AdS_3/CFT_2 to AdS_2/CFT_1

Rajesh Kumar Gupta and Ashoke Sen

Harish-Chandra Research Institute
Chhatnag Road, Jhusi, Allahabad 211019, INDIA

E-mail: rajesh@mri.ernet.in, sen@mri.ernet.in, ashokesen1999@gmail.com

Abstract

It has been suggested that the quantum generalization of the Wald entropy for an extremal black hole is the logarithm of the ground state degeneracy of a dual quantum mechanics in a fixed charge sector. We test this proposal for supersymmetric extremal BTZ black holes for which there is an independent definition of the quantum entropy as the logarithm of the degeneracy of appropriate states in the dual 1+1 dimensional superconformal field theory. We find that the two proposals agree. This analysis also suggests a possible route to deriving the OSV conjecture.

Wald's formula for black hole entropy [1,2,3,4], when applied to extremal black holes, leads to the entropy function formalism [5,6]. Since extremal black holes have an AdS_2 factor in their near horizon geometry [7,8], one expects that the underlying quantum gravity theory in this background will have a dual description in terms of a conformal quantum mechanics (CQM) living at the boundary of AdS_2 [9,10,11,12,13,14,15,16,17,18,19,20]. In [19] it was shown that in the classical limit, when Wald's formula is expected to be valid, the Wald entropy computed from the entropy function can be interpreted as the logarithm of the ground state degeneracy of this dual CQM in a fixed charge sector. This suggested that the latter should be taken as the definition of the entropy of extremal black holes in the full quantum theory.

In this paper we shall test this proposal for a special class of black holes, – the BTZ black holes [21]. The latter are rotating black hole solutions in AdS_3 characterized by their mass M and angular momentum J. We shall assume that the BTZ black hole solution has been embedded in a string theory with certain amount of supersymmetry where we have sufficient control on the system [22, 23]. In particular in this case via AdS_3/CFT_2 correspondence [24,25,26,27] one can identify the BTZ black holes as states in the superconformal field theory (CFT) living on the boundary of AdS_3, with the identification[1]

$$L_0 = \frac{M+J}{2}, \qquad \bar{L}_0 = \frac{M-J}{2}\,. \tag{1}$$

Extremal supersymmetric BTZ black holes, corresponding to $M = \pm J$, correspond to states with $\bar{L}_0 = 0$ and $L_0 = 0$ respectively. For definiteness we shall consider black holes with $M = J$, i.e. with $\bar{L}_0 = 0$. In order that the state preserves supersymmetry it must belong to the Ramond sector of the anti-holomorphic part of the superconformal algebra of the CFT, so that the condition $\bar{L}_0 = 0$ forces the state to be in the supersymmetric ground state of the Ramond sector [28, 23, 29].

The identification of the BTZ black hole with a state in the dual CFT suggests a natural definition of the entropy of this black hole, – it is simply the logarithm of the degeneracy of the corresponding states in the CFT [23]. For large L_0 where we can use Cardy formula to estimate the degeneracy of states, the entropy defined this way agrees with the one computed via Wald's formula [30, 31, 32, 33]. Our goal will be to compare the definition of the quantum entropy of the black hole based on the degeneracies in the dual CFT with the one suggested by the AdS_2/CFT_1 correspondence, where we identify the entropy as the logarithm of the

[1] L_0 and $\bar{L}_0$ denote the Virasoro generators on the cylinder; thus in their definition we include the contributions $-c/24$ and $-\bar{c}/24$ of the central charges.

degeneracy of certain states in the dual CQM. Thus for this comparison we need to study the relationship between the CQM and the CFT. The comparison is not completely straightforward since the CFT lives on the boundary of the AdS_3 space in which the black hole is embedded, whereas the CQM lives on the boundary of AdS_2 that appears in the near horizon geometry of the black hole.

The general BTZ black hole solution in an AdS_3 space with scalar curvature $-6/l^2$ is given by

$$ds_3^2 = -\frac{(\rho^2-\rho_+^2)(\rho^2-\rho_-^2)}{l^2\rho^2}d\tau^2 + \frac{l^2\rho^2}{(\rho^2-\rho_+^2)(\rho^2-\rho_-^2)}d\rho^2 + \rho^2\left(dy - \frac{\rho_+\rho_-}{l\rho^2}d\tau\right)^2 , \tag{2}$$

where τ denotes the time coordinate, ρ is the radial variable, y is the azimuthal angle with period 2π and $\rho_\pm$ are parameters labelling the black hole solution satisfying $\rho_+ > \rho_-$. M and J are determined in terms of $\rho_\pm$, but the precise relation requires the knowledge of higher derivative terms. Nevertheless the extremal limit always corresponds to $\rho_+ \to \rho_-$. Following [19] we take this limit by first defining new variables λ, t, r, ϕ and R through

$$\rho_+ - \rho_- = 2\lambda, \quad \rho - \rho_+ = \lambda(r-1), \quad \tau = l^2\, t/(4\lambda), \quad y = \phi + \frac{l}{4\lambda}\left(1 - \frac{2\lambda}{\rho_+}\right)t, \quad \rho^+ = \frac{lR}{2}, \tag{3}$$

and then taking $\lambda \to 0$ with t, r, ϕ and R fixed. In this limit the metric (2) takes the form

$$ds_3^2 = \frac{l^2}{4}\left[-(r^2-1)dt^2 + \frac{dr^2}{r^2-1} + R^2\left(d\phi + \frac{1}{R}(r-1)dt\right)^2\right] . \tag{4}$$

The metric (4) is locally AdS_3. Thus by the standard rules of AdS/CFT correspondence any quantum theory of gravity in the background (4) has a dual (1+1) dimensional conformal field theory. Since locally this AdS_3 space is the same as the one in which we embed the BTZ black hole, we expect that as a local field theory the (1+1) dimensional CFT living on the boundary of the near horizon geometry of the BTZ black hole must be identical to that living on the boundary of the AdS_3 in which the full BTZ black hole solution is embedded. The conformal structure of the two dimensional space in which the theory lives will however be quite different for the theory dual to AdS_3 and the one dual to the near horizon geometry of the black hole.

Now via a dimensional reduction we can also regard the three dimensional metric (4) as a two dimensional field configuration [9,34]. For this we introduce a two dimensional metric ds_2^2, a scalar field χ and a gauge field a_μ via the relation:

$$ds_3^2 = ds_2^2 + \chi\,(d\phi + a_\mu dx^\mu)^2 , \tag{5}$$

where $\{x^\mu\}$ for $\mu = 0, 1$ represent the two dimensional coordinates (t, r). From the two dimensional viewpoint, the background (4) takes the form

$$ds_2^2 = \frac{l^2}{4}\left[-(r^2-1)dt^2 + \frac{dr^2}{r^2-1}\right], \quad \chi = \frac{l^2\,R^2}{4}, \quad a_\mu dx^\mu = \frac{1}{R}(r-1)dt\,. \tag{6}$$

$$e \equiv F_{rt} = 1/R\,. \tag{7}$$

This describes an AdS_2 space-time with background scalar and electric field. Then via the rules of AdS/CFT correspondence the theory is dual to a CQM living on the boundary of AdS_2. In particular we can relate the partition function of the quantum gravity theory on AdS_2 to the partition function of the CQM living on the boundary of AdS_2 [19].

Since (4) and (6) describe the same background, the quantum theories dual to them must also be identical. Consequently the CQM living on the boundary of (6) and the (1+1) dimensional CFT living on the boundary of (4) are also different descriptions of the same quantum theory. Our goal will be to exploit this equivalence to learn about the CQM living on the boundary of AdS_2.

First consider the two dimensional viewpoint. The metric is that of AdS_2, and the boundary is located at $r = r_0$. The induced metric, scalar and gauge field on the boundary are

$$ds_B^2 = -\frac{l^2}{4}(r_0^2-1)dt^2, \qquad \chi_B = \frac{l^2R^2}{4}, \qquad a_t|_B = \frac{1}{R}(r_0-1)\,. \tag{8}$$

We shall denote by H_t the total Hamiltonian of the CQM living on the boundary of AdS_2 including the effect of the background gauge fields and by Q the conserved charge in the CQM conjugate to the gauge field a_μ in the bulk.[2]

We now turn to the three dimensional viewpoint. The dual (1+1) dimensional CFT lives on the two dimensional boundary labelled by (t, ϕ) with induced metric

$$ds_B^2 = \frac{l^2}{4}\left[-(r_0^2-1)dt^2 + R^2\left(d\phi + \frac{1}{R}(r_0-1)dt\right)^2\right]\,. \tag{9}$$

[2]In the analysis of [19] the Hamiltonian was split into two parts, one due to the background gauge fields given by $-a_tQ$ and the other due to the rest of the fields. We shall not need to use this split. Also the analysis of [19] was carried out using the rescaled time coordinate $\widetilde{t} = r_0 t$ so that the metric on the boundary remains finite in the $r_0 \to \infty$ limit, but the span of the time coordinate becomes infinite in this limit. This corresponded to taking the infrared cut-off to infinity keeping the ultraviolet cut-off fixed. In this paper we shall use the opposite (and more conventional) viewpoint where we take t as the time coordinate. In this case the induced metric (8) on the boundary goes to infinity as $r_0 \to \infty$ but the range of t remains fixed. This corresponds to taking the ultraviolet cut-off to zero keeping the infrared cut-off fixed.

To get some insight into this theory we introduce new coordinates

$$\widetilde{t} = R^{-1}\sqrt{r_0^2 - 1}\, t, \quad \widetilde{\phi} = \phi + \frac{1}{R}(r_0 - 1)t\,, \tag{10}$$

so that the metric (9) becomes

$$ds_B^2 = \frac{l^2 R^2}{4}[-d\widetilde{t}^2 + d\widetilde{\phi}^2]\,. \tag{11}$$

Thus up to the overall scale factor the metric is the standard Minkowski metric, and the space coordinate $\widetilde{\phi}$ is compact with period 2π. This gives a standard 1+1 dimensional CFT on a cylinder, and the generators $i\partial_{\widetilde{t}}$ and $-i\partial_{\widetilde{\phi}}$ are identified as

$$i\partial_{\widetilde{t}} = L_0 + \bar{L}_0, \qquad -i\partial_{\widetilde{\phi}} = L_0 - \bar{L}_0\,. \tag{12}$$

In order that in the extremal limit we get a supersymmetric black hole, we impose Ramond boundary condition along $\widetilde{\phi}$ on the anti-holomorphic part of the superconformal algebra.

In relating this (1+1) dimensional CFT to the CQM living on the boundary of AdS_2, we must identify the total Hamiltonian H_t of the CQM as the generator of t-translation in the CFT. On the other hand the charge Q of the CQM can be identified as the generator of ϕ translation. This gives

$$\begin{aligned} H_t &= i\partial_t = iR^{-1}\sqrt{r_0^2-1}\frac{\partial}{\partial\widetilde{t}} + i\frac{r_0-1}{R}\frac{\partial}{\partial\widetilde{\phi}} = 2\,R^{-1} r_0 \bar{L}_0 + R^{-1}(L_0 - \bar{L}_0) + \mathcal{O}(r_0^{-1})\,, \\ Q &= -i\partial_\phi = -i\partial_{\widetilde{\phi}} = L_0 - \bar{L}_0\,. \end{aligned} \tag{13}$$

Thus in the $r_0 \to \infty$ limit, the only states with finite H_t eigenvalues are those with minimal value of $\bar{L}_0$. Since we have Ramond boundary condition, the minimal value of $\bar{L}_0$ is 0. In other words the states of the CQM living on the boundary of AdS_2 are described by the $\bar{L}_0 = 0$ states of the 1+1 dimensional CFT living on the boundary of AdS_3.[3] In particular the ground state degeneracy $d(q)$ of the CQM, carrying a given charge q, can be identified as the degeneracy of the states of the CFT which are in the ground state of the Ramond sector in the anti-holomorphic sector and carries $(L_0 - \bar{L}_0)$ eigenvalue q. The former is the quantity that appears in the definition of the entropy via AdS_2/CFT_1 correspondence [19] whereas the latter appears in the definition of the entropy of the extremal BTZ black hole via AdS_3/CFT_2

[3]This is in accordance with the expectation that the CQM dual to gravity in AdS_2 is described by the chiral half of the (1+1) dimensional CFT dual to gravity in AdS_3 [39, 9, 34, 18].

correspondence. Thus we see that the two definitions of entropy agree up to subtleties involving ultraviolet cut-off of the CFT to be discussed below (29)

Using the identification of the CQM as a specific compactification of the CFT we can compute the partition function of the theory. For this we make the Euclidean continuation $t \to -iu$. Regularity of the metric (4) (or (6)) at the horizon $r = 1$ requires u to be a periodic coordinate with period 2π. From the point of view of the CQM, the partition function of the theory will be given by $Tr(e^{-2\pi H_t})$. Using (13) this can be reinterpreted as an appropriate trace over the Hilbert space of the (1+1) dimensional CFT dual to gravity in AdS_3. It is however instructive to do this computation directly in the CFT. For this we note that under the replacement $t \to -iu$ the boundary metric (9) takes the form

$$ds_B^2 = \frac{l^2}{4}\left[(r_0^2-1)du^2 + R^2\left(d\phi - \frac{i}{R}(r_0-1)du\right)^2\right] = \frac{l^2R^2}{4}[\tau_2^2 du^2 + (d\phi + \tau_1 du)^2]\,, \tag{14}$$

where

$$\tau_1 = -\frac{i}{R}(r_0-1), \quad \tau_2 = \frac{\sqrt{r_0^2-1}}{R}\,. \tag{15}$$

The metric is complex, but we can nevertheless go ahead and compute the partition function. Since u and ϕ both have period 2π, the partition function of the CFT with this background metric will be given by

$$Z = Tr\left[e^{2\pi i(\tau_1+i\tau_2)L_0 - 2\pi i(\tau_1-i\tau_2)\bar{L}_0}\right] = Tr\left[e^{-4\pi r_0 R^{-1}\bar{L}_0 - 2\pi R^{-1}(L_0-\bar{L}_0)} + \mathcal{O}(r_0^{-1})\right]\,. \tag{16}$$

This agrees with $Tr(e^{-2\pi H_t})$ with H_t given in (13). Eq.(16) again shows that in the $r_0 \to \infty$ limit only the $\bar{L}_0 = 0$ states contribute to the trace. We also see that in this limit the contribution to the partition function from states with a given charge $Q = q$ is given by

$$d(q)\, e^{-2\pi eq}\,, \tag{17}$$

where q is the $L_0 - \bar{L}_0$ eigenvalue, $e = 1/R$ is the near horizon electric field, and $d(q)$ is the degeneracy of the states with charge q. Eq.(17) agrees with eq.(24) of [19], where this result was also derived both from the microscopic computation in the CQM and a computation of the partition function in the bulk theory in the semiclassical limit.

A similar dimensional reduction from AdS_3 to AdS_2 was carried out in [40] in the context of extremal black holes in type IIA string theory on a Calabi-Yau manifold. However in that paper the authors interpreted the ϕ coordinate as the euclidean time direction and the u

coordinate as the spatial circle, thereby arriving at a modular transformed version of eq.(16). Since our goal is to identify the CQM living on the boundary of AdS_2, we must choose u as the time coordinate on the boundary of AdS_3 so that it matches the time coordinate of the CQM.

So far in our analysis we have considered neutral BTZ black holes. Let us now suppose that the three dimensional theory has additional $U(1)$ gauge fields $A^{(i)}_M$ with Chern-Simons action of the form

$$\frac{1}{2}\int d^3x\,\epsilon^{MNP}\,C_{ij}\,A^{(i)}_M F^{(j)}_{NP}, \qquad F^{(i)}_{NP} \equiv \partial_N A^{(i)}_P - \partial_P A^{(i)}_N\,, \tag{18}$$

where M, N, P run over the three coordinates of AdS_3 and C_{ij} are constants. Then we can construct charged black hole solutions by superimposing on the original BTZ solution (2) constant gauge fields:

$$A^{(i)}_M dx^M = w_i\left[dy - \frac{1}{l}\,\frac{\rho_-}{\rho_+}\,d\tau\right]\,. \tag{19}$$

Here w_i are constants. The term proportional to $d\tau$ has been chosen so as to make the gauge fields non-singular at the horizon. Even though the gauge field strength vanishes, the background (19) induces a charge on the black hole since the latter, being proportional to $\delta S/\delta F^{(i)}_{\rho t}$ (in the classical limit), is given by $C_{ij}A^{(j)}_y$ up to a constant of proportionality. Taking the near horizon limit as in (3) we arrive at the background

$$ds_3^2 = \frac{l^2}{4}\left[-(r^2-1)dt^2 + \frac{dr^2}{r^2-1} + R^2\left(d\phi + \frac{1}{R}(r-1)dt\right)^2\right]\,, \qquad A^{(i)}_M dx^M = w_i d\phi\,. \tag{20}$$

In order to make contact with the two dimensional viewpoint we define two dimensional gauge fields $a^{(i)}_\mu$ and scalar fields $\chi^{(i)}$ via the relations:

$$A^{(i)}_M dx^M = \chi^{(i)}(d\phi + a_\mu dx^\mu) + a^{(i)}_\mu\, dx^\mu\,, \tag{21}$$

where a_μ has been defined in (5). For the background (20) we have $a_\mu dx^\mu = \frac{1}{R}(r-1)dt$, and hence [41]

$$\chi^{(i)} = w_i, \quad a^{(i)}_\mu dx^\mu = e^{(i)}(r-1)dt, \quad e^{(i)} \equiv -\frac{w_i}{R}\,. \tag{22}$$

$e^{(i)}$ is the near horizon electric field associated with the two dimensional gauge fields $a^{(i)}_\mu$.

We shall now compute the partition function of the CQM living on the boundary of AdS_2 in the presence of these background gauge fields. This is equivalent to computing the partition function of the CFT living on the boundary of the space-time given in (20). Let $(J^\phi_{(i)}, J^t_{(i)})$ be the currents in the CFT dual to the gauge fields $A^{(i)}_M$ in the bulk. Then in the presence of the gauge field background given in (20) we have an insertion of

$$\exp\left[iw_i\int dt d\phi\sqrt{-\det g}J^{\phi}_{(i)}\right] , \tag{23}$$

in the boundary theory. To proceed further we need to assume some properties of the currents $J_{(i)}$. Typically in AdS_3/CFT_2 correspondence the currents dual to gauge fields are either holomorphic or anti-holomorphic depending on the sign of the Chern-Simons term in the bulk theory [42]. We shall assume for simplicity that all our gauge fields are dual to holomorphic currents; if the state carries charge associated with anti-holomorphic currents then in general we shall not be able to satisfy the $\bar{L}_0 = 0$ condition and the analysis will be more complicated.[4] This gives a relation between $J^{\phi}_{(i)}$ and $J^{t}_{(i)}$. To determine this relation we note from (14) that in the euclidean theory the holomorphic coordinate z is given by $\phi + \tau_1 u + i\tau_2 u$. Using the relation $u = it$ and the values of τ_1, τ_2 given in (15) we get

$$z = \phi - \frac{1}{R}t + \mathcal{O}(r_0^{-1}) . \tag{24}$$

Requiring holomorphicity gives $J^{z}_{(i)} = 0$ since by virtue of current conservation $\partial_z J^{z}_{(i)} = 0$, $J^{z}_{(i)}$ would have described an anti-holomorphic current. Thus we have

$$J^{\phi}_{(i)} - \frac{1}{R}J^{t}_{(i)} = 0 . \tag{25}$$

Substituting this into (23) and using the definition of the charge $Q_{(i)}$,

$$Q_{(i)} = \int d\phi\sqrt{-\det g}J^{t}_{(i)} , \tag{26}$$

we can express (23) as

$$\exp\left[iw_i\int dt\, Q_{(i)}/R\right] = \exp(2\pi\, w_i\, Q_{(i)}/R) = \exp(-2\pi\, e^{(i)}Q_{(i)}) , \tag{27}$$

where in the last step we have used (22). Insering this into (16) and using $e = 1/R$ we get

$$Z = Tr\left[e^{-4\pi r_0 R^{-1}\bar{L}_0 - 2\pi\sum_I e^I Q_I}\right] , \tag{28}$$

[4] If there are gauge fields dual to anti-holomorphic currents, then an analysis identical to that for the holomorphic currents shows that in the first term in the exponent in eq.(28), $\bar{L}_0$ will be replaced by $\bar{L}_0 + \sum'_i w_i Q_{(i)}$, with the sum over i in $\sum'$ running over the anti-holomorphic currents. The finite part retains the same form as the holomorphic currents, i.e. $-2\pi\sum' e^{(i)}Q_{(i)}$, in agreement with the results of [19].

where the index I now runs over all the two dimensional gauge fields, – the one coming from the dimensional reduction of the three dimensional metric as well as the ones coming from the three dimensional gauge fields. From (28) we see that in the $r_0 \to \infty$ limit we are still restricted to the $\bar{L}_0 = 0$ states. The contribution from the sector with charge $\vec{q}$ is given by

$$d(\vec{q})\, e^{-2\pi \sum_I q_I e^I}\,, \tag{29}$$

in agreement with eq.(24) of [19]. Here $d(\vec{q})$ denotes the degeneracy of $\bar{L}_0 = 0$ states in the CFT carrying charge $\vec{q}$. It can also be interpreted as the degeneracy of the lowest energy states in the CQM carrying charge $\vec{q}$.

One issue that we have not completely resolved is the following. From (11) we see that in the $(\widetilde{t}, \widetilde{\phi})$ coordinate system the conformal factor in front of the metric remains finite as $r_0 \to \infty$, suggesting that we have a finite ultraviolet cut-off. In particular the size of the $\widetilde{\phi}$ circle is of the order of the cut-off. We do not have a direct understanding of the role of this cut-off in the CFT. However studying the effect of this cut-off in the bulk gives us some insight. First of all note that in conventional AdS_3, it is more natural to define the partition function by summing over states of all charges with a fixed value of the chemical potential. However in AdS_2 the modes representing fluctuation of the total charge represent non-normalizable deformations and hence it is more natural to define the partition function by summing over a fixed charge sector [19]. Thus it would seem that the effect of the finite ultraviolet cut-off in the CFT must be to restrict the Hilbert space of a given CFT to a fixed charge sector. There are also other effects of this finite cut-off in the bulk when we embed the BTZ black hole in a supersymmetric theory with additional moduli scalars and vector fields. When we view the extremal BTZ black hole from the point of view of the asymptotically AdS_3 space-time by setting $\rho_+ = \rho_-$ in (2) then the ultraviolet cut-off is small compared to the size of the y circle since the latter approaches ∞ as $\rho \to \infty$, but such asymptotic space-time could admit other multi-centered black hole solutions [35]. On the other hand when we view the same extremal black hole from the point of view of its near horizon geometry as in (4), then the size of the ϕ circle becomes comparable to the ultra-violet cut-off, but this space-time geometry no longer admits the other multi-centered black hole solutions in AdS_2 since the values of the various scalar fields are fixed at their attractor values.[5] Thus it would seem that the ultraviolet cut-off weeds out the contribution due to the multi-centered black hole configurations of the type

[5]Possible exceptions are multi-centered black holes with mutually local charges [36, 11, 37], i.e. charges satisfying $(\vec{q}_i \cdot \vec{p}_j - \vec{q}_j \cdot \vec{p}_i) = 0$ where $(\vec{q}_i, \vec{p}_i)$ denote the electric and magnetic charge vectors of the ith black hole. But they do not contribute to the degeneracy [38, 67].

discussed in [35] from the CFT spectrum. In support of this speculation we would like to note that for large R the size of the ϕ circle is large compared to the ultra-violet cut-off and hence effect of the cut-off is expected to be small. This is precisely the region in which the entropy of a single centered black hole gives the dominant contribution to the entropy [35].

Even though it is more natural to work in a fixed charge sector of AdS_2, one can get some insight into the OSV conjecture if one does sum over the contribution from different charge sectors. After summing over charges the full partition function is given by

$$Z(\vec{e}) = \sum_{\vec{q}} d(\vec{q})\, e^{-2\pi \vec{e}\cdot\vec{q}} \,. \tag{30}$$

For large charges the dominant contribution to this sum comes from $\vec{q}$ satisfying $\partial \ln d(\vec{q})/\partial q_I = 2\pi e^I$, in agreement with the classical relation between the electric field and the charge. The right hand side of (30) has the flavor of the black hole partition function defined in [43]. On the other hand, using AdS/CFT correspondence, the left hand side can be expressed as a functional integral over the fields in the bulk theory.[6] Now, as was shown in [19], after ignoring terms linear in r_0 in the exponent – which must cancel among themselves – the classical result for the partition function in the $r_0 \to \infty$ limit is given by

$$Z = e^{-2\pi f} \,, \tag{31}$$

where f is the classical Lagrangian density evaluated in the near horizon geometry. One might expect that the effect of quantum corrections would be to replace the classical Lagrangian density by some effective Lagrangian density. As we shall now review, if we assume that the effective Lagrangian density that contributes to the partition function is governed only by the F-type terms, i.e. terms which can be encoded in the prepotential $\mathcal{F}$ [51], then Z takes the form predicted in the original OSV conjecture.

In $\mathcal{N} = 2$ supergravity theories in four dimensions the information about the 'F-type terms' can be encoded in a function $F(\{X^I\}, \widehat{A})$ – known as the prepotential – of a set of

[6]Note that we have switched back from the three dimensional viewpoint to the two dimensional viewpoint. The black hole partition function has been analyzed using AdS/CFT correspondence earlier (see *e.g.* [44,45,46]). Also various other approaches to relating the entropy function formalism to Euclidean action formalism and / or OSV conjecture can be found in [47, 48, 49]. The advantage of our approach lies in the fact that since we apply AdS/CFT correspondence on the near horizon geometry, the chemical potentials dual to the charges are directly related to the near horizon electric field, and hence, via the attractor mechanism, to other near horizon field configuration. Furthermore the path integral needs to be performed only over the near horizon geometry where we have enhanced supersymmetry and hence stronger non-renormalization properties. The approach closest to ours is the one given in [40]; we shall comment on it later. A different approach to deriving the OSV conjecture using AdS/CFT correspondence can be found in [50].

complex variables X^I which are in one to one correspondence with the gauge fields and an auxiliary complex variable $\widehat{A}$ related to the square of the graviphoton field strength [51, 52]. Supersymmetry demands that F is a homogeneous function of degree two in its arguments:

$$F(\{\lambda X^I\}, \lambda^2 \widehat{A}) = \lambda^2 F(\{X^I\}, \widehat{A})\,. \tag{32}$$

For a given choice of electric field one finds that the extremum of the effective Lagrangian density computed with the F-term effective action occurs at the attractor point where [53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64]

$$\widehat{A} = -4w^2, \quad 4(\bar{w}^{-1}\bar{X}^I + w^{-1}X^I) = e^I, \quad 4(\bar{w}^{-1}\bar{X}^I - w^{-1}X^I) = -ip^I\,. \tag{33}$$

Here w is an arbitrary complex parameter and p^I are the magnetic charges carried by the black hole. These magnetic charges have not appeared explicitly in our discussion so far because from the point of view of the near horizon geometry they represent fluxes through compact two cycles and appear as parameters labelling the two (or three) dimensional field theory describing the near horizon dynamics. The value of the effective Lagrangian density at the extremum (33) is given by [64]

$$f = 16\,i\,(w^{-2}F - \bar{w}^{-2}\bar{F})\,. \tag{34}$$

Note that (33) determines X^I in terms of the unknown parameter w. However due to the scaling symmetry (32), f given in (34) is independent of w. Using this scaling symmetry we can choose

$$w = -8\,i\,, \tag{35}$$

and rewrite (33), (34) as

$$\widehat{A} = 256, \quad X^I = -i(e^I + ip^I)\,, \tag{36}$$

$$f = -\frac{i}{4}(F(\{X^I\}, 256) - \overline{F(\{X^I\}, 256)})\,. \tag{37}$$

Thus we have

$$Z(\vec{e}) = e^{-\pi\, Im\, F(\{p^I - ie^I\}, 256)}\,. \tag{38}$$

This is precisely the original OSV conjecture [43].

It has however been suggested in subsequent papers that agreement with statistical entropy requires modifying this formula by including additional measure factors on the right hand side of (38) [65, 66, 67]. A careful analysis of the path integral keeping track of the holomorphic anomaly [68,69,70] may be able to reproduce these corrections, but we shall not undertake that task here. Some of these corrections are in fact necessary for restoring the duality invariance of the final result for the entropy [66].

Ref. [40] presented an argument as to why the partition function of type IIA string theory on $AdS_2 \times S^2 \times CY_3$ may be related to $|Z_{top}|^2$. In this analysis the divergence due to the integration over AdS_2 was regulated by supersymmetry. This argument led to $Z_{AdS_2} = |Z_{top}|^{2C}$, where C is a constant that was not calculated directly from first principles. In our interpretation of the AdS_2 partition function there is a clear understanding of the divergent parts that is independent of supersymmetry, – terms linear in r_0 in the exponent represent the effect of ground state energy and the r_0 independent piece encodes information about the ground state spectrum. In particular the classical partition function calculated with F-type terms in our approach agrees with $|Z_{top}|^2$ *after we remove the terms linear in* r_0 *from the exponent.* Thus combining this regularization scheme with the analysis of [40] may lead to a complete understanding of Z_{AdS_2}. In particular there may be additional finite pieces from the interference between order r_0 divergent terms and order r_0^{-1} terms which reproduce the measure factors described in [65, 66, 67].

Our attempt to justify the OSV conjecture from a macroscopic viewpoint makes it clear that $d(\vec{q})$ appearing in the expression for the black hole partition function counts only the states associated with single centered black holes.[7] Thus OSV formula should have nothing to say about the contribution to the entropy from the multi-centered black holes. This in particular would explain why we do not see the effect of wall crossing or the entropy enigma discussed in [67] in the OSV formula.

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References

[1] R. M. Wald, "Black hole entropy in the Noether charge," Phys. Rev. D **48**, 3427 (1993) [arXiv:gr-qc/9307038].

[2] T. Jacobson, G. Kang and R. C. Myers, "On Black Hole Entropy," Phys. Rev. D **49**, 6587 (1994) [arXiv:gr-qc/9312023].

[3] V. Iyer and R. M. Wald, "Some properties of Noether charge and a proposal for dynamical black hole entropy," Phys. Rev. D **50**, 846 (1994) [arXiv:gr-qc/9403028].

[7] An operational definition of such a $d(\vec{q})$ can be taken, for example, as the degeneracy of microstates evaluated at the attractor point corresponding to $\vec{q}$.

[4] T. Jacobson, G. Kang and R. C. Myers, "Black hole entropy in higher curvature gravity," arXiv:gr-qc/9502009.

[5] A. Sen, "Black hole entropy function and the attractor mechanism in higher derivative gravity," JHEP **0509**, 038 (2005) [arXiv:hep-th/0506177].

[6] D. Astefanesei, K. Goldstein, R. P. Jena, A. Sen and S. P. Trivedi, "Rotating attractors," JHEP **0610**, 058 (2006) [arXiv:hep-th/0606244].

[7] H. K. Kunduri, J. Lucietti and H. S. Reall, "Near-horizon symmetries of extremal black holes," Class. Quant. Grav. **24**, 4169 (2007) [arXiv:0705.4214 [hep-th]].

[8] P. Figueras, H. K. Kunduri, J. Lucietti and M. Rangamani, "Extremal vacuum black holes in higher dimensions," arXiv:0803.2998 [hep-th].

[9] A. Strominger, "AdS(2) quantum gravity and string theory," JHEP **9901**, 007 (1999) [arXiv:hep-th/9809027].

[10] M. Cadoni and S. Mignemi, "Entropy of 2D black holes from counting microstates," Phys. Rev. D **59**, 081501 (1999) [arXiv:hep-th/9810251].

[11] J. M. Maldacena, J. Michelson and A. Strominger, "Anti-de Sitter fragmentation," JHEP **9902**, 011 (1999) [arXiv:hep-th/9812073].

[12] M. Spradlin and A. Strominger, "Vacuum states for AdS(2) black holes," JHEP **9911**, 021 (1999) [arXiv:hep-th/9904143].

[13] J. Navarro-Salas and P. Navarro, "AdS(2)/CFT(1) correspondence and near-extremal black hole entropy," Nucl. Phys. B **579**, 250 (2000) [arXiv:hep-th/9910076].

[14] M. Caldarelli, G. Catelani and L. Vanzo, "Action, Hamiltonian and CFT for 2D black holes," JHEP **0010**, 005 (2000) [arXiv:hep-th/0008058].

[15] M. Cadoni, P. Carta, D. Klemm and S. Mignemi, "AdS(2) gravity as conformally invariant mechanical system," Phys. Rev. D **63**, 125021 (2001) [arXiv:hep-th/0009185].

[16] A. Giveon and A. Sever, "Strings in a 2-d extremal black hole," JHEP **0502** (2005) 065 [arXiv:hep-th/0412294].

[17] T. Azeyanagi, T. Nishioka and T. Takayanagi, "Near Extremal Black Hole Entropy as Entanglement Entropy via AdS2/CFT1," Phys. Rev. D **77** (2008) 064005 [arXiv:0710.2956 [hep-th]].

[18] T. Hartman and A. Strominger, "Central Charge for AdS_2 Quantum Gravity," arXiv:0803.3621 [hep-th].

[19] A. Sen, "Entropy Function and AdS_2/CFT_1 Correspondence," arXiv:0805.0095v4 [hep-th].

[20] M. Alishahiha and F. Ardalan, "Central Charge for 2D Gravity on AdS(2) and AdS(2)/CFT(1) Correspondence," arXiv:0805.1861 [hep-th].

[21] M. Banados, C. Teitelboim and J. Zanelli, "The Black hole in three-dimensional space-time," Phys. Rev. Lett. **69**, 1849 (1992) [arXiv:hep-th/9204099].

[22] K. Sfetsos and K. Skenderis, "Microscopic derivation of the Bekenstein-Hawking entropy formula for non-extremal black holes," Nucl. Phys. B **517**, 179 (1998) [arXiv:hep-th/9711138].

[23] A. Strominger, "Black hole entropy from near-horizon microstates," JHEP **9802**, 009 (1998) [arXiv:hep-th/9712251].

[24] J. M. Maldacena, "The large N limit of superconformal field theories and supergravity," Adv. Theor. Math. Phys. **2**, 231 (1998) [Int. J. Theor. Phys. **38**, 1113 (1999)] [arXiv:hep-th/9711200].

[25] S. S. Gubser, I. R. Klebanov and A. M. Polyakov, "Gauge theory correlators from non-critical string theory," Phys. Lett. B **428**, 105 (1998) [arXiv:hep-th/9802109].

[26] E. Witten, "Anti-de Sitter space and holography," Adv. Theor. Math. Phys. **2**, 253 (1998) [arXiv:hep-th/9802150].

[27] M. Henningson and K. Skenderis, "The holographic Weyl anomaly," JHEP **9807**, 023 (1998) [arXiv:hep-th/9806087].

[28] O. Coussaert and M. Henneaux, "Supersymmetry of the (2+1) black holes," Phys. Rev. Lett. **72**, 183 (1994) [arXiv:hep-th/9310194].

[29] E. Witten, "Three-Dimensional Gravity Revisited," arXiv:0706.3359 [hep-th].

[30] H. Saida and J. Soda, "Statistical entropy of BTZ black hole in higher curvature gravity," Phys. Lett. B **471**, 358 (2000) [arXiv:gr-qc/9909061].

[31] P. Kraus and F. Larsen, "Microscopic black hole entropy in theories with higher derivatives," arXiv:hep-th/0506176.

[32] B. Sahoo and A. Sen, "BTZ black hole with Chern-Simons and higher derivative terms," JHEP **0607**, 008 (2006) [arXiv:hep-th/0601228].

[33] S. Nampuri, P. K. Tripathy and S. P. Trivedi, "Duality Symmetry and the Cardy Limit," arXiv:0711.4671 [hep-th].

[34] J. H. Cho, T. Lee and G. W. Semenoff, "Two dimensional anti-de Sitter space and discrete light cone quantization," Phys. Lett. B **468**, 52 (1999) [arXiv:hep-th/9906078].

[35] J. de Boer, F. Denef, S. El-Showk, I. Messamah and D. Van den Bleeken, "Black hole bound states in AdS_3xS^2," arXiv:0802.2257 [hep-th].

[36] D. Brill, "Splitting of an extremal Reissner-Nordstrom throat via quantum tunneling," Phys. Rev. D **46**, 1560 (1992) [arXiv:hep-th/9202037].

[37] R. Dijkgraaf, R. Gopakumar, H. Ooguri and C. Vafa, "Baby universes in string theory," Phys. Rev. D **73**, 066002 (2006) [arXiv:hep-th/0504221].

[38] F. Denef, "Quantum quivers and Hall/hole halos," JHEP **0210**, 023 (2002) [arXiv:hep-th/0206072].

[39] H. J. Boonstra, B. Peeters and K. Skenderis, "Brane intersections, anti-de Sitter spacetimes and dual superconformal theories," Nucl. Phys. B **533**, 127 (1998) [arXiv:hep-th/9803231].

[40] C. Beasley, D. Gaiotto, M. Guica, L. Huang, A. Strominger and X. Yin, "Why Z(BH) = —Z(top)—**2," arXiv:hep-th/0608021.

[41] A. Sen, "Black Hole Entropy Function, Attractors and Precision Counting of Microstates," arXiv:0708.1270 [hep-th].

[42] P. Kraus, "Lectures on black holes and the AdS(3)/CFT(2) correspondence," arXiv:hep-th/0609074.

[43] H. Ooguri, A. Strominger and C. Vafa, "Black hole attractors and the topological string," arXiv:hep-th/0405146.

[44] R. Dijkgraaf, J. M. Maldacena, G. W. Moore and E. P. Verlinde, "A black hole farey tail," arXiv:hep-th/0005003.

[45] P. Kraus and F. Larsen, "Partition functions and elliptic genera from supergravity," JHEP **0701**, 002 (2007) [arXiv:hep-th/0607138].

[46] J. de Boer, M. C. N. Cheng, R. Dijkgraaf, J. Manschot and E. Verlinde, "A farey tail for attractor black holes," JHEP **0611**, 024 (2006) [arXiv:hep-th/0608059].

[47] M. Alishahiha and H. Ebrahim, "New attractor, entropy function and black hole partition function," JHEP **0611**, 017 (2006) [arXiv:hep-th/0605279].

[48] N. V. Suryanarayana and M. C. Wapler, "Charges from Attractors," Class. Quant. Grav. **24**, 5047 (2007) [arXiv:0704.0955 [hep-th]].

[49] O. J. C. Dias and P. J. Silva, "Euclidean analysis of the entropy functional formalism," Phys. Rev. D **77**, 084011 (2008) [arXiv:0704.1405 [hep-th]].

[50] D. Gaiotto, A. Strominger and X. Yin, "From AdS(3)/CFT(2) to black holes / topological strings," JHEP **0709**, 050 (2007) [arXiv:hep-th/0602046].

[51] B. de Wit, "N = 2 electric-magnetic duality in a chiral background," Nucl. Phys. Proc. Suppl. **49**, 191 (1996) [arXiv:hep-th/9602060].

[52] B. de Wit, "N=2 symplectic reparametrizations in a chiral background," Fortsch. Phys. **44**, 529 (1996) [arXiv:hep-th/9603191].

[53] S. Ferrara, R. Kallosh and A. Strominger, "N=2 extremal black holes," Phys. Rev. D **52**, 5412 (1995) [arXiv:hep-th/9508072].

[54] A. Strominger, "Macroscopic Entropy of $N = 2$ Extremal Black Holes," Phys. Lett. B **383**, 39 (1996) [arXiv:hep-th/9602111].

[55] S. Ferrara and R. Kallosh, "Supersymmetry and Attractors," Phys. Rev. D **54**, 1514 (1996) [arXiv:hep-th/9602136].

[56] J. M. Maldacena, A. Strominger and E. Witten, "Black hole entropy in M-theory," JHEP **9712**, 002 (1997) [arXiv:hep-th/9711053].

[57] K. Behrndt, G. Lopes Cardoso, B. de Wit, D. Lust, T. Mohaupt and W. A. Sabra, "Higher-order black-hole solutions in N = 2 supergravity and Calabi-Yau string backgrounds," Phys. Lett. B **429**, 289 (1998) [arXiv:hep-th/9801081].

[58] G. Lopes Cardoso, B. de Wit and T. Mohaupt, "Corrections to macroscopic supersymmetric black-hole entropy," Phys. Lett. B **451**, 309 (1999) [arXiv:hep-th/9812082].

[59] G. Lopes Cardoso, B. de Wit and T. Mohaupt, "Deviations from the area law for supersymmetric black holes," Fortsch. Phys. **48**, 49 (2000) [arXiv:hep-th/9904005].

[60] G. Lopes Cardoso, B. de Wit and T. Mohaupt, "Area law corrections from state counting and supergravity," Class. Quant. Grav. **17**, 1007 (2000) [arXiv:hep-th/9910179].

[61] T. Mohaupt, "Black hole entropy, special geometry and strings," Fortsch. Phys. **49**, 3 (2001) [arXiv:hep-th/0007195].

[62] G. Lopes Cardoso, B. de Wit, J. Kappeli and T. Mohaupt, "Stationary BPS solutions in N = 2 supergravity with R**2 interactions," JHEP **0012**, 019 (2000) [arXiv:hep-th/0009234].

[63] G. L. Cardoso, B. de Wit, J. Kappeli and T. Mohaupt, "Examples of stationary BPS solutions in N = 2 supergravity theories with R**2-interactions," Fortsch. Phys. **49**, 557 (2001) [arXiv:hep-th/0012232].

[64] B. Sahoo and A. Sen, "Higher derivative corrections to non-supersymmetric extremal black holes in N = 2 supergravity," JHEP **0609**, 029 (2006) [arXiv:hep-th/0603149].

[65] D. Shih and X. Yin, "Exact black hole degeneracies and the topological string," arXiv:hep-th/0508174.

[66] G. L. Cardoso, B. de Wit, J. Kappeli and T. Mohaupt, "Black hole partition functions and duality," arXiv:hep-th/0601108.

[67] F. Denef and G. W. Moore, "Split states, entropy enigmas, holes and halos," arXiv:hep-th/0702146.

[68] M. Bershadsky, S. Cecotti, H. Ooguri and C. Vafa, "Holomorphic anomalies in topological field theories," Nucl. Phys. B **405**, 279 (1993) [arXiv:hep-th/9302103].

[69] I. Antoniadis, E. Gava, K. S. Narain and T. R. Taylor, "Topological amplitudes in string theory," Nucl. Phys. B **413**, 162 (1994) [arXiv:hep-th/9307158].

[70] M. Bershadsky, S. Cecotti, H. Ooguri and C. Vafa, "Kodaira-Spencer theory of gravity and exact results for quantum string amplitudes," Commun. Math. Phys. **165**, 311 (1994) [arXiv:hep-th/9309140].