\phi) = \frac{1}{2}a(R)\phi^2 + \frac{1}{4}b(R)\phi^4 \,, \quad b(R) > 0 \,. \tag{6.2}$$

Now we know that for $R > R_c$, $a(R)$ must be negative because the field $\phi$ is tachyonic. Hence besides the maximum at $\phi = 0$, (6.2) admits two minimia at $\pm\phi_0$ with

$$\phi_0 = \sqrt{-a(R)/b(R)} \tag{6.3}$$

which we can identify as a BPS Dp-brane or $\bar{\text{D}}$p-brane placed as $y = \pi R$. As we reduce $R$, $a(R)$ vanishes at the critical radius $R_c$. At this radius $\phi_0$ vanishes and the G and U type branes become identical.[11] As we decrease $R$ further $a(R)$ becomes positive and the two solutions continue to be

[10]Nevertheless we must admit that many aspects of this conformal field theory are not understood and there may be subtle effects which invalidate our analysis.

[11]In the absence of NS 5-branes the point where $a(R)$ vanishes the whole potential vanishes and the two types of branes, instead of being identical, are related by a marginal deformation. However this is a highly non-generic situation and we are implicitly assuming that the presence of NS 5-branes turns this into a generic situation [15].

identical. Thus below this critical value of $R$ there is no distinction between the G and U type branes.

Let us now consider the case of a general potential. We shall continue to denote by $a(R)$ and $b(R)$ the coefficients of the quadratic and quartic terms in the potential. Thus again we have $a(R) < 0$ for $R > R_c$ and $a(R) > 0$ for $R < R_c$. It is easy to see that a general potential will produce the same results if the following conditions are satisfied:

1. For $R > R_c$ the Dp-brane / $\bar{\text{D}}$p-brane must correspond to the minima of the potential closest to the origin and this feature should continue all the way to the critical radius where $a(R)$ vanishes. In other words the potential should not have any additional extrema corresponding to new (unstable) D-brane configurations.

2. At the critical radius where $a(R)$ vanishes, $b(R)$ must be positive. For negative $b(R)$, instead of the minima at $\pm\phi_0$ merging with the maxima at 0, there will be new maxima developing around $\phi = 0$ as $R$ goes below the critical radius.

We see that if either of the above conditions is violated then there will be new unstable D-branes in the spectrum of the theory. Thus the most economical solution is to have the minima at $\pm\phi_0$ merge with the maximum at $\phi = 0$ at the critical point.

If the picture described above is correct then it would seem that certain discrete symmetry associated with $\phi \to -\phi$ transformation, which was broken at the vacua $\phi = \pm\phi_0$, is being restored below the critical value of $R$. Can we identify this symmetry? In fact this is just the $(-1)^{F_L}$ symmetry. Above the critical value of $R$ the non-BPS D(p+1)-brane wrapped along $y$, represented by the solution $\phi = 0$, is $(-1)^{F_L}$ invariant, but neither a BPS Dp nor a BPS $\bar{\text{D}}$p-brane situated at the point $y = \pi R$, represented by the solutions $\phi = \pm\phi_0$, is invariant under $(-1)^{F_L}$. If the picture described above is correct, then it would imply that below the critical value of $R$ a BPS Dp and a BPS $\bar{\text{D}}$p-brane situated at the point $y = \pi R$ describe identical configurations and become $(-1)^{F_L}$ invariant. This in particular will imply that the Ramond-Ramond part of the boundary state describing the G-type branes should vanish below the critical value of $R$.

Note that this analysis also applies to other values of $k$ except that the potential will have different behaviour in those cases.[12] For large $k$ the analysis of §4 shows that $a(R)$ remains negative as $R \to 0$ and hence the solutions describing the two types of Dp-branes remain distinct. If we believe that eqs.(4.5) are not renormalized then this should continue to hold till $k = 3$, so that the G-type brane

[12]Since for two or more coincident NS 5-branes the dual geometry is singular, we cannot apply the previous argument for the existence of an effective potential. However one can carry out the analysis by first separating the NS 5-branes in a $\sigma$-symmetric fashion and at the end of the analysis take the coincident limit.

always remains lighter than the U-type brane. On the other hand for $k = 2$, $a(R)$ should vanish precisely at $R = 0$ so that the two types of branes become identical at that point.

This analysis also suggests a mechanism that would modify the results (4.5) for $k = 1$ without modifying them for $k \geq 2$. This happens essentially because for $k = 1$ the G-type D-brane undergoes a second order phase transition at the critical radius $R_c$. For $R < R_c$ these D-branes merge with the U-type D-branes and hence the relevant formulæ to use are those in (4.6). A naive analytic continuation of the results from the $R > R_c$ region will give us the corresponding quantities for the unphysical (complex) solutions. The same reasoning should apply for the unphysical answers gotten in (5.2) for the G-type D-brane in the presence of a pair of separated NS 5-branes. Since at zero separation $c$ the branches describing the G and U/S type D-branes meet exactly at $R = 0$, it is quite likely that for non-zero $c$ the branches will meet at non-zero $R$ and the G-type D-brane will encounter a second order phase transition where it merges with the S-type D-brane. Below this critical radius the relevant formulæ are those given in (5.1). Thus in the $(R, c)$ plane there will be a line of second order fixed points passing through the $(R = 0, c = 0)$ point. In fact this phase transition is not disconnected from the one we encountered in the $k = 1$ case. For $c = \pi/2$ the configuration of the pair of NS 5-branes is just the double cover of the $k = 1$ configuration, and the G and S type D-branes sees background identical to the G and U/S type D-branes in the $k = 1$ case. Thus at $c = \pi/2$ the line of critical point in the $(R, c)$ plane should reach the $k = 1$ critical point $R = 2R_c$, the factor of 2 accounting for the fact that the configuration for $k = 2$ is the double cover of the configuration for $k = 1$. The same reasoning can also be applied to the case of $k$ NS 5-branes. In this case we should have a codimension one critical surface in the full moduli space labelled by the radius $R$ and the possible separation of the NS 5-branes maintaining the $\sigma$ symmetry. This critical curve does not pass through any physical value of $R$ when all the branes are coincident, but passes through the point $R = kR_c$ when the $k$ NS 5-branes are situated as equal intervals $2\pi R/k$ along the transverse circle.

If the non-renormalization theorems are violated, then the precise details of the critical surface will be different from the one given above. However we would expect the general features to remain, assuming that the quartic term in the tachyon effective potential for $k = 1$ has the correct sign. In this case we can start at the critical point corresponding to $R = kR_c$ and equally separated NS 5-branes, and follow its fate in the moduli space of NS 5-brane configurations.

If it turns out that the non-renormalization theorems do hold, and the identification of the non-BPS Dp-branes with geometrically unstable Dp-branes in the T-dual theory hold both for two coincident NS 5-branes as well as a single NS 5-brane at $R = 0$, then we have two possible description of the open string tachyon on a non-BPS D-brane as a geometric tachyon. Which one is more useful? Our first thought may be that the $k = 1$ case is more useful because it lands us directly on non-BPS Dp-branes in flat space-time. The reverse however is true. For the $k = 1$ case we have seen that even if our picture of branch merger holds, there is a switch of branch for the geometrically unstable D-branes at the critical radius. In particular if we had made a naive analytic continuation of various physical quantities of this system to values of $R$ below the critical value, we would get the wrong answer because it will land us into the wrong branch of (possibly complex) solutions which do not correspond to any physical D-brane. On the other hand if for $k = 2$ the two branches precisely meet at $R = 0$, then all the universal properties of the tachyon on the non-BPS Dp-brane can be derived by analytic continuation of the results for the geometrically unstable Dp-brane to $R = 0$.

One aspect of this correspondence may seem puzzling. One might wonder how the boundary state of a Dp-brane that is localized at $y = \pi R$ could coincide with that of a D(p+1)-brane that spreads out along the $y$ circle. This however is not a serious problem since the tidal forces on the Dp-brane due to the $y$ dependent dilaton will tend to spread out the boundary state away from $y = 0$. Indeed this has been observed even in simpler cases of hairpin brane boundary state [37, 38] where we have a linear dilaton background.

# 7 Discussion

In this paper we have argued that under certain conditions a BPS D-brane with geometric instability due to being placed at a saddle point of the potential may be identified with a non-BPS D-brane with the usual tachyonic instability. This would give a geometric interpretation of the usual open string tachyons. It will be interesting to see if this geometric picture can provide some insight into the analytic solutions of string field equations describing the tachyon vacuum and various solitons, – a problem whose bosonic counterpart has only been solved recently [39–47]. Another aspect of tachyon condensation where the present analysis may throw some light is vacuum string field theory [48], – string field theory around the tachyon vacuum. For a geometrically unstable D-brane the tachyon vacuum represents a D-brane sitting at the core of an NS 5-brane. This may provide some insight into the nature of the open string tachyon vacuum.

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