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Descent Relations Among Bosonic D-branes

Ashoke Sen [1]

*Mehta Research Institute of Mathematics
and Mathematical Physics
Chhatnag Road, Jhoosi, Allahabad 211019, INDIA*

Abstract

We show that the tachyonic kink solution on a pair of D-p-branes in the bosonic string theory can be identified with the D-$(p-1)$-brane of the same theory. We also speculate on the possibility of obtaining the D-$(p-1)$-brane as a tachyonic lump on a single D-p-brane. We suggest a possible reinterpretation of the first result which unifies these two apparently different descriptions of the D-$(p-1)$ brane.

[1]E-mail: asen@thwgs.cern.ch, sen@mri.ernet.in

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1 Introduction and Summary

During the last year various relationships between the D-branes of type II and type I string theories have been discovered. In particular, it was found that quite often a D-brane can be realised as a soliton solution associated with the tachyon field on a brane-antibrane pair of higher dimension[1, 2, 3, 4]. This fact has been used to show that the D-branes of type II and type I string theories can be classified by elements of the appropriate K-group of space-time[5, 6, 7, 8]. Various other applications / generalisations of these results have also been proposed[9, 10, 11].

In this paper we shall extend these results to the D-branes of bosonic string theory[12]. This theory contains D-branes of all dimensions, unlike type IIA (IIB) string theory which contains D-branes of even (odd) dimensions only. There is a tachyonic mode even on a single D-brane in the bosonic string theory; but when we bring two parallel D-branes close to each other, we get extra tachyonic modes from open strings stretched between the two D-branes. It is this tachyonic mode which will be of interest to us. The potential involving this tachyonic mode is even; and according to one possible interpretation of our results, it has a (local) minimum at non-zero value of the tachyon field. Thus there are two degenerate minima, and we can have tachyonic kink solution which interpolates between the two minima. We shall show that this kink solution describes a D-brane of one lower dimension. Thus if we start with a pair of D-p-branes, the tachyonic kink solution describes a D-$(p-1)$-brane.

Before we outline the steps involved in our analysis, let us briefly discuss the motivation. Since bosonic string theory itself is unstable due to the presence of the tachyon in the closed string sector; and since the D-branes themselves are also unstable due to the presence of the tachyon in the open string sector, it is natural to question the usefulness of the results of this paper. We propose the following reasons for our study:

- Quite often bosonic string theory provides a simpler setting compared to superstring theories for the study of various issues. Since there are several issues which are ill understood in regarding D-branes as solitons in type II / type I string theories, studying these issues in bosonic string theory might be of help. One such issue involves studying the fate of the diagonal U(1) gauge field living on the brane-antibrane pair after tachyon condensation[9, 5]. Yet another issue is an explicit construction of the soliton / vacuum configuration on the brane-antibrane pair as a classical solution of the open string field theory living on the pair. These issues might be simpler to study in the bosonic string theory.

- On a more speculative side, we would like to point out that in view of recent developments in the subject of string duality, it seems unlikely that the bosonic string theory will forever remain outside the scheme that unifies all string theories as different limits of the same underlying theory. Already some explicit proposals have been made which relate bosonic string theory to other theories with world-sheet supersymmetry[13]. Study of D-branes in the bosonic string theory is certainly going to be important if we are to study such duality relations; although due to lack of supersymmetry our task will be much more difficult.

The steps involved in proving the equivalence of a tachyonic kink solution and a lower dimensional D-brane are very similar to the ones used in [3], where we derived a similar result for type I / type II string theory.[2] However, there is a crucial difference: unlike in [3] where the boundary conformal field theory interpolating between the p-brane pair and the $(p-1)$-brane was a free fermionic field theory, in the present case it corresponds to a level one SU(2) current algebra theory. We now outline the basic idea of the proof:

1. First we compactify one of the directions (x) tangential to the p-brane pair along a circle of radius R, and switch on half a unit of Wilson line along x on one of the p-branes. This makes the open string states with two ends lying on two different

[2] Related issues have been discussed in [14, 15, 16].

branes anti-periodic under $x \to x + 2\pi R$. As a result, the zero x-momentum mode is absent from this open string sector, and at a critical radius R_c, the lowest momentum mode of the tachyon associated with such an open string becomes massless. The critical radius turns out to be half of the self-dual radius where the conformal field theory associated with the scalar field X develops an $SU(2)_L \times SU(2)_R$ current algebra.

2. By exploiting the $SU(2)_L \times SU(2)_R$ current algebra, one can show that the vertex operator associated with the massless mode of the tachyon represents an exactly marginal deformation. Switching on a vacuum expectation value α of this mode gives rise to a solvable boundary conformal field theory (BCFT), so that the spectrum and correlation functions in this BCFT can be calculated for all values of α. In particular we show that at $\alpha = 1$, the BCFT is identical to the one describing a D-$(p-1)$-brane located on a circle of radius R_c (with the directions tangential to the D-$(p-1)$-brane being orthogonal to the circle.) On the other hand, by explicitly examining the form of the tachyon background, we find that it corresponds to a tachyonic kink on a circle of radius R_c. This allows us to identify the tachyonic kink on a circle of radius R_c to a D-$(p-1)$-brane on a circle of radius R_c.

3. In the final step, we increase the radius R back to infinity. Although at $R = R_c$ all values of α are allowed, we find that as soon as R increases beyond R_c, the tachyonic mode which was massless at $R = R_c$ develops a tadpole for a generic value of α. This shows that a generic α is no longer a solution of the equations of motion. However, there are two inequivalent values of α where the tadpole vanishes $-$ $\alpha = 0$ and $\alpha = 1$. If we take the radius back to infinity keeping $\alpha = 0$, we recover the original D-p-brane pair, whereas if we do so at $\alpha = 1$, we expect to recover the tachyonic kink solution. On the other hand, by examining the BCFT corresponding to this configuration, we find that it describes a D-$(p-1)$-brane of the bosonic string theory in 26 dimensional Minkowski space. This proves the identification of the tachyonic kink solution on the D-p-brane pair with the D-$(p-1)$-brane.

The paper is organised as follows. For simplicity of notation we shall consider the case $p = 1$, although generalisation to arbitrary p is completely straightforward. Also we shall work in units where the fundamental string tension is given by $1/(2\pi)$, *i.e.* $\alpha' = 1$. In section 2 we discuss some general properties of a pair of D-branes wrapped on a circle, and carry out the step 1 of our analysis. We also outline in slightly more detail the logic of our analysis in steps 2 and 3. In sections 3 and 4 we explicitly carry out steps 2 and 3 of our analysis. In section 5 we speculate on the possibility of constructing the D0-brane as a tachyonic lump on a single D-string. Again, this result can be easily generalized to the case of a D-p-brane. In section 6 we suggest a possible reinterpretation of the results of sections 3 and 4 which unifies the two apparently different descriptions of the lower dimensional D-brane.

2 Pair of D-strings Wrapped on a Circle

Our starting point will be a coincident pair of D-p-branes in bosonic string theory. The open strings living on this system are described by 2×2 Chan Paton (CP) factors. The massless sector of the open string contains a U(2) gauge field living on the brane world-volume. All open string states transform in the adjoint representation of U(2) $-$ *i.e.* they are neutral under the U(1) and transform in the singlet plus a triplet representation of SU(2). States with CP factor I (the 2×2 identity matrix) are in the singlet representation of SU(2), whereas those with CP factors σ_i (the Pauli matrices) are in the triplet representation of SU(2).

The ground state of the open string in each sector describes a tachyon field with

$$m^2 = -1\,. \tag{2.1}$$

The first excited states correspond to the U(2) gauge fields as well as $(25-p)$ massless scalars in the adjoint representation of U(2). Higher excited states correspond to massive modes. We shall denote the tachyonic state with CP factor σ_1 by T. Let $V(T)$ denote the *classical* effective potential for this tachyon obtained after integrating out the other massive string modes. Since the SU(2) gauge transformation corresponding to the group element $\exp(i\pi\sigma_3/2)$ takes T to $-T$, $V(T)$ must be an even function of T.[3] We shall argue shortly that $V(T)$ has (local) minimum at some values $\pm T_0$ such that

$$V(\pm T_0) + 2\mathcal{T}_p = 0\,, \tag{2.2}$$

where $\mathcal{T}_p$ denotes the tension of the D-p-brane. Thus for $T = \pm T_0$, the total energy density on the brane pair vanishes and the system is indistinguishible from vacuum.

[3] If we also consider the tachyonic modes coming from CP factors σ_2 and σ_3, then there is an SU(2) triplet of tachyon field, and the potential is invariant under the SU(2) transformation.

If $T_0 \neq 0$, we can define the tachyonic kink solution on the D-p-brane pair as a solution of the equations of motion of classical open string field theory, subject to the following conditions:

1. Only those open string fields which carry CP factors I and σ_1 are present as background.

2. The configuration is time independent, as well as independent of $(p-1)$ of the spatial directions along the brane.

3. T depends on the remaining spatial direction $-$ which we shall denote by x $-$ such that

$$
\begin{aligned}
T(x) \quad &\to \quad T_0 \qquad \text{as} \qquad x \to \infty \,, \\
&\to \quad -T_0 \qquad \text{as} \qquad x \to -\infty \,.
\end{aligned}
\tag{2.3}
$$

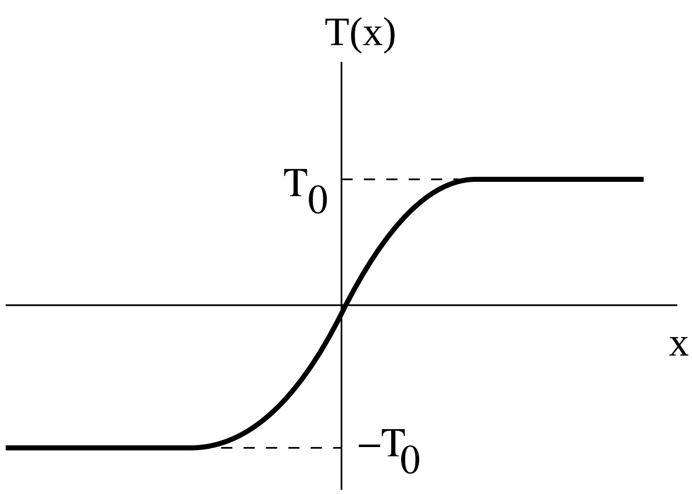

Figure 1: The tachyonic kink on the pair of D-p-branes.

This has been illustrated in Fig.1 and describes a kink. From eqs.(2.2) and (2.3) we see that the energy density vanishes as $x \to \pm\infty$, and is concentrated near the core at $x = 0$. Thus this describes a $(p-1)$ dimensional brane. We shall argue that this kink actually describes the D-$(p-1)$-brane of the bosonic string theory.

Note that if (2.2) had not been true, the tachyonic kink defined this way would have infinite tension when regarded as a $(p-1)$-brane, since the energy density, integrated along x, will not give a finite answer. Thus proving that the tachyonic kink describes the D-$(p-1)$-brane $-$ which is known to have finite tension $-$ automatically proves (2.2).

A somewhat different scenario, which is also consistent with the results of sections 3, 4 will be suggested in section 6. In this description T_0 vanishes and the D-$(p-1)$-brane is regarded as a tachyonic lump on a pair of D-p-branes. The argument leading to the vanishing of $V(T_0)$ is still valid, but in this case the negative contribution to the potential energy, cancelling the tension of the pair of D-p-branes, comes from the vev of the tachyon associated with the CP factor I.[4]

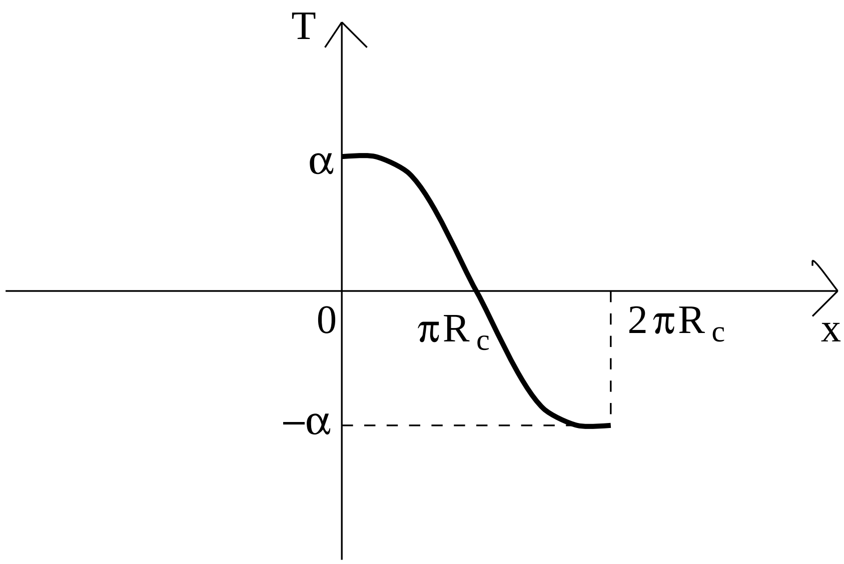

Figure 2: The tachyonic kink on the pair of D-strings on a circle.

From now on we shall focus our attention on the case $p = 1$, and also compactify the direction x tangential to the D-string pair on a circle of radius R. Associated with each D-string is a U(1) gauge field which forms part of the full U(2) gauge group. We now switch on half a unit of Wilson line on one of the D-strings. This breaks the U(2) gauge symmetry to U(1)×U(1). Presence of this Wilson line does not affect the spectrum of

[4]Since $V(T)$ denotes the *effective action* obtained after integrating out the other fields, including the tachyon field from the identity sector, it is automatically minimized with respect to this tachyon.

open string states with CP factors I and σ_3, which represent strings with both ends lying on the same D-string. But for open strings with CP factors σ_1 and σ_2, corresponding to open strings with two ends lying on two different D-strings, the wave function is now required to be anti-periodic instead of periodic under $x \to x + 2\pi R$. In particular the tachyon T coming from CP factor σ_1 now has a mode expansion of the form:

$$T(x) = \sum_{n \in Z} T_{n+\frac{1}{2}} e^{i(n+\frac{1}{2})\frac{x}{R}} . \tag{2.4}$$

The effective mass^2 of $T_{n+\frac{1}{2}}$ is given by:

$$m^2_{n+\frac{1}{2}} = \frac{(n+\frac{1}{2})^2}{R^2} - 1 . \tag{2.5}$$

This shows that at the critical radius

$$R_c = \frac{1}{2} , \tag{2.6}$$

the modes $T_{\pm\frac{1}{2}}$ become massless. In the next section we shall show that the combination

$$S \equiv T_{\frac{1}{2}} + T_{-\frac{1}{2}} \tag{2.7}$$

has vanishing potential as well, so that it represents an exactly marginal deformation of the BCFT describing the D-string pair. We can parametrise a general vacuum expectation value (vev) of this marginal operator as:[5]

$$T_{\frac{1}{2}} + T_{-\frac{1}{2}} = \alpha, \qquad T_{\frac{1}{2}} - T_{-\frac{1}{2}} = 0, \qquad T_r = 0 \quad \text{for} \quad |r| > \frac{1}{2} . \tag{2.8}$$

From (2.4) we see that this corresponds to

$$T(x) = \alpha \cos \frac{x}{2R_c} = \alpha \cos x . \tag{2.9}$$

This has been plotted in Fig.2, and clearly represents a tachyonic kink on a circle of radius $R_c = \frac{1}{2}$.[6]

After switching on the tachyon vev, we would like to take the radius back to infinity. We shall see in section 4 that as soon as we make $R > R_c$, the field S develops a tadpole

[5]Once the interactions are taken into account, the higher modes of the tachyon will also acquire non-zero vev, but $T(x)$ will continue to have the shape of a kink.

[6]It actually represents an anti-kink, but since bosonic D-branes do not carry any charge, there is no distinction between a kink and an anti-kink.

for a generic value of α. This is not surprising since S represents an exactly marginal deformation only for $R = R_c$. However, we find that the tadpole vanishes for two inequivalent values of α, namely, $\alpha = 0$ and $\alpha = 1$. Thus it is natural to identify the $\alpha = 1$ configuration as the tachyonic kink on a circle of radius R. We show in section 4 that the BCFT corresponding to this configuration is identical to the one describing a D-particle on a circle of radius R. This equivalence continues to hold in the $R \to \infty$ limit as well.

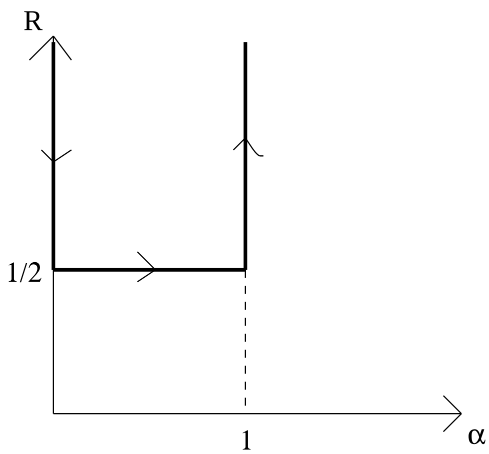

Figure 3: The marginal flow in the $R - \alpha$ plane interpolating between a pair of D-strings and a D-particle.

Thus we see that there is a series of marginal deformations which relate a pair of D-strings to the D-particle. In the $R - \alpha$ plane, this marginal flow has been depicted in Fig.3.

As a consistency check for this proposal, let us note that the mass of the wrapped pair of D-strings on a circle of radius $\frac{1}{2}$ is given by,

$$2 \cdot 2\pi \cdot \frac{1}{2} \cdot \mathcal{T}_1 \, . \tag{2.10}$$

Using the relation[12]

$$\mathcal{T}_p = \frac{1}{2\pi} \mathcal{T}_{p-1} \, , \tag{2.11}$$

wee see that (2.10) is identical to the mass $\mathcal{T}_0$ of a single D-particle.

3 CFT at $R = \frac{1}{2}$

The world-sheet field that will be important for our analysis is the scalar field X associated with the coordinate along the D-string. Besides this, there are 25 other scalar fields $X^0, \ldots X^{24}$, and the ghost fields $b, c, \bar{b}, \bar{c}$. Although X has radius $\frac{1}{2}$, it will be more convenient for our analysis to regard this as a scalar field of radius 1, and then mod out the theory by the transformation:

$$h_X : X \to X + \pi \tag{3.1}$$

All closed string states, as well as open string states with CP factors I and σ_3 are required to be even under h_X, whereas open string states with CP factors σ_1 and σ_2 are required to be odd under h_X.

We shall express X as a sum of the left and the right-moving parts:

$$X \equiv X_L + X_R\,, \tag{3.2}$$

with h_X acting on X_L and X_R as

$$h_X : \quad X_L \to X_L + \frac{\pi}{2}, \quad X_R \to X_R + \frac{\pi}{2}\,. \tag{3.3}$$

At unit radius, the CFT describing X possesses an $\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R$ current algebra. This allows us to introduce two other bosons:

$$\phi \equiv \phi_L + \phi_R, \qquad \phi' \equiv \phi'_L + \phi'_R\,, \tag{3.4}$$

through the relations:

$$e^{2iX_L} = \partial\phi_L + i\partial\phi'_L, \qquad e^{2iX_R} = \partial\phi_R + i\partial\phi'_R\,. \tag{3.5}$$

The set of left-moving currents $(\partial X_L, \partial\phi_L, \partial\phi'_L)$ transform as a triplet of $\mathrm{SU}(2)_L$; similarly the right-moving currents transform as a triplet of $\mathrm{SU}(2)_R$. From (3.5) we can also write down the SU(2) rotated version of these relations:[7]

$$\begin{aligned} e^{2i\phi_L} &= \partial X_L - i\partial\phi'_L, \qquad e^{2i\phi_R} = \partial X_R - i\partial\phi'_R, \\ e^{2i\phi'_L} &= \partial X_L + i\partial\phi_L, \qquad e^{2i\phi'_R} = \partial X_R + i\partial\phi_R\,. \end{aligned} \tag{3.6}$$

[7]There is some freedom in writing down these relations, since eq.(3.5) only defines ϕ, ϕ' up to a constant shift.

We have already introduced the transformation h_X. We now introduce a new transformation:

$$g_X: \quad X \to -X\,. \tag{3.7}$$

We also introduce the transformations g_ϕ, h_ϕ, $g_{\phi'}$ and $h_{\phi'}$ in an identical manner. Then, using eqs.(3.5), (3.6) we see that,

$$h_X = g_\phi = g_{\phi'}, \qquad g_X = h_\phi = h_{\phi'} g_{\phi'}\,. \tag{3.8}$$

We shall use the convention where the open string world-sheet corresponds to the upper half plane spanned by a complex coordinate z, and the right-moving currents are holomorphic in z. The operator product expansion of the currents are given by,

$$\partial X_R(z)\partial X_R(w) \simeq \partial\phi_R(z)\partial\phi_R(w) \simeq \partial\phi'_R(z)\partial\phi'_R(w) \simeq -\frac{1}{2(z-w)^2}\,, \tag{3.9}$$

where $\simeq$ denotes equality up to non-singular terms. There are similar relations involving the left-moving currents.

Since both the D-strings are stretched along x, we impose Neumann boundary condition on X:

$$(X_L)_B = (X_R)_B \equiv \frac{1}{2} X_B\,, \tag{3.10}$$

where the subscript B denotes boundary value. Using eq.(3.5) this translates into Neumann boundary condition on ϕ and ϕ':

$$(\phi_L)_B = (\phi_R)_B \equiv \frac{1}{2}\phi_B, \qquad (\phi'_L)_B = (\phi'_R)_B \equiv \frac{1}{2}\phi'_B\,. \tag{3.11}$$

The vertex operator for the mode $T_{n+\frac{1}{2}}$ defined in (2.4) is given by:

$$V_{n+\frac{1}{2}} = i e^{2i(n+\frac{1}{2})X_B} \otimes \sigma_1\,. \tag{3.12}$$

Thus the vertex operator for $S \equiv (T_{\frac{1}{2}} + T_{-\frac{1}{2}})$ is given by:

$$V_S = i(e^{iX_B} + e^{-iX_B}) \otimes \sigma_1 = i\partial\phi_B \otimes \sigma_1\,. \tag{3.13}$$

In deriving the right hand side of (3.13) we have used eq.(3.5). At this stage, the overall normalization of V_S is arbitrary. From (3.13) we see that V_S represents the vertex operator of a zero momentum gauge field $\mathcal{A}_\phi$ along ϕ with CP factor σ_1, – in other words a Wilson line along ϕ. This clearly is an exactly marginal deformation. We parametrize it by a parameter α, normalized such that it corresponds to inserting an operator

$$\exp(i\frac{\alpha}{4}\sigma_1 \oint \partial_t \phi_B dt)\,, \tag{3.14}$$

on the boundary of the world-sheet. t denotes a parameter along this boundary.

We shall now study the effect of switching on this Wilson line on the open string states. Using the relations

$$[\sigma_1, I] = 0 = [\sigma_1, \sigma_1]\,, \tag{3.15}$$

we see that open string states with CP factors I and σ_1 are neutral under this gauge field, and hence the spectrum of these open string states is not modified upon switching on the tachyon vev. On the other hand, since

$$[\sigma_1, \sigma_3 \mp i\sigma_2] = \pm 2(\sigma_3 \mp i\sigma_2)\,, \tag{3.16}$$

open string states in these sectors carry charge ± 2 under $\mathcal{A}_\phi$. Thus the momentum p_ϕ along the ϕ direction gets shifted;

$$p_\phi \to p_\phi \pm \frac{\alpha}{2}\,. \tag{3.17}$$

Eq.(3.17) can be restated by saying that the h_ϕ quantum number of the state is multiplied by a factor:

$$\exp(\pm i\pi\frac{\alpha}{2})\,. \tag{3.18}$$

Using the identification of h_ϕ with g_X (eq.(3.8)) and the fact that the open string spectrum has no g_X projection, we see that before switching on the tachyon vev, the open string states in each CP factor contains both h_ϕ even and h_ϕ odd states. If $\alpha = 2$, then, as seen from eq.(3.18), the h_ϕ quantum numbers get multiplied by -1. But this means that the complete spectrum of open strings in each CP sector remains unchanged. Thus the BCFT at $\alpha = 2$ is equivalent to that at $\alpha = 0$, and we conclude that α is a periodic variable with period 2.

Although at the critical radius $R = R_c$, all values of α describe consistent BCFT, in anticipation of the results of the next section let us pay special attention to the spectrum at $\alpha = 1$. We begin by tabulating the h_ϕ and g_ϕ eigenvalues of various open string states at $\alpha = 0$:

CP factors	$g_X = h_\phi$	$h_X = g_\phi$
I	± 1	1
σ_1	± 1	-1
σ_2	± 1	-1
σ_3	± 1	1

Table 1: Spectrum at $\alpha = 0$

For $\alpha = 1$, the quantum numbers in sectors I and σ_1 remain unchanged, but the h_ϕ eigenvalues in sectors $\sigma_3 \mp i\sigma_2$ get multiplied by $\pm i$. Thus the above table is modified to:

CP factors	$h_\phi = h_{\phi'} g_{\phi'}$	$g_\phi = g_{\phi'}$
I	± 1	1
σ_1	$\pm\, 1$	-1
σ_2	$\pm i$	-1
σ_3	$\pm i$	1

Table 2: Spectrum at $\alpha = 1$

Combining the open string spectrum from all sectors we see that *in the Fock space*,

- there is no $g_{\phi'}$ projection, and
- all ϕ' momentum of the form:

$$p_{\phi'} = \frac{n}{2}, \qquad n \in Z\,, \tag{3.19}$$

  are allowed.

Let us define a new field ϕ'' related to ϕ' by T-duality transformation:

$$\phi''_L = \phi'_L, \qquad \phi''_R = -\phi'_R\,, \tag{3.20}$$

If $w_{\phi''}$ denotes the winding charge along ϕ'' (defined as $\Delta\phi''/2\pi$), and $g_{\phi''}$ denotes the transformation $\phi'' \to -\phi''$, then we have the relations:

$$p_{\phi'} = w_{\phi''}, \qquad g_{\phi'} = g_{\phi''}\,. \tag{3.21}$$

We now note that in ϕ'' coordinate,

1. The boundary condition (3.11) takes the form:

$$(\phi''_L)_B = -(\phi''_R)_B\,. \tag{3.22}$$

   In other words, we have Dirichlet boundary condition along ϕ''.

2. The full spectrum of open strings from all CP sectors has no $g_{\phi''}$ projection in the Fock space.

3. The ϕ'' winding charge is quantized as:

$$w_{\phi''} = \frac{n}{2}, \qquad n \in Z\,. \tag{3.23}$$

This is precisely the spectrum of open strings living on a single D-particle on a circle of radius $\frac{1}{2}$.

One can ask whether the interaction among these open strings is also identical to that among open strings living on a D-particle. The difference between the interaction rules in the two theories comes from the CP factors. In the BCFT under consideration various open string vertex operators are accompanied by CP factors, and there could be extra selection rules, as only those products of CP factors with non-vanishing trace will give non-zero amplitude. This requires, for example, that either each σ_i come in pairs, or they appear in the combination $\sigma_1\sigma_2\sigma_3$. But upon examining the $h_{\phi'}$ and $g_{\phi'}$ quantum numbers carried by these states, we see that these selection rules are automatically imposed by $h_{\phi'}$ and $g_{\phi'}$ conservation laws, which are present also for the open string living on D-particle on a circle. This shows that non only the spectrum, but also the correlation functions of the BCFT obtained here agrees with those in the BCFT describing D-particle on a circle of radius $\frac{1}{2}$.

This shows the equivalence between the tachyonic kink on a pair of D-strings on a circle of radius $\frac{1}{2}$, and the D-particle on a circle of the same radius. In the next section we shall show that this identification of the two boundary conformal field theories persists even when we increase the radius back to ∞.

4 Taking the Radius Back to ∞

The analysis in this section will follow closely that of ref.[3]. The effect of increasing the radius is achieved by perturbing by the closed string vertex operator:

$$\int d^2z \partial X_L \partial X_R \equiv \int d^2z V_r\,. \tag{4.1}$$

First we shall show that even at first order in $(R - R_c)$, the field S develops a tadpole at a generic value of α. This one point function is proportional to the two point function at

$R = R_c$ of the open string vertex operator V_S and the closed string vertex operator V_r in the presence of the tachyonic background parametrised by α. This is given by[8]

$$\langle V_S V_r \rangle_\alpha \propto Tr_{CP} \Big[\Big\langle \partial X_L \partial X_R(P) \partial\phi_B(Q) \otimes \sigma_1 \exp(i\frac{\alpha}{4}\sigma_1 \oint \partial_t \phi_B dt) \Big\rangle \Big] . \tag{4.2}$$

In the left hand side of this equation $\langle \ \rangle_\alpha$ denotes the correlation function in the presence of the tachyonic background. In the right hand side $\langle \ \rangle$ denotes the correlation function at $\alpha = 0$; the effect of the tachyonic background has been taken into account by explicitly putting the exponential factor inside the correlator. P denotes a point in the interior of the world-sheet, Q denotes a point on the boundary, and Tr_{CP} denotes trace over the Chan Paton factors. By explicitly carrying out the trace over the CP factors, and using the relations (3.6), we can rewrite the right hand side of (4.2) as

$$\frac{1}{2} \Big\langle \sin(\frac{\alpha}{2}\pi w_\phi) \Big(e^{2i(\phi_L+\phi_R)} + e^{-2i(\phi_L+\phi_R)} + e^{2i(\phi_L-\phi_R)} + e^{-2i(\phi_L-\phi_R)} \Big)(P) \partial\phi_B(Q) \Big\rangle , \tag{4.3}$$

where,

$$w_\phi = \frac{1}{2\pi} \oint \partial_t \phi_B dt . \tag{4.4}$$

w_ϕ measures the total ϕ winding charge carried by the closed string vertex operators inserted in the interior. Since $\exp(\pm 2i(\phi_L + \phi_R))$ has $w_\phi = 0$, whereas $\exp(\pm 2i(\phi_L - \phi_R))$ has $w_\phi = \pm 2$, we can rewrite (4.3) as

$$\frac{1}{2} (\sin \alpha\pi) \Big\langle \Big(e^{2i(\phi_L-\phi_R)} - e^{-2i(\phi_L-\phi_R)} \Big)(P) \partial\phi_B(Q) \Big\rangle . \tag{4.5}$$

Using the fact that ϕ satisfies Neumann boundary condition, one can easily show that the correlation function appearing in (4.5) is non-zero. This shows that the one point function of S for $R > R_c$ is non-zero. It vanishes at $\alpha = 0$ and at $\alpha = 1$, as stated earlier.

We shall now focus our attention on the $\alpha = 1$ point, and analyse a general correlation function of open string vertex operators at a general radius R. This would require summing over arbitrary number of insertions of V_r with appropriate weight factors. Thus a typical correlation function to be analysed has the form:

$$\Big\langle \prod_i V^i_{open}(Q_i) \prod_m V_r(P_m) \exp\Big(i\frac{\pi}{2} w_\phi \sigma_1\Big) \Big\rangle , \tag{4.6}$$

[8]Ghost number conservation requires that we also insert appropriate number of ghost fields in the vertex operators. But the correlation function factors into a matter part and the ghost part, and non-trivial information comes from analysing the matter part. Hence we focus on the matter part of the correlator.

where w_ϕ has been defined in eq.(4.4). Here Q_i are points on the boundary of the world-sheet and P_m are points in the interior. The effect of the exponential factor on the open string vertex operators is to shift their ϕ momentum, which has already been taken into account in the previous section. Thus we can interprete w_ϕ appearing in (4.6) as the sum of the ϕ winding charges of all the closed string vertex operators inserted in the interior. Expressing V_r as

$$V_r = \frac{1}{4}\Big(e^{2i(\phi_L+\phi_R)} + e^{-2i(\phi_L+\phi_R)} + e^{2i(\phi_L-\phi_R)} + e^{-2i(\phi_L-\phi_R)}\Big)\,, \tag{4.7}$$

we see that the first two terms carry $w_\phi = 0$ whereas the third and the fourth terms carry $w_\phi = \pm 2$. Since all terms have w_ϕ even, we have the relation:

$$\exp(i\frac{\pi}{2}w_\phi\sigma_1) = (-1)^{w_\phi/2}\,. \tag{4.8}$$

This transforms V_r given in (4.7) to

$$\frac{1}{4}\Big(e^{2i(\phi_L+\phi_R)} + e^{-2i(\phi_L+\phi_R)} - e^{2i(\phi_L-\phi_R)} - e^{-2i(\phi_L-\phi_R)}\Big) = -\partial\phi'_L\partial\phi'_R = \partial\phi''_L\partial\phi''_R\,, \tag{4.9}$$

where ϕ'' has been defined in eq.(3.20). Perturbing by the operator $\partial\phi''_L\partial\phi''_R$ has the effect of increasing the ϕ'' radius (or, equivalently, decreasing the ϕ' radius). Thus we see that the effect of increasing R in the presence of a tachyon background is achieved by increasing the ϕ'' radius in the same proportion, and ignoring the tachyon background. Since for $R = R_c = \frac{1}{2}$, the open string spectrum corresponds to a D-particle on the ϕ'' circle of radius $\frac{1}{2}$, we see that if we increase R to $\lambda\cdot\frac{1}{2}$, this would correspond to a D-particle on the ϕ'' circle of radius $\lambda\cdot\frac{1}{2} = R$. As $R\to\infty$, this corresponds to a D-particle in the (25+1) dimensional Minkowski space.

This proves the equivalence of the BCFT describing the tachyonic kink solution on a pair of D-strings, and that describing a D-particle. Note that the marginal deformation interpolating between the two configurations does not involve the fields $X^0,\ldots X^{24}$ at any step. Thus by putting Neumann boundary condition on $(p-1)$ of the fields $X^1,\ldots X^{24}$ we can make our initial configuration into a pair of D-p-branes, and the final configuration into a D-$(p-1)$-brane.

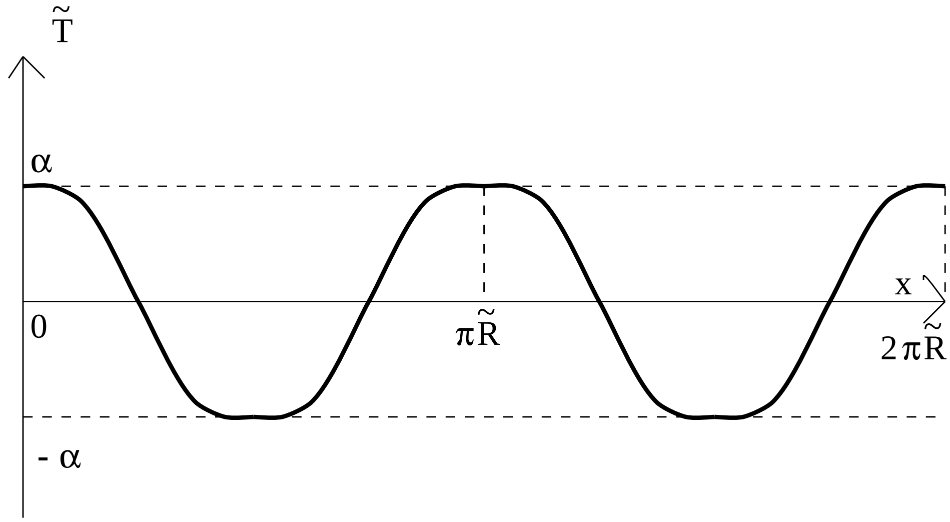

Figure 4: The tachyon field on the D-string which produces a pair of D-particles.

5 D-particle as a Tachyonic Lump on a Single D-string?

Following ref.[4], we can study a T-dual version of our analysis.[9] In this case the massless mode of the tachyon interpolates between a pair of D-particles situated at diametrically opposite points on a circle of radius $\widetilde{R} = 2$, and a single D-string wrapped on a circle of the same radius. Running the marginal flow backwards, we can conclude that tachyon condensation on a D-string on a circle of radius $\widetilde{R}$ produces a pair of D-particles at diametrically opposite points on the same circle. From the point of view of the D-string, the particular mode which condenses corresponds to:

$$\widetilde{T}(x) = \tilde{\alpha}\cos x = \tilde{\alpha}\cos\frac{2x}{\widetilde{R}}\,, \tag{5.1}$$

where $\widetilde{T}$ is the tachyonic field living on the D-string. This has been plotted in Fig.4. This can be viewed as a pair of lumps on the circle, one spanning the range $0 \le x \le \pi\widetilde{R}$, and the other spanning the range $\pi\widetilde{R} \le x \le 2\pi\widetilde{R}$. Although this configuration is symmetric under $\widetilde{T} \to -\widetilde{T}$ (together with a translation along x), this symmetry will be destroyed once interactions are taken into account.

[9] Many of the results of this section have been discussed earlier in refs.[14, 15, 16].

This suggests that we can identify a single D-particle in 26 dimensional Minkowski space as a tachyonic lump on an infinite D-string as shown in Fig.5. In this figure $\widetilde{T}_0$ denotes the minimum of the tachyonic potential $\widetilde{V}(\widetilde{T})$ satisfying,

$$\widetilde{V}(\widetilde{T}_0) + \mathcal{T}_1 = 0\,. \tag{5.2}$$

This relation guarantees that the lump has a finite mass.

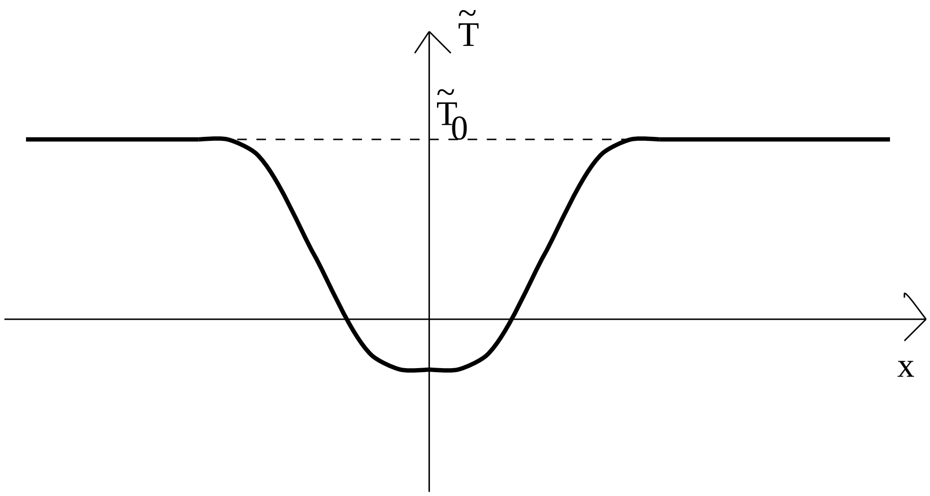

Figure 5: The tachyonic lump on a D-string, representing a single D-particle.

This analysis can be generalised by starting from a D-string compactified on a circle of radius n for any integer n (including 1), and condensing the mode proportional to $\cos x$. This will produce n lumps on the circle, which can be identified as n uniformly spaced D-particles on a circle of radius n[14, 15, 16].

6 Reconciling the Two Different Descriptions of D-branes

The analysis of the previous sections suggests two different ways of viewing a D0-brane (D-($p-1$)-brane), – as a kink on a D-string anti-D-string (D-p-brane anti-D-p-brane) pair, and also as a lump on a single D-string (D-p-brane). In this section we shall show that we can reinterprete the results of sections 3 and 4 in such a way that the second description can explain all the results of this paper. To do this let us denote by $f_L(x)$ the function shown in Fig.5. Thus on a single D1-brane, the configuration

$$\widetilde{T}(x) = f_L(x) \tag{6.1}$$

denotes a D0-brane. On the other hand, $\widetilde{T} = \widetilde{T}_0$ denotes the vacuum solution. Thus if $\widehat{T}$ denotes the 2×2 matrix valued tachyon field on a pair of coincident D-branes, then the configuration:

$$\widehat{T}(x) = \begin{pmatrix} \widetilde{T}_0 & \\ & f_L(x) \end{pmatrix} \tag{6.2}$$

will denote a single D0-brane. (6.2) can be rewritten as,

$$\widehat{T}(x) = \frac{1}{2}(f_L(x) + \widetilde{T}_0)I + \frac{1}{2}(\widetilde{T}_0 - f_L(x))\sigma_3\,. \tag{6.3}$$

Finally, using the U(2) gauge symmetry on the coincident D-string pair, we can replace σ_3 in eq.(6.2) by σ_1. This gives,

$$\widehat{T}(x) = \frac{1}{2}(f_L(x) + \widetilde{T}_0)I + \frac{1}{2}(\widetilde{T}_0 - f_L(x))\sigma_1\,. \tag{6.4}$$

The coefficient of σ_1 is the tachyon field $T(x)$ of sections 2-4. From Fig.5 we see that this has the form given in Fig.6. This appears to be a lump rather than a kink, as suggested by the analysis of sections 4 and 5.

We shall now show that it is in principle possible to reinterprete the tachyonic configuration of sections 4, 5 to be a lump rather than a kink. For this we need to note that Fig.2 can be redrawn as Fig.7 by changing the range of x parametrizing S^1 from $(0, 2\pi R_c)$ to $(-\pi R_c, \pi R_c)$. In this form the tachyon configuration does appear as a lump rather than a kink. The main question is: as we take the $R \to \infty$ limit, does most of the circle get covered by the configuration $T \simeq \pm T_0$ for some $T_0 \neq 0$ (shown in Fig.1), or does it get covered by the configuration $T \simeq 0$ (as shown in Fig.8)? In the first case the configuration is to be interpreted as a kink, whereas in the second case it is to be interpreted as a lump as shown in Fig.6.

Note that in the second case $T(x) \to 0$ as $x \to \pm\infty$. In the case of the D-string - anti-D-string pair of type IIB string theory this was unacceptable, since for $T = 0$ the energy per unit length of the pair is finite, and hence the resulting solution would not represent a localised lump of energy. In the present case however there is a tachyon from the identity sector as well, and condensation of this tachyon can make the energy density

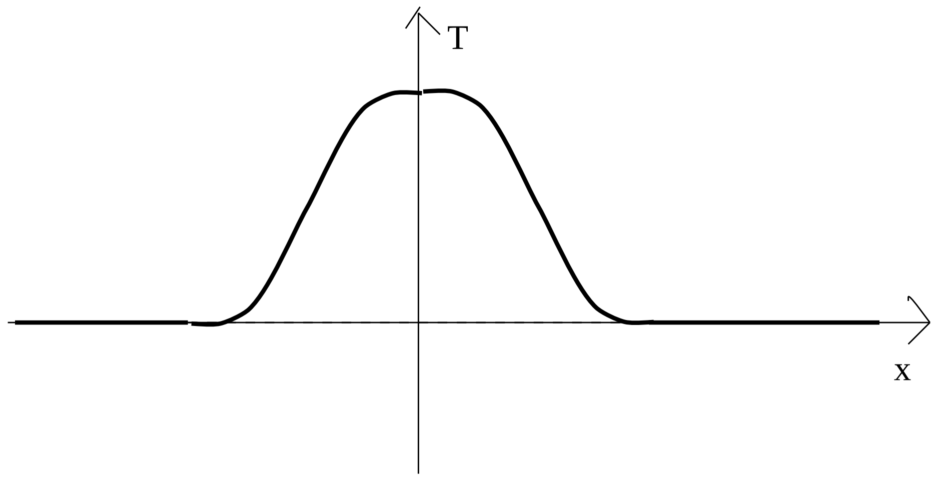

Figure 6: Coefficient of σ_1 in eq.(6.4).

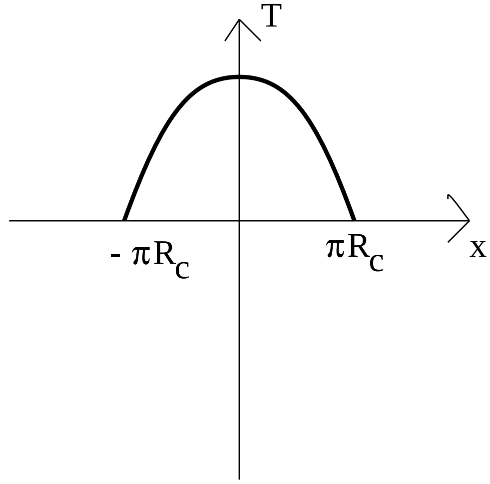

Figure 7: A redrawing of the configuration of Fig.2.

vanish far away from the core even though the tachyon associated with the CP factor σ_1 has no vev in this region. Indeed, this is precisely what happens for the configuration of eq.(6.4). Thus if we reinterprete the results of section 3 and 4 this way, all the results of this paper would be consistent with the hypothesis that the tachyonic lump on a single

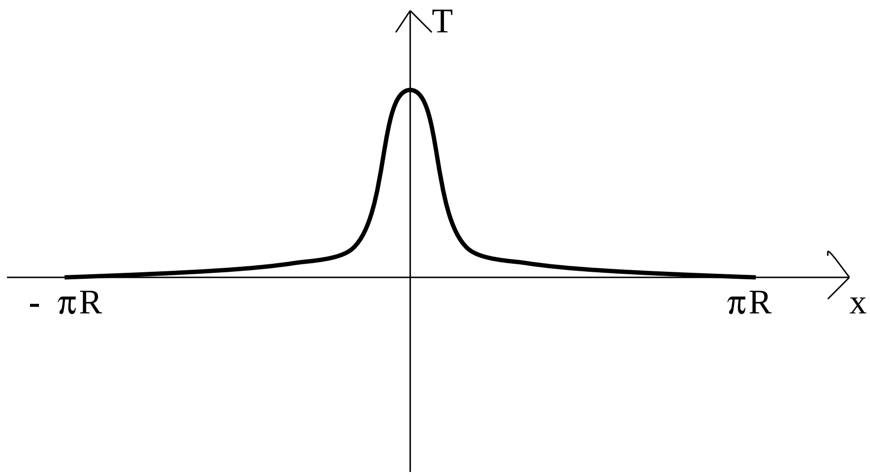

Figure 8: A possible $R \to \infty$ limit of the configuration of Fig.7. This provides an alternative to the possibility shown in Fig.1.

D-string represents a D0-brane.

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