hep-th/0002211
CTP-MIT-2951
IFT-P.018/2000
MRI-PHY/P20000203

Tachyon Condensation in Superstring Field Theory

Nathan Berkovits[a], Ashoke Sen[b] and Barton Zwiebach[c]

[a] *Instituto de Física Teórica, Universidade Estadual Paulista*
Rua Pamplona 145, 01405-900, São Paulo, SP, BRASIL
E-mail: nberkovi@ift.unesp.br

[b] *Mehta Research Institute of Mathematics and Mathematical Physics*
Chhatnag Road, Jhoosi, Allahabad 211019, INDIA
E-mail: asen@thwgs.cern.ch, sen@mri.ernet.in

[c] *Center for Theoretical Physics, Massachussetts Institute of Technology*
Cambridge, MA 02139, USA
E-mail: zwiebach@mitlns.mit.edu

Abstract

It has been conjectured that at the stationary point of the tachyon potential for the D-brane-anti-D-brane pair or for the non-BPS D-brane of superstring theories, the negative energy density cancels the brane tensions. We study this conjecture using a Wess-Zumino-Witten-like open superstring field theory free of contact term divergences and recently shown to give 60% of the vacuum energy by condensation of the tachyon field alone. While the action is non-polynomial, the multiscalar tachyon potential to any fixed level involves only a finite number of interactions. We compute this potential to level three, obtaining 85% of the expected vacuum energy, a result consistent with convergence that can also be viewed as a successful test of the string field theory. The resulting effective tachyon potential is bounded below and has two degenerate global minima. We calculate the energy density of the kink solution interpolating between these minima finding good agreement with the tension of the D-brane of one lower dimension.

Contents

1 Introduction

The spectrum of open strings living on a D-brane anti-D-brane pair of type IIA or IIB string theory contains a pair of tachyonic modes from the Neveu-Schwarz (NS) sector, indicating that the system is unstable [1, 2]. There are general arguments [3, 4, 5] which indicate that the tachyonic potential has a minimum, and this minimum represents the usual vacuum of the closed string theory without any D-brane.[1] For this to be true, the negative energy density contribution from the tachyon potential at the minimum must exactly cancel the sum of the tensions of the brane-antibrane pair.

[1]Some of these arguments used earlier gauge theory analysis of brane-antibrane systems, see [6, 7].

During the last few years it has been realised that type IIA (IIB) string theory contains unstable non-BPS Dp-branes for odd (even) p [8, 5, 9, 10, 11, 12]. Any of these D-branes has a tachyonic mode, indicating that the brane is unstable. A straightforward extension of the general arguments for the brane-antibrane system can be used to argue that the tachyonic potential has a minimum, and this minimum represents the usual vacuum of the closed string theory without any D-brane. For this to be true, the negative energy density contribution from the tachyon potential at this minimum must exactly cancel the tension of the non-BPS D-brane.

A similar conjecture exists also for D-branes of bosonic string theory [13, 14]. On any bosonic string D-brane there is a tachyonic open string mode. Indirect arguments, similar to those for the brane-antibrane pair of type II string theories, indicate that the tachyon potential has an extremum whose negative energy density contribution cancels the tension of the D-brane, so that this particular extremum represents the vacuum of closed bosonic string theory without any D-branes. In refs.[15, 16] this phenomenon was studied directly in open bosonic string field theory [17], following earlier work of ref.[18]. Using the level truncation scheme of ref.[18], ref.[16] showed that including scalars up to level four, the value of the potential at the extremum cancels almost 99% of the D-brane tension. This is a strong indication that this extremum indeed represents vacuum without D-branes. This remarkable cancellation has now been verified to an accuracy of 99.9% by including scalars up to level ten [19]. Evidence to the validity of the level expansion was recently given in [20]. Further evidence for the identification of the tachyonic vacuum has been found in ref.[21] who considered tachyonic lump solutions of the string equations of motion.

In a recent paper [22], the zeroth order contribution to the tachyon potential on a non-BPS D-brane of type II string theory was computed using the open string field theory action formulated in refs.[23, 24], and was found to contain a minimum at which the potential cancels 60% of the D-brane tension. Unlike the cubic action proposed in [25], the Wess-Zumino-Witten-like action used in ref.[22] has no problems with contact term divergences [26].[2] Although it is not known how to include Ramond (R) sector states into this action in a manifestly SO(9,1) covariant manner, this is not a problem here since the phenomenon of tachyon condensation involves NS sector states only. Of course, it involves the full unprojected NS sector, namely both GSO(+) and GSO(−) states. While there is

[2]An early attempt at generalizing the analysis of ref.[18] to superstring theory was made in ref.[27].

a superspace version of the Wess-Zumino-Witten-like action which is manifestly SO(3,1) super-Poincaré invariant and includes all GSO(+) states in the NS and R sectors, it is not yet known how to incorporate GSO(−) states into it.

In this paper, we compute the first-order correction to the tachyon potential on a non-BPS D-brane of type II string theory using the same action as [22], and find a minimum of the potential at which 85% of the D-brane tension is cancelled by the potential. This provides strong evidence that the approximation scheme is converging as in the bosonic string computation and that the exact tachyon potential has a minimum where the D-brane tension is exactly cancelled. Alternatively, this result can also be viewed as a successful test of the correctness of this superstring field theory action.

Although we carry out the explicit analysis for the non-BPS D-brane, the result also holds for the brane-antibrane system. Indeed, the tachyon potential on the non-BPS D-brane of type IIA (IIB) string theory can be obtained from the tachyon potential on a brane-antibrane system of type IIB (IIA) string theory after restricting the field configuration to a Z_2 invariant subspace[10, 12]; so the existence of a minimum of the tachyon potential for a non-BPS D-brane corresponding to the vacuum without D-branes also establishes the corresponding result for the brane-antibrane system. This can also be made self-evident by comparing the structure of the string field theory action on the non-BPS D-brane and that on a brane-antibrane pair, both of which we write down explicitly.

Since the tachyon potential on a non-BPS D-brane is invariant under a change of sign of the tachyon field, there are doubly degenerate minima of the potential, and we can construct a kink solution interpolating between these two minima. It has been conjectured that this represents a BPS brane of one lower dimension[10, 11] following a similar conjecture for the brane-antibrane system[3, 5]. We compute numerically the energy density of the kink solution using the tachyon potential, but ignoring string field theory corrections to the tachyon kinetic term, in the same spirit as in a recent paper for bosonic string field theory [21]. The result is 1.03 times the expected answer. Although such a close agreement is likely to be accidental, it is encouraging to note that the mass of the kink even in this crude approximation has the correct order of magnitude. We should also note that the effect of the non-zero tachyon background should be to reduce the kinetic term, since at the minimum of the potential the kinetic term is expected to vanish, so that we have no physical excitations. Thus we expect that once we take into account corrections to the kinetic term, the energy of the kink should be lowered. In this context, it is also encouraging to note that in the analysis of ref.[21] the mass of the lump *decreased* after taking into account corrections to the tachyon kinetic energy term.

The paper is organised as follows. In section 2 of this paper, we shall review this WZW-like action, discuss in detail its basic ingredients, its gauge invariance, and its application to describe the non-BPS D-brane as well as the brane anti-brane system. In section 3, we shall use the action to compute the zeroth and first order contributions to the tachyon potential and show that the potential has a minimum at 85% of the D-brane tension. In section 4 we discuss the tachyonic kink solution and calculate its mass. We offer some perspective on our results and discuss open questions in section 5. Important details have been provided in the appendices. Appendix A establishes the cyclicity properties of the amplitudes appearing in the string action– this cyclicity is essential for gauge invariance. Appendix B explains the twist properties of the amplitudes– such properties allows us to restrict the multiscalar tachyon field (the space $\mathcal{H}_1$ defined in section 3) to the twist even sector. Appendix C gives a self-contained derivation of the mass of the D-brane described by a string field theory action. Finally, in appendix D we provide details on the computation of the tachyon potential.

2 Open superstring field theory

In this section we shall explain and analyze the superstring field theory that describes the dynamics of a non-BPS D-brane of type II string theory. As it will be clear, this string field theory is readily modified to discuss the D-brane anti-D-brane system in superstring theory. In fact, the same calculations give the tachyon potential for both physical situations. As in the case of refs.[15, 16, 22], we shall not restrict our analysis to any specific background, but will assume, for convenience, that all the directions tangential to the D-brane are compact, so that the system has a finite mass.

We will begin by discussing the GSO projected, or GSO(+) sector of the open superstring theory formulated in refs.[23, 24], $-$ this would describe the dynamics of NS sector open strings living on a single BPS D-brane. Here the *basic structure* of the theory will be elaborated. Then we turn to the non-BPS D-brane whose formulation requires incorporating both the GSO($-$) sector and the GSO(+) sector of the theory. This can be done by attaching *internal* Chan-Paton matrices to the GSO plus and minus sectors in such a way that the complete string field and the relevant operators satisfy the basic structure of the original GSO(+) theory. This device was used in [22] for the analysis of the non-BPS D-brane. Finally, in the last subsection we show how, in addition to the internal Chan-Paton matrices, *external* Chan-Paton matrices must be tensored to describe the brane-antibrane system.

2.1 Superstring field theory on a BPS D-brane

In the formalism of refs.[23, 24, 22], a general off-shell string field configuration in the GSO(+) NS sector corresponds to a *Grassmann even* open string vertex operator Φ of ghost number 0 and picture number[28] 0 in the combined conformal field theory of a $c = 15$ superconformal matter system, and the b, c, β, γ ghost system with $c = -15$. In terms of the bosonized ghost fields ξ, η, ϕ related to β, γ through the relations

$$\beta = \partial\xi e^{-\phi}, \qquad \gamma = \eta e^{\phi}, \tag{2.1}$$

the ghost number (n_g) and the picture number (n_p) assignments are as follows:

$$\begin{aligned} &b: \quad n_g = -1, n_p = 0, \qquad c: \quad n_g = 1, n_p = 0\,, \\ &e^{q\phi}: \quad n_g = 0, n_p = q\,, \\ &\xi: \quad n_g = -1, n_p = 1, \qquad \eta: \quad n_g = 1, n_p = -1\,. \end{aligned} \tag{2.2}$$

The SL(2,R) invariant vacuum carries zero ghost and picture number. Note that this definition of ghost number agrees with the definition of [28] for states with zero picture, but unlike the definition of [28], it allows the spacetime-supersymmetry generators to carry zero ghost number[24]. One notable difference from other formulations of open string field theory is that here the string field correspond to vertex operators in the 'large Hilbert space' containing the zero mode of the field ξ.

We shall denote by $\langle \prod_i A_i \rangle$ the correlation function of a set of vertex operators in the combined matter-ghost conformal field theory on the unit disk with open string vertex operators inserted on the boundary of the disk, without including trace over CP factors. These correlation functions are to be computed with the normalization

$$\langle \xi(z) c\partial c\partial^2 c(w) e^{-2\phi(y)} \rangle = 2\,. \tag{2.3}$$

Throughout this paper we shall be working in units where $\alpha' = 1$. The nilpotent BRST operator of this theory is given by

$$Q_B = \oint dz j_B(z) = \oint dz \Big\{ c(T_m + T_{\xi\eta} + T_\phi) + c\partial c b + \eta\, e^{\phi}\, G_m - \eta\partial\eta e^{2\phi} b \Big\}\,, \tag{2.4}$$

where

$$T_{\xi\eta} = \partial\xi\,\eta, \quad T_\phi = -\frac{1}{2}\partial\phi\partial\phi - \partial^2\phi\,, \tag{2.5}$$

T_m is the matter stress tensor and G_m is the matter superconformal generator. G_m is a dimension 3/2 primary field and satisfies:

$$G_m(z)G_m(w) \simeq \frac{10}{(z-w)^3} + \frac{2T_m}{(z-w)}\,. \tag{2.6}$$

The normalization of ϕ, ξ, η, b and c are as follows:

$$\xi(z)\eta(w) \simeq \frac{1}{z-w}, \quad b(z)c(w) \simeq \frac{1}{z-w}, \quad \partial\phi(z)\partial\phi(w) \simeq -\frac{1}{(z-w)^2}\,. \tag{2.7}$$

We denote by $\eta_0 = \oint dz\eta(z)$ the zero mode of the field η acting on the Hilbert space of matter ghost CFT.

The string field theory action is given by[22]

$$S = \frac{1}{2g^2}\Big\langle\Big\langle (e^{-\Phi}Q_B e^{\Phi})(e^{-\Phi}\eta_0 e^{\Phi}) - \int_0^1 dt (e^{-t\Phi}\partial_t e^{t\Phi})\{(e^{-t\Phi}Q_B e^{t\Phi}), (e^{-t\Phi}\eta_0 e^{t\Phi})\}\Big\rangle\Big\rangle\,, \tag{2.8}$$

where $\{A, B\} \equiv AB + BA$, and $e^{-t\Phi}\partial_t e^{t\Phi} = \Phi$ but has been written this way for convenience. This action is defined by expanding all exponentials in formal Taylor series carefully preserving the order of all operators and letting $\langle\langle\cdots\rangle\rangle$ of an ordered sequence of arbitrary vertex operators $A_1, \ldots A_n$ be defined as:

$$\langle\langle A_1 \ldots A_n \rangle\rangle = \Big\langle f_1^{(n)} \circ A_1(0) \cdots f_n^{(n)} \circ A_n(0) \Big\rangle\,. \tag{2.9}$$

Here, $f \circ A$ for any function $f(z)$, denotes the conformal transform of A by f, and

$$f_k^{(n)}(z) = e^{\frac{2\pi i(k-1)}{n}} \Big(\frac{1+iz}{1-iz}\Big)^{2/n} \quad \text{for} \quad n \geq 1. \tag{2.10}$$

In particular if φ denotes a primary field of weight h, then

$$f \circ \varphi(0) = (f'(0))^h \varphi(f(0))\,. \tag{2.11}$$

Q_B (or η_0) acting on a set of vertex operators inside $\langle\langle\ \rangle\rangle$ corresponds to a contour integral of j_B (or η) around the insertion points of these vertex operators on the right hand side of eq.(2.9).

Since we have, in general, non-integer weight vertex operators, we should be more careful in defining $f \circ A$ for such vertex operators. Noting that

$$f_k^{(N)\prime}(0) = \frac{4i}{N} e^{2\pi i \frac{k-1}{N}} \equiv \frac{4}{N} e^{2\pi i (\frac{k-1}{N} + \frac{1}{4})}, \tag{2.12}$$

we adopt the following definition of $f_k^{(N)} \circ \varphi(0)$ for a primary vertex operator $\varphi(x)$ of conformal weight h:

$$f_k^{(N)} \circ \varphi(0) = \left| \left(\frac{4}{N}\right)^h \right| e^{2\pi i h (\frac{k-1}{N} + \frac{1}{4})} \varphi(f_k^{(N)}(0)) . \tag{2.13}$$

Since all secondary vertex operators can be obtained as products of derivatives of primary vertex operators, this uniquely defines $f_k^{(N)} \circ A(0)$ for all vertex operators.

The geometry of the interaction described in (2.10) is simple. The function $f_1^{(n)}$ maps the upper half disk $|z| \le 1, \Im(z) > 0$ into the wedge $|\mathrm{Arg}(f_1^{(n)})| \le \pi/n$, $|f_1^{(n)}| \le 1$, with the puncture $z = 0$ ending at $f_1^{(n)} = 1$. With $k = 1, \cdots n$, we end up gluing n such wedges together to form a full unit disk where the n vertex operators are inserted at equally spaced points on the boundary. By a further SL(2,C) transformation F (*e.g.* $F(w) = i(1-w)/(1+w)$) we can map the interior of the unit disk onto the upper half plane. We could use the functions $g_k^{(n)}(z) = F(f_k^{(n)}(z))$ instead of $f_k^{(n)}(z)$ to define the string field theory action. $\langle\ \rangle$ will now denote the correlation function of the conformal field theory on the upper half plane, with open string vertex operators inserted on the real axis. As will be shown in appendix D, by a convenient choice of the SL(2,C) transformation F we can ensure that $g_1^{(n)}(0), \dots g_n^{(n)}(0)$ are ordered from left to right on the real axis. Also one finds that $g_k^{(n)\prime}(0)$ is real and positive for all k. The prescription (2.13) then corresponds to choosing real, positive values of $(g_k^{(n)\prime}(0))^h$ in the expression for the conformal transform of a field Φ of weight h. As a double-check of our computations, we shall compute the tachyon potential using both the disk and the UHP prescriptions and compare answers.

The correlator $\langle\langle\ \rangle\rangle$ defined in eq.(2.9) satisfies cyclicity properties. Let Φ denote any component of the string field, and $A_1, \dots A_{n-1}$ denote arbitrary vertex operators (*i.e.* arbitrary grassmanality, ghost number, etc.). Then

$$\langle\langle A_1 \dots A_{n-1} \Phi \rangle\rangle = \langle\langle \Phi A_1 \dots A_{n-1} \rangle\rangle,$$

$$\langle\langle A_1 \dots A_{n-1}(Q_B\Phi)\rangle\rangle = -\langle\langle (Q_B\Phi)A_1 \dots A_{n-1}\rangle\rangle,$$
$$\langle\langle A_1 \dots A_{n-1}(\eta_0\Phi)\rangle\rangle = -\langle\langle (\eta_0\Phi)A_1 \dots A_{n-1}\rangle\rangle\,. \tag{2.14}$$

The proof of these relations has been given in appendix A.

Note that in this notation the BPZ inner product is given by:

$$\langle A|B\rangle = \langle\langle AB\rangle\rangle\,, \tag{2.15}$$

which uses the two punctured disk (eqn.(2.10) with $n = 2$). We now define the multilinear products $|A_1A_2\dots A_n\rangle$ of n vertex operators $A_1, A_2, \dots A_n$ through the relation:

$$\langle B|A_1 \dots A_n\rangle = \langle\langle BA_1 \dots A_n\rangle\rangle\,, \tag{2.16}$$

for any state $\langle B|$. The product $|A_1A_2\rangle$, computed with (2.10) and $n = 3$, is simply the associative (non-commutative) star product $|A_1 * A_2\rangle$ of [17]. It follows from the geometry of the interaction that the higher products are equivalent to iterated multiplication using the star product: namely, $|A_1A_2\cdots A_n\rangle = |A_1 * A_2 * \cdots * A_n\rangle$. While the order of the sequence of operators must be preserved, the multiplications in this ket can be done in any order, thanks to the associativity of the star product. It follows that all products associate. From now on we shall denote the product of a set of vertex operators $A_1, A_2, \dots A_n$ by $A_1A_2\dots A_n$.

It will now be shown that this action is invariant under the gauge transformation[22]

$$\delta e^{\Phi} = (Q_B\Omega)e^{\Phi} + e^{\Phi}(\eta_0\Omega')\,, \tag{2.17}$$

where the gauge transformation parameters Ω and Ω' are Grassmann odd, GSO(+) vertex operators with (n_g, n_p) values $(-1, 0)$ and $(-1, 1)$ respectively. The proof will use the cyclicity relations (2.14) and the following identities:

$$\{Q_B, \eta_0\} = 0, \quad Q_B^2 = \eta_0^2 = 0, \tag{2.18}$$

$$Q_B(\Phi_1\Phi_2) = (Q_B\Phi_1)\Phi_2 + \Phi_1(Q_B\Phi_2), \quad \eta_0(\Phi_1\Phi_2) = (\eta_0\Phi_1)\Phi_2 + \Phi_1(\eta_0\Phi_2), \tag{2.19}$$

$$\langle\langle Q_B(...)\rangle\rangle = \langle\langle \eta_0(...)\rangle\rangle = 0. \tag{2.20}$$

Note that in the identities of the second line there are no minus signs necessary as Q_B or η_0 go through the string field because the string field is Grassmann even. The identities in the last last line hold because Q_B and η_0 are integrals of dimension one currents which can be "pulled" off the boundary and collapsed inside the disk.

Defining $G = e^{\Phi}$ and using the above identities, one finds that under an arbitrary variation δG,

$$\delta S = \frac{1}{g^2}\langle\langle G^{-1}\delta G \eta_0 (G^{-1}Q_B G)\rangle\rangle \tag{2.21}$$

where the first term of S contributes $(2g^2)^{-1}G^{-1}\delta G[\eta_0(G^{-1}Q_B G) - Q_B(G^{-1}\eta_0 G)]$ to the variation and the second term of S contributes $(2g^2)^{-1}G^{-1}\delta G[\eta_0(G^{-1}Q_B G)+Q_B(G^{-1}\eta_0 G)]$ to the variation. Using the fact that S goes to $-S$ after switching η_0 with Q_B and G with G^{-1}, (2.21) can also be written as

$$\delta S = -\frac{1}{g^2}\langle\langle G\delta G^{-1}Q_B(G\eta_0 G^{-1})\rangle\rangle \tag{2.22}$$

To prove gauge invariance under $\delta G = G\eta_0\Omega'$, use (2.21) and pull η_0 off the $(G^{-1}Q_B G)$ term. Since $\eta_0(G^{-1}\delta G) = 0$, $\delta S = 0$. To prove gauge invariance under $\delta G = (Q_B\Omega)G$, use (2.22) and pull Q_B off of the $(GQ_B G^{-1})$ term. Since $Q_B(G\delta G^{-1}) = -Q_B(\delta G G^{-1}) = 0$, $\delta S = 0$. So we have proven invariance of the action under the gauge transformations of (2.17).

The equation of motion for the action is easily derived from (2.21) to be

$$\eta_0(e^{-\Phi}Q_B e^{\Phi}) = 0. \tag{2.23}$$

As stated earlier, the string field in the present theory corresponds to vertex operators in the 'large Hilbert space' which includes the zero mode of ξ. However, using the Ω' gauge invariance, we can choose the gauge $\xi_0\Phi = 0$. In that gauge, the string field configuration Φ is in one to one correpondence with vertex operators in the 'small Hilbert space' which does not include the zero mode of ξ. This will be discussed in some detail in section 3.

2.2 Superstring field theory on a Non-BPS D-brane

The open string states living on a single non-BPS D-brane are divided into two classes, GSO(+) states and GSO($-$) states. Since the GSO($-$) states are Grassmann odd they cannot be incorporated directly into a string field preserving the algebraic structure reviewed in the previous subsection. This structure can be recovered by tensoring 2×2 matrices carrying *internal* Chan-Paton (CP) indices. These are added both to the vertex operators and to Q_B and η_0.

We attach the 2×2 identity matrix I on the usual GSO(+) sector (recall that the Neveu-Schwarz (NS) sector ground state is odd under the projection operator $(-1)^F$) and the Pauli matrix σ_1 to the GSO(−) sector. The complete string field is thus written as

$$\widehat{\Phi} = \Phi_+ \otimes I + \Phi_- \otimes \sigma_1 \,, \tag{2.24}$$

where the subscripts denote the $(-)^F$ eigenvalue of the vertex operator. In addition, we define:

$$\widehat{Q}_B = Q_B \otimes \sigma_3, \qquad \widehat{\eta}_0 = \eta_0 \otimes \sigma_3 \,. \tag{2.25}$$

Note that this definition shows that these matrices do not really carry conventional CP indices; had it been so, both Q_B and η_0 should have been tensored with the identity matrix, as such operators should not change the sector the strings live in [29]. Finally, we define

$$\langle\langle \widehat{A}_1 \ldots \widehat{A}_n \rangle\rangle = Tr\Big\langle f_1^{(n)} \circ \widehat{A}_1(0) \cdots f_n^{(n)} \circ \widehat{A}_n(0) \Big\rangle \,, \tag{2.26}$$

where the trace is over the internal CP matrices. We shall adopt the convention that fields or operators with internal CP factors included are denoted by symbols with a hat on them, and fields or operators without internal CP factors included are denoted by symbols without a hat, as in the previous subsection.

Indeed, with these definitions the cyclicity relations (2.14) given in the previous section now hold as

$$\begin{aligned}
&\langle\langle \widehat{A}_1 \ldots \widehat{A}_{n-1} \widehat{\Phi} \rangle\rangle = \langle\langle \widehat{\Phi} \widehat{A}_1 \ldots \widehat{A}_{n-1} \rangle\rangle, \\
&\langle\langle \widehat{A}_1 \ldots \widehat{A}_{n-1} (\widehat{Q}_B \widehat{\Phi}) \rangle\rangle = -\langle\langle (\widehat{Q}_B \widehat{\Phi}) \widehat{A}_1 \ldots \widehat{A}_{n-1} \rangle\rangle, \\
&\langle\langle \widehat{A}_1 \ldots \widehat{A}_{n-1} (\widehat{\eta}_0 \widehat{\Phi}) \rangle\rangle = -\langle\langle (\widehat{\eta}_0 \widehat{\Phi}) \widehat{A}_1 \ldots \widehat{A}_{n-1} \rangle\rangle \,,
\end{aligned} \tag{2.27}$$

where $\widehat{\Phi}$ denotes any component of the the string field, and $\widehat{A}_1, \ldots \widehat{A}_{n-1}$ denote arbitrary vertex operators. The proof of these relations, as well as those for the unhatted case have been given in appendix A. In addition, we have the analogs of (2.18) holding

$$\{\widehat{Q}_B, \widehat{\eta}_0\} = 0, \quad \widehat{Q}_B^2 = \widehat{\eta}_0^2 = 0, \tag{2.28}$$

$$\widehat{Q}_B(\widehat{\Phi}_1 \widehat{\Phi}_2) = (\widehat{Q}_B \widehat{\Phi}_1)\widehat{\Phi}_2 + \widehat{\Phi}_1(\widehat{Q}_B \widehat{\Phi}_2), \quad \widehat{\eta}_0(\widehat{\Phi}_1 \widehat{\Phi}_2) = (\widehat{\eta}_0 \widehat{\Phi}_1)\widehat{\Phi}_2 + \widehat{\Phi}_1(\widehat{\eta}_0 \widehat{\Phi}_2), \tag{2.29}$$

$$\langle\langle \widehat{Q}_B(\ldots) \rangle\rangle = \langle\langle \widehat{\eta}_0(\ldots) \rangle\rangle = 0. \tag{2.30}$$

The reason no extra signs appear in the middle equation is clear, when the string field is Grassmann odd the sign arising by moving Q_B across the vertex operator is cancelled by having to move σ_3 across σ_1.

Given that the relations satified by the hatted objects are the same as those of the unhatted ones, the string field action for the non-BPS D-brane takes the same structural form as that in (2.8) and is given by[22]

$$S = \frac{1}{4g^2}\Big\langle\Big\langle(e^{-\widehat{\Phi}}\widehat{Q}_B e^{\widehat{\Phi}})(e^{-\widehat{\Phi}}\widehat{\eta}_0 e^{\widehat{\Phi}}) - \int_0^1 dt(e^{-t\widehat{\Phi}}\partial_t e^{t\widehat{\Phi}})\{(e^{-t\widehat{\Phi}}\widehat{Q}_B e^{t\widehat{\Phi}}),(e^{-t\widehat{\Phi}}\widehat{\eta}_0 e^{t\widehat{\Phi}})\}\Big\rangle\Big\rangle\,, \quad (2.31)$$

where we have divided the overall normalization by a factor of two in order to compensate for the trace operation on the internal matrices. This action is invariant under the gauge transformation[22]

$$\delta e^{\widehat{\Phi}} = (\widehat{Q}_B\widehat{\Omega})e^{\widehat{\Phi}} + e^{\widehat{\Phi}}(\widehat{\eta}_0\widehat{\Omega}')\,, \quad (2.32)$$

where, as before the gauge transformation parameters $\widehat{\Omega}$ and $\widehat{\Omega}'$ are vertex operators with (n_g, n_p) values $(-1,0)$ and $(-1,1)$ respectively. The internal CP indices carried by the gauge parameters are as follows

$$\widehat{\Omega} = \Omega_+\otimes\sigma_3 + \Omega_-\otimes i\sigma_2\,, \quad (2.33)$$

with a similar relation holding for $\widehat{\Omega}'$. The GSO even Ω_+ is Grassmann odd, while the GSO odd Ω_- is Grassmann even. This makes the overall gauge parameters $\widehat{\Omega}$, $\widehat{\Omega}'$ odd relative to $\widehat{Q}_B$, $\widehat{\eta}_0$. The proof of gauge invariance is formally identical to the one given in the earlier section. The equation of motion is just (2.23) with hats on fields and operators. Again, the gauge parameter $\widehat{\Omega}'$ can be used to choose the gauge $\xi_0\widehat{\Phi} = 0$ so we can restrict to string states which are proportional to ξ_0.

For future use, we shall now give the expansion of the action (2.31) in power series in $\widehat{\Phi}$. Expanding the exponentials in a power series, we get,

$$e^{-\widehat{\Phi}}\mathcal{O}e^{\widehat{\Phi}} = \sum_{M,N=0}^{\infty}\frac{1}{(M+N+1)!}\binom{M+N}{M}(-1)^M\widehat{\Phi}^M(\mathcal{O}\widehat{\Phi})\widehat{\Phi}^N\,, \quad (2.34)$$

valid for $\mathcal{O}$ equal to $\widehat{Q}_B$ or $\widehat{\eta}_0$. Using the cyclicity relations eq.(2.27), and the identity (2.34), we can express the action (2.31) as

$$S = \frac{1}{2g^2}\sum_{M,N=0}^{\infty}\frac{1}{(M+N+2)!}\binom{M+N}{N}(-1)^N\langle\langle(\widehat{Q}_B\widehat{\Phi})\widehat{\Phi}^M(\widehat{\eta}_0\widehat{\Phi})\widehat{\Phi}^N\rangle\rangle\,. \quad (2.35)$$

As in refs.[15, 16] we shall find it convenient to take the time direction to be periodic with period 1, so that for a static configuration we can identify the potential with the negative of the action. In this case, an analysis analogous to that in ref.[15] shows that the string field action (2.31) describes a D-brane with mass

$$M = \frac{1}{2\pi^2 g^2} \,. \tag{2.36}$$

The details of this calculation have been outlined in appendix C. We shall calculate the tachyon potential and attempt to show that at the minimum it exactly cancels the mass M given in eq.(2.36).

2.3 Superstring field theory on a D-brane anti-D-brane pair

This system incorporates a GSO(+) sector in the form of vertex operators that represent strings that live on the brane or on the antibrane. With conventional Chan-Paton indices these would use the 2×2 matrices I and σ_3. In addition there is a GSO($-$) sector representing strings stretched between the brane and antibrane. With conventional Chan-Paton indices these would use the 2×2 matrices σ_1 and σ_2. We will call the conventional Chan-Paton matrices *external* CP matrices, to distinguish them from the internal CP matrices used in the previous subsection. Since we still have the complication of including two GSO types in the string field, we will not dispense of the internal CP matrices, and thus the brane-antibrane system will use *both* internal and external CP matrices. This time we therefore write:

$$\widehat{\Phi} = \Phi_+^{(1)} \otimes I \otimes I + \Phi_+^{(2)} \otimes I \otimes \sigma_3 + \Phi_-^{(3)} \otimes \sigma_1 \otimes \sigma_1 + \Phi_-^{(4)} \otimes \sigma_1 \otimes \sigma_2 \,, \tag{2.37}$$

where the first set of matrices are the internal ones and the second set are the external ones. In computing products of fields the two sets of matrices are defined to commute. For the operators and gauge parameters we have

$$\widehat{Q}_B = Q_B \otimes \sigma_3 \otimes I, \qquad \widehat{\eta}_0 = \eta_0 \otimes \sigma_3 \otimes I \,, \tag{2.38}$$

$$\widehat{\Omega} = \Omega_+^{(1)} \otimes \sigma_3 \otimes I \, + \, \Omega_+^{(2)} \otimes \sigma_3 \otimes \sigma_3 \, + \, \Omega_-^{(3)} \otimes i\sigma_2 \otimes \sigma_1 \, + \, \Omega_-^{(4)} \otimes i\sigma_2 \otimes \sigma_2 \,. \tag{2.39}$$

We are still writing all fields and operators with hats, for simplicity. The structure found earlier (eqs. (2.27)-(2.30), in particular) survives when the correlators $\langle\langle \cdots \rangle\rangle$ now include the double trace $Tr \otimes Tr$. The action takes then the same form as in (2.31) with the same normalization factor. If we restrict $\widehat{\Phi}$ to be of the form $\Phi \otimes I \otimes \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$, we recover the open string field theory action (2.8) on a single BPS D-brane.

As discussed elsewhere[5], in analyzing the tachyon potential we can restrict ourselves to the external CP sector I in the GSO(+) sector, and to the external CP sector σ_1 in the GSO(−) sector. Thus it is clear that there is a one to one correspondence between the component fields of the open string field theory on the non-BPS brane and that on the brane anti-brane system in this restricted sector. In fact since GSO(−) fields must appear always in even numbers, the external CP factors with their trace will simply produce an extra factor of two for every interaction. Thus the computations of the tachyon potential are identical. For the same value of the open string coupling constant g^2, in the brane anti-brane system the potential is twice as large compared to that of the non-BPS D-brane due to the trace over the external CP factors. On the other hand now the mass of the brane or the anti-brane is given by $(1/2\pi^2 g^2)$, so that the total mass of the brane-antibrane system is also twice the mass of the non-BPS D-brane. Thus if for a non-BPS brane the potential energy at the bottom of the well cancels the tension, then for the brane-antibrane system the potential energy at the bottom of the well will also cancel the total tension of the brane-antibrane system. We will therefore use in this paper the simpler notation required for the analysis of the non-BPS brane.

3 Computation and analysis of the tachyon potential

We shall be interested here in the phenomenon of tachyon condensation on the non-BPS D-brane. As mentioned before, the analysis applies also to the D-brane anti-D-brane problem. We begin by setting up the level expansion of the full tachyon string field relevant to the condensation. We then discuss the expansion of the action. Finally, relegating some computations to an appendix, we calculate the tachyon potential, find its minimum and test the brane annihilation conjecture.

3.1 The tachyon string field

In the present case the zero momentum tachyon corresponds to the vertex operator $\xi c e^{-\phi} \otimes \sigma_1$. Let us denote by $\mathcal{H}_1$ the subset of vertex operators of ghost number 0 and picture

L_0	*level*	*GSO*$(+)$	*GSO*$(-)$
$-1/2$	0	$---$	$\vert\widetilde{\Omega}\rangle$
0	1/2	$c_0\beta_{-\frac{1}{2}}\vert\widetilde{\Omega}\rangle$	$---$
1/2	1	$---$	$\beta_{-\frac{1}{2}}\gamma_{-\frac{1}{2}}\vert\widetilde{\Omega}\rangle$
1	3/2	$\{c_{-1}\beta_{-\frac{1}{2}}\,,\, b_{-1}\gamma_{-\frac{1}{2}},\, G^m_{-\frac{3}{2}}\}\vert\widetilde{\Omega}\rangle$	$---$

Table 1: Zero-momentum Lorentz scalar states of ghost number one living in the "small Hilbert space". Here $|\widetilde{\Omega}\rangle \equiv c_1 e^{-\phi(0)}|0\rangle$ is the GSO($-$) tachyon state in the conventional minus one picture. It satisfies $L_0|\widetilde{\Omega}\rangle = -\frac{1}{2}|\widetilde{\Omega}\rangle$.

number 0, created from the matter superstress tensor $(G_m(z), T_m(z))$,[3] and the ghost fields b, c, ξ, η, ϕ. It can be easily seen that by restricting the string field $\widehat{\Phi}$ to be in $\mathcal{H}_1$ gives a consistent truncation of the action, and hence we can look for a solution of the equations of motion, representing tachyon condensation, by restricting the string field $\widehat{\Phi}$ to be in this subspace $\mathcal{H}_1$. Thus from now on we shall always take the string field to lie in this restricted subspace.

We now expand the string field $\widehat{\Phi}$ in a basis of L_0 eigenstates, and write the action (2.35) in terms of component fields, which are the coefficients of expansion of the string field in this basis. As in [18, 16], we shall define the level of a string field component multiplying a vertex operator of conformal weight h to be $(h + \frac{1}{2})$, so that the tachyon field, multiplying the vertex operator $\xi c e^{-\phi} \otimes \sigma_1$, has level 0. We also define the level of a given term in the string field action to be the sum of the levels of the individual fields appearing in that term, and define a level $2n$ approximation to the action to be the one obtained by including fields up to level n and terms in the action up to level $2n$. Thus for example, a level 3 approximation to the action will involve string field components up to level (3/2). This is the approximation we shall be using to compute the action.

Using gauge invariance (2.32) of string field theory action, we can choose gauge conditions

$$b_0\widehat{\Phi} = 0, \qquad \xi_0\widehat{\Phi} = 0\,. \tag{3.1}$$

[3]For convenience of notation, we shall denote the kth oscillator mode of G_m and T_m by G^m_k and L^m_k respectively.

As in ref.[16], the legitimacy of this gauge condition can be proved at the linearized level. We then assume that string field configuration under consideration is not too large, so that such a gauge choice is also possible for the configuration under study. Also, as discussed in ref.[16], the gauge choice $b_0\widehat{\Phi} = 0$ can be made only for states with non-zero L_0 eigenvalue.

We can build systematically the relevant expansion of the string field by recalling that the string field $\widehat{\Phi}$ satisfying the gauge condition $\xi_0\widehat{\Phi} = 0$ is related to the NS string field $\widehat{V}$ of [25] by the relation $\widehat{\Phi} = \xi_0\widehat{V}$. The string field $\widehat{V}$ is built on the tachyon vacuum $|\widetilde{\Omega}\rangle \equiv c_1 e^{-\phi(0)}|0\rangle$. This vacuum state is GSO odd, it has ghost number $+1$ and $L_0 = -1/2$. Being in the minus one picture, it is annihilated by all positively moded oscillators $\{\gamma_r, \beta_r\}$. In addition, it is annihilated by b_0, L^m_{-1} and $G^m_{-\frac{1}{2}}$. All relevant states of ghost number one are now obtained by acting with ghost number zero combinations of oscillators $\{b, c, \beta, \gamma, L^m, G^m\}$ on $|\widetilde{\Omega}\rangle$. The $b_0\widehat{\Phi} = 0$ gauge condition allows us to ignore states with a c_0 oscillator in them. The states one finds up to L_0 eigenvalue 1 are given in Table 1. For ease of notation we have not included the CP factor. Note that we have included at $L_0 = 0$ a state which is not annihilated by b_0. This is the case because having $L_0 = 0$ this state cannot be gauged away.

The string field we need, which uses the "large" Hilbert space, is obtained by acting on the states of the table with ξ_0. This operation, however, does not change the dimension of the operators. As shown in appendix B, the string field theory action in the restricted subspace $\mathcal{H}_1$ has a Z_2 twist symmetry under which string field components associated with a vertex operator of dimension h carry charge $(-1)^{h+1}$ for even $2h$, and $(-1)^{h+\frac{1}{2}}$ for odd $2h$. The tachyon vertex operator, having dimension $-\frac{1}{2}$, is even under this twist transformation. Thus we can consider a further truncation of the string field theory by restricting the string field $\widehat{\Phi}$ to be twist even. This, in particular, means that the $L_0 = 0$ vertex operator mentioned above is to be omitted from the the string field. The same is true for the $L_0 = +1/2$ state in the GSO($-$) sector. Therefore, in addition to the tachyon, we will include the three scalar fields appearing in the GSO(+) sector at level 3/2.

In the language of vertex operators $|\widetilde{\Omega}\rangle$ is $ce^{-\phi}$, and the three states in table 1 at level (3/2) are

$$c\,\partial^2 c\,\partial\xi\, e^{-2\phi}, \quad \eta\,, \quad G_m\, c\, e^{-\phi}\,, \tag{3.2}$$

as can be seen with the help of eq.(2.1). We readily pass to the string field $\widehat{\Phi}$ by acting the above operators with ξ_0, thus guaranteeing that both gauge conditions (3.1) are satisfied. Denoting the tachyon operator by $\widehat{T}$ and the three other operators by $\widehat{A}, \widehat{E}$ and $\widehat{F}$ respectively, we have:

$$\begin{aligned}
\widehat{T} \quad &= \xi\, c\, e^{-\phi} \otimes \sigma_1 \\
\widehat{A} \quad &= c\, \partial^2 c\, \xi \partial \xi\, e^{-2\phi} \otimes I \\
\widehat{E} \quad &= \xi\, \eta \otimes I \\
\widehat{F} \quad &= \xi\, G_m\, c\, e^{-\phi} \otimes I
\end{aligned} \tag{3.3}$$

Therefore, the general twist even string field up to level (3/2), satisfying the gauge condition (3.1), has the following form:[4]

$$\widehat{\Phi} = t\, \widehat{T} + a\, \widehat{A} + e\, \widehat{E} + f\, \widehat{F}\,. \tag{3.4}$$

As explained above, the tachyon vertex operator $\widehat{T}$ of $L_0 = -1/2$ is a GSO odd operator of level zero. The operators $\widehat{A}, \widehat{E}$ and $\widehat{F}$ of $L_0 = +1$, and thus level 3/2, are in the GSO even sector.

3.2 Level expansion of the string action

We shall now substitute (3.4) into the action (2.35) and keep terms to all orders in t, but only up to quadratic order in a, e and f. Although the string field action contains vertices of arbitrarily high order, it can be shown that the truncated action to any given level only has a finite number of terms. To see this, let us first note that for a term in the action of a given level, the number of fields of level > 0 must be finite. Since all components of the string field other than the tachyon t has level > 0, we only need to show that there cannot be arbitrarily large number of tachyon fields. This is easily seen by noting that the tachyon vertex operator $\widehat{T}$ has -1 unit of ϕ momentum. Since in order to get a non-zero correlation function, the total ϕ momentum of all the vertex operators must add up to -2, it is clear that for a fixed set of other vertex operators, we can only insert a finite number of tachyon vertex operators in order to have a non-vanishing correlation function.

Each term in the action has one $\widehat{\eta}_0$ and one $\widehat{Q}_B$, each acting on a string field. While $\widehat{\eta}_0$ carries no ϕ-momentum, the BRST operator $\widehat{Q}_B$ can supply zero, one or two units

[4] Ref.[22] had a factor of i in front of the $t\widehat{T}$ term. In this paper we have used slightly different set of conformal maps $f_k^{(N)}$ in defining the string field theory action; these map the upper half plane into the inside of the unit disk rather than outside. With this choice, the kinetic term for the tachyon field has the standard sign provided there is no factor of i multiplying $t\widehat{T}$ in eq.(3.4).

of ϕ-momentum (see (2.4)). The operators $\widehat{A}$, $\widehat{E}$ and $\widehat{F}$ carry -2, 0 and -1 units of ϕ momentum respectively. Since the operator E entering in the string field carries no ϕ momentum, this is the field that can appear together with the largest number of tachyon fields. For example, the string action term coupling E with four T's is nonvanishing since the tachyons give (-4) units of ϕ-momentum and the BRST operator can supply $(+2)$ units. Since we are going to compute the action to level three we can have a term in the string action with two E's and four T's. This is the term with the largest possible number of fields that can contribute to level three. This means we need the expansion of the string action (2.35) up to terms with six string fields. This is given by,

$$
\begin{aligned}
S = \frac{1}{2g^2}\Big\langle\Big\langle\, & \frac{1}{2}(\widehat{Q}_B\widehat{\Phi})\,(\widehat{\eta}_0\widehat{\Phi}) + \frac{1}{6}(\widehat{Q}_B\widehat{\Phi})\Big(\widehat{\Phi}\,(\widehat{\eta}_0\widehat{\Phi}) - (\widehat{\eta}_0\widehat{\Phi})\,\widehat{\Phi}\Big) \\
&+\frac{1}{24}(\widehat{Q}_B\widehat{\Phi})\Big(\widehat{\Phi}^2\,(\widehat{\eta}_0\widehat{\Phi}) - 2\widehat{\Phi}\,(\widehat{\eta}_0\widehat{\Phi})\,\widehat{\Phi} + (\widehat{\eta}_0\widehat{\Phi})\,\widehat{\Phi}^2\Big) \\
&+\frac{1}{120}(\widehat{Q}_B\widehat{\Phi})\Big(\widehat{\Phi}^3\,(\widehat{\eta}_0\widehat{\Phi}) - 3\widehat{\Phi}^2\,(\widehat{\eta}_0\widehat{\Phi})\,\widehat{\Phi} + 3\widehat{\Phi}\,(\widehat{\eta}_0\widehat{\Phi})\,\widehat{\Phi}^2 - (\widehat{\eta}_0\widehat{\Phi})\,\widehat{\Phi}^3\Big) \\
&+\frac{1}{720}(\widehat{Q}_B\widehat{\Phi})\Big(\widehat{\Phi}^4\,(\widehat{\eta}_0\widehat{\Phi}) - 4\widehat{\Phi}^3\,(\widehat{\eta}_0\widehat{\Phi})\,\widehat{\Phi} + 6\widehat{\Phi}^2\,(\widehat{\eta}_0\widehat{\Phi})\,\widehat{\Phi}^2 - 4\widehat{\Phi}\,(\widehat{\eta}_0\widehat{\Phi})\,\widehat{\Phi}^2 + (\widehat{\eta}_0\widehat{\Phi})\,\widehat{\Phi}^4\Big)\Big\rangle\Big\rangle
\end{aligned}
\tag{3.5}
$$

Since our computation will be restricted to twist even fields in $\mathcal{H}_1$, the above result can be simplified further by use of (B.15) and cyclicity. We find:

$$
\begin{aligned}
S = \frac{1}{2g^2}\Big\langle\Big\langle\, & \frac{1}{2}(\widehat{Q}_B\widehat{\Phi})\,(\widehat{\eta}_0\widehat{\Phi}) + \frac{1}{3}(\widehat{Q}_B\widehat{\Phi})\,\widehat{\Phi}\,(\widehat{\eta}_0\widehat{\Phi}) + \frac{1}{12}(\widehat{Q}_B\widehat{\Phi})\Big(\widehat{\Phi}^2\,(\widehat{\eta}_0\widehat{\Phi}) - \widehat{\Phi}\,(\widehat{\eta}_0\widehat{\Phi})\,\widehat{\Phi}\Big) \\
&+\frac{1}{60}(\widehat{Q}_B\widehat{\Phi})\Big(\widehat{\Phi}^3\,(\widehat{\eta}_0\widehat{\Phi}) - 3\widehat{\Phi}^2\,(\widehat{\eta}_0\widehat{\Phi})\,\widehat{\Phi}\Big) \\
&+\frac{1}{360}(\widehat{Q}_B\widehat{\Phi})\Big(\widehat{\Phi}^4\,(\widehat{\eta}_0\widehat{\Phi}) - 4\widehat{\Phi}^3\,(\widehat{\eta}_0\widehat{\Phi})\,\widehat{\Phi} + 3\widehat{\Phi}^2\,(\widehat{\eta}_0\widehat{\Phi})\,\widehat{\Phi}^2\Big)\Big\rangle\Big\rangle\,.
\end{aligned}
\tag{3.6}
$$

This expansion suffices for the present computation. All we need to do is to substitute (3.4) into this expression and evaluate the correlation functions appearing in various terms. The required calculations are relatively straightforward, and involve correlation functions of appropriate conformal transforms of the operators $\widehat{T}$, $\widehat{A}$, $\widehat{E}$, $\widehat{F}$, the BRST current j_B, and the field η. Here we shall only state the result; some of the details have been discussed in appendix D.[5]

[5] We used the symbolic manipulation program Mathematica to carry out some of these computations.

3.3 The tachyon potential

We shall now give the result for the action and the potential by truncating it to level 3. Not all terms allowed by level counting are non-vanishing. Several vanish because they fail to satisfy ϕ-momentum conservation, for example, there is no a^2t^2 term.

The result, with S_k denoting the level k terms in the action, is

$$
\begin{aligned}
g^2 S_0 &= \frac{1}{4}\,t^2 - \frac{1}{2}\,t^4 \ , \\
g^2 S_{\frac{3}{2}} &= a\,t^2 + \frac{1}{4}\,e\,t^2 + \frac{5}{96}\sqrt{50+22\sqrt{5}}\ e\,t^4 \ , \\
g^2 S_3 &= \quad -2\,a\,e - 5\,f^2 \\
&\qquad +\Big(\frac{1}{\sqrt{2}} - \frac{1}{24}\Big)\,e^2\,t^2 - \frac{5}{18}\,e^2\,t^4 \\
&\qquad -\frac{5}{4}(4\sqrt{2}\ - 1)\,f^2\,t^2 - \frac{1}{12}\,(\,3+40\sqrt{2}\,)\,ae\,t^2 + \frac{5}{12}\,(\,10\sqrt{2}-1\,)ef\,t^2\,.
\end{aligned}
\tag{3.7}
$$

The action to level three is given by $S^{(3)} = S_0 + S_{\frac{3}{2}} + S_3$. Collecting the terms above, using the relation (2.36), and expressing the various radicals as approximate decimals, we have,

$$
\begin{aligned}
V = -S \ \ = \ \ -M\,(2\pi^2)\Big(& 0.25\,t^2 - 0.5\,t^4 + a\,t^2 + 0.25\,e\,t^2 + 0.519\,e\,t^4 \\
& -2\,ae - 5f^2 + 0.665\,e^2t^2 - 0.278\,e^2t^4 \\
& -5.82\,f^2t^2 \ - 4.96\,aet^2 \ + \ 5.476\,eft^2 \ \Big)
\end{aligned}
$$

The potential has extrema at $(\pm t_0, a_0, e_0, f_0)$ with

$$
t_0 = 0.58882, \quad a_0 = 0.056363, \quad e_0 = 0.093175, \quad f_0 = 0.012603\,. \tag{3.8}
$$

At these extrema,

$$
V = -0.85446M\,. \tag{3.9}
$$

The expected exact answer for the value of the potential at the extrema is $-M$, so that it can cancel the mass of the D-brane exactly. Thus we see that the level three approximation produces 85% of the exact answer. Note that one finds 60% of the exact answer at level zero [22], so the approximation scheme appears to be converging to the exact answer.

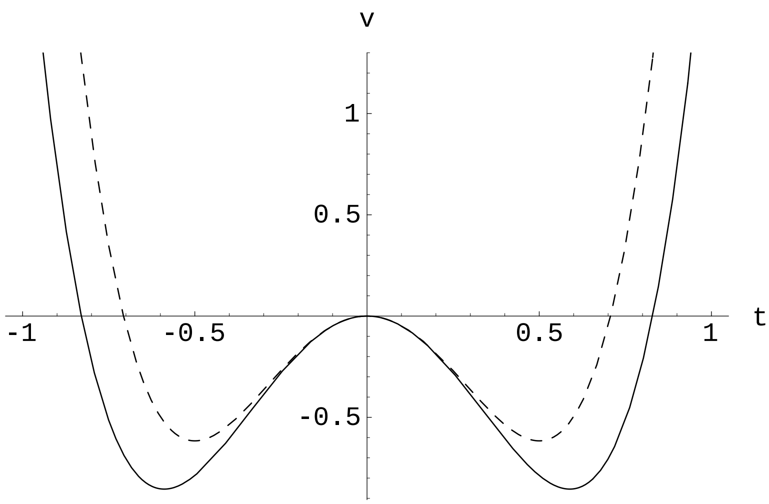

Figure 1: The tachyon potential $v(t) = V(t)/M$ given in (3.10) (solid line). For reference we also show the zeroeth order potential (dashed line).

The potential computed to this approximation (level three) includes the fields (a, e, f) only quadratically. So they can be integrated out exactly to find an effective potential $V(t)$ for the tachyon. One obtains

$$v(t) \equiv \frac{1}{M} \cdot V(t) = -4.93\, t^2 \; \frac{(1 + 4.63\, t^2 + 3.21\, t^4 - 9.48\, t^6 - 11.67\, t^8)}{(1 + 1.16\, t^2)(1 + 2.48\, t^2)^2} \tag{3.10}$$

Several important properties are manifest from this expression. Since the denominators never vanish, there is no singularity in $V(t)$ to this approximation. The small tachyon instability for $t \to 0$ is manifest. Since $V \sim +t^4$, when t is large, this potential is clearly bounded below. It can be easily checked that only critical points are $\pm t_0$ with $t_0 = 0.58882$; they are (equivalent) *global* minima of the presently computed effective potential.[6] The effective tachyon potential $v(t)$ has been displayed in fig. 1.

4 Tachyonic kink configuration

In the last section we analysed the tachyon potential on a non-BPS D-brane in a background independent fashion. In this section we focus on a specific case where the non-BPS

[6] The critical point, however, is not a global minima of the full multiscalar potential $V(t, a, e, f)$. This is a reflection of the fact that some of the fields a, e, f are auxiliary fields.

D-brane corresponds to a non-BPS D-string of type IIA string theory wrapped on a circle of radius R. If $\mathcal{T}_1$ denotes the tension of the non-BPS D-string, then $M = 2\pi R\mathcal{T}_1$. We denote by t the (1+1) dimensional tachyon field living on the non-BPS D-string. Using the definition of $v(t)$ given in (3.10), we may express the potential $V(t)$ as $2\pi R\mathcal{T}_1 v(t) = \mathcal{T}_1 \int_0^{2\pi R} dx v(t)$. From this we arrive at the conclusion that the potential energy of the D-string is given by

$$V(t) = \mathcal{T}_1 \int_0^{2\pi R} dx v(t) \,. \tag{4.1}$$

We denote by x^μ for $\mu = 0, 1$ the world volume coordinates of the D-string, and $x \equiv x^1$ is the spatial coordinte.

Since $v(t)$ given in (3.10) has doubly degenerate minima at $\pm t_0$, we can consider a kink solution which interpolates between these two minima. It has been conjectured that this kink describes a BPS D0-brane of type IIA string theory[10, 11]. In this section we shall compute the mass of this kink solution, and compare it with the mass of a BPS D0-brane of type IIA string theory.

Computing the mass of the kink also requires knowledge of the kinetic term of the tachyon.[7] We use the kinetic term obtained from the quadratic term of the action, ignoring corrections from the higher order terms in the presence of background t. There is no justification for ignoring these corrections, and we should not expect this calculation to yield more than an order of magnitude estimate. Using the action (2.31), eq.(2.36) relating $M = 2\pi R\mathcal{T}_1$ and g^2, and the fact that the length of D-string is $2\pi R$, we see that the Lagrangian contains a term

$$-\frac{1}{2}(2\pi^2\mathcal{T}_1) \int_0^{2\pi R} dx \partial_\mu t \partial^\mu t \,. \tag{4.2}$$

We shall now take the limit $R \to \infty$, *i.e.* we consider infinitely long D-string. From eqs.(4.1) and (4.2) we get the following equations of motion for a *static* tachyonic configuration:

$$2\pi^2 \partial_x^2 t = v'(t) \,. \tag{4.3}$$

Using the boundary condition that as $x \to \pm\infty$, $t \to \pm t_0$ and $\partial_x t \to 0$, the solution to this equation is implicitly given by:

$$\partial_x t \;=\; \frac{1}{\pi}\sqrt{v(t) - v(t_0)} \,,$$

[7] By kinetic term we refer to the terms involving derivatives of the tachyon field t, including spatial derivatives.

$$x = \pi \int_0^{t(x)} dy \frac{1}{\sqrt{v(y) - v(t_0)}} . \tag{4.4}$$

The total energy associated with this solution (measured above the $t = t_0$ solution), obtained by adding the kinetic and the potential terms is given by:

$$E = 2\pi \mathcal{T}_1 \int_{-t_0}^{t_0} dy \sqrt{v(y) - v(t_0)} . \tag{4.5}$$

Let $\mathcal{T}_0$ denote the mass of a BPS D0-brane of type IIA string theory. Then we have the relation:

$$\mathcal{T}_1 = \sqrt{2} \frac{\mathcal{T}_0}{2\pi} . \tag{4.6}$$

Using this, eq.(4.5) can be written as

$$E = \sqrt{2} \mathcal{T}_0 \int_{-t_0}^{t_0} dy \sqrt{v(y) - v(t_0)} . \tag{4.7}$$

If we use the zeroeth order approximation[22] for the potential

$$v(t) = 2\pi^2 (-\frac{1}{4} t^2 + \frac{1}{2} t^4) , \tag{4.8}$$

then (4.7) can be evaluated analytically, and we get

$$E = \frac{1}{6} \sqrt{2} \pi \mathcal{T}_0 . \tag{4.9}$$

This is about 74% of the expected answer $\mathcal{T}_0$.[8]

For the potential given in eq.(3.10), we can calculate the right hand side of eq.(4.7) numerically. The answer is

$$E = 1.03 \mathcal{T}_0 . \tag{4.10}$$

Considering the crude approximation that we have used, this close agreement with the expected answer is likely to be accidental. However it is encouraging to note that the numerical answer is close to the expected answer.

This analysis can be easily extended to the case of a tachyonic kink on a non-BPS D-p brane for any value of p.

[8] A similar result was independently obtained by Bergman[30], and also by Iqbal and Naqvi[31].

5 Concluding remarks and open questions

There are two main points to the present paper. Point one: we seem to have a consistent NS open string field theory [23] in which calculations are feasible. Point two: the calculations performed here with this string field theory give good direct evidence for the tachyon condensation phenomenon and its implications for unstable non-BPS D-branes as well as for the D-brane anti- D-brane system.

Let us first focus on the string field theory itself. While the cubic open string field theory of [17] gives a consistent classical theory of bosonic open strings, its extension to superstrings [25] was recognized early on to be problematic [33]. Problems arise because the NS string vertex carries a picture changing operator at the interaction point, and in testing the associativity of the star product one induces the collision of two picture changing operators, upon which a divergence is encountered. It is believed that contact terms with infinite coefficients must be added to the action to restore gauge invariance. One may wonder if these complications are just irrelevant to the problem of computing the tachyon potential. We are not optimistic on this point. Indeed, in this theory, the potential of the tachyon field alone is purely quadratic. The absence of a cubic term (because of $(-)^F$ conservation), and the absence of a quartic term (as the theory is cubic) imply that the potential for the tachyon field alone has no critical points. It would therefore be necessary for the interactions of the tachyon with the other scalars to generate stabilizing terms of the right magnitude. However, the experience in this paper, as well as that in open bosonic string theory [16] indicate that massive fields rather than stabilizing the tachyon, tend to lower the critical point that is generated by the tachyon field alone. Given the uncertainty in such arguments, it would be desirable to carry out the direct computation of the tachyon potential in this cubic theory. Since this theory is expressed in the "small Hilbert space" the table given in section 3.1 lists the relevant states. Just as it was the case in our present work, we expect that a twist analysis will show that the three fields at level 3/2 are the ones that must be used for a lowest level nontrivial computation.

On the other hand the WZW-like NS string field theory used here is free of divergences and its gauge invariance is manifest. Given that it seems now to provide a consistent framework for dealing with the tachyon potential in the relevant brane systems, much of our work here has focused in the detailed setup of the action for the non-BPS brane, as well as for the brane anti-brane systems. We have also given very explicit consideration to the cyclicity and twist properties of the string action, and we have explained in detail how to work out branch cuts for dealing with the fractional dimension operators of the GSO odd sector. While this string field theory is non-polynomial, the level expansion is workable and the higher interactions are relatively simple to compute since they do not involve integration over the moduli space of Riemann surfaces; they are finite contact interactions. In contrast to bosonic string field theory, where gauge invariance was directly related to the covering of the moduli space of Riemann surfaces (see, for example, [34] and [35]), something different and subtle is going on here as moduli spaces would be covered without the help of the higher interactions.[9]

Turning now to the tachyon conjectures, the results obtained here are consistent with convergence to the expected values. While the condensation of the tachyon field alone gave about 60% of the desired value, the first nontrivial correction computed here (level 3) gave about 60% of the remaining energy. It should not be very hard to use the setup of this paper to carry the computation to level four, and perhaps to automate the computation further to deal with higher levels. Further evidence of convergence would be desirable. It would also be of interest to investigate further the properties of the tachyonic kink solution representing a lower dimensional brane. It should be noted that the convergence of the level approximation scheme to the answers seems slower than in the case of the bosonic open string, where the tachyon field alone gave about 70% of the desired energy, and inclusion of two additional scalars gave 95% of the expected answer.

Of course, at a deeper level the most intriguing questions remain those that were already apparent in the bosonic case [16]: (i) Is there a closed form solution for the tachyon condensate? and, (ii) what is the physics of the vacuum around the tachyon condensate? Insight into any of these two questions would open up exciting new possibilities.

Acknowledgements: N.B. would like to thank Oren Bergman and Ion Vancea for useful discussions, Caltech, Harvard University and Massachusetts Institute of Technology for

[9]Other approaches have been suggested to deal with the difficulties of [25]. One possibility is to use string fields of non-canonical picture number [36, 37]. Another possibility is to make the superstring theory non-polynomial in the same way as must be done to incorporate closed strings off-shell [38]. In such approach the region of moduli space where the collision of picture changing operators happens is within the interaction terms, which could be modified to prevent such collisions. Since the interactions in such theory would not be of contact type the level approximation would appear to be difficult to implement.

their hospitality, and CNPq grant 300256/94-9 for partial financial support. A.S. would like to thank Caltech for hospitality during part of this work. B.Z. would like to thank N. Moeller and W. Taylor for useful discussions.

A Cyclicity property of string amplitudes

In this appendix we shall prove eqs.(2.14), (2.27). Since the trace over the Chan Paton matrices satisfy the cyclicity property without any extra sign, we can work with the unhatted vertex operators, and prove (2.14) and (2.27) simultaneously. First we shall prove this for string fields belonging to the restricted subspace $\mathcal{H}_1$, and then indicate its generalization for general string fields.

The cyclicity properties of the conformal field theory correlation functions are analyzed by using the property:

$$\begin{aligned} T\circ f_i^{(n)}\circ A &= f_{i+1}^{(n)}\circ A \quad \text{for} \quad 1\le i\le (n-1) \\ T\circ f_n^{(n)}\circ A &= T^n\circ f_1\circ A\equiv R\circ f_1\circ A, \end{aligned} \tag{A.1}$$

for any vertex operator A. Here $T(w)=e^{2\pi i/n}w$, and $R=T^n$ denotes rotation by 2π. While the transformation R acts trivially on the complex plane, it must be viewed in general as the composition T^n of n transformations by T. Thus R affects the transformation of fields with non-integer dimension. Since T maps unit disk to itself in a one to one fashion, it corresponds to an SL(2,R) transformation. Using $SL(2,R)$ invariance of the correlation functions on the disk, we can write

$$\langle (f_1^{(n)}\circ A_1)\cdots(f_{n-1}^{(n)}\circ A_{n-1})(f_n^{(n)}\circ\Phi)\rangle = \langle (f_2^{(n)}\circ A_1)\cdots(f_n^{(n)}\circ A_{n-1})(R\circ f_1^{(n)}\circ\Phi)\rangle \tag{A.2}$$

In the subspace $\mathcal{H}_1$, the conformal weight of Φ is integer if Φ is Grassmann even (GSO(+)), and half integer if it is Grassman odd (GSO($-$)). Thus the transformation by R gives a factor of 1 if Φ is Grassmann even, and -1 if Φ is Grassmann odd. As can be seen from eq.(2.3), the product of all the operators inside the correlation function must be Grassmann even in order to get a non-vanishing correlator. Thus we pick up a factor of 1 (-1) in moving the $R\circ f_1^{(n)}\circ\Phi$ factor on the right hand side of eq.(A.2) to the first place if Φ is Grassmann even (odd). Thus the right hand side of eq.(A.2) may be written as

$$\langle (f_1^{(n)}\circ\Phi)(f_2^{(n)}\circ A_1)\cdots(f_n^{(n)}\circ A_{n-1})\rangle \tag{A.3}$$

irrespective of whether Φ is Grassmann even or Grassmann odd.

If we replace Φ by $(Q_B\Phi)$ or $(\eta_0\Phi)$, eq.(A.2) still holds, and transformation by R still gives a factor of 1 (-1) if Φ is Grassmann even (Grassmann odd). But now, since $(Q_B\Phi)$ and $(\eta_0\Phi)$ have statistics opposite to that of Φ, we pick up a factor of -1 $(+1)$ in moving the $R\circ f_1^{(n)}\circ(Q_B\Phi)$ or $R\circ f_1^{(n)}\circ(\eta_0\Phi)$ factor to the first place if Φ is Grassmann even (odd). This gives,

$$\begin{aligned}\langle (f_1^{(n)}\circ A_1)\cdots(f_{n-1}^{(n)}\circ A_{n-1})(f_n^{(n)}\circ(Q_B\Phi))\rangle &= -\langle (f_1^{(n)}\circ(Q_B\Phi))(f_2^{(n)}\circ A_1)\cdots(f_n^{(n)}\circ A_{n-1})\rangle\\ \langle (f_1^{(n)}\circ A_1)\cdots(f_{n-1}^{(n)}\circ A_{n-1})(f_n^{(n)}\circ(\eta_0\Phi))\rangle &= -\langle (f_1^{(n)}\circ(\eta_0\Phi))(f_2^{(n)}\circ A_1)\cdots(f_n^{(n)}\circ A_{n-1})\rangle\end{aligned} \tag{A.4}$$

This proves eqs.(2.14) and (2.27).

The cyclicity rules derived above also hold for a general string field $\widehat{\Phi}$ not necessarily inside $\mathcal{H}_1$, and are in fact needed for the proof of gauge invariance of the action. The proof of these relations for a general D-brane system, however, requires using appropriate cyclicity axioms for the correlation functions of a general boundary conformal field theory. In the present context this axiom states that if Φ denotes a vertex operator of conformal weight h, then in moving $R\circ f_1^{(n)}\circ\Phi$ from the extreme right to the extreme left in the right hand side of eq.(A.2), we pick up a factor of $e^{-2\pi ih}$. On the other hand, from eq.(2.13) one can easily check that $R=T^n$ acting on Φ gives a factor of $e^{2\pi ih}$. Thus these two factors cancel each other, and we recover the cyclicity rules given in the first of eqs.(2.14), (2.27) for a general string field component Φ or $\widehat{\Phi}$. The other two equations of (2.14), (2.27) can be proved along similar lines.

B Twist invariance of the restricted action

In this appendix we shall show that the superstring field theory action (2.31), or equivalently (2.35), has a Z_2 twist invariance when we restrict the string field $\widehat{\Phi}$ to lie in the subspace $\mathcal{H}_1$ defined in section 3. Using the form (2.35) of the action and the cyclicity relations given in eq.(2.27), it is easy to verify that a vertex with even number of string fields ($(M+N)$ even terms of eq.(2.35)) is odd under $Q_B\leftrightarrow\eta_0$, whereas a vertex with odd number of string fields is even under $Q_B\leftrightarrow\eta_0$. Let us now consider a typical pair of terms in the string field theory action:

$$\langle\langle\widehat{\Phi}^{i-1}(\widehat{Q}_B\widehat{\Phi})\widehat{\Phi}^{n-i-1}(\widehat{\eta}_0\widehat{\Phi})\rangle\rangle+(-1)^{n+1}\langle\langle\widehat{\Phi}^{i-1}(\widehat{\eta}_0\widehat{\Phi})\widehat{\Phi}^{n-i-1}(\widehat{Q}_B\widehat{\Phi})\rangle\rangle \tag{B.1}$$

Let now $\widehat{\Phi}_1, \ldots \widehat{\Phi}_n$ denote n arbitrary components of the string field $\widehat{\Phi}$. In the expansion of the string action, the first term of (B.1) will give rise to a term of the form

$$(I) \equiv \langle\langle \widehat{\Phi}_1 \cdots \widehat{\Phi}_{i-1}(\widehat{Q}_B \widehat{\Phi}_i)\widehat{\Phi}_{i+1} \cdots \widehat{\Phi}_{n-1}(\widehat{\eta}_0 \widehat{\Phi}_n)\rangle\rangle\,, \tag{B.2}$$

while the second term in (B.1) will give rise to a term of the form

$$(II) \equiv (-1)^{n+1}\langle\langle \widehat{\Phi}_{i-1} \cdots \widehat{\Phi}_2 \widehat{\Phi}_1 (\widehat{\eta}_0 \widehat{\Phi}_n)\widehat{\Phi}_{n-1} \cdots \widehat{\Phi}_{i+1}(\widehat{Q}_B \widehat{\Phi}_i)\rangle\rangle\,. \tag{B.3}$$

In fact, when expanding the string field in arbitrary components all terms in the action arising from (B.1) can be paired just as (I) and (II). Note that up to a cyclic transformation, the order of inputs in (I) and (II) are precisely reversed (twisted). We will relate (I) to (II) up to a sign, and this relation will enable us to derive a selection rule based on twist.

Let $M(z) = -z$, $\widetilde{I}(z) = (1/z)$, and $R = T^n$ denote rotation by 2π. R leaves vertex operators with integral conformal weight unchanged, and changes the sign of the vertex operators with half-integral conformal weight. As in the case of $f_k^{(N)}$'s, the definition of M and $\widetilde{I}$ are not complete unless we specify how to choose the sign when these transformations act on an half integral weight vertex operator. We adopt the following convention. Acting on a primary field φ of weight h,

$$M \circ \varphi(z) = e^{i\pi h}\varphi(-z), \qquad \widetilde{I} \circ \varphi(z) = (iz)^{-2h}\varphi(\frac{1}{z})\,. \tag{B.4}$$

Note that since h is either an integer or a half-integer, $(iz)^{-2h}$ is well defined. We can now verify the relations:

$$\begin{aligned} f_i^{(n)} \circ M \circ \varphi &= \widetilde{I} \circ f_{n-i+2}^{(n)} \circ \varphi \quad \text{for} \quad n \geq i \geq 2 \\ f_1^{(n)} \circ M \circ \varphi &= \widetilde{I} \circ R \circ f_1^{(n)} \circ \varphi, \end{aligned} \tag{B.5}$$

where the second equation is clearly the natural generalization of the first once we note that $f_{n+1}^{(n)}$ is identified with $R \circ f_1^{(n)}$. Since secondary vertex operators are obtained from products of derivatives of primary vertex operators, these relations also hold if we replace φ by a secondary vertex operator.

We now consider (I) which explicitly reads

$$\begin{aligned} (I) \;\; = \;\; & Tr\langle (f_1^{(n)} \circ \widehat{\Phi}_1) \cdots (f_{i-1}^{(n)} \circ \widehat{\Phi}_{i-1})(f_i^{(n)} \circ (\widehat{Q}_B \widehat{\Phi}_i))(f_{i+1}^{(n)} \circ \widehat{\Phi}_{i+1}) \\ & \cdots (f_{n-1}^{(n)} \circ \widehat{\Phi}_{n-1})(f_n^{(n)} \circ (\widehat{\eta}_0 \widehat{\Phi}_n))\rangle\,, \end{aligned} \tag{B.6}$$

where for simplicity we have omitted the zeroes from the arguments of $\widehat{\Phi}_i$. Since M preserves the origin of the coordinate system, using (B.4), we can replace each vertex operator $\widehat{\Phi}_i(0)$ in the above correlator by $e^{-i\pi h_i} M \circ \widehat{\Phi}_i(0)$, where h_i is the conformal dimension of $\widehat{\Phi}_i$. We then use (B.5) to bring (B.6) into the form:

$$\begin{aligned} (I) = (-1)^{\sum h_i} Tr\langle (\widetilde{I} \circ R \circ f_1^{(n)} \circ \widehat{\Phi}_1)(\widetilde{I} \circ f_n^{(n)} \circ \widehat{\Phi}_2) \cdots (\widetilde{I} \circ f_{n-i+3}^{(n)} \circ \widehat{\Phi}_{i-1}) \\ (\widetilde{I} \circ f_{n-i+2}^{(n)} \circ (\widehat{Q}_B \widehat{\Phi}_i))(\widetilde{I} \circ f_{n-i+1}^{(n)} \circ \widehat{\Phi}_{i+1}) \cdots (\widetilde{I} \circ f_3^{(n)} \circ \widehat{\Phi}_{n-1})(\widetilde{I} \circ f_2^{(n)} \circ (\widehat{\eta}_0 \widehat{\Phi}_n))\rangle \, . \end{aligned} \tag{B.7}$$

We now use the following results:

- In the restricted sector in which we are working, the SL(2,C) transformation $\widetilde{I}$ is a symmetry of the correlation functions. Thus we can remove all factors of $\widetilde{I}$ from eq.(B.7).
- Acting on $\widehat{\Phi}_1$, R gives a factor of $(-1)^{2h_1}$.
- If we reverse the ordering of $f_n^{(n)} \circ \widehat{\Phi}_2 \ldots f_{n-i+2}^{(n)} \circ (\widehat{Q}_B \widehat{\Phi}_i) \ldots f_2^{(n)} \circ (\widehat{\eta}_0 \widehat{\Phi}_n)$ in eq.(B.7), then we pick up a factor of $(-1)(-1)^{n'_o(n'_o-1)/2}$, where n'_o is the number of odd string fields in the set $\widehat{\Phi}_2, \ldots \widehat{\Phi}_n$. The first minus sign in this expression comes from passing $\widehat{Q}_B$ through $\widehat{\eta}_0$; the other factor comes from passing the odd components of the string field through each other. Note that due to the internal CP matrices there is no extra sign in passing $\widehat{Q}_B$ or $\widehat{\eta}_0$ through $\widehat{\Phi}_j$, irrespective of whether $\widehat{\Phi}_j$ is Grassmann even or odd.

Thus (B.7) can be written as

$$\begin{aligned} (I) &= (-1)^{\sum h_i} (-1)^{1+2h_1+n'_o(n'_o-1)/2} \, Tr\langle (f_1^{(n)} \circ \widehat{\Phi}_1)(f_2^{(n)} \circ (\widehat{\eta}_0 \widehat{\Phi}_n))(f_3^{(n)} \circ \widehat{\Phi}_{n-1}) \\ &\quad \cdots (f_{n-i+1}^{(n)} \circ \widehat{\Phi}_{i+1})(f_{n-i+2}^{(n)} \circ (\widehat{Q}_B \widehat{\Phi}_i))(f_{n-i+3}^{(n)} \circ \widehat{\Phi}_{i-1}) \cdots (f_n^{(n)} \circ \widehat{\Phi}_2)\rangle \\ &= (-1)^{\sum h_i} (-1)^{1+2h_1+n'_o(n'_o-1)/2} \langle\langle \widehat{\Phi}_1 (\widehat{\eta}_0 \widehat{\Phi}_n) \widehat{\Phi}_{n-1} \cdots \widehat{\Phi}_{i+1} (\widehat{Q}_B \widehat{\Phi}_i) \widehat{\Phi}_{i-1} \cdots \widehat{\Phi}_2 \rangle\rangle \, . \end{aligned} \tag{B.8}$$

Let n_e and n_o denote the total number of even and odd fields in the set $\widehat{\Phi}_1, \ldots \widehat{\Phi}_n$. Since n_o is always even, we may write $n_o = 2m$ for some integer m. We now analyse two cases separately.

1. $\widehat{\Phi}_1$ odd. In this case $n'_o = 2m - 1$ and $(-1)^{2h_1} = -1$. Thus:

   $$(-1)^{2h_1} (-1)^{n'_o(n'_o-1)/2} = (-1)^{1+(m-1)(2m-1)} = (-1)^m \, , \tag{B.9}$$

   for integer m.

2. $\widehat{\Phi}_1$ is even. In this case $n'_o = 2m$, $(-1)^{2h_1} = 1$, and we have

$$(-1)^{2h_1}(-1)^{n'_o(n'_o-1)/2} = (-1)^{m(2m-1)} = (-1)^m \,. \tag{B.10}$$

Thus in both cases $(-1)^{2h_1}(-1)^{n'_o(n'_o-1)/2} = (-1)^m$. Using eqs.(B.9)-(B.10), and the cyclicity property (2.27), we can finally express the right hand side of (B.8) as

$$(I) = (-1)^{\frac{n_o}{2}+1}(-1)^{\sum h_i}\langle\langle \widehat{\Phi}_{i-1}\cdots\widehat{\Phi}_2\widehat{\Phi}_1(\widehat{\eta}_0\widehat{\Phi}_n)\widehat{\Phi}_{n-1}\cdots\widehat{\Phi}_{i+1}(\widehat{Q}_B\widehat{\Phi}_i)\rangle\rangle \,. \tag{B.11}$$

We now recognize that the operators inside the correlator are ordered just as in (B.3). Since the total contribution to the action is given by the addition of (I) and (II), combining (B.11) and (B.3) we see that we get a non-zero contribution only if

$$(-1)^{n+\frac{n_o}{2}}(-1)^{\sum_i h_i} = 1 \,. \tag{B.12}$$

Since n_o is always even, we have $(-1)^n = (-1)^{n_o+n_e} = (-1)^{n_e}$. Thus we may rewrite (B.12) as

$$(-1)^{\sum_{even}(h_i+1)}(-1)^{\sum_{odd}(h_i+\frac{1}{2})} = 1 \,. \tag{B.13}$$

This can be interpreted by saying that the action has a Z_2 twist invariance under which even fields carry twist charge $(-1)^{h+1}$ and odd fields carry twist charge $(-1)^{h+\frac{1}{2}}$. This means, in particular, that the tachyon $\widehat{T}$, being Grassmann odd and of dimension $-1/2$ has twist charge $+1$. In the computation of the tachyon potential we can therefore restrict $\mathcal{H}_1$ to twist even fields.

The results of this appendix can be used to relate terms in the string action. It follows from our discussion above that for a vertex involving n string fields in $\mathcal{H}_1$:

$$\langle\langle \widehat{\Phi}_1\cdots(\widehat{Q}_B\widehat{\Phi}_k)\cdots(\widehat{\eta}_0\widehat{\Phi}_l)\cdots\widehat{\Phi}_n\rangle\rangle = (-)^{n+1}\Big(\prod_{i=1}^n \Omega_i\Big)\langle\langle \widehat{\Phi}_n\cdots(\widehat{\eta}_0\widehat{\Phi}_l)\cdots(\widehat{Q}_B\widehat{\Phi}_k)\cdots\widehat{\Phi}_1\rangle\rangle \tag{B.14}$$

where Ω_i is the twist eigenvalue of $\widehat{\Phi}_i$. When we restrict to twist even fields in $\mathcal{H}_1$ the above equation is even simpler:

$$\langle\langle \widehat{\Phi}_1\cdots(\widehat{Q}_B\widehat{\Phi}_k)\cdots(\widehat{\eta}_0\widehat{\Phi}_l)\cdots\widehat{\Phi}_n\rangle\rangle = (-)^{n+1}\langle\langle \widehat{\Phi}_n\cdots(\widehat{\eta}_0\widehat{\Phi}_l)\cdots(\widehat{Q}_B\widehat{\Phi}_k)\cdots\widehat{\Phi}_1\rangle\rangle \,. \tag{B.15}$$

C Mass of the D-brane

In this appendix we shall show that the mass of the D-brane, whose world volume theory is given by the action (2.31), is given by $(2\pi^2 g^2)^{-1}$. The strategy that we shall be following is as follows. As in ref.[15], we assume that there is a set of (at least one) non-compact flat directions transverse to the D-brane; we shall denote these coordinates by x^i. Then the open string modes living on the D-brane will include the location of the D-brane along the directions x^i. Let Y^i denote the coordinate of the D-brane along x^i. The string field theory action contains a term proportional to $(\partial_t Y^i)^2$, where ∂_t denotes time derivative. The coefficient of the $(\partial_t Y^i)^2$ can be identified as half of the D-brane mass.

Let X^i be the free world-volume scalar field associated with the coordinate x^i, and ψ^i, $\widetilde{\psi}^i$ its left- and right-handed supersymmetric partners. We denote by $x^0 \equiv t$ the time coordinate, X^0 the corresponding world-volume scalar field, and k_0 the quantum number labelling momentum conjugate to X^0. If we write $X^\mu = X^\mu_L + X^\mu_R$ with L and R denoting left and right-moving components, then,

$$\partial X^\mu_L(z)\partial X^\nu_L(w) \simeq -\frac{\eta^{\mu\nu}}{2(z-w)^2}\,, \quad \psi^\mu(z)\psi^\nu(w) \simeq \frac{\eta^{\mu\nu}}{2(z-w)}\,, \tag{C.1}$$

with $\eta^{\mu\nu} = diag(-1, 1, \ldots, 1)$. With this normalization,

$$T_m = -\partial X_L \cdot \partial X_L - \psi \cdot \partial\psi + \cdots, \quad G_m = 2i\,\psi \cdot \partial X_L + \cdots\,. \tag{C.2}$$

There is a similar set of relations for the right-moving (anti-holomorphic) fields.

Since the time direction has been taken to be periodic with period 1, k_0 is quantized in units of 2π. Let us now consider the following term in the expansion of the string field $\widehat{\Phi}$;

$$\widehat{\Phi} = \sum_{k_0} \phi^i(k_0)\sqrt{2}\xi c e^{-\phi}\psi^i e^{ik_0X^0} \otimes I + \cdots\,. \tag{C.3}$$

The $\sqrt{2}$ factor in this expansion has been included to compensate for the factor of $(1/2)$ in the operator product of ψ^i with itself. Although $X^0 = X^0_L + X^0_R$, using the Neumann boundary condition $X_L = X_R$ at the boundary we can replace $e^{ik_0X^0}$ by $e^{2ik_0X^0_L}$. This facilitates computation of various correlation functions. In particular, using eqs.(C.1), (C.2) we see that this vertex operator has L^m_0 eigenvalue equal to $\frac{1}{2} - (k_0)^2$, where L^m_k denotes the kth mode of the matter Virasoro generator.

We shall now examine the quadratic term in the action involving the mode $\phi^i(k_0)$. Only the cT_m term of the BRST current j_B contributes to the k_0 dependent part of the quadratic term involving this mode, and the result is given by

$$\frac{1}{2g^2}\sum_{k_0}(k_0)^2\phi^i(k_0)\phi^i(-k_0)\,, \tag{C.4}$$

in the $\alpha' = 1$ unit. If $\chi^i(t) \equiv \sum_{k_0} e^{ik_0 t}\phi^i(k_0)$ denotes the Fourier transform of $\phi^i(k_0)$, then the above action can be rewritten as

$$\frac{1}{2g^2}\int dt \partial_t \chi^i \partial_t \chi^i\,, \tag{C.5}$$

where $t \equiv x^0$ denotes the time variable conjugate to k_0.

Up to an overall normalization factor, χ^i has the interpretation of the location Y^i of the D-brane in the x^i direction. We shall now determine the normalization factor between χ^i and Y^i. For this, instead of taking a single D-brane, let us take a pair of identical D-branes, separated by a distance b^i along the X^i direction. Then each state in the open string Hilbert space carries a 2×2 Chan Paton factor, besides the usual CP factor carried by a single non-BPS D-brane; we shall call these external CP factors. States with off diagonal external CP factors, representing open strings stretched between the two branes, are forced to carry an amount of winding charge b^i along X^i. For $\alpha' = 1$, *i.e.* string tension= $(2\pi)^{-1}$, the classical contribution to the mass of these open string states due to the tension of the string is equal to $|\vec{b}|/(2\pi)$. If we now move one of the branes by an amount Y^i along X^i, the change in the (mass)^2 of the open string with Chan Paton factors $\begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$ and $\begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}$ should be given by:

$$\frac{1}{(2\pi)^2}\{(\vec{b}+\vec{Y})^2 - \vec{b}^2\} = \frac{1}{2\pi^2}\vec{b}\cdot\vec{Y} + O(\vec{Y}^2)\,. \tag{C.6}$$

On the other hand, since χ^i denotes the mode which translates the brane, moving *one of the branes* along X^i will correspond to switching on a constant χ^i. This is represented by a string field background

$$\sqrt{2}\chi^i \xi c e^{-\phi}\psi^i \otimes I \otimes \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}\,. \tag{C.7}$$

We shall now explicitly use the string field theory action (2.31) to calculate the change of the (mass)^2 of states with Chan Paton factors $\begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$ and $\begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}$ due to the presence of this background string field, and compare with eq.(C.6). For this we note that the vertex operator for the lowest mass open string with internal CP factor I and external CP factors $\begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$ and $\begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}$ are given by, respectively,

$$\begin{aligned}
&\xi c e^{-\phi}(\vec{\epsilon}\cdot\vec{\psi}) e^{i\frac{b^i}{2\pi}(X^i_L - X^i_R)} e^{2ik_0 X^0_L} \otimes I \otimes \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}\,, \quad \text{and} \\
&\xi c e^{-\phi}(\vec{\epsilon}\cdot\vec{\psi}) e^{-i\frac{b^i}{2\pi}(X^i_L - X^i_R)} e^{2ik_0 X^0_L} \otimes I \otimes \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}\,,
\end{aligned} \tag{C.8}$$

where $\vec{\epsilon}$ is a polarization vector. Using Dirichlet boundary condition on X^i, we can write $X^i_L - X^i_R = 2X^i_L$. Requiring BRST invariance of these vertex operators gives,

$$\vec{\epsilon} \cdot \vec{b} = 0, \qquad (k_0)^2 = \frac{\vec{b}^2}{(2\pi)^2}\,. \tag{C.9}$$

Thus they represent states of mass $|\vec{b}|/(2\pi)$. We shall normalize $\vec{\epsilon}$ such that

$$|\vec{\epsilon}|^2 = 2\,. \tag{C.10}$$

Let us now consider the following expansion of the string field

$$\widehat{\Phi} = \chi^i \widehat{P}^i + \sum_{k_0} (u(k_0)\widehat{U}(k_0) + u^*(k_0)\widehat{V}(k_0)) + \ldots \tag{C.11}$$

where

$$\widehat{P}^i = \sqrt{2}\xi c e^{-\phi}\psi^i \otimes I \otimes \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, \tag{C.12}$$

$$\widehat{U}(k_0) = \xi c e^{-\phi}(\vec{\epsilon}\cdot\vec{\psi}) e^{2i\frac{b^i}{2\pi}X^i_L} e^{2ik_0 X^0_L} \otimes I \otimes \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}, \tag{C.13}$$

$$\widehat{V}(k_0) = \xi c e^{-\phi}(\vec{\epsilon}\cdot\vec{\psi}) e^{-2i\frac{b^i}{2\pi}X^i_L} e^{2ik_0 X^0_L} \otimes I \otimes \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}. \tag{C.14}$$

χ^i, $u(k_0)$ and $u^*(k_0)$ are specific components of the string field. We can now evaluate the string field theory action as a function of these fields. We shall be interested in the quadratic term involving u, u^*, as well as the $\chi^i u u^*$ coupling. The quadratic term is given by

$$\frac{1}{g^2}\sum_{k_0} u^*(-k_0)u(k_0)(k_0^2 - \frac{\vec{b}^2}{4\pi^2})\,. \tag{C.15}$$

The computation of the $\chi^i u u^*$ coupling can be simplified if we work on-shell at $k_0^2 = \vec{b}^2/(2\pi)^2$. (This suffices for computing the shift in mass^2 of the state to order χ^i.) We now note that:

- Using the three point vertex $(12g^2)^{-1}(\langle\langle(\widehat{Q}_B\widehat{\Phi})\widehat{\Phi}(\widehat{\eta}_0\widehat{\Phi})\rangle\rangle - \langle\langle\widehat{\Phi}(\widehat{Q}_B\widehat{\Phi})(\widehat{\eta}_0\widehat{\Phi})\rangle\rangle)$ we get twelve terms contributing to the $\chi^i u u^*$ coupling. Half of these terms vanish due to the trace identity:

$$Tr\Big(\begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \Big) = 0\,. \tag{C.16}$$

  The other cyclic ordering of these matrices produce a non-zero answer (equal to unity) for this trace.

- Each of the vertex operators $\widehat{P}^i$, $\widehat{U}$ and $\widehat{V}$ is annihilated by $\widehat{Q}_B\widehat{\eta}_0$ if $k_0^2 = \vec{b}^2/(2\pi)^2$. Using this result we can manipulate each of the remaining six terms so that $\widehat{Q}_B$ acts on $\widehat{P}^i$, and $\widehat{\eta}_0$ acts on $\widehat{U}$. Finally, using the cyclicity relations (2.27) we can show that each of these six terms gives identical result proportional to $\langle\langle(\widehat{Q}_B\widehat{P}^i)(\widehat{\eta}_0\widehat{U})\widehat{V}\rangle\rangle$.

After performing the trace over CP factors, and restricting to only on-shell components of u and u^*, we may express the $\chi^i uu^*$ term in the action as:

$$-\frac{1}{g^2}\sum_{k_0}{}'\chi^i u^*(-k_0)u(k_0)\langle f_1\circ(Q_B P^i) f_2\circ(\eta_0 U(k_0)) f_3\circ V(-k_0)\rangle\,, \tag{C.17}$$

where $\sum'$ denotes sum over on-shell momenta $k_0 = \pm|\vec{b}|/(2\pi)$. This correlation function is easily evaluated and the result is

$$\frac{1}{g^2}\frac{1}{\sqrt{2}\pi}\vec{b}\cdot\vec{\chi}\sum_{k_0}{}'u^*(-k_0)u(k_0)\,. \tag{C.18}$$

Combining this with eq.(C.15) we see that the shift in the mass^2 of the u, u^* field due to the presence of χ^i background is given by

$$-\frac{1}{\sqrt{2}\pi}\vec{b}\cdot\vec{\chi} + O(\vec{\chi}^2)\,. \tag{C.19}$$

Comparing eqs.(C.6) and (C.19) we get

$$\chi^i = -\frac{Y^i}{\sqrt{2}\pi}\,. \tag{C.20}$$

Once we have determined the relative normalization between χ^i and Y^i, we can return to the system containing a single brane.[10] Substituting eq.(C.20) into eq.(C.5), we get,

$$(4\pi^2 g^2)^{-1}\int dt\partial_t Y^i\partial_t Y^i\,. \tag{C.21}$$

This contribution to the D-brane world-volume action can be interpreted as due to the kinetic energy associated with the collective motion of the D-brane in the non-compact transverse directions. This allows us to identify the D-brane mass as

$$M = (2\pi^2 g^2)^{-1}\,. \tag{C.22}$$

[10] This can be done, for example, by moving the other brane infinite distance away by taking the limit $|\vec{b}|\to\infty$.

D Details on the calculation of the tachyon potential

We first consider some of the ingredients of the calculation, then do a particular example in detail. First of all, computation of $\langle\langle\ \rangle\rangle$ involving the various vertex operators $T, A, \ldots$ requires knowledge of $f \circ T(0)$, $f \circ A(0)$ etc., for a conformal map f. If $f(0) = w$, then we have the following relations:

$$\begin{aligned}
f \circ T(0) &= (f'(0))^{-\frac{1}{2}}\, T(w) \\
f \circ A(0) &= f'(0)\Big(A(w) - \frac{f''(0)}{(f'(0))^2} c\,\partial c\,\xi\partial\xi\, e^{-2\phi}(w)\Big) \\
f \circ E(0) &= f'(0)\Big(E(w) - \frac{f''(0)}{2(f'(0))^2}\Big) \\
f \circ F(0) &= f'(0) F(w)
\end{aligned} \tag{D.1}$$

Since the action involves Q_B and η_0 acting on string fields, we need to evaluate those on T, A, E and F and the result of conformal transform of these operators. However the analysis can be simplified by noting that

$$f \circ (\mathcal{O}A) = \mathcal{O}(f \circ A)\,, \tag{D.2}$$

where $\mathcal{O}$ can be either Q_B or η_0. This is due to the fact that the BRST current j_B and η are dimension 1 primary fields. Thus for example, in calculating correlation function involving $f \circ (Q_B A(0))$ we need to calculate the correlation function involving $j_B(w) f \circ A(0)$ and pick up the residue of the pole at $w = f(0)$. A similar procedure holds for $f \circ (\eta_0 A(0))$.

These relations, together with eq.(2.6), and the identity

$$\begin{aligned}
& \langle \prod_{i=1}^{n+1} \xi(x_i) \prod_{j=1}^{n} \eta(y_j) \prod_{k=1}^{m} b(u_k) \prod_{l=1}^{m+3} c(v_l) \prod_{s=1}^{p} e^{q_s\phi(z_s)} \rangle \\
= \ & -\prod_{i<i'} (x_i - x_{i'}) \prod_{j<j'} (y_j - y_{j'}) \prod_{i,j} (x_i - y_j)^{-1} \prod_{k<k'} (u_k - u_{k'}) \prod_{l<l'} (v_l - v_{l'}) \prod_{k,l} (u_k - v_l)^{-1} \\
& \times \prod_{s<s'} (z_s - z_{s'})^{-q_s q_{s'}}\,,
\end{aligned} \tag{D.3}$$

allows us to compute the relevant terms which appear in the computation of the tachyon potential. Eq.(D.3) follows from the normalization convention (2.3), and the operator products (2.7).

In evaluating correlation functions involving the operator E, we need to exercise special care, as it involves product of ξ and η at the same point. This has to be interpreted as:

$$\xi\eta(w) = \lim_{z \to w} \left(\xi(z)\eta(w) - \frac{1}{z - w} \right). \tag{D.4}$$

Let us give as an example the computation of the quartic term in the tachyon potential. From the expansion of the action (3.6), focusing on the terms with four string fields, we find:

$$\begin{aligned} g^2 S\Big|_{t^4} &= -\frac{t^4}{24}\Big\{\langle\langle\, (\widehat{Q}_B\widehat{T})\,\widehat{T}\,(\widehat{\eta}_0\widehat{T})\,\widehat{T}\rangle\rangle - \langle\langle\, (\widehat{Q}_B\widehat{T})\,\widehat{T}\,\widehat{T}(\widehat{\eta}_0\widehat{T})\,\rangle\rangle\Big\}\,, \\ &= -\frac{t^4}{12}\Big\{\langle\langle\, (Q_B T)\,T\,(\eta_0 T)\,T\rangle\rangle + \langle\langle\, (Q_B T)\,T\,T(\eta_0 T)\,\rangle\rangle\Big\}\,. \end{aligned} \tag{D.5}$$

In the second step we evaluated the trace over the internal CP matrices. We therefore have two correlators to compute. Using the fact that T correspond to a dimension $-(1/2)$ primary field, and that both $j_B(w)$ and $\eta(w)$ have only single poles near a T, the first correlator in the above equation can be written as:

$$\begin{aligned} C(f_1,f_2,f_3,f_4) &\equiv \langle\, f_1\circ(Q_B T(0))\; f_2\circ T(0)\; f_3\circ(\eta_0 T(0))\; f_4\circ T(0)\rangle \\ &= \lim_{y_1\to w_1}\lim_{y_2\to w_3}(y_1-w_1)(y_2-w_3)\frac{\langle\, j_B(y_1)T(w_1)\,T(w_2)\,\eta(y_2)T(w_3)\,T(w_4)\rangle}{(f_1')^{\frac12}(f_2')^{\frac12}(f_3')^{\frac12}(f_4')^{\frac12}}\,, \end{aligned} \tag{D.6}$$

where $w_i = f_i(0)$. We have, for simplicity of notation, defined $f_i \equiv f_i^{(4)}$. This correlation function can be easily evaluated, and the answer is

$$C(f_1,f_2,f_3,f_4) = \frac{w_{13}w_{24}}{(f_1')^{\frac12}(f_2')^{\frac12}(f_3')^{\frac12}(f_4')^{\frac12}}\,, \tag{D.7}$$

where $w_{ij} = (w_i - w_j)$. We now recognize that the second correlator in (D.5) is simply $C(f_1,f_2,f_4,f_3)$ with no extra sign factor because the last two vertex operators do not induce a sign factor when they are transposed. We can therefore write the complete answer as

$$g^2 S\Big|_{t^4} = -\,\frac{w_{13}w_{24}+w_{14}w_{23}}{12\,(f_1')^{\frac12}(f_2')^{\frac12}(f_3')^{\frac12}(f_4')^{\frac12}}\,t^4\,. \tag{D.8}$$

This off-shell amplitude is PSL(2,C) invariant[11]. Indeed letting

$$w \to \frac{aw+b}{cw+d}\,, \quad ad-bc=1, \tag{D.9}$$

we readily find that

$$w_{ij} \to \frac{w_{ij}}{(cw_i+d)(cw_j+d)}\,, \qquad f_i' \to \frac{f_i'}{(cw_i+d)^2} \tag{D.10}$$

[11] See [32] for a Riemann surface interpretation of invariant off-shell amplitudes.

and therefore we get PSL(2,C) invariance if we choose the branch[12]

$$(f_i')^{1/2} \to \frac{(f_i')^{1/2}}{cw_i + d}\,. \tag{D.11}$$

We evaluate now the term. Our first choice of coordinates is that of the unit disk, described in detail in section 2.1. The prescription for dealing with the square roots there ((2.13)) is used to find

$$g^2 S\Big|_{t^4} = -\frac{2(2i) + (1+i)^2}{12 \cdot e^{i\pi/2}}\, t^4 = -\frac{1}{2}\, t^4 \tag{D.12}$$

which is the result obtained in [22].

We shall now do the computation in the upper half plane (UHP) using the maps $g_k^{(n)}$, related to $f_k^{(n)}$ by an SL(2,C) transformation which maps the disk to UHP. But before we proceed, we need to derive the analog of eq.(2.13) for half integer h, *i.e.* the presription for choosing the sign of $(g_k^{(n)\prime}(0))^{\frac{1}{2}}$ appearing in the conformal transform of half-integer weight fields. This will be done by starting with the presciption (2.13) and then using prescription (D.11) for an appropriate SL(2,C) transformation relating $f_k^{(n)}$ to $g_k^{(n)}$. First note that for fixed n and k, $f_k^{(n)}(z)$ moves anti-clockwise along the boundary of the unit disk as z moves along the positively oriented real line.[13] In addition, since the map from the disk to the UHP takes the anti-clockwise oriented boundary of the disk to the positively oriented real line, it is clear that the $g_k^{(n)\prime}$'s map to positive real values at the punctures on the real line. In computing $(g_k^{(n)\prime})^{\frac{1}{2}}$ we have a sign ambiguity. We shall now show that *if the conformal map relating $f_k^{(n)}$ to $g_k^{(n)}$ is such that the points $g_1^{(n)}(0), \ldots g_n^{(n)}(0)$ are ordered from the left to the right on the real axis, then we should choose the positive sign for all the $(g_k^{(n)\prime}(0))^{\frac{1}{2}}$.*

We prove this as follows. As a first step it is convenient to rotate the punctures on the disk to a new position. For this, we define:

$$\widetilde{f}_k^{(n)}(z) = e^{\frac{2\pi i}{n}(-\frac{n}{2}+1-\epsilon)} f_k^{(n)}(z) = e^{\frac{2\pi i}{n}(k-\frac{n}{2}-\epsilon)} \left(\frac{1+iz}{1-iz}\right)^{\frac{2}{n}}, \tag{D.13}$$

where ϵ is a small positive number; in fact any $0 < \epsilon < 1$ will do. In this case

$$(\widetilde{f}_k^{(n)\prime}(0))^{\frac{1}{2}} = e^{\frac{i\pi}{n}(-\frac{n}{2}+1-\epsilon)} (f_k^{(n)\prime}(0))^{\frac{1}{2}} = \left|\left(\frac{4}{n}\right)^{\frac{1}{2}}\right| e^{\frac{i\pi}{n}(k-\frac{n}{4}-\epsilon)}\,. \tag{D.14}$$

[12]In the chosen SL(2,C) transformation there is a sign ambiguity in which all coefficients a, b, c, d of the transformation are changed in sign. Since this transformation must be used for an even number of punctures, this is not a problem.

[13]This is related to the fact that the canonical half disks representing the strings are all mapped analytically into the interior of the disk and the boundary of the canonical half-disks is oriented in the direction of increasing real values.

Next we define

$$g_k^{(n)}(z) = F(\widetilde{f}_k^{(n)}(z))\,, \tag{D.15}$$

with

$$F(u) = i\,\frac{1-u}{1+u} \equiv \frac{au+b}{cu+d}\,, \tag{D.16}$$

where we use our freedom to fix the signs of a, b, c, d to write:

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} \frac{1}{\sqrt{2}}e^{-\frac{i\pi}{4}} & -\frac{1}{\sqrt{2}}e^{-\frac{i\pi}{4}} \\ \frac{1}{\sqrt{2}}e^{\frac{i\pi}{4}} & \frac{1}{\sqrt{2}}e^{\frac{i\pi}{4}} \end{pmatrix}, \qquad ad - bc = 1\,. \tag{D.17}$$

F describes an SL(2,C) map from the disk to the UHP. With this,

$$g_k^{(n)}(0) = \tan\Big(\frac{\pi}{n}(k - \frac{n}{2} - \epsilon)\Big)\,. \tag{D.18}$$

For $k = 1, \ldots n$, the $g_k^{(n)}(0)$'s given above are arranged from left to right on the real axis. We also have

$$(g_k^{(n)\prime}(0))^{\frac{1}{2}} = (c\widetilde{f}_k^{(n)}(0) + d)^{-1}(\widetilde{f}_k^{(n)\prime}(0))^{\frac{1}{2}} = \frac{1}{\sqrt{2}}\left|\Big(\frac{4}{n}\Big)^{\frac{1}{2}}\right| \sec\Big(\frac{\pi}{n}(k - \frac{n}{2} - \epsilon)\Big)\,. \tag{D.19}$$

This is manifestly positive for $1 \le k \le n$.

This gives one set of $g_k^{(n)}$'s for which the square root rules stated above hold, but we need to show that this holds for any other set of functions $\widetilde{g}_k^{(n)}(z)$, related to $g_k^{(n)}(z)$ by an SL(2,R) transformation. For this, let us consider another set of functions $\widetilde{g}_k^{(n)}$'s related to the $g_k^{(n)}$'s via an SL(2,R) transformation $\begin{pmatrix} p & q \\ r & s \end{pmatrix}$ with the property that $\widetilde{g}_1^{(n)}(0), \ldots \widetilde{g}_n^{(n)}(0)$ are arranged from the left to the right on the real axis. In that case, if $v_k = g_k^{(n)}(0)$, then for $k > l$,

$$v_k > v_l, \qquad \frac{pv_k + q}{rv_k + s} - \frac{pv_l + q}{rv_l + s} = \frac{(v_k - v_l)}{(rv_k + s)(rv_l + s)} > 0\,. \tag{D.20}$$

Thus

$$(rv_k + s)(rv_l + s) > 0\,. \tag{D.21}$$

This shows that $(rv_k + s)$ has the same sign for all k. Using the freedom of changing the sign of p, q, r, s, we can take $(rv_k + s)$ to be positive. Then

$$(\widetilde{g}_k^{(n)\prime}(0))^{\frac{1}{2}} = (rv_k + s)^{-1}(g_k^{(n)\prime}(0))^{\frac{1}{2}} > 0\,. \tag{D.22}$$

This proves the desired result.

Let us now get back to the computation of (D.8) using maps to UHP. For this we map the disk, punctured at $1, i, -1, -i$, into the UHP with the real boundary punctured at $-4, -1, 0, 2$. These are particularly nice points that give coordinates without radicals:

$$\begin{aligned} g_1^{(4)}(z) \quad &= -4 + 6\,z \;\; - 9\,z^2 + \cdots \\ g_2^{(4)}(z) \quad &= -1 + \tfrac{3}{4}\,z - \tfrac{3}{16}\,z^2 + \cdots \\ g_3^{(4)}(z) \quad &= \;\; 0 + \tfrac{2}{3}\,z \; + \tfrac{1}{9}\,z^2 + \cdots \\ g_4^{(4)}(z) \quad &= \;\; 2 + 3\,z + \; 3\,z^2 + \cdots \end{aligned} \tag{D.23}$$

In this presentation, all computations are manifestly real. In addition all $g_i^{(4)\prime}(0)$'s are positive as expected and we simply take their positive square roots in evaluating (D.8) with f_i replaced by g_i. We get:

$$g^2 S \Big|_{t^4} = -\, \frac{(-4)(-3) + (-6)(-1)}{12 \cdot 3}\, t^4 = -\frac{1}{2}\, t^4 \tag{D.24}$$

This agrees with the result of the disk computation.

References

[1] T. Banks and L. Susskind, “Brane - Antibrane Forces,” [hep-th/9511194].

[2] M. B. Green and M. Gutperle, “Light-cone supersymmetry and D-branes,” Nucl. Phys. **B476**, 484 (1996) [hep-th/9604091].

[3] A. Sen, “Stable non-BPS bound states of BPS D-branes,” JHEP **9808**, 010 (1998) [hep-th/9805019].

[4] A. Sen, “Tachyon condensation on the brane antibrane system,” JHEP **9808**, 012 (1998) [hep-th/9805170].

[5] A. Sen, “SO(32) spinors of type I and other solitons on brane-antibrane pair,” JHEP **9809**, 023 (1998) [hep-th/9808141].

[6] E. Gava, K.S. Narain, M.H. Sarmadi, “On the Bound States of p- and (p+2)-Branes”, Nucl.Phys. **B504** (1997) 214-238 [hep-th/9704006].

[7] A. Hashimoto and W. Taylor, “Fluctuation Spectra of Tilted and Intersecting D-branes from the Born-Infeld Action”, Nucl.Phys. **B503** (1997) 193-219, [hep-th/9703217]

[8] O. Bergman and M. R. Gaberdiel, "Stable non-BPS D-particles," Phys. Lett. **B441**, 133 (1998) [hep-th/9806155].

[9] A. Sen, "Type I D-particle and its interactions," JHEP **9810**, 021 (1998) [hep-th/9809111].

[10] A. Sen, "BPS D-branes on non-supersymmetric cycles," JHEP **9812**, 021 (1998) [hep-th/9812031].

[11] P. Horava, "Type IIA D-branes, K-theory, and matrix theory," Adv. Theor. Math. Phys. **2**, 1373 (1999) [hep-th/9812135].

[12] J. A. Harvey, P. Horava and P. Kraus, "D-sphalerons and the topology of string configuration space," [hep-th/0001143].

[13] A. Recknagel and V. Schomerus, "Boundary deformation theory and moduli spaces of D-branes," Nucl. Phys. **B545**, 233 (1999) hep-th/9811237;
C.G. Callan, I.R. Klebanov, A.W. Ludwig and J.M. Maldacena, "Exact solution of a boundary conformal field theory," Nucl. Phys. **B422**, 417 (1994) hep-th/9402113;
J. Polchinski and L. Thorlacius, "Free fermion representation of a boundary conformal field theory," Phys. Rev. **D50**, 622 (1994) hep-th/9404008.

[14] A. Sen, "Descent relations among bosonic D-branes," Int. J. Mod. Phys. **A14**, 4061 (1999) [hep-th/9902105].

[15] A. Sen, "Universality of the Tachyon Potential," [hep-th/9911116].

[16] A. Sen and B. Zwiebach, "Tachyon Condensation in String Field Theory," [hep-th/9912249].

[17] E. Witten, "Noncommutative Geometry And String Field Theory," Nucl. Phys. **B268**, 253 (1986).

[18] V.A. Kostelecky and S. Samuel, "The Static Tachyon Potential in the Open Bosonic String Theory," Phys. Lett. **B207** (1988) 169.

[19] N. Moeller and W. Taylor, "Level truncation and the tachyon in open bosonic string field theory", to appear.

[20] W. Taylor, "D-brane effective field theory from string field theory", hep-th/0001201.

[21] J. A. Harvey and P. Kraus, "D-Branes as Unstable Lumps in Bosonic Open String Field Theory," hep-th/0002117.

[22] N. Berkovits, "The Tachyon Potential in Open Neveu-Schwarz String Field Theory," [hep-th/0001084].

[23] N. Berkovits, "Super-Poincare Invariant Superstring Field Theory," Nucl. Phys. **B450** (1995) 90, [hep-th/9503099].

[24] N. Berkovits, "A New Approach to Superstring Field Theory," proceedings to the 32^{nd} International Symposium Ahrenshoop on the Theory of Elementary Particles, Fortschritte der Physik (Progress of Physics) **48** (2000) 31, [hep-th/9912121].

[25] E. Witten, "Interacting Field Theory of Open Superstrings," Nucl. Phys. **B276** (1986) 291.

[26] N. Berkovits and C.T. Echevarria, "Four-Point Amplitude from Open Superstring Field Theory," [hep-th/9912120].

[27] I. Y. Arefeva, P. B. Medvedev and A. P. Zubarev, "Nonperturbative vacuum for superstring field theory and supersymmetry breaking," Mod. Phys. Lett. **A6**, 949 (1991).

[28] D. Friedan, E. Martinec, and S. Shenker, "Conformal Invariance, Supersymmetry, and String Theory," Nucl. Phys. B**271** (1986) 93.

[29] M. R. Gaberdiel and B. Zwiebach, "Tensor constructions of open string theories I: foundations", Nucl. Phys. **B505**, 569 (1997) hep-th/9705038.

[30] O. Bergman, private communication.

[31] A. Iqbal and A. Naqvi, private communication.

[32] A. Belopolsky and B. Zwiebach. "Off-shell closed string amplitudes: Towards a computation of the tachyon potential", Nucl. Phys. **B442**, 494 (1995) hep-th/9409015.

[33] C. Wendt, “Scattering amplitudes and contact interactions in Witten's superstring field theory”, Nucl. Phys. **B 314** 209 (1989).
J. Greensite and F.R. Klinkhamer, “Superstring Amplitudes and Contact Interactions,” Nucl. Phys. **B304** (1988) 108.

[34] S. Giddings, E. Martinec and E. Witten, “Modular invariance in string field theory”, Phys. Lett. **B226** 362 (1986).
B. Zwiebach, “A proof that Witten's open string theory gives a single cover of moduli space”, Commun. Math. Phys. **142** 193 (1991).

[35] B. Zwiebach, “Closed string field theory: Quantum action and the BV master equation”. Nucl.Phys. **B390**(1993) 33-152, hep-th/9206084.

[36] C. Preitschopf, S. Yost and C. Thorn,“ Superstring field theory”, Nucl. Phys. **B337** 363 (1990).

[37] I. Y. Arefeva, P. B. Medvedev and A. P. Zubarev, “Background Formalism For Superstring Field Theory,” Phys. Lett. **B240**, 356 (1990);
“New Representation For String Field Solves The Consistence Problem For Open Superstring Field,” Nucl. Phys. **B341**, 464 (1990).

[38] B. Zwiebach, “Oriented open-closed string theory revisited,” Annals Phys. **267**, 193 (1998) hep-th/9705241. “Quantum open string theory with manifest closed string factorization,” Phys. Lett. **B256**, 22 (1991). “Interpolating String Field Theories,”, Mod. Phys. Lett. A7 (1992) 1079-1090, hep-th/9202015.