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Orbifolds of M-Theory and String Theory

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Abstract

It is shown that many of the conjectured dualities involving orbifold compactification of M-theory follow from the known dualities involving M-theory and string theory in ten dimensions, and the ansatz that orbifolding procedure commutes with the duality transformation. This ansatz also leads to a new duality conjecture, namely that M-theory compactified on T^8/Z_2 is dual to type I string theory on T^7. In this case the 'twisted sector states' in M-theory live on sixteen membranes transverse to the internal manifold.

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Introduction: In recent past it has been realised that the moduli space of string theories contains a special point, known as the M-theory, whose low energy limit corresponds to the eleven dimensional supergravity theory[1, 2, 3, 4]. Compactification of M-theory on various orbifolds have been studied, and have been conjectured to be dual to various known string theory compactifications[5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15]. In this paper we shall try to get a systematic understanding of some of these duality conjectures in terms of other duality conjectures involving M-theory and ten dimensional string theories; namely the equivalence of M-theory on S^1 and type IIA string theory, and the self-duality of type IIB string theory.

The basic idea is an old one. Suppose two theories A and B are equivalent, and suppose further that the theory A has a symmetry σ_A which gets mapped to a symmetry σ_B under the equivalence relation. Then naively we would expect that the theory A modded out by the symmetry σ_A will be equivalent to the theory B modded out by the symmetry σ_B. Unfortunately, this naive expectation does not always work, and there are specific instances where this procedure gives nonsensical answer[16]. It works when the equivalence relation between the two theories involves a T-duality transformation rather than a more general U-duality transformation; it also works when the adiabatic argument given in ref.[16] can be applied. Nevertheless, there are many interesting examples of dual pairs constructed by this (or closely related) method where neither of these conditions hold[17, 18, 16, 10, 19]. We shall start with the assumption that this procedure of obtaining dual pairs works even when one of the theories corresponds to M-theory compactification rather than string theory compactification, and show that in most cases it leads to sensible answers.[3] One of the exceptions is M-theory compactification on S^1/Z_2, where this procedure would indicate that the dual theory is type IIB string theory, whereas in actual practice it is known to be the $E_8 \times E_8$ heterotic string theory[5].

Notation: To begin with let us set up some notations. For either M-theory or string theory compactified on an n dimensional torus T^n, we shall denote by $\mathcal{I}_n$ the transformation that changes the sign of all the coordinates on the torus. This is usually not a symmetry for odd n. For type IIA theory, and for M-theory, this can be made into a symmetry by combining it with an internal transformation, which, besides other effects, changes the sign of the three form gauge field. We shall denote this combined transformation by $\mathcal{J}_n$. For type IIA theory, the internal part of this transformation can be identified to world-sheet parity transformation. For the type IIB theory, the world-sheet parity transformation is itself a symmetry of the theory. We shall denote this transformation by Ω. Finally, both type IIA and type IIB theory possesses a symmetry which is easiest to describe in the light-cone gauge Green-Schwarz formalism. In this formalism this transformation changes the sign of all the left-moving fermions on the world-sheet. We shall denote this transformation by $(-1)^{F_L}$, where F_L stands for the space-time fermion number arising in the left-moving sector of the world-sheet. Acting on the massless bosonic fields

[3] Some related observations were made in ref.[15].

in the theory, the effect of this transformation is to change the sign of all the fields arising from the Ramond-Ramond (RR) sector.

The only other Z_2 symmetry that we shall encounter in this paper is a Z_2 symmetry transformation acting on a special class of $K3$ surfaces[20, 21, 22, 8]. This has the following properties:

1. Modding out the $K3$ surface by this symmetry preserves $SU(2)$ holonomy.
2. Acting on the lattice of second cohomology elements of $K3$, it exchanges the two E_8 factors leaving the rest of the lattice invariant.
3. It has eight fixed points.

We shall denote this symmetry by σ.

We begin our discussion with M-theory on $S^1/\{1, \mathcal{J}_1\}$. According to ref.[5] this theory is equivalent to $E_8 \times E_8$ heterotic string theory. On the other hand, by using the known duality transformation between M-theory on S^1 and type IIA string theory, one can easily verify that the transformation $\mathcal{J}_1$ in M-theory gets mapped to the transformation $(-1)^{F_L}$ in the type IIA theory. Thus naively we would conclude that M-theory on $S^1/\{1, \mathcal{J}_1\}$ should be equivalent to type IIA theory modded out by $\{1, (-1)^{F_L}\}$. However, the latter theory is known to be equivalent to the type IIB theory. Thus we see that the naive procedure of getting dual pairs through orbifolding breaks down in this case. However, as we shall see, this procedure does work in most of the other cases.

M-theory on $T^5/\{1, \mathcal{J}_5\}$: M-theory on T^5 is equivalent to type IIA on T^4. In M-theory the transformation $\mathcal{J}_5$ is equivalent to $\mathcal{J}_1 \cdot \mathcal{I}_4$. This goes over to the symmetry $(-1)^{F_L} \cdot \mathcal{I}_4$ in the type IIA theory. Thus we would conclude that M-theory on $T^5/\{1, \mathcal{J}_5\}$ is equivalent to the type IIA theory on $T^4/\{1, (-1)^{F_L} \cdot \mathcal{I}_4\}$. It is clear that the spectrum of massless states coming from the untwisted sector in the two theories will be identical, since by construction it is guaranteed that the fields that are even (odd) in M-theory on T^5 under $\mathcal{J}_5$ get mapped to fields that are even (odd) in type IIA on T^4 under $(-1)^{F_L} \cdot \mathcal{I}_4$. Thus we only need to verify that the spectra from the 'twisted sector' agree. In M-theory the twisted sector contributes 16 tensor multiplets[6, 7]. On the type IIA side, the space-time part $\mathcal{I}_4$ of the Z_2 symmetry has sixteen fixed points on T^4. Thus in order that the spectrum of massless states in the two theories agree, one must verify that at each fixed point, the twisted sector of the type IIA theory contains a single tensor multiplet.

It is easy to verify this result by working in the light cone gauge Green-Schwarz formalism. First of all, one can analyse the surviving supersymmetry generators, and verify that they belong to the chiral $N = 2$ supersymmetry algebra in six dimensions, as is the case for M-theory on $T^5/\{1, \mathcal{J}_5\}$. The spectrum may be analysed as follows. Both on the left and the right hand sector there are four periodic bosons and fermions and four anti-periodic bosons and fermions. Thus the total vacuum energy vanishes. Quantization of the eight fermionic zero modes (four from the left and four from the right) gives a sixteen fold degenerate state. Through careful analysis of the transformation laws of the fermion zero modes under the six dimensional Lorentz group one can verify that there sixteen states indeed belong to a tensor multiplet of the chiral $N = 2$ supersymmetry algebra.

Instead of presenting the details of this calculation, we shall map this type IIA orbifold into a type IIB orbifold via a T-duality transformation, which would make this equivalence obvious. Let us denote by $6, 7, 8, 9$ the compact directions, and make an $R \to 1/R$ duality transformation on the 6th coordinate. If X^m_L, X^m_R $(6 \le m \le 9)$ denote the left and right moving components of the bosonic coordinates in the type IIA theory, and Y^m_L, Y^m_R denote the left and right moving components of the bosonic coordinates in the type IIB theory, then the two sets of variables are related as

$$Y^6_L = -X^6_L, \qquad Y^m_L = X^m_L \quad \text{for} \quad 7 \le m \le 9, \qquad Y^m_R = X^m_R \quad \text{for} \quad 6 \le m \le 9\,. \tag{1}$$

Now both the transformations $\mathcal{I}_4$, as well as $(-1)^{F_L}$, in the type IIA theory can be represented as a T-duality rotation of the form[4]

$$\begin{aligned} & (X^6_L + iX^7_L, X^8_L + iX^9_L, X^6_R + iX^7_R, X^8_R + iX^9_R) \\ \to \quad & (e^{i\theta_L}(X^6_L + iX^7_L), e^{i\phi_L}(X^8_L + iX^9_L), e^{i\theta_R}(X^6_R + iX^7_R), e^{i\phi_R}(X^8_R + iX^9_R))\,. \end{aligned} \tag{2}$$

In particular the transformation $\mathcal{I}_4$ corresponds to

$$(\theta_L, \phi_L; \theta_R, \phi_R) = (-\pi, \pi; -\pi, \pi)\,, \tag{3}$$

and the transformation $(-1)^{F_L}$ corresponds to

$$(\theta_L, \phi_L; \theta_R, \phi_R) = (2\pi, 0; 0, 0)\,. \tag{4}$$

Note that the rotation by 2π has no effect on the bosonic coordinates which transform as vectors, but acts on the left moving fermions transforming in the spinor representation as a change of sign. This is precisely the effect of $(-1)^{F_L}$. Thus the combined effect of $(-1)^{F_L}$ and $\mathcal{I}_4$ is given by

$$(\theta_L, \phi_L; \theta_R, \phi_R) = (\pi, \pi; -\pi, \pi)\,, \tag{5}$$

From eq.(1) we see that the transformation (2) can be rewritten in terms of the coordinates in the type IIB theory as

$$\begin{aligned} & (Y^6_L + iY^7_L, Y^8_L + iY^9_L, Y^6_R + iY^7_R, Y^8_R + iY^9_R) \\ \to \quad & (e^{i\theta'_L}(Y^6_L + iY^7_L), e^{i\phi'_L}(Y^8_L + iY^9_L), e^{i\theta'_R}(Y^6_R + iY^7_R), e^{i\phi'_R}(Y^8_R + iY^9_R))\,, \end{aligned} \tag{6}$$

where

$$(\theta'_L, \phi'_L; \theta'_R, \phi'_R) = (-\theta_L, \phi_L; \theta_R, \phi_R)\,. \tag{7}$$

[4]This notation is similar to the one used in [23].

From eqs.(5) and (7) we see that the transformation $(-1)^{F_L} \cdot \mathcal{I}_4$ in the type IIA theory corresponds to

$$(\theta'_L, \phi'_L; \theta'_R, \phi'_R) = (-\pi, \pi; -\pi, \pi) \tag{8}$$

in the type IIB theory. This can easily be identified to be the transformation $\mathcal{I}_4$ in type IIB theory.[5] This leads us to conclude that type IIA on $T^4/\{1, (-1)^{F_L} \cdot \mathcal{I}_4\}$ is equivalent to type IIB on $T^4/\{1, \mathcal{I}_4\}$.

Using the earlier equivalence between M-theory on $T^5/\{1, \mathcal{J}_5\}$ and type IIA theory on $T^4/\{1, (-1)^{F_L} \cdot \mathcal{I}_4\}$ we then conclude that M-theory on $T^5/\{1, \mathcal{J}_5\}$ is equivalent to type IIB on $T^4/\{1, \mathcal{I}_4\}$. The latter is a special case of type IIB theory on a K3 surface, and is known to give one tensor multiplet from each of the fixed points in the twisted sector. In fact the duality between type IIB on $K3$ and M-theory on $T^5/\{1, \mathcal{J}_5\}$ has already been conjectured in refs[6, 7]. Thus we have reproduced the conjectured duality of refs.[6, 7] by assuming that the orbifolding procedure commutes with the duality transformation.

M-theory on $T^9/\{1, \mathcal{J}_9\}$: This case may be analyzed along more or less similar lines as the previous model. Using identical logic we see that this theory is expected to be dual to type IIA theory on $T^8/\{1, (-1)^{F_L} \cdot \mathcal{I}_8\}$. Under a further $R \to 1/R$ duality in one of the eight compact directions, this theory reduces to type IIB on $T^8/\{1, \mathcal{I}_8\}$. The duality between M-theory on $T^9/\{1, \mathcal{J}_9\}$ and type IIB on $T^8/\{1, \mathcal{I}_8\}$ has already been conjectured[24], where it was shown that each of the 256 fixed points in the type IIB theory gives one left-moving chiral boson neutral under supersymmetry, which upon fermionization maps to 512 chiral fermions, one associated with each of the 512 fixed points of T^9 under $\mathcal{J}_9$ in the M-theory.

M-theory on $(S^1 \times K3)/\{1, \mathcal{J}_1 \cdot \sigma\}$: This model was analyzed in ref.[8]. The spectrum of massless states in this model corresponds to that of a chiral $N = 1$ supergravity theory with nine tensor-, eight vector- and twenty huper-multiplets. Using the duality between M theory on S^1 and the type IIA theory, and noting that the symmetries $\mathcal{J}_1$ and σ in M-theory corresponds to $(-1)^{F_L}$ and σ respectively in the type IIA theory, we can map this model to type IIA on $T^4/\{1, (-1)^{F_L} \cdot \sigma\}$. The spectra of massless states in the untwisted sector match trivially. The spectra of massless states in the twisted sector can be seen to match in the following way. In the M-theory orbifold, the 'twisted sector states' consist of eight tensor- and eight hyper-multiplets of the chiral $N = 1$ supersymmetry algebra[8], thus we need to show that the twisted sector states in the type IIA orbifold also consist of eight tensor multiplets. Since $K3/\{1, \sigma\}$ has eight fixed points, this amounts to proving that there is one tensor- and one hyper- multiplet coming from each fixed point. The spectrum of massless states coming from the twisted sector of the type IIA orbifold can be computed by noting that locally the space $K3/\{1, \sigma\}$ near the fixed point has the same structure as the space $T^4/\{1, \mathcal{I}_4\}$. Thus the spectrum of massless states per

[5] One can also verify explicitly, by using the standard map between the massless fields in the type IIA and the type IIB theories under duality transformation, that the action of $(-1)^{F_L} \cdot \mathcal{I}_4$ on the massless fields in the type IIA theory is identical to that of $\mathcal{I}_4$ on the massless fields in the type IIB theory.

fixed point from the twisted sector must be the same as that in type IIA on $T^4/\{1,\mathcal{I}_4\}$. As has already been argued, the latter theory has one tensor multiplet of the $N=2$ supersymmetry algebra per fixed point. This corresponds to one tensor- and one hyper-multiplet of the $N=1$ supersymmetry algebra. Thus type IIA on $K3/\{1,(-1)^{F_L}\cdot\sigma\}$ does have eight hyper- and eight tensor- multiplets from the twisted sector.[6] This shows that the massless spectrum of M-theory on $(S^1\times K3)/\{1,\mathcal{J}_1\cdot\sigma\}$ agrees with that of type IIA string theory on $K3/\{1,(-1)^{F_L}\cdot\sigma\}$.

We would also like to relate this to a type IIB compactification. Conventional T-duality transformation on type IIA on $K3$ will give us back type IIA on $K3$, so we need to use a different strategy. For this we shall choose a special $K3$ surface, namely an orbifold $T^4/\{1,\mathcal{I}_4\}$. We do not suffer from any loss of generality this way, since once we establish the duality at one point in the moduli space, it holds at all other points as well. Also for convenience we shall take the T^4 to be the product of four circles, each at the self-dual radius. Let η denote a transformation

$$(X^6,X^7,X^8,X^9)\to(X^6,X^7,X^8,X^9+\pi R)\,, \tag{9}$$

where X^m denote the coordinates on T^4 and R denotes the radius of the ninth circle.[7] Then if we define our $K3$ surface to be $T^4/\{1,\mathcal{I}_4\}$, the transformation σ on this $K3$ can be identified to $\mathcal{I}_4\cdot\eta$.[8] There are sixteen fixed points of this transformation on T^4, but modding out by $\mathcal{I}_4$ identifies them pairwise. This gives eight fixed points on $K3$ as expected.

Thus our starting point, type IIA on $K3/\{1,(-1)^{F_L}\cdot\sigma\}$, is now represented as type IIA on $T^4/\{1,\mathcal{I}_4,(-1)^{F_L}\cdot\mathcal{I}_4\cdot\eta,(-1)^{F_L}\cdot\eta\}$. The transformation $\mathcal{I}_4$, in the notation of eq.(2), is represented by the transformation given in (3). Let us now make an $R\to 1/R$ duality transformation in the sixth coordinate. This would convert the type IIA theory to type IIB. Using eqs.(3), (4) and (7) we see that the transformation $(-1)^{F_L}$ and $\mathcal{I}_4$ in the type IIA theory are mapped to $(-1)^{F_L}$ and $(-1)^{F_L}\cdot\mathcal{I}_4$ in the type IIB theory respectively. The transformation η remains the same in the type IIB theory, as this represents a shift in the 9th direction whereas the duality transformation is being performed in the 6th direction. This leads us to the conclusion that the theory under consideration is equivalent to type IIB on $T^4/\{1,(-1)^{F_L}\cdot\mathcal{I}_4,\mathcal{I}_4\cdot\eta,(-1)^{F_L}\cdot\eta\}$. Let us now trade in the coordinates Y^m labelling the T^4 of the type IIB theory in favour of new coordinates Z^m defined as

$$Z^m=Y^m \qquad \text{for} \qquad 6\le m\le 8\,, \qquad\qquad Z^9=Y^9+(\pi R/2)\,. \tag{10}$$

In terms of the new coordinates Z^m, the symmetries $\{1,(-1)^{F_L}\cdot\mathcal{I}_4,\mathcal{I}_4\cdot\eta,(-1)^{F_L}\cdot\eta\}$ map onto $\{1,(-1)^{F_L}\cdot\mathcal{I}_4\cdot\eta,\mathcal{I}_4,(-1)^{F_L}\cdot\eta\}$ with η denoting shift of Z^9 by πR. Noting

[6] Incidentally, this provides us with a new class of string compactification with more than one tensor multiplets in six dimensions.

[7] Note that η commutes with $(-1)^{F_L}$ and $\mathcal{I}_4$, since translation of X^9 by $2\pi R$ is an identity transformation on T^4.

[8] This is a slightly different notation from ref.[9].

that $T^4/\{1, \mathcal{I}_4\}$ denotes a $K3$ surface, and furthermore, that the symmetry $\mathcal{I}_4 \cdot \eta$ denotes the transformation σ on this $K3$ surface, we can identify the model as type IIB on $K3/\{1, (-1)^{F_L} \cdot \sigma\}$.

Note that during the course of the manipulation that we have performed in going from the type IIA compactification to the type IIB compactification, the Z_2 transformation $\mathcal{I}_4$ in the type IIA theory that was responsible for creating the $K3$ out of T^4 has been mapped to $\sigma \cdot (-1)^{F_L}$ acting on the $K3$ surface on which the type IIB theory is compactified. On the other hand, the Z_2 transformation that generated $(-1)^{F_L} \cdot \sigma$ on the original $K3$ surface has been mapped to the Z_2 transformation that generates the final $K3$ out of T^4. As a result, viewed as compactification on an orbifold of $K3$, the twisted sector states in the original theory will get mapped to the untwisted sector states of the final theory. On the other hand, the twisted sector states in the final theory will be mapped onto the untwisted sector states of the original theory.

We can now map this model to another model by using the S-duality of the type IIB theory in ten dimension[25]. Under this, the transformation $(-1)^{F_L}$ gets mapped to the world-sheet parity transformation Ω[16], whereas the transformation σ is unchanged. Thus one would expect that the type IIB compactification that we have obtained is also equivalent to type IIB on $K3/\{1, \Omega \cdot \sigma\}$. Note that this implicitly assumes that the S-duality transformation commutes with the orbifolding procedure, and since this S-duality is not part of a T-duality group, we must explicitly check that the spectrum from the twisted sector matches between the two theories. However, the latter model is precisely the one that was analysed by Dabholkar and Park[9] and was shown to have identical spectrum of massless states as M-theory on $(S^1 \times K3)/\{1, \mathcal{J}_1 \cdot \sigma\}$ at a generic point in the moduli space. Thus we see that following this chain of dualities we have been able to 'prove' the equivalence between the M-theory compactification on $(S^1 \times K3)/\{1, \mathcal{J}_1 \cdot \sigma\}$ and type IIB compactification on $K3/\{1, \Omega \cdot \sigma\}$. We have also seen that at one stage in this chain of dualities, the twisted sector states and part of the untwisted sector states got exchanged. This would explain why the eight tensor multiplets that arise from the 'twisted sector' of M-theory on $(S^1 \times K3)/\{1, \mathcal{J}_1 \cdot \sigma\}$ appear in the untwisted sector in type IIB on $K3/\{1, \Omega \cdot \sigma\}$, and the eight vector multiplets that arise in the 'twisted sector' in type IIB on $K3/\{1, \Omega \cdot \sigma\}$ appear in the untwisted sector in M-theory on $(S^1 \times K3)/\{1, \mathcal{J}_1 \cdot \sigma\}$.

M-theory on $T^4/\{1, \mathcal{I}_4\}$: This is a special case of M-theory on $K3$, and so we expect the spectrum of massless states to be identical to that of M-theory on $K3$. Using the relationship between the massless fields in M-theory on S^1 and type IIA theory, we can easily identify the symmetry $\mathcal{I}_4$ in M-theory on T^4 as the symmetry $\mathcal{J}_3 \cdot (-1)^{F_L}$ in the type IIA theory.[9] Thus this particular M-theory compactification is expected to be dual to type IIA on $T^3/\{1, \mathcal{J}_3 \cdot (-1)^{F_L}\}$. This is an orientifold[26] since $\mathcal{J}_3$ involves a world-sheet parity transformation. One could analyse the spectrum of massless states in this theory

[9]One way to see this is to note that upon rewriting T^4 as $S^1 \times T^3$, the transformation $\mathcal{I}_4$ in the M-theory can be regarded as the product $\mathcal{J}_1 \cdot \mathcal{J}_3$. $\mathcal{J}_1$ maps to $(-1)^{F_L}$ in the type IIA theory, whereas $\mathcal{J}_3$ of M-theory is mapped to $\mathcal{J}_3$ of the type IIA theory.

directly, but we can simplify the analysis by making a T-duality transformation that inverts the radii of *all the three circles* of T^3. This converts the type IIA theory to type IIB theory. Using the standard relationship between the massless fields in the two theories it is easy to see that the transformation $\mathcal{J}_3 \cdot (-1)^{F_L}$ gets mapped to the world sheet parity transformation Ω in the type IIB theory. Thus this orbifold compactification is equivalent to type IIB on $T^3/\{1, \Omega\}$. This is nothing but the type I string theory compactified on T^3. Thus we conclude that M-theory on $T^4/\{1, \mathcal{I}_4\}$ is equivalent to type I theory on T^3. Using the conjectured duality between type I and SO(32) heterotic string theory in ten dimensions, this can be further related to heterotic string theory compactified on T^3. The equivalence between M-theory on $K3$ and heterotic string theory on T^3 has already been conjectured in ref.[2].

M-theory on $T^8/\{1, \mathcal{I}_8\}$: The analysis in this case proceeds exactly as in the previous case, leading us to the conclusion that this theory is equivalent to type IIA on $T^7/\{1, \mathcal{J}_7 \cdot (-1)^{F_L}\}$, which, in turn, is equivalent to the type I theory on T^7. In the type I theory, viewed as an orientifold of the type IIB theory, the 'twisted sector states' are open strings living on 32 nine-branes[26] and give rise to SO(32) gauge fields and their superpartners. The duality transformation that maps this to the type IIA theory on $T^7/\{1, (-1)^{F_L} \cdot \mathcal{J}_7\}$ involves inverting the radius of each of the seven circles of T^7, and converts the 32 nine-branes into 32 membranes. At a generic point in the moduli space these 32 membranes can be grouped into 16 pairs, the membranes in each pair being related by the transformation $(-1)^{F_L} \cdot \mathcal{J}_7$. Since the map from the type IIA theory to M-theory on S^1 converts a membrane to a membrane, we see that in the M-theory the twisted sector states live on 16 membranes moving on the internal manifold $T^8/\{1, \mathcal{I}_8\}$ (which would appear as 32 membranes on T^8).[10] Each of these 16 membranes contributes one vector multiplet of the supersymmetry algebra. This is analogous to the situation for M theory on $T^5/\{1, \mathcal{J}_5\}$, where the 'twisted sector states' come from the sixteen five-branes moving on the internal manifold, each of which correspond to one tensor multiplet of the chiral $N = 2$ supersymmetry algebra.

Conclusion: We have seen that in many cases the duality involving M-theory orbifolds may be understood by assuming that duality transformations commute with orbifolding. There are, however, specific examples where this procedure breaks down, notably when one gets extra supersymmetry generators in the 'twisted sector' of one of the theories. It will be extremely interesting to have a systematic understanding of when and why duality transformation commutes with the orbifolding procedure, since this will give us a global understanding of the interconnections between various dualities in string theory.

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[10]This, in turn, implies that each of the 256 fixed points on $T^8/\{1, \mathcal{I}_8\}$ acts as a source of $-1/16$ units of anti-symmetric tensor field charge.

References

[1] P. Townsend, Phys. Lett. **350B** (1995) 184 [hep-th/9501068].

[2] E. Witten, Nucl. Phys. **B443** (1995) 85 [hep-th/9503124].

[3] J. Schwarz, Phys. Lett. **B360** (1995) 13 [hep-th/9508143], Phys. Lett. **B367** (1996) 97 [hep-th/9510086].

[4] P. Aspinwall, hep-th/9508154.

[5] P. Horava and E. Witten, Nucl. Phys. **B460** (1996) 506 [hep-th/9510209].

[6] K. Dasgupta and S. Mukhi, hep-th/9512196.

[7] E. Witten, hep-th/9512219.

[8] A. Sen, hep-th/9602010.

[9] A. Dabholkar and J. Park, hep-th/9602030.

[10] C. Vafa, hep-th/9602022;
D. Morrison and C. Vafa, hep-th/9602114.

[11] M. Duff, J. Minasian and E. Witten, hep-th/9601036.

[12] E. Witten, hep-th/9602070.

[13] N. Seiberg and E. Witten, hep-th/9603003.

[14] A. Kumar and K. Ray, hep-th/9602144.

[15] B. Acharya, hep-th/9603033.

[16] C. Vafa and E. Witten, hep-th/9507050.

[17] S. Kachru and C. Vafa, Nucl. Phys. **B450** (1995) 69 [hep-th/9505105].

[18] S. Ferrara, J. Harvey, A. Strominger and C. Vafa, Phys. Lett. **B361** (1995) 59 [hep-th/9505162].

[19] S. Kachru and E. Silverstein, hep-th/9511228.

[20] V. Nikulin, Trans. Moscow Math. Soc. **2** (1980) 71.

[21] J. Schwarz and A. Sen, Phys. Lett. **B357** (1995) 323 [hep-th/9507027].

[22] S. Chaudhuri and D. Lowe, Nucl. Phys. **B459** (1996) 113 [hep-th/9508144].

[23] A. Sen and C. Vafa, Nucl. Phys. **B455** (1995) 165 [hep-th/9508064].

[24] K. Dasgupta and S. Mukhi, private communication.

[25] C. Hull and P. Townsend, Nucl. Phys. **B438** (1995) 109 [hep-th/9410167].

[26] E. Gimon and J. Polchinski, hep-th/9601038;
J. Polchinski, S. Choudhuri and C. Johnson, hep-th/9602052;
and references therein.